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Organizers: Marcelo Disconzi, Giusy Mazzone, Gieri Simonett

Magdalena Czubak
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The fluid equations on negatively curved manifolds

Abstract: In this talk we survey results on how the curvature of the underlying domain can affect the solutions of the equations of fluid mechanics. We compare and contrast with the classical counterparts.

Mikhail Feldman
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Uniqueness for shock reflection problem

Abstract: We discuss shock reflection problem for compressible gas dynamics, von Neumann conjectures on transition between regular and Mach reflections, and existence of regular reflection solutions for potential flow equation. Then we will talk about recent results on uniqueness of regular reflection solutions for potential flow equation in a natural class of self-similar solutions. The approach is to reduce the shock reflection problem to a free boundary problem for a nonlinear elliptic equation, and prove uniqueness by a version of method of continuity. A property of solutions important for the proof of uniqueness is convexity of the free boundary.

This talk is based on joint work with G.-Q. Chen and W. Xiang.

Susan Friedlander
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Asymptotics for magnetostrophic turbulence in the Earth's fluid core

Abstract: We consider the three dimensional magnetohydrodynamics (MHD) equations in the presence of stochastic forcing as a model for magnetostrophic turbulence. For scales relevant to the Earth's fluid core this MHD system is very rich in small parameters. We discuss results concerning the asymptotics of the stochastically forced PDEs in the limit of vanishing parameters. In particular we establish that the system sustains ergodic statistically steady states thus providing a rigorous foundation for magnetostrophic turbulence.

This is joint work with Juraj Foldes, Nathan Glatt-Holtz and Geordie Richards.

Giovanni P. Galdi
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Self-oscillations of a Navier-Stokes liquid past a body

Abstract: We provide a rather complete mathematical analysis of branching out of a time-periodic solutions from steady-state solutions to the two-dimensional Navier-Stokes equations in the exterior of a body, in two- and three-dimensional cases. More precisely, we begin to provide necessary and sufficient conditions for such a phenomenon to happen. Successively, we furnish a detailed study of their asymptotic spatial behavior that shows, in particular, that the purely oscillatory motion occurs downstream, essentially only in a neighborhood of the body, whose size increases with the relevant Reynolds number Re. We also show that the original steady-state solution looses stability to the time periodic one, at least in a suitable spectral sense. Finally, we give sufficient conditions for stability of the time-periodic bifurcating solution as Re is further increased.

Philip Isett
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A proof of Onsager's conjecture for the Incompressible Euler equations

Abstract: In an effort to explain how anomalous dissipation of energy occurs in hydrodynamic turbulence, Onsager conjectured in 1949 that weak solutions to the incompressible Euler equations may fail to exhibit conservation of energy if their spatial regularity is below 1/3-Hölder. I will discuss a proof of this conjecture that shows that there are nonzero, $(1/3\text{-}\epsilon)$-Hölder Euler flows in 3D that have compact support in time. The construction is based on a method known as "convex integration," which has its origins in the work of Nash on isometric embeddings with low codimension and low regularity. A version of this method was first developed for the incompressible Euler equations by De Lellis and Szkelyhidi to build Hölder-continuous Euler flows that fail to conserve energy, and was later improved by Isett and by Buckmaster-De Lellis-Szkelyhidi to obtain further partial results towards Onsager's conjecture. The proof to be discussed of the full conjecture combines a new idea in the convex integration scheme due to Daneri-Szkelyhidi with a new "gluing approximation" technique. The latter technique exploits a special structure in the linearization of the incompressible Euler equations.

Hans Lindblad
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The motion of the free surface of slightly compressible liquid and the incompressible limit

Abstract: We prove energy bounds for the free boundary problem for a compressible liquid and study the incompressible limit. This is joint work with Chenyun Luo.

Eugene Wayne
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Stability and metastability in two-dimensional fluids

Abstract: Two dimensional fluids exhibit behavior on a variety of time scales. The origin of some of these time scales, like the viscous time scale, is obvious, but the origin of others is much less clear. In this talk I'll discuss some results on the long-time and intermediate-time behavior of such fluids and describe some of the mathematical techniques used to analyze them.