Scheduling Parallel Tasks under Multiple Resources: List vs. Pack

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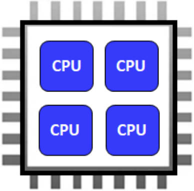
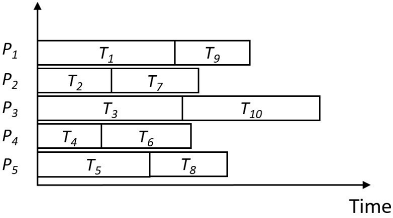
Introduction

Single-resource scheduling

- Most traditional scheduling problems target a single type of resource (e.g., CPUs)



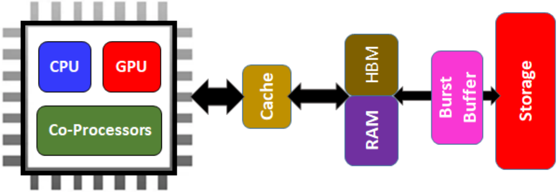


- For example: classic NP-complete problem of makespan minimization on identical machines ($P||C_{\max}$). List scheduling is $(2 - \frac{1}{P})$-approx. [Graham 1969]

Introduction

The case for multi-resource scheduling

- HPC systems embrace more heterogeneous components (e.g., CPU, GPU, FPGA, MIC, APU)
- Data-intensive applications drive architecture enhancement for better data-transfer efficiency (e.g., High-Bandwidth Memory, Partitionable Cache, Burst Buffers)



To achieve optimal system/application performance, multiple types of resources (e.g., CPU, GPU, memory, cache, I/O) should be scheduled simultaneously

Models and Objective

A multi-resource scheduling model:

- System with d resource types; i-th type has $P^{(i)}$ identical resources
- Set $\{1, 2, \cdots, n\}$ of independent, moldable tasks released at time 0
- Each task j's execution time $t_j(\vec{p}_j)$ depends on its resource allocation vector $\vec{p}_j = (p_j^{(1)}, p_j^{(2)}, \cdots, p_j^{(d)})$
- Assumption: *non-increasing execution time*

$$\vec{p}_j \preceq \vec{q}_j \ \left(\text{or } p_j^{(i)} \leq q_j^{(i)}, \forall i\right) \implies t_j(\vec{p}_j) \geq t_j(\vec{q}_j)$$

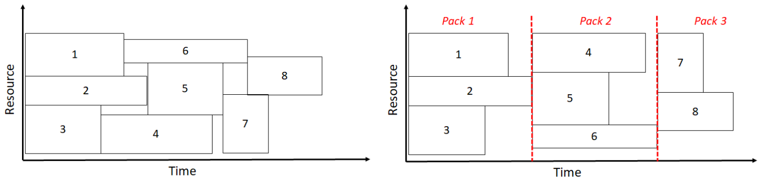
Scheduling objective:

- Find a moldable schedule, i.e., resource allocation vector $\vec{p}_j$ and starting time s_j for each task j
  - minimize makespan: $T = \max_j(s_j + t_j(\vec{p}_j))$
  - subject to resource constraint: $\sum_{j \text{ active at time } t} p_j^{(i)} \leq P^{(i)}, \forall i, t$

Focus of This Work

Two scheduling paradigms:

- List: greedily schedule tasks in a list on first available resources
- Pack: partition tasks in packs to be scheduled one after another



(a) list scheduling (b) pack scheduling

- Simple yet efficient schedules favored by practical runtime systems
- Easily adopted to online or heterogeneous scheduling environments

Main Results

Theoretically:

- Approximation ratios that increase linearly with number d of resource types
  - List-scheduling: $2d$-approx.
  - Pack-scheduling: $(2d+1)$-approx.
- Strategy to transform multi-resource problem to single-resource problem to reduce computational complexity

Empirically:

- Experiments on Intel Xeon Phi Knights Landing (KNL) with 2 resource types (cores + high-bandwidth memory)
- Simulations with up to 4 resource types using synthetic workloads that extend classical speedup profiles

Outline

Preliminaries

Definitions: for a given resource allocation $\mathbf{p} = (\vec{p}_1, \vec{p}_2, \cdots, \vec{p}_n)^T$

- Total task area (normalized): $A(\mathbf{p}) = \sum_{j=1}^{n} \sum_{i=1}^{d} \frac{p_j^{(i)}}{P^{(i)}} \cdot t_j(\vec{p}_j)$
- Maximum task execution time: $t_{\max}(\mathbf{p}) = \max_j t_j(\vec{p}_j)$

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Analogous to *area bound* (T_1/P) and *depth bound* (T_∞) in single-resource scheduling

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Analogous to *area bound* (T_1/P) and *depth bound* (T_∞) in single-resource scheduling

Lower bound (on makespan): $L(\mathbf{p}, d) = \max\left(\frac{A(\mathbf{p})}{d}, t_{\max}(\mathbf{p})\right)$

Proposition

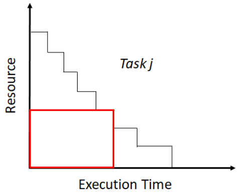
*The **optimal makespan** satisfies*

$$T_{\text{OPT}} \geq L_{\min}(d) = \min_{\mathbf{p}} L(\mathbf{p}, d)$$

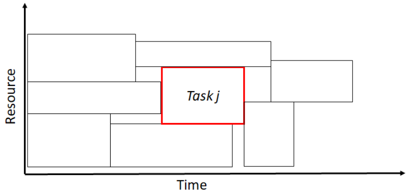
Moldable Scheduling

Two-phase approach [Turek et al. 1992]:

- *Phase 1*: Determines a resource allocation for each moldable task



- *Phase 2*: Constructs a rigid schedule based on the fixed resource allocations of all tasks

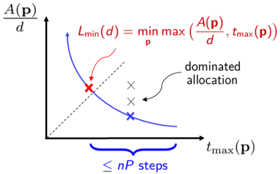


Phase 1: Resource Allocation

Goal: find allocation $\mathbf{p}_{\min}^{d}$ matching lower bound $L_{\min}(d) = \min_{\mathbf{p}} L(\mathbf{p}, d)$

Resource Allocation (RA_d)

- Step (1). For each task j:
  - Linearize all $P = \prod_{i=1}^{d}(P^{(i)} + 1)$ allocations
  - Remove ones with both higher execution time and larger area
  - Sort in order of increasing execution time and decreasing area
- Step (2). For all n tasks:
  - Traverse the n lists in $\leq nP$ steps by tracing $t_{\max}(\mathbf{p})$ at each step until dominated by $\frac{A(\mathbf{p})}{d}$ (v.s. exhaustive search in P^n time)



Complexity: $O(nP(\log P + \log n + d))$

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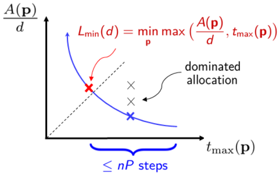


Complexity: $O(nP(\log P + \log n + d))$

Proposition

*If a **rigid scheduling algorithm** R_d that uses $\mathbf{p}_{\min}^d$ produces a makespan*

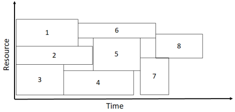
$$T_{\mathrm{R}_d}(\mathbf{p}_{\min}^d) \leq c \cdot L_{\min}(d)$$

*then the **two-phase algorithm** $\mathrm{RA}_d + \mathrm{R}_d$ is c-approximation.*

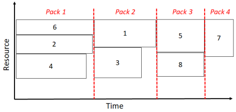
Phase 2: Rigid Scheduling

For a fixed resource allocation:

- List Scheduling (LS_d): 2-approx. for $d = 1$
  - Arrange all tasks in a list. Whenever an existing task completes, scan the list and schedule first task that fits (i.e., with sufficient resources in all dimensions)



- Pack Scheduling (PS_d): 3-approx. for $d = 1$
  - Sort all tasks in decreasing order of exec. time. Assign each task in sequence to last pack if fits (i.e., with sufficient resources in all dimensions). Otherwise, create a new pack.



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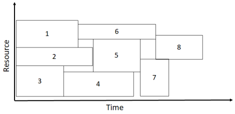

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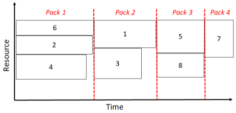

Proposition

For a set of rigid tasks with fixed resource allocation $\mathbf{p}$, *we have*

$$\textbf{\textit{List Scheduling}}: \quad T_{\mathrm{LS}_d}(\mathbf{p}) \leq 2d \cdot L(\mathbf{p}, s)$$

$$\textbf{\textit{Pack Scheduling}}: \quad T_{\mathrm{PS}_d}(\mathbf{p}) \leq (2d+1) \cdot L(\mathbf{p}, s)$$

$\Rightarrow$ $\mathrm{RA}_d + \mathrm{LS}_d$ is $2d$-approx. and $\mathrm{RA}_d + \mathrm{PS}_d$ is $(2d+1)$-approx. Moreover, the bounds are tight for the two algorithms

Transformation

Transformation (TF):

- Step (1). *d*-**resource instance** $I \Longrightarrow$ 1-**resource instance** I'
  - I' has same number n of tasks and total resource $Q = \text{lcm}_{i=1\cdots d} P^{(i)}$
  - For any task j' in I': execution time $t_{j'}(q) = t_j((\lfloor \frac{q \cdot P^{(i)}}{Q} \rfloor)_{i=1\cdots d})\ \forall q$
- Step (2). **Solve the 1-resource instance** I'
- Step (3). 1-**resource solution** $S' \Longrightarrow$ *d*-**resource solution** S
  - For any task j in I: starting time is same $s_j = s_{j'}$
    resource allocation is $\vec{p}_j = (\lfloor \frac{q_{j'} \cdot P^{(i)}}{Q} \rfloor)_{i=1\cdots d}$

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Performance: $\mathrm{TF} + \mathrm{RA}_1 + \mathrm{LS}_1$ is $2d$-approx.
$\mathrm{TF} + \mathrm{RA}_1 + \mathrm{PS}_1$ is $(2d+1)$-approx.

Complexity: Transform $Q = \text{lcm}_i P^{(i)}$ v.s. Direct $P = \prod_i (P^{(i)}+1)$
If $P^{(i)} = p\ \forall i \quad \Rightarrow \quad O(p)$ v.s. $O(p^d)$

Outline

Introduction

Theoretical Analysis

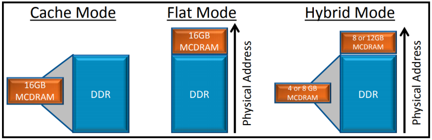
Experimental Evaluation

Future Work

Experimental Setup

Platform: Intel Xeon Phi 7230 Knights Landing (KNL)

- 64 cores
- 96GB slow memory (DDR)
- 16GB fast memory (MCDRAM)
  - 4-5x the bandwidth
  - 3 configuration modes



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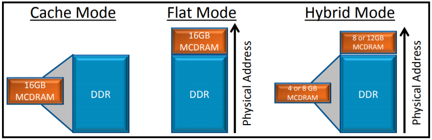

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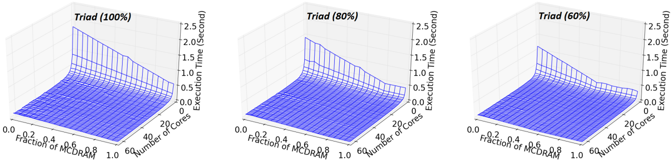
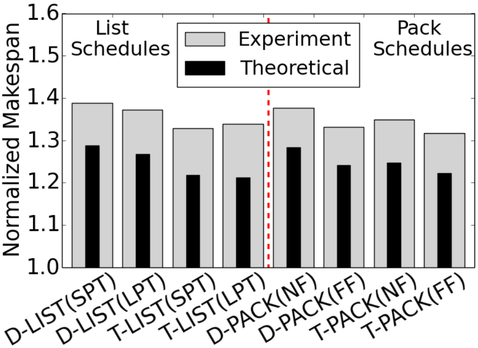
Benchmarks: STREAM (*triad, write, ddot*)

- Create tasks of different sizes by varying array length and thus memory footprint as % of MCDRAM size



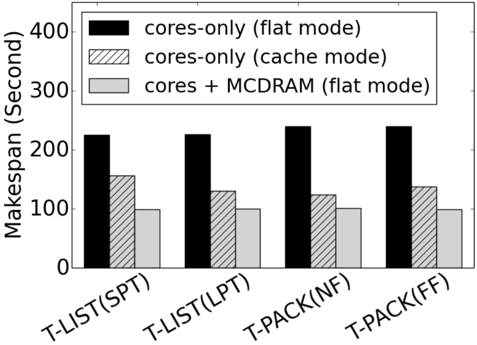
Experimental Results



Comparing different algorithms:

- Comparable performance for list- and pack-based solutions
- LPT (list) and FF (pack) perform generally better
- Transform-based solutions perform just as well

Experimental Results



Flat mode vs. cache mode:

- Managing fast memory directly as a resource (in flat mode) result in better performance than treating it as a cache for co-scheduled applications (due to possible interference)

Simulation Setup

Resources:

- Up to four different types (e.g., CPU, GPU, cache, memory, I/O)
- Amount of resources for each type: $(64, 32, 16, 8)$

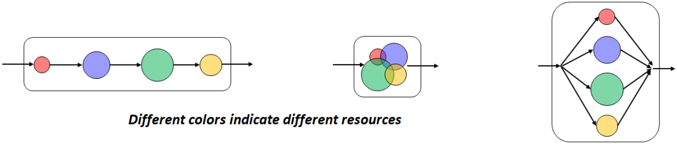
Workload (synthetic):

- Extended Amdahl's law: $s_0 \sim \mathcal{U}(0, 0.2)$

  (i) $1/\left(s_0 + \sum_{i=1}^{d} \frac{s_i}{p^{(i)}}\right)$; (ii) $1/\left(s_0 + \frac{1-s_0}{\prod_{i=1}^{d} p^{(i)}}\right)$; (iii) $1/\left(s_0 + \max_{i=1..d} \frac{s_i}{p^{(i)}}\right)$

- Extended power law: $\alpha_i \sim \mathcal{U}(0.3, 1)$

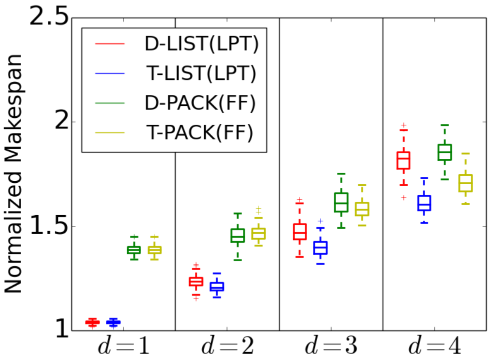
  (i) $1/\left(\sum_{i=1}^{d} \frac{s_i}{(p^{(i)})^{\alpha_i}}\right)$; (ii) $\prod_{i=1}^{d} (p^{(i)})^{\alpha_i}$; (iii) $1/\left(\max_{i=1..d} \frac{s_i}{(p^{(i)})^{\alpha_i}}\right)$



Different colors indicate different resources

(i) sequential *(ii) collaborative* *(iii) concurrent*

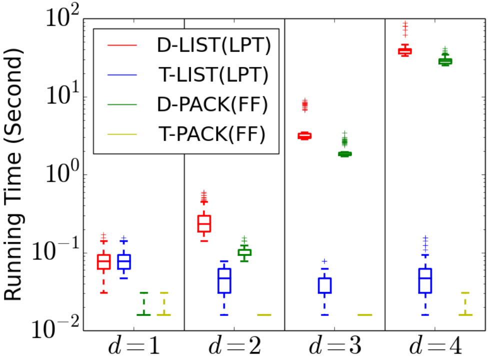
Simulation Results



Performance (makespan normalized w.r.t lower bound):

- Ratios increase with d, but far below theoretical bounds
- List algorithms perform better, but gap reduces as d increases
- Transform-based solutions perform slightly better

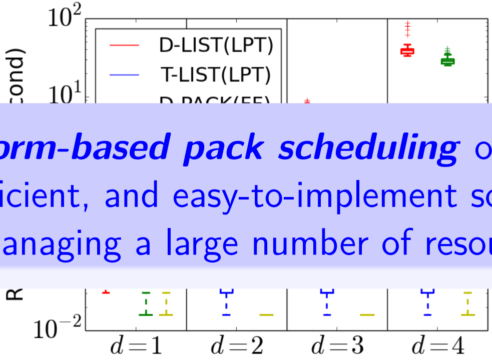
Simulation Results



Complexity (running time of algorithms):

- Pack algorithms run slightly faster than list algorithms
- Direct solutions increase drastically with d
- Transform-based solutions orders of magnitude faster (esp. $d \geq 3$)

Simulation Results



Transform-based pack scheduling offers fast, efficient, and easy-to-implement solutions when managing a large number of resources

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- Transform-based solutions orders of magnitude faster (esp. $d \geq 3$)

Outline

Open Questions

Performance of list-scheduling under multi-resources

- Rigid jobs: $(d+1)$-approx. [Garey and Graham, 1975]
- Moldable jobs: $2d$-approx. [This work, with algo. lower bound]
- Malleable jobs: $(d+1)$-approx. [He et al. 2007]
  (Represented as DAGs containing unit-size tasks of different types)

- Can we achieve $(d+1)$-approx. for moldable jobs (possibly with a more coupled design/analysis of resource allocation and rigid scheduling), or is it inherently harder?

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Performance of general models for moldable task scheduling

- 2-Pack Sol.: $(1.5+\epsilon)$-approx. [Mounié et al. 2004, Jansen 2012]
- Precedence constraints: e.g., $(3+\sqrt{5})$-approx. [Lepère et al. 2001]

- Could these results be extended to multi-resource scheduling?

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Other practical applications of multi-resource scheduling

- e.g., cache partitioning, bandwidth allocation, burst buffer sharing?