Question: Calculate lim sup x_n for $x_n = (-1)^n(\frac{n+5}{n})$

We need to find the supremum of the subsequences.

Let's try an substitute natural numbers for "n" to get an idea of what the elements of x_n look like:

$x_n = \{\frac{-6}{1}, \frac{7}{2}, \frac{-8}{3}, \frac{9}{4}, \frac{-10}{5}, \frac{11}{6}, \frac{-12}{7}, \frac{13}{8}...\}$

Now let's look at subsequences of x_n:

$x_1 = \{\frac{-6}{1}, \frac{7}{2}, \frac{-8}{3}, \frac{9}{4}, \frac{-10}{5}, \frac{11}{6}, \frac{-12}{7}, \frac{13}{8}...\} \rightarrow inf(x_1) = \frac{7}{2}$

$x_2 = \{\frac{7}{2}, \frac{-8}{3}, \frac{9}{4}, \frac{-10}{5}, \frac{11}{6}, \frac{-12}{7}, \frac{13}{8}...\} \rightarrow inf(x_2) = \frac{7}{2}$

$x_3 = \{\frac{-8}{3}, \frac{9}{4}, \frac{-10}{5}, \frac{11}{6}, \frac{-12}{7}, \frac{13}{8}...\} \rightarrow inf(x_3) = \frac{9}{4}$

$x_4 = \{\frac{9}{4}, \frac{-10}{5}, \frac{11}{6}, \frac{-12}{7}, \frac{13}{8}...\} \rightarrow inf(x_4) = \frac{9}{4}$

$\vdots$

Now taking the limits of all those supremums, we get: $lim\,sup\,\{\frac{7}{2}, \frac{7}{2}, \frac{9}{4}, \frac{11}{6}, ...\} = 1.$

Thus the final answer is 1.

If you want to convince yourself further, try and make a plot of the sequence in R.

Question: Find lim inf A_n where:

$$A_n = \begin{cases} 0 \leq x < 1 & n\,is\,odd \\ 1 \leq x \leq 2 & n\,is\,even \end{cases}$$

Recall that $lim\,inf A_n = \bigcup_{k=1}^{\infty} \bigcap_{n=k}^{\infty} A_n$

Let's start off by writing out some of the subsets:

$A_1 = [0, 1)$

$A_2 = [1, 2]$

$A_3 = [0, 1)$

$\vdots$

Now let's take intersections of different combinations of subsets:

$A_1 \cap A_2 \cap A_3 \cap ... \rightarrow \emptyset$

$A_2 \cap A_3 \cap A_4 \cap ... \rightarrow \emptyset$

$A_3 \cap A_4 \cap A_5 \cap ... \rightarrow \emptyset$

$\vdots$

And the union of all the above intersections: $\{\emptyset, \emptyset, \emptyset, ...\} = \emptyset$

Thus the final answer is: $\emptyset$