Induction Example

Question: Prove that $\sum_{i=1}^{n}(8i-5) = 4n^2 - n$ for all $n \in N$

The question asks to show a statement is true for all $n \in N$ which suggests the use of induction.

- **Define** $I(n)$

  Let $I(n) : \sum_{i=1}^{n}(8i-5) = 4n^2 - n$

- **Prove the base case**

  $I(1) : \sum_{i=1}^{1}(8i-5) \stackrel{?}{=} 4(1)^2 - 1$

  $\Rightarrow I(1) : (8(1)-5) \stackrel{?}{=} 4(1)^2 - 1$

  $\Rightarrow 3 = 3$

  $\Rightarrow$ Thus $I(1)$ is true

- **Given that the k^{th} case is true, show that the $k+1^{th}$ case is true**

  Suppose $I(k)$ is true, that is $\sum_{i=1}^{k}(8i-5) = 4k^2 - k$

  Remember that you want to show that the $k+1^{th}$ case is true.
  That is, you want to show the following equation is true:

$$\sum_{i=1}^{k+1}(8i-5) = 4(k+1)^2 - (k+1) \tag{1}$$

  Start with a common algebra trick: add the same quantitiy to both sides. What may be difficult is figuring out what that quantity should be. Since we want to arrive at (1), start with what you know is true, *I(k)*, and add *(8(k+1)-5)* so the left hand side of *I(k)* will look like (1)

  $\Rightarrow 3 + 11 + ... + (8k-5) + (8(k+1)-5) = 4k^2 - k + (8(k+1)-5)$

  $= 4k^2 - k + 8k + 8 - 5$

  Let's see if we can use algebra to re-arrange the above expression to make it look like the right hand side of (1)

  $= 4k^2 + 7k + 3$

  $= 4k^2 + (8k - k) + (4 - 1)$

  $= 4k^2 + 8k + 4 + (-k) + (-1)$

  $= 4(k^2 + 2k + 1) - (k+1)$

$= 4(k+1)^2 - (k+1)$

Thus we have: $3+11+...+(8k-5)+(8(k+1)-5) = 4(k+1)^2-(k+1)$ This is (1), this is what we want to show is true.

And we have shown $I(k) \Rightarrow I(k+1)$. By induction, we have can conclude $I(n)$ holds for all $n \in N$.

Note: We could also take a direct proof approach using the identity $\sum_{i=1}^{n} i = n(n+1)/2$.