

## Direct Proof Example

**Question:** Show  $a + b = |a - b| \Rightarrow ab = 0$

Start with what is given:

$$a + b = |a - b|$$

$$\Rightarrow (a + b)^2 = (|a - b|)^2 = (a - b)^2 \text{ (since } |a|^2 = a^2 \text{)}$$

$$\Rightarrow (a + b)^2 = (a - b)^2$$

$$\Rightarrow a^2 + 2ab + b^2 = a^2 - 2ab + b^2$$

$$\Rightarrow ab = 0$$

Note: This is an example of an algebraic proof, only algebraic properties are required to solve this problem.

Note: You could also answer this question using cases:

$$\text{Case 1: " } a \geq b \text{ " : } a + b = a - b \Rightarrow b = 0 \Rightarrow ab = 0$$

$$\text{Case 2: " } a < b \text{ " : } a + b = b - a \Rightarrow a = 0 \Rightarrow ab = 0$$

Since we have shown  $a + b = |a - b| \Rightarrow ab = 0$  is true for all  $a, b$  we can conclude the  $a + b = |a - b| \Rightarrow ab = 0$  is true.