

# Counterexample Example

**True or False: The sum of two divergent sequences is divergent.**

Something to think about while you use a proof by counterexample: which is easier, proving a statement to be true, or proving it to be false? If you want to prove it to be false, you only need to find one case where the statement fails. In this example we can think of one very basic sequence that diverges.

Choose the divergent sequence  $a_n = (-1)^n$ .

Choose another divergent sequence  $b_n = (-1)^{n+1}$

The sum of these two sequences is:  $a_n + b_n = (-1 + 1), (1 + -1), (-1 + 1), \dots$

$= 0, 0, 0, \dots$

The sum of the two divergent sequences is a constant sequence  $\{0, 0, \dots\}$  which converges, trivially, to 0

Thus we have found a counterexample to disprove the statement. (The statement is FALSE)

Note: This question asks to show if this statement is true for all divergent sequences, which suggests use of a counterexample.