Continuous model theory and operator algebras

Bradd Hart

Abstract

This short course will focus on the interaction between continuous model theory and functional analysis. I will review the basics of metric logic with particular emphasis on the ultraproduct and then describe how to see the categories of Banach spaces, C^*-algebras and II_1 factors as examples of elementary classes in this logic. The role of definable sets will be highlighted.

Groups of piecewise linear and piecewise projective homeomorphisms

Justin Moore

Abstract

The group of piecewise linear homeomorphisms of $[0,1]$ and its subgroups have long been an important example in group theory, especially in the context of amenability. Still, many basic questions around this group remain unresolved. For instance, is Richard Thompson's group F amenable? If G is a subgroup of F, must G be either elementary amenable or else contain a copy of F? While these questions remain open, there have been a number of developments related to them. The purpose of this minicourse is to introduce Thompson's group F as well as Monod's group of piecewise projective homeomorphisms of the unit interval and to describe some of these recent developments.

On the classification of Type III von Neumann Algebras

Roman Sasyk

Abstract

In this series of lectures I will comment on the complexity of the classification problem for certain classes of well known von Neumann algebras of type III: The Araki-Woods factors and the Free Araki-Woods factors.

A descriptive view of unitary group representations

Simon Thomas

Abstract

I will discuss the problem of determining the relative complexity of the unitary duals of countable groups.

Countable Borel equivalence relations

Anush Tserunyan

Abstract

We will begin with a general overview of equivalence relations on Polish spaces, their Borel reducibility, and the main classification benchmarks. Focusing on countable Borel equivalence relations, we will introduce basic tools such as the LuzinNovikov theorem and the marker lemma, and give some immediate applications to Borel graph colorings and hyperfinite equivalence relations, as well as a short proof of the pointwise ergodic theorem. In the probability measure preserving context, we will discuss orbit equivalence and focus on the theory of cost, treating its foundations as well as newer results.

Ultraproducts of Betti numbers

Alessandro Carderi

Abstract

I will report on a work in progress in which we study ultraproducts of probability measure preserving actions of locally compact groups. Even though the standard ultraproduct construction gives us a highly non measurable action, the ultraproduct often admits a non-trivial factor on which the action is continuous. Moreover this construction commutes with cross sections: a cross section of the ultraproduct can be naturally identified with the ultraproduct of the cross sections. Therefore we will be able to use a joint work with D. Gaboriau and M. de la Salle (also in progress) to obtain a different approach to some of the results of the 7 samurai.

II_1 factors with non-isomorphic ultrapowers

Ionut Chifan

Abstract

In this talk I will show that there exists uncountably many separable II_1 factors whose ultrapowers (with respect to arbitrary free ultrafilters) are non-isomorphic. In fact it will be shown that the families of non-isomorphic II_1 factors introduced by Dusa McDuff in the late sixties are such examples. This entails the existence of a continuum of non-elementarily equivalent II_1 factors, thus settling a well-known open problem in the continous model theory of operator algebras. This is based on a joint work with Remi Boutonnet and Adrian Ioana.

Følner tilings via matchings

Clinton Conley

Abstract

In establishing their celebrated result characterizing amenable groups as those whose free probability-measure-preserving actions are orbit equivalent to actions of the integers, Ornstein and Weiss developed the technique of *quasi-tiling* groups. An ε-quasi-tiling of an amenable group is a cover of all but measure ε of the group (as measured by a witness to amenability) by translates of finitely many ε-Følner sets which are ε-disjoint. Recently, Downarowicz, Huczek, and Zhang have improved this to exhibit a genuine tiling of any amenable group by Følner sets. This talk will discuss a dynamical analogue of this result: any free, probability-measure-preserving action of an amenable group admits a measurable tiling by Følner sets (off a set of measure zero). In addition to techniques used in the previous tiling arguments, this incorporates ideas from Borel combinatorics, namely, the existence of measurable matchings. Time permitting, we also mention applications to the genericity of $\mathcal{Z}$-stability

This is joint work with Steve Jackson, David Kerr, Andrew Marks, Brandon Seward, and Robin Tucker-Drob.

Model theory of abelian C^*-algebras

Isaac Goldbring

Abstract

Usually C^*-algebraists view abelian C^*-algebras as boring (they are just topology). From the model-theoretic point of view, the same philosophy is more or less true: the theory of (unital) abelian C^*-algebras has a separably categorical model-completion, namely the theory of continuous functions on Cantor space. However, the subclass of projectionless abelian C^*-algebras (that is, the algebras of continuous functions on connected compact Hausdorff spaces) has a less well-understood model theory. After discussing the class of abelian C^*-algebras in general, I will discuss what is known about the model theory of projectionless abelian C^*-algebras, some of which is due to Bankston (in the guise of co-model theory of continua) and some of which represents joint work with Christopher Eagle and Alessandro Vignati. The pseudo-arc will play a central role in the latter story. I will also discuss some open questions.

Properties of groups of measurable functions

Aleksandra Kwiatkowska

Abstract

For a topological group G we consider the topological group $L_0(G)$ that consists of measurable functions defined on $([0,1],\lambda)$, where λ is the Lebesgue measure, with values in G. The multiplication is pointwise and the metric is of convergence in measure. We will focus on non-archimedean Polish groups G and on concepts and properties related to conjugacy classes, such as topological similarity and cyclically dense conjugacy classes. In the talk, I will state and explain a number of results on $L_0(G)$ groups obtained in a joint work with Maciej Malicki.

On the Tarski numbers of groups of piecewise projective homeomorphisms

Yash Lodha

Abstract

The Tarski number of a non amenable group is the smallest number of pieces needed for a paradoxical decomposition of the group. Non amenable groups of piecewise projective homeomorphisms were introduced by Monod, and non amenable finitely presented groups of piecewise projective homeomorphisms were introduced by me in joint work with Justin Moore. These groups do not contain non abelian free subgroups. In this talk I will explain why the Tarski number of all groups in both families is at most 25.

A notion of dimension for inclusion of C^*-algebras, and applications (joint work with Eusebio Gardella and Mehrdad Kalantar)

Martino Lupini

Abstract

I will present a notion of dimension for equivariant inclusions of C^*-algebras endowed with a G-action for a given compact (quantum) group G. I will explain how this notion allows one to 1) give a unified conceptual approach to several preservation results for actions with finite Rokhlin dimension, and 2) generalize them to the setting of compact quantum group actions.

Borel circle squaring

Andrew Marks

Abstract

We give a completely constructive solution to Tarski's circle squaring problem. More generally, we prove a Borel version of an equidecomposition theorem due to Laczkovich. If $k > 0$ and $A, B \subset R^k$ are bounded Borel sets with the same positive Lebesgue measure whose boundaries have upper Minkowski dimension less than k, then A and B are equidecomposable by translations using Borel pieces. This answers a question of Wagon. Our proof uses ideas from the study of flows in graphs, and a recent result of Gao, Jackson, Krohne, and Seward on special types of witnesses to the hyperfiniteness of free Borel actions of Z^d. This is joint work with Spencer Unger.

Rigidity and nonrigidity of corona algebras

Paul McKenney

Abstract

Shelah proved in the 70's that there is a model of set theory where every automorphism of the C^*-algebra of bounded sequences modulo the sequences converging to zero must be induced by a function on the natural numbers. In 2006, Farah proved that in the same model, every automorphism of the Calkin algebra on a separable Hilbert space is inner. I will discuss a common generalization of these rigidity results for a large class of corona algebras, as well as related results on nonrigidity under other models of set theory.

Coarse equivalence, topological couplings and a theorem of Gromov

Christian Rosendal

Abstract

A seminal theorem of M. Gromov states that two finitely generated groups are quasi-isometric if and only if they admit a topological coupling, thus establishing a link between the geometry and topological dynamics of groups. Much work has been done recently on expanding the tools and results of geometric group theory to locally compact groups and beyond and we shall explain how Gromov's theorem admits generalisations to all locally compact groups and even a special class of non-locally compact topological transformation groups. (The initial part of the talk will be based on joint work with U. Bader.)

Hyperfiniteness of boundary actions of cubulated hyperbolic groups

Marcin Sabok

Abstract

An old result of Dougherty, Jackson and Kechris implies that the boundary action of the free group F2 induces a hyperfinite equivalence relation. During the talk, I will discuss generalizations of this theorem to the class of hyperbolic groups. The examples discussed will include groups acting properly and cocompactly on CAT(0) cube complexes. This is joint work with Jingyin Huang and Forte Shinko

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Cardinal invariants for von Neumann algebras

David Sherman

Abstract

A cardinal invariant is an answer to the question, "How big is it?" I will show that most common cardinal invariants for von Neumann algebras can be expressed in terms of two particular ones. Among the applications are some reformulations of the generator problem. Analysts may be flabbergasted by the enormous quantities that come up, while set theorists probably will yawn.

Robinson forcing in C^*-algebras

Thomas Sinclair

Abstract

Several long-standing open problems in the theory of C^*-algebras reduce to whether for a given class of C^*-algebras there is a locally universal one among them with certain nice properties. I will discuss how techniques from model theory, in particular model-theoretical forcing, can be used to shed light on these problems. This is joint work with Isaac Goldbring.

Dynamics of monoid actions on compact semigroups (and Ramsey theory)

Slawek Solecki

Abstract

I will present a theorem on dynamics of actions of monoids by endomorphisms of semigroups. This theorem connects with algebraic structures suitable for formalizing infinitary Ramsey statements and makes it possible to prove some general Ramsey theorems for sequences. Among more concrete statements, this approach gives a generalization of the Furstenberg–Katznelson Ramsey theorem and yields a negative answer to a question of Lupini on possible extensions of Gowers' Ramsey theorem.

On the classification of Rokhlin flows

Gabor Szabo

Abstract

The classification problem for flows on operator algebras is a fundamental problem that has historically attracted much attention. This includes some recent notable achievements on the von Neumann algebraic side. Although the C^*-algebraic side has proven itself to be notoriously difficult more than anything else, the relentless pioneering work of Kishimoto stands out as an invaluable contribution to the field. A notable body of his work concerns those flows that satisfy the so-called Rokhlin property, which is a strong freeness condition introduced and studied by him and his collaborators. It is expected that this type of flows ought to be the most natural class of objects accessible to classification, and in fact it is conjectured that Rokhlin flows on Kirchberg algebras are unique. The main result of this talk is that Kishimoto?s conjecture is true. If time permits, I will speculate towards more far-reaching classification results for Rokhlin flows that may not be far behind.

Quasidiagonality

Aaron Tikuisis

Abstract

Quasidiagonality is an external approximation property for C*-algebras. The concept was original introduced by Halmos in the context of operator theory, and subsequently developed, particularly by Voiculescu. A closely related idea is an MF C*-algebra.

The allure of quasidiagonality is that there are only two known obstructions: a coarse, obvious one about stable finiteness, and a slightly subtler one relating to amenability. For a C*-algebra to be MF, only the stable finiteness obstruction persists. There are a few intriguing open questions, centering around the question of whether any other obstructions are possible.

Recent developments in the classification and structure of C*-algebras have created a resurgence in interest in quasidiagonality.

I will discuss all these things in my talk.

The complexity of the isomorphism problem for nuclear separable C^*-algebras

Andrew Toms

Abstract

In this talk we will survey the work of several researchers which ultimate leads to a full understanding of the Borel complexity of the isomorphism problem for nuclear separable C^*-algebras. The talk is intended to be broadly accessible.

Characterizing Polish groups of finite type

Asger Törnquist

Abstract

A Polish group is said to be of "finite type" if it is embeddable (as a closed subgroup) into the unitary group of a separably acting II_1 factor. These groups show up as target groups in Popa's well-known cocycle superrigidity theorems, and Popa asked if the Polish groups of finite type are characterized by the two (necessary) conditions of (1) being unitarily representable (i.e., a closed subgroup of the unitary group of separable Hilbert space), and (2) having a two-sided compatible metric.

About a year ago, Ando, Matsuzawa, Thom and the speaker gave a family of counterexamples to the above question of Popa's. In this talk, I will discuss these counterexamples, and then discuss the general (and unsolved, as far as I know) question of characterizing the Polish groups of finite type.

Model theory of measure-preserving actions

Todor Tsankov

Abstract

I will discuss a continuous logic approach to the study of measure-preserving actions of countable groups. For actions of Z (or, more generally, an amenable group G), the theory is well-understood: all free actions of G are elementarily equivalent and the theory is stable and eliminates quantifiers. (This is a result of Ben Yaacov, Berenstein, Henson, and Usvyatsov.) For non-amenable groups, the situation is more complicated and no naturally axiomatizable complete theories are known. Nonetheless, many dynamical notions such as weak containment of actions are naturally expressed in model-theoretic language and this leads to a new approach to some rigidity results. This is joint work in progress with Tomás Ibarlucía and François Le Maître.

Inner amenable groupoids and compact actions

Robin Tucker-Drob

Abstract

We introduce the notion of inner amenability for discrete p.m.p. groupoids which generalizes the notion of inner amenability of groups. In the special case of of p.m.p. equivalence relations, this gives a new orbit equivalence invariant. We show that the orbit equivalence relation associated to any free compact action of an inner amenable group is itself inner amenable as a groupoid. Conversely, any group which freely generates an inner amenable p.m.p. equivalence relation must itself be inner amenable.

RSK, representation of infinite symmetric group and Plancherel measure

Anatoly Vershik

Abstract

The isomorphism between space of classical Bernoulli scheme, Plancherel measure on the space of Young tableaux and space of linear ordering of countable (infinite space of Weyl cameras) will be explained.

The generalization of RSK (Robinson-Shenstead-Knuth correspondence) on infinite symmetric group was defined in our paper with S. Kerov in 1981.

Recently P. Sniady used this generalization, the Jeu de Taquin and limit shape theorem (VK-LS) in order to establish the isomorphism mentioned above. The direct proof of this fact and various corollaries for representation theory will be discussed.