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Talent Versus Payroll as Strategic Variables in Game Theoretic Models of Sports Leagues: Response to Madden

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Abstract

This is a response to Madden's comment on our original article: It's Not Over 'til the Fat Lady Sings: Game-Theoretic Analyses of Sports Leagues.

Keywords

game theory, duopsony, strategy space, sports league models

In the first actual duopsony analysis of a two-team sports league, Madden (2011) initially argued that the specified wage would be multivalued or *indeterminant* when the supply of talent was perfectly inelastic. To insure that his duopsony analysis would apply to this extreme case of a vertical supply curve—a case that had complicated preceding work on two-team sports leagues—Madden defaulted to *talent expenditure* as the strategic choice variable rather than the more familiar game-theoretic choice of talent itself.

Driskill and Vrooman (DV, 2014) also provided a duopsony analysis of a two-team model and like Madden's *strategic market game* (SMG) approach, the DV approach realized that team owners behave as duopsonists rather than passive price takers in the talent market. Madden described traditional duopsonists:

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> In the Cournot model of a talent market clubs would choose quantities of talent as strategic variables, anticipating the way that the wage for talent would subsequently adjust to clear the market. Since each club is non-negligible relative to the market (one of just two buyers here), they know that their strategy (quantity) choice will affect the wage, thus capturing their market power; in the Nash equilibrium, given other club quantities, no club wishes to change its quantity choice (and hence the wage). (2011, p. 409)

Like Madden (2011), DV also emphasized how their analysis applied to the extreme case of perfectly inelastic talent supply. This extreme case had led earlier researchers to invoke the since-discredited conjectural variations approach or to avoid duopsony analysis altogether. The problem appeared to be what the best responses were in the case of perfectly inelastic talent supply. Szymanski and Kesenne explained,

> The choice of one team automatically constrains the other in a two-team model, and so every possible choice of talent is a Nash equilibrium, because the other team has only one feasible response, which is therefore "best." However, this clearly makes little sense as an economic model. (2004, p. 176)

More recently, Szymanski wrote:

> Teams also cannot choose talent directly if the supply of talent is fixed and fully employed, since the quantity allocated to Team 1 will also depend on the quantity allocated to Team 2 (if talent is not fully employed, then it would be possible to choose a quantity independently of the other team). (2013, p. 322)

The perfectly inelastic supply problem was simply about multiple equilibria. DV sought a selection criterion to help choose the "most reasonable" one of many possible equilibria. Moreover, the perfectly inelastic supply assumption is an abstract extreme case of the more-realistic assumption of a "very" inelastic supply. Extreme case assumptions in economics usually are made to simplify an analysis, but a perfectly inelastic supply curve in duopsony analysis complicates more than it simplifies.

The DV approach zeroes in on equilibria associated with "very inelastic" supply arbitrarily close to perfectly inelastic supply. As noted in DV,

> This seems a natural selection criterion because it captures the notion that a perfectly inelastic inverse supply function is really an abstraction designed to be close to a very inelastic function. The notion that a parametric change from very inelastic to perfectly inelastic should lead to a change in the number of equilibria from one to infinity suggests that these multiple equilibria are simply an artifact of this perfectly inelastic abstraction. The only one of these equilibria that are reflective of the underlying economic forces is the one associated with the limit as the supply moves from very inelastic to arbitrarily close to perfectly elastic. (DV 2014, p. 6n)

In contrast to Madden (2011), DV assumed that teams chose talent levels, not talent expenditures, as their strategic variable. Madden (2015) offers a multipronged critique of the choice of talent rather than talent expenditure in DV duopsony analysis. At the theoretical level, Madden (2015) criticizes our treatment of the perfectly inelastic inverse talent supply function as perhaps letting the audience see us put the rabbit in the hat. DV argues that in duopsony analysis, wages are set at the reservation level when talent demanded by the two teams just equals total talent. Madden argues that wages should be indeterminate—anywhere between the reservation wage (normalized to 1) and $+\infty$.

Our primary response to this is based on the intersection of two ideas. First, our view (which seems to be shared by Madden) is that the important innovation of a duopsony analysis is the acknowledgment that teams infer what the effects of their talent choices will have on the subsequent equilibrium price of talent. How to specifically model what teams infer when demand equals supply in the perfectly inelastic case is problematic, but the choice of strategy space should not be the tail that wags the dog.

Our implicit assumption was that teams assumed the "auctioneer" would choose the lowest price when total demand equaled total supply. This seems to us to be plausible because, as we established, that lowest price was arbitrarily close to the equilibrium price for the "almost" perfectly inelastic case. But suppose we invoked a different assumption. Suppose the auctioneer randomly picked (from a known or perceived distribution) one of the feasible prices. For any choice of talent t_i by team i, team j's best response would then be a choice of talent arbitrarily close, but less than, to what is left over. This could be made more precise if talent is assumed to be composed of discrete (but very fine) units. Then, best responses would be such that $t_1 + t_2$ is always one (very small) unit less than 1. This concept of an approximate equilibrium has often been used to analyze models with indivisibilities.

Madden also raises the question about the realism of the implication of the DV model that predicts that wages in the perfectly inelastic case are at their reservation level. He points to North American professional leagues as evidence that this cannot be true. On the contrary, all four major North American sports leagues currently have collective bargaining agreements (CBAs) that all began with the creation of Major League Baseball Players' Association (MLBPA) in 1968.

Before collective bargaining, most professional ballplayers were paid just above their reservation wage. Major League Baseball (MLB) average and minimum player salaries are compared to the U.S. national average wage index (AWI) used by the Social Security Administration in 1964-1984 (Figure 1) and 1964-2015 (Table 1). In the absence of countervailing bargaining power, the MLB market for talent was dominated by the oligopsony power of owners. Under the reserve system, players were paid a minimum salary just above reservation AWI. Bilateral monopoly in the MLB players' labor market began with the advent of free agency in the 1976 CBA but vestiges of unbridled monopsony power continue to exist in the lowest seniority tier of the labor market, where players are still paid the MLB minimum.[1]

Madden also argues that his choice of expenditure as the strategic variable, which provides global uniqueness, "avoids the arbitrariness" associated with choosing a particular selection mechanism. This seems to be a matter of taste concerning where

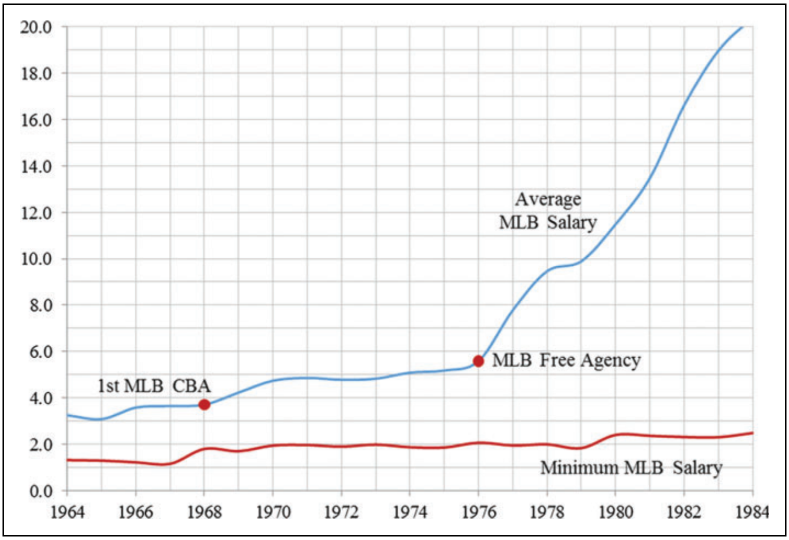

Figure 1. Major League Baseball player salaries indexed to National Average Wage 1964-1984.

and when you like your arbitrariness. The choice of a strategy space is certainly arbitrary in the same way as our choice of a selection mechanism. An argument about choice of strategy space must be based on context, much like the traditional choices in duopoly theory between quantities (Nash–Cournot) or prices (Bertrand).

Finally, Madden argues that the choice of expenditures as the strategy space is arguably a more realistic description of reality: “Team owners instructing coaches and those involved in player acquisition that they can spend $50 million (SMG), seems to resonate more than the instruction that they must land the 3 biggest stars, no matter what the cost (Cournot).”Again, this is a hard claim to verify.

Madden is making a distinction that doesn’t really make a difference. General managers are often given a dollar budget to spend, but if owners (or general partners) of teams indeed satisfy the underlying assumption that they understand the connection between talent choices and associated wages, then giving managers a money budget is indistinguishable from given them a talent level to achieve. In *Moneyball,* Oakland A’s infamous general manager Billy Bean successfully convinced team owner Steve Schott that he needed to go beyond his budget to get talent. “After Billy acquired Ricardo Rincon and Ray Durham, the team went from good to great” (Lewis, 2003, p. 217).

Finally, we must ask why team owners would play a game in which their strategy choices of *talent expenditures* (or payrolls) would leave them with less profits than would a game in which they chose *talent levels*. A comparison of the two models in terms of the specific example used in DV (2014) and in Madden SMG (2015) elaborates this point.

Table 1. MLB Average and Minimum Salaries Indexed 1964-2015 (US$).

Season	Average	Minimum	AWI	AVG	MIN
1964	$14,863	$6,000	$4,576	3.2	1.3
1965	$14,341	$6,000	$4,659	3.1	1.3
1966	$17,664	$6,000	$4,938	3.6	1.2
1967	$19,000	$6,000	$5,213	3.6	1.2
1968[a]	$20,632	$10,000	$5,572	3.7	1.8
1969	$24,909	$10,000	$5,894	4.2	1.7
1970[a]	$29,303	$12,000	$6,186	4.7	1.9
1971	$31,543	$12,750	$6,497	4.9	2.0
1972	$34,092	$13,500	$7,134	4.8	1.9
1973[a]	$36,566	$15,000	$7,580	4.8	2.0
1974	$40,839	$15,000	$8,031	5.1	1.9
1975	$44,676	$16,000	$8,631	5.2	1.9
1976[a]	$51,501	$19,000	$9,226	5.6	2.1
1977	$76,066	$19,000	$9,779	7.8	1.9
1978	$99,876	$21,000	$10,556	9.5	2.0
1979	$113,558	$21,000	$11,479	9.9	1.8
1980[a]	$143,756	$30,000	$12,513	11.5	2.4
1981	$185,651	$32,500	$13,773	13.5	2.4
1982	$241,497	$33,500	$14,531	16.6	2.3
1983	$289,194	$35,000	$15,239	19.0	2.3
1984	$329,408	$40,000	$16,135	20.4	2.5
1985[a]	$371,571	$60,000	$16,823	22.1	3.6
1986	$412,520	$60,000	$17,322	23.8	3.5
1987	$412,454	$62,500	$18,427	22.4	3.4
1988	$438,729	$62,500	$19,334	22.7	3.2
1989	$512,804	$68,000	$20,100	25.5	3.4
1990[a]	$578,930	$100,000	$21,028	27.5	4.8
1991	$891,188	$100,000	$21,812	40.9	4.6
1992	$1,084,408	$109,000	$22,935	47.3	4.8
1993	$1,120,254	$109,000	$23,133	48.4	4.7
1994[b]	$1,188,679	$109,000	$23,754	50.0	4.6
1995[a]	$1,071,029	$109,000	$24,706	43.4	4.4
1996	$1,176,967	$122,667	$25,914	45.4	4.7
1997[a]	$1,383,578	$150,000	$27,426	50.4	5.5
1998	$1,441,406	$170,000	$28,861	49.9	5.9
1999	$1,720,050	$200,000	$30,470	56.5	6.6
2000	$1,998,034	$200,000	$32,155	62.1	6.2
2001	$2,264,403	$200,000	$32,922	68.8	6.1
2002	$2,383,235	$200,000	$33,252	71.7	6.0
2003[a]	$2,555,476	$300,000	$34,065	75.0	8.8
2004	$2,486,609	$300,000	$35,649	69.8	8.4
2005	$2,632,655	$316,000	$36,953	71.2	8.6
2006	$2,866,544	$327,000	$38,651	74.2	8.5
2007[a]	$2,944,556	$380,000	$40,405	72.9	9.4

(continued)

Table 1. (continued)

Season	Average	Minimum	AWI	AVG	MIN
2008	$3,154,845	$390,000	$41,335	76.3	9.4
2009	$3,240,206	$400,000	$40,712	79.6	9.8
2010	$3,297,828	$400,000	$41,674	79.1	9.6
2011	$3,305,393	$414,000	$42,980	76.9	9.6
2012[a]	$3,440,000	$480,000	$44,322	77.6	10.8
2013	$3,650,000	$490,000	$44,888	81.3	10.9
2014	$3,950,000	$500,000			
2015	$4,250,000	$507,500			

Note. MLB = Major League Baseball; AWI = U.S. Social Security Administration national average wage index; AVG = average MLB salary divided by AWI; MIN = minimum MLB salary divided by AWI. Adapted from MLBPA and U.S. Social Security Administration.
[a] MLB Collective Bargaining Agreements beginning in 1968. [b]1990-1993 CBA terms extended through 1994-1995 player strike.

For simplicity of exposition, consider the symmetric case where the two clubs have equal home–market size. The twin revenue functions for DV and SMG are

$$R_i = m\left(\frac{\frac{1}{2}t_i^2 + t_j^2}{\left(t_i + t_j\right)^2}\right) = m\left(\frac{\frac{1}{2}e_i^2 + e_j^2}{\left(e_i + e_j\right)^2}\right), \tag{1}$$

where $e_i = ct_i$ and c is the price per unit of talent. The inverse talent supply function is

$$c(T) = (1 - T)^{-\theta}, \quad \theta \geq 0, \quad T = t_1 + t_2 \tag{2}$$

With talent expenditure as the variable of SMG choice, the first-order conditions are

$$\frac{dR_i^{\mathrm{SMG}}}{de_i} = m\left(\frac{e_j^2}{\left(e_i + e_j\right)^3}\right) = 1 \tag{3}$$

In SMG equilibrium,

$$e_i = e_j \equiv e = \frac{m}{8} \tag{4}$$

Note that the SMG expenditure equilibrium e is solved recursively, with no need for information about the talent supply function. This implies

$$t_i = t_j \equiv t = \frac{m}{8c} \tag{5}$$

Equilibrium wages are the solution to:

$$2t(c) = T^S(c) \tag{6}$$

Equation 6 shows what Madden (2011, 2015) observed with this homogenous degree zero revenue function: The SMG model makes identical predictions to a model, where teams take wages as parametrically given. In other words, the SMG model needs a nonhomogenous revenue function to generate duopsony effects.[2]

For comparison, consider the wage when $\theta = 1$ in the inverse talent supply function:

$$c = \frac{1}{(1-2t)}, \quad 2t = \frac{c-1}{c} \tag{7}$$

Together Equations 5 and 6 yield

$$\frac{m}{4c} = \frac{c-1}{c} \tag{8}$$

or

$$c^{\mathrm{SMG}} = 1 + \frac{m}{4} \tag{9}$$

Equilibrium SMG profits are also independent of any feature of the inverse-supply function of talent:

$$\pi_i^{\mathrm{SMG}} = R_i^{\mathrm{SMG}} - C_i^{\mathrm{SMG}} = \frac{3}{8}m - \frac{1}{8}m = \frac{1}{4}m \tag{10}$$

Now contrast SMG with the DV approach. As noted in our original article, general closed-form results are hard to get, but some insight can be gained by considering the solvable case for $\theta = 1$. The DV equilibrium is described by:

$$(2t)^{\mathrm{DV}} = 1 - \sqrt{\frac{2}{2+m}} \tag{11}$$

$$c^{\mathrm{DV}} = 1 - \sqrt{\frac{2+m}{2}} \tag{12}$$

$$\pi_i^{\mathrm{DV}} = R_i^{\mathrm{DV}} - C_i^{\mathrm{DV}} = \frac{3}{8}m - \frac{\frac{1}{2}\left(1 - \sqrt{\frac{2}{2+m}}\right)}{\sqrt{\frac{2}{2+m}}} \tag{13}$$

It is straightforward to show that if $m > 0$, then:

$$(2t)^{\mathrm{DV}} < (2t)^{\mathrm{SMG}}$$

$$c^{\mathrm{DV}} < c^{\mathrm{SMG}}$$

$$\pi^{\mathrm{DV}} > \pi^{\mathrm{SMG}} \tag{14}$$

As expected, the equilibrium wage is lower and the talent hired is lower for DV, but perhaps most telling are the greater profits for DV. It is hard to imagine that team owners would end up with a strategy space of expenditures, when higher profits are readily available with a strategy space of talent levels.

The choice of strategy space is the core element of game theory, and it must be based on context and guided by reality. Making talent levels, the strategic variable of choice is consistent with actual evidence from professional sports leagues and team decision making, and the choice of talent level as the strategic variable seems quite defensible.

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Notes

1. Under the modified reserve system adopted in 1976 Collective Bargaining Agreement, the Major League Baseball labor market is segmented into three seniority tiers. Tier 1 players in their first 2+ seasons remain subject to the reserve clause, Tier 2 players with 2+ years and less than six seasons are eligible for salary arbitration, and Tier 3 players with 6 full years of service (about one third) are eligible for free agency. The top 22% of players with 2 years' service (super twos) are currently arbitration eligible. For discussion of this labor market twist, see Vrooman (1996).
2. Madden (2011) correctly introduces total talent as an argument in each team's revenue function. The advantage of DV approach is that duopsony effects exist even with a traditional zero-homogeneity revenue function.

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