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SPORTSMAN LEAGUES

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Abstract

This paper compares duopsony profit-maximization and sportsman leagues and analyzes the effects of revenue sharing in both leagues. This involves formulation of a duopsony model that compares game-theoretic approaches and price-taking models. This duopsony game is played in open and closed talent markets with a supply function that approaches perfect inelasticity in the limit. The analysis explores welfare optimality of competitive balance, fan preference and revenue sharing. Revenue sharing minimizes payrolls and reduces overall talent in profit-max leagues. This leads to the conclusion that a sportsman league with optimal revenue sharing is welfare superior.

> The goal is to win. It's not about making money.
>
> —Roman Abramovich, owner Chelsea FC

I Introduction

According to received theory, the perfect game is a symbiotic contest between equally matched opponents. The practical economic problem is that professional sports leagues form imperfectly competitive natural cartels where games are played between teams with asymmetric market power. The natural duality of sports leagues implies that dominant teams may only be as strong as their weakest opponents and that competitive balance is welfare superior. The success of unbalanced leagues throughout Europe that are perennially dominated by a few powerful clubs raises the empirical question that optimal competitive balance may obtain at less than absolute equality of teams.

The economics of sports has been preoccupied with two empirical propositions that have been deemed to be true *a priori* (Rottenberg, 1956). The first truth is the *invariance proposition* that free agency for baseball players would yield the same talent distribution as the reserve (transfer) system that bound a player to one team for life. The *revenue-sharing paradox* holds that revenue sharing among asymmetric clubs has no effect on talent distribution among teams and that it serves only to deepen player exploitation.[1]

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[1] "A market in which freedom is limited by a reserve rule such as that which now governs the baseball players labor market distributes players among about as a free market would" (Rottenberg, 1956, p. 255).

In theory, the most efficient way to defeat a large-market club is to increase product market competition by adding teams to monopoly markets. Another way is for the large-market clubs to internalize diseconomies of their dominance. According to the *Yankee paradox* fans prefer winning close contests and therefore large-market dominance could be self-defeating.[2] The Yankee paradox rests on the second assumed truth that fans prefer balanced competition, when they may in fact prefer perennially dominant clubs. In reality there a several ways to defeat large-market clubs, including the possibility that club owners could be sportsmen whose ultimate goal is to win, rather than maximizing profit.[3]

The revenue-sharing paradox was formalized in two adaptations of sports league theory to the changing American sport-scape (Fort and Quirk, 1995; Vrooman, 1995) (QFV). European theorists (Szymanski, 2004; Szymanski and Kesenne, 2004) (SK) used a contest success function (CSF) to show that the invariance proposition does not hold in the *open* markets of European football, and that revenue sharing leads to *less* competitive balance. The distinction between open and closed markets may not make any difference, because both models assume that club owners maximize profits. It is possible that owners are utility maximizing *sportsmen* who sacrifice profit in order to win.

In his 1971 article celebrated in this *Journal,* Sloane observed that, "Rottenberg's argument rests on his assumption that teams are profit maximisers. However, if the clubs are utility maximizers, this (*Coasian*) result may not follow and star players will not be equally distributed between teams (Sloane, 1971, p. 138). Following a classic argument by Scitovszky (1943), Vrooman (1997, 2000) formalizes the sportsman proposition:

> The optimization problem facing the sportsman owner concerns the joint maximization of franchise value and the satisfaction derived from winning. The sportsman owner sacrifices franchise value for winning and expands the talent of his club beyond its value maximum. The resulting undervaluation of the franchise is the *sportsman effect* (1997a, p. 596).

In the limit, sportsman owners become win-maximizers, who are only constrained by zero profit. Win-max leagues are less balanced than profit-max leagues and revenue sharing *increases* competitive balance (Késenne, 1996; Vrooman, 2007, 2009).

Following QFV the basic sports-league model generally assumes a two-team league where owners are seen as monopolists in product markets, but then

[2] Also called *uncertainty of outcome hypothesis*: "No team can be successful unless its competitors also survive and prosper. Two teams opposing each other in play are like two firms producing a single product" (Rottenberg, 1956, p. 254). Glasgow rivals Celtic and Rangers are collectively called the "Old Firm".

[3] Rottenberg anticipated the sportsman effect. "Let franchises be distributed so that the size of the product market is equal for all teams (6 teams in New York, 3 in Chicago). If attendance is a unique function of the size of the market then such a distribution of teams may equalize revenues among teams. But attendance is a function of several variables. *If the psychic income is not zero for all team owners or if it is not zero for all owners differences in revenues will still occur*" (Rottenberg, 1956, p. 257–258, *italics added*).

viewed as passive price takers in a talent market where they hold duopsony power. These models generally make one of two simplifying assumptions about the elasticity of talent supply. In a closed-market, the supply of talent is perfectly inelastic, where one team's talent gain results in another's zero-sum talent loss. In an open-market, talent supply is perfectly elastic and one club's talent choice has no effect on its opponent's choice. In either case the solutions are the same as models that assume price-taking behavior. Driskill and Vrooman (DV, 2014) offer an alternative game-theoretic approach that synthesizes the limiting cases of *European* perfectly elastic supply and *American* perfectly inelastic talent supply. The DV approach leads to major differences in the price of talent in the duopsony case compared to the price-taking model.

This paper compares duopsony profit-max and sportsman leagues and analyzes the effects of revenue sharing on competitive balance (talent distribution) in both leagues. This involves a formulation of a duopsony model that shows the differences between game-theoretic approaches and conventional price-taking models. The DV duopsony game is played out in both open and closed labor markets with a talent supply function that approaches perfect inelasticity in the limit. This leads to the conclusion that without revenue sharing the talent distribution in a non-cooperative duopsony is too balanced, and that sportsman league competition is too unbalanced for a welfare optimum.

It is also shown that with revenue sharing, a cooperative profit-max cartel maximizes league revenues and fan welfare, but that it is welfare inferior because it also minimizes payrolls and reduces the overall level of league talent. Revenue sharing leads to progressive cartelization and maximum total revenue and profit at the expense of player salaries in a profit-max league, and increased competitive balance and maximum players' share at the expense of owner profit in a sportsman league. This leads to the conclusion that a sportsman league with optimal revenue sharing is welfare superior to a classic cartel. A revenue-sharing sportsman league can maximize payroll, league revenue and fan welfare, but it also can optimize the distribution of a superior level of talent.

II Talent Demand

A standard model of the demand side in sports economics is one where a team owner faces a revenue function that depends on own-market size m_i and quality of a match q_{ij} with an opponent with the market size m_j.

$$R_i = m_i q_{ij}, m_i > 0, \tag{1}$$

where quality is a quadratic function of the probability of winning w_{ij}[4]

$$q_{ij} = w_{ij} - \frac{1}{2} w_{ij}^2 \tag{2}$$

[4] This concave revenue function reflects the *Yankee paradox* that fans prefer winning close contests. Fan preference for competitive balance is a more general empirical question about the parameter $(1 - \varphi)$ in the game quality function $q_{ij} = \varphi\ w_{ij} + (1 - \varphi) w_{ij} w_{ji}$. In this case $\varphi = .5$ for $w_{ij} = 1 - w_{ji}$.

The probability of winning is determined by a contest success function CSF that depends on the relative amount of talent purchased by team i and team j

$$w_{ij} = \frac{t_i}{t_i + t_j} \tag{3}$$

This CSF implies that the relative probabilities of winning are identical to relative talent levels or competitive balance $w_1/w_2 = t_1/t_2$. The revenue function for team i becomes:

$$R_i = m_i \left[\frac{\frac{1}{2} t_i^2 + t_i t_j}{(t_i + t_j)^2} \right], \quad i,j = 1,2, i \neq j \tag{4}$$

In a profit-max league, this implies the marginal revenue product (MRP_i) for team i in terms of t_i and t_j[5]

$$MRP_i = \frac{m_i t_j^2}{(t_i + t_j)^3} \tag{5}$$

The revenue and marginal revenue product functions reflect an externality where each team's functions depend on both t_i and t_j. This externality is absent in early non-duopsony models and the simple contest success function of $w_i = t_i$ makes price-taking behavior and strategic behavior appear identical. In these models, the marginal cost of an additional unit of talent is viewed by team owners as an exogenous constant c. If they are price takers then they choose t_i to maximize profits given c and t_j.

The two-first-order conditions for each team can be interpreted as demand curves for talent with each team's talent demand conditional on the other team. But because t_j shows up in the first-order condition for team i, these first-order conditions can also be interpreted as best-response functions. For any value of t_j, the first-order condition for team i reveals the best choice of t_i. Game theory and price-taking models lead to identical conclusions without an analysis of duopsony.

III Talent Supply

The supply of talent is specified by assuming that the price of talent is an increasing function of the total amount of talent. Consider an upward-sloping inverse supply function for talent that parametrically approaches a perfectly inelastic supply of talent at the limit. Let c denote the cost of a unit of talent and total talent be $T = t_1 + t_2$. The increasing inverse supply function is

$$c(T) = (1 - T)^{-\theta}; \quad \theta > 0, \tag{6}$$

As $\theta \to 0$ in Figure 1 this inverse supply function approaches a right-angle that becomes perfectly inelastic at $T = 1$ and perfectly elastic for $T \in [0, 1]$. The classic duopsony talent solution is shown at point A in Figure 1 with the associated wage rate A'. These duopsony solutions approach B and B' as

[5] Using the chain rule: $MRP_i = MR_i MP_i = \frac{\partial R_i}{\partial t_i} = \frac{\partial R_i}{\partial w_i}\frac{\partial w_i}{\partial t_i} = \frac{m_i t_j}{(t_i+t_j)}\frac{t_j}{(t_i+t_j)^2} = \frac{m_i t_j^2}{(t_i+t_j)^3}$

$\theta \to 0$. By comparison the win-max sportsman pays talent its average revenue product $ARP_i = AC_i = c$. This solution is shown at point C in Figure 1.

IV Duopsony Solution

Team i payroll expenditure is simply ct_i and therefore:

$$C_i = ct_i = (1 - T)^{-\theta} t_i \tag{7}$$

This yields the marginal cost function for team i in a profit-max duopsony:

$$MC_i = c(T) + t_i c'(T) = (1 - T)^{-\theta-1}(1 - T + \theta t_i) \tag{8}$$

Profit maximization from equating (5) and (8) leads to $MRP_i = MC_i$:

$$MRP_i = MC_i = \frac{m_i t_j^2}{T^3} = (1 - T)^{-\theta-1}(1 - T + \theta t_i) \tag{9}$$

This implicitly defines the ith team's best-response function. In the duopsony solution the ratio of the reaction curves yields:

$$\frac{m_1 t_2^2}{m_2 t_1^2} = \frac{(1 - T + \theta t_1)}{(1 - T + \theta_2)} \tag{10}$$

In the limit where $\theta \to 0$ this also implies:

$$\frac{t_1}{t_2} = \sqrt{\frac{m_1}{m_2}} \tag{11}$$

Asymmetric solutions are shown in Figure 2 for reaction curves for $m_1 = 6$ and $m_2 = 4$ and decreasing from $\theta = 1$ to $\theta = .01$ to show the effects of $\theta \to 0$. The upward-sloping straight line is $t_1 = t_2$ and the downward-sloping straight line is $t_1 = 1 - t_2$ and its transpose $t_2 = 1 - t_1$. At the limit $\theta \to 0$ these reaction curves approach multiple equilibria along a straight line $t_1 = 1 - t_2$ analogous to the vertical supply curve. As $\theta \to 0$ the asymmetric

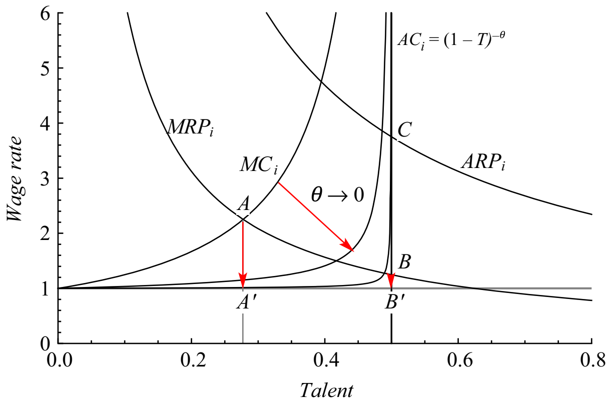

Figure 1. Duopsony limit solution.

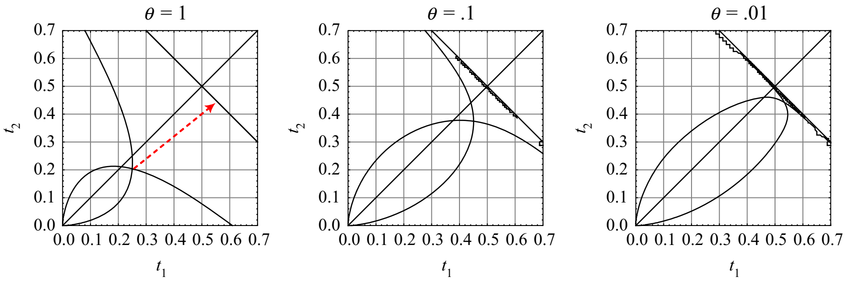

Figure 2. symmetric reaction curve limit solutions for $\theta \to 0$.

talent solution approaches $\sqrt{m_1/m_2} = \sqrt{1.5} = 1.225 = .550/.450$ infinitesimally close to the infinitely inelastic vertical range of the supply function.

In conventional price-taking models the first-order conditions for both teams are:

$$MRP_1 = \frac{m_1 t_2^2}{(t_1 + t_2)^3} = c, \quad i = 1, 2, i \neq j, \tag{12}$$

where c is exogenous for both clubs. Simultaneous profit maximization yields:

$$m_1 t_2^2 = m_2 t_1^2 \tag{13}$$

It is easy to see that relative talent ratio in the simultaneous price-taking solution is the square root of the market size ratio, which is the same as the duopsony solution in (11).

V Wage Solution

The price-taking wage rate can be determined by assuming that $T = t_1 + t_2 = 1$ when $\theta \to 0$. From (11) the price-taking solution is:

$$t_1 = \sqrt{\frac{m_1}{m_2}} \quad t_2 = \sqrt{\frac{m_2}{m_1}} \tag{14}$$

$$t_1 = \frac{1}{(1 + \sqrt{m_2/m_1})} \quad t_2 = \frac{1}{(1 + \sqrt{m_1/m_2})} \tag{15}$$

In the limiting case the price-taking wage becomes:

$$c = \frac{m_1}{(1 + \sqrt{m_1/m_2})^2} \tag{16}$$

This implicitly defines the market size combinations that are sufficient for all talent to be employed in the vertical portion of the talent supply curve: $\frac{m_1}{(1+\sqrt{m_1/m_2})^2} \geq 1$

In the limit $\theta \to 0$ the duopsony solution is identical to price-taking solution with an important exception. Talent is paid the minimum wage regardless if all talent is used. MC always lies above c and therefore duopsony talent is always paid less than its MRP.

VI Cartel Solution

Total league cartel profit π^* is the sum of club revenues minus total talent cost:

$$\pi^* = R_1 + R_2 - cT = \frac{m_1\left(\frac{1}{2}t_1^2 + t_1t_2\right) + m_2\left(\frac{1}{2}t_2^2 + t_1t_2\right)}{(t_1+t_2)^2} - c(1-T)^{-\theta} \tag{17}$$

Total league profit and revenue each reach a maximum when: $\frac{\partial\pi^*}{\partial t_1} = \frac{\partial\pi^*}{\partial t_2}$

$$\frac{t_2(m_1t_2 - m_2t_1)}{(t_1+t_2)^3} = \frac{t_1(m_2t_1 - m_1t_2)}{(t_1+t_2)^3} = (1-T)^{-\theta-1}(1 - T + \theta T) \tag{18}$$

In the limit:

$$t_2(m_1t_2 - m_2t_1) = t_1(m_2t_1 - m_1t_2) \tag{19}$$

and for $m_1 > m_2$ the cartel profit maximum occurs when[6] :

$$\frac{t_1}{t_2} = \frac{m_1}{m_2} \tag{20}$$

This leads to an initial duopsony proposition:

Proposition 1: The profit-max league equilibrium is characterized by:

(1) Talent distribution in both price-taking and duopsony models is $\frac{t_1}{t_2} = \sqrt{\frac{m_1}{m_2}}$.

(2) Talent distribution in a league cartel is less balanced than duopsony $\frac{t_1}{t_2} = \frac{m_1}{m_2}$.

(3) The wage rate in the conventional price-taking model is $c = \frac{m_1}{(1+\sqrt{m_1/m_2})^2}$.

(4) The duopsony price of talent is the reservation wage $c = 1$ and $T = 1$ in a closed league when $\frac{m_1}{(1+\sqrt{m_1/m_2})^2} \geq 1$.

(5) The duopsony price of talent is the reservation wage $c = 1$ and $T < 1$ in an open league when $\frac{m_1}{(1+\sqrt{m_1/m_2})^2} < 1$

VII Revenue Sharing in Profit-Max League

A major controversy in the modeling sports leagues concerns the effects of revenue sharing and the so-called *invariance proposition*. Intuition suggests that revenue sharing should improve competitive balance between a large and a small-market team. The existence of a counter-intuitive *revenue-sharing paradox* arises from virtually all models with profit-maximizing teams and revenue functions. This result remains in the limiting case of this duopsony model as $\theta \to 0$. It can also be shown that as revenue sharing approaches the pure syndicate $\alpha \to 0$ relative talent (competitive balance) approaches the cooperative

[6] Vrooman (1995) solved for the win ratio $w_1/w_2 = [m_1/m_2]^{(\alpha - \gamma)/(\delta - \beta)}$. "Diminishing marginal returns to talent would imply that the *MP* of talent acquired by Team 1 would be greater than it was for Team 2 for winning percentages above .500. As a result the actual competitive balance solution under profit maximization will be more balanced than that predicted by QF's league revenue maximization solution (1995, p. 976)".

cartel solution $t_1/t_2 = m_1/m_2$. Ironically it is also true that at the revenue sharing limit $\alpha \to 0$ the absolute talent levels approach zero for both clubs.

If α is the home team revenue share, and $(1-\alpha)$ is the visiting team share of revenue then pooled revenue sharing modifies the revenue function for team 1 for $\alpha \in [0, 1]$[7]:

$$R'_1 = \alpha R_1 + (1-\alpha)(R_1 + R_2)/2 \tag{21}$$

The revenue-sharing function for team 1 in terms of t_1 and t_2:

$$R'_1 = \frac{(1+\alpha)m_1\left(\frac{1}{2}t_1^2 + t_1t_2\right) + (1-\alpha)m_2\left(\frac{1}{2}t_2^2 + t_1t_2\right)}{2(t_1+t_2)^2} \tag{22}$$

implicitly yields the best-response function for team 1 talent t_1 in terms of t_2:

$$MRP'_1 = \frac{(1+\alpha)m_1t_2^2 - (1-\alpha)m_2t_1t_2}{2(t_1+t_2)^3} = (1-t_1-t_2)^{-\theta-1}(1-t_1-t_2+\theta t_1) \tag{23}$$

This implies the MRP' ratio for both clubs:

$$\frac{MRP'_1}{MRP'_2} = \frac{(1+\alpha)m_1t_2^2 - (1-\alpha)m_2t_1t_2}{(1+\alpha)m_2t_1^2 - (1-\alpha)m_1t_1t_2} = \frac{(1-t_1-t_2+\theta t_1)}{(1-t_1-t_2+\theta t_2} \tag{24}$$

In the limit $\theta \to 0$ and:

$$(1+\alpha)m_1t_2^2 - (1-\alpha)m_2t_1t_2 = (1+\alpha)m_2t_1^2 - (1-\alpha)m_1t_1t_2 \tag{25}$$

Dividing both sides by t_2^2 yields the quadratic in terms of the relative talent ratio t_1/t_2:

$$m_2\left(\frac{t_1}{t_2}\right)^2 - \frac{(1-\alpha)}{(1+\alpha)}(m_1-m_2)\frac{t_1}{t_2} - m_1 = 0 \tag{26}$$

At the revenue-sharing limit $\alpha \to 0$, equation (26) can be solved for the limiting case:

$$\frac{t_1}{t_2} = \frac{m_1}{m_2} \tag{27}$$

As the *relative* talent of the large-market club increases in a revenue-sharing duopsony league and approaches the cartel profit-max ratio in the limit.[8] The interesting result from DV game-theoretic approach is that the *MRP* demand for talent for both clubs approaches zero at the revenue-sharing limit. The vanishing talent effect can be easily seen in the in the best-response solutions in Figure 5 for (m_1 = 6, m_2 = 4 and θ = .01). As best-response functions shift downward as $\alpha \to 0$ left to right, absolute talent vanishes while relative talent

[7] In a pooled sharing scheme the visiting team share is pooled and divided equally among clubs $(1-\alpha)/n$. This formula has been used in *MLB* since 1999 and *NFL* since 2002 because it equalizes the shared amount.

[8] As $\alpha \to 0$, equation (25) becomes equation (16). This result can be found in Vrooman (2007, 2009).

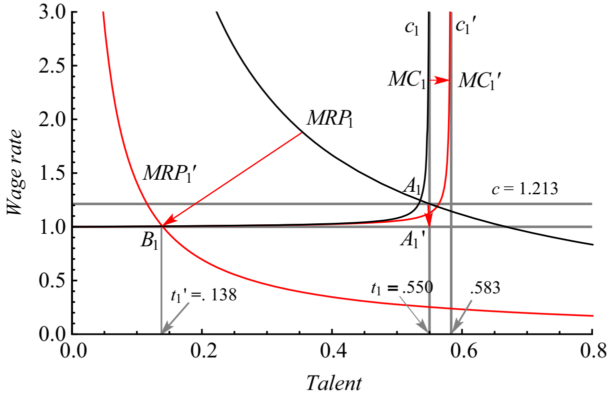

Figure 3. Profit-max revenue sharing for large-market club.

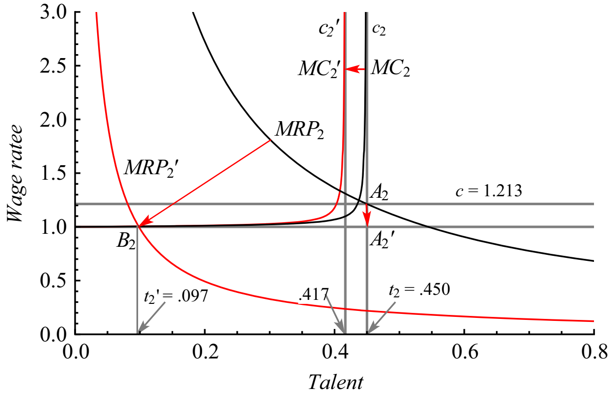

Figure 4. Profit-max revenue sharing in small-market club.

approaches the cartel max-profit solution $t_1/t_2 = 1.5$. This occurs because cartel revenue depends only on the ratio of talent, and not the level of talent. For any relative ratio t_1/t_2, the cartel would maximize profits by scaling down operations.

The vanishing talent effect is shown in Figure 3 for the large-market club and Figure 4 for the small-market club. Compare price-taking solutions at A_1 and A_2 with the duopsony solutions at A_1' and A_2' and the revenue-sharing solutions at B_1 and B_2 for the large-market and small-market clubs. The *MRP* curves are rectangular hyperbolae that asymptotically approach their respective axes as the league becomes the perfect syndicate.[9] As $\alpha \to 0$ the t_1/t_2 ratio approaches the cartel profit maximum but both t_1 and t_2 approach zero. Revenue sharing shifts the league from the non-cooperative solution $t_1/t_2 = \sqrt{m_1/m_2} = \sqrt{1.5}$ with $t_1 = .550$ and $t_2 = .450$ toward $t_1/t_2 = m_1/m_2 = 1.5$.

[9] As $\alpha \to 0$ talent demand (*MRP*) curves each approach L-shape of the two axes, and in the limit $\theta \to 0$ the talent supply function approaches a reverse L-shape along $c = 1$; $T = 1$ for a corner solution $c = 1$; $T = 0$.

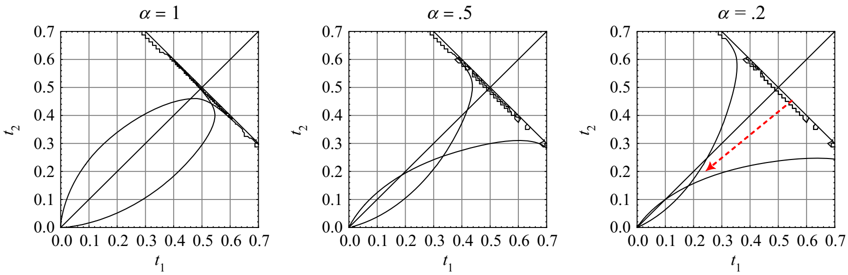

Figure 5. Revenue-sharing paradox.

Figures 3 and 4 can be combined in the zero-sum restrictions of Figure 5 (from QFV and Vrooman (2007, 2009)) where talent level of team 1 is shown moving from left to right and team 2′s is talent moving from right to left. As $\alpha \to 0$ MRP_1 and MRP_2 ($\alpha = 1$) shift to MRP'_1 and MRP'_2 ($\alpha = .2$) and the league moves from the non-cooperative duopoly solution $t_1/t_2 = \sqrt{m_1/m_2}$ at A (price-taking) and A' (duopsony) toward the less-balanced cooperative cartel solution $t_1/t_2 = m_1/m_2$. For $\alpha = .2$ talent levels are cut for team 1 from $t_1 = .550$ at A to $t'_1 = .138$ at B_1 and for team 2 from $t_2 = .450$ to $t'_2 = .097$ at B_2.[10] Simultaneously, their talent ratio $t'_1/t'_2 = 1.42$ is approaching the cooperative cartel profit-max solution $t_1/t_2 = m_1/m_2 = 1.5$. As $\alpha \to 0$ the league is progressively being converted from a strategic duopsony into a cooperative cartel that maximizes league profit by downscaling talent while it approaches the talent *ratio* that maximizes league profit.[11] This leads to a revenue-sharing proposition for profit-max leagues:

Proposition 2: Revenue sharing in profit-max league is characterized by:

(1) The revenue-sharing paradox still holds that there is an inverse relationship between pooled revenue sharing $\alpha \to 0$ and competitive balance t_1/t_2

(2) Competitive balance approaches the cartel profit maximum $t_1/t_2 \to m_1/m_2$ as the league becomes a perfect syndicate $\alpha \to 0$.

(3) Exploitation of talent $c < \frac{m_1}{(1+\sqrt{m_1/m_2})^2}$ in both the price-taking model and the duopsony model where talent is paid the reservation wage $c = 1$.

(4) Talent exodus for both teams $T \to 0$ as the duopsony morphs into a perfect syndicate where $\alpha \to 0$.

[10] An example of the talent drain occurred in 2001, when French Ligue 1 was ranked last of the European Big 5 leagues while FIFA had the French national team ranked first in the world. Before 2005, French Ligue 1 TV revenue sharing allocated 83 percent for solidarity, ten percent merit and seven percent appearances. Increased merit sharing under *Charte 2002 des clubs de football* was justified on the premise that Ligue 1 clubs were at a disadvantage in international competition (UEFA Champions League) because of solidarity sharing. Ligue 1 changed the formula to 50 percent solidarity, 30 percent league finish and 20 percent appearances.

[11] The cartel is internalizing the Yankee paradox externality and revenue sharing is a tax and subsidy payment from team 1 to team 2 to recapture the revenue loss incurred under non-cooperative duopsony.

VIII Sportsman Win-Max League

In *sportsman leagues*, team owners are willing to sacrifice profit for winning. At the limit, a *pure sportsman* becomes a win-maximizer, constrained by zero profit rather than maximum profit such that $R_1 = ct_1$ and $R_1/t_1 = c$. The *sportsman league* win-max solution obtains where talent is paid its average revenue product from (4) and (6):

$$ARP_1 = AC_1 = \frac{m_1\left(\frac{1}{2}t_1 + t_2\right)}{(t_1 + t_2)^2} = (1 - T)^{-\theta} \tag{28}$$

The *win-max* equilibrium is shown at C in Figure 1 and A in Figure 6. The ratio of ARP curves implies that in league equilibrium:

$$m_1\left(\frac{1}{2}t_1 + t_2\right) = m_2\left(\frac{1}{t_2 + t_1}\right) \tag{29}$$

Dividing both sides by t_2 yields the general win-max league solution:

$$\frac{t_1}{t_2} = \frac{m_1 - \frac{1}{2}m_2}{m_2 - \frac{1}{2}m_1} = \frac{\sigma - \frac{1}{2}}{1 - \frac{\sigma}{2}} \tag{30}$$

where $\sigma \equiv \frac{m_1}{m_2}$. If the interior solution requirement of $\sigma < 2$ is not satisfied then the win-max league results in a corner solution, $t_1 = 1$ and $t_2 = 0$. In Figure 7 the sportsman league equilibrium $ARP_1 = ARP_2$ at B is compared to the duopsony solution at A. A large-market sportsman club is more dominant than in a duopsony or cartel, because win-max owners spend all revenue on talent and are constrained by zero profit rather than max profit.[12]

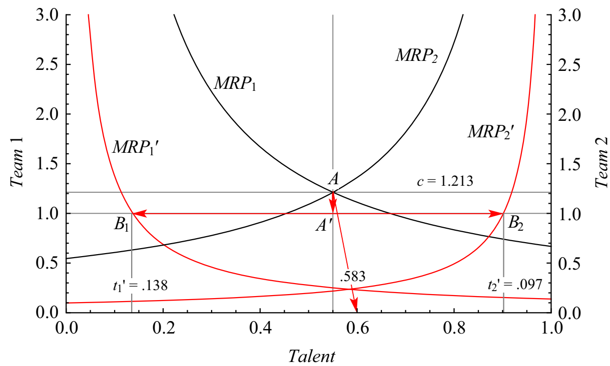

Figure 6. Revenue sharing in profit-max league.

[12] An unconstrained (profits can be negative) asymmetric league with symmetric sportsman preferences (not always win-max) is more balanced than a profit max league: $t_1/t_2 = [(\delta m_1 - 1) + 1]/[(\delta m_2 - 1) + 1]$, where δ is a preference for profit and $1 - \delta$ is a preferences for wins. This yields a pure profit-max $t_1/t_2 = m_1/m_2$ and sportsman ratio of .500 at the unconstrained sugar-daddy limit. (See, Vrooman, 1997, 2000).

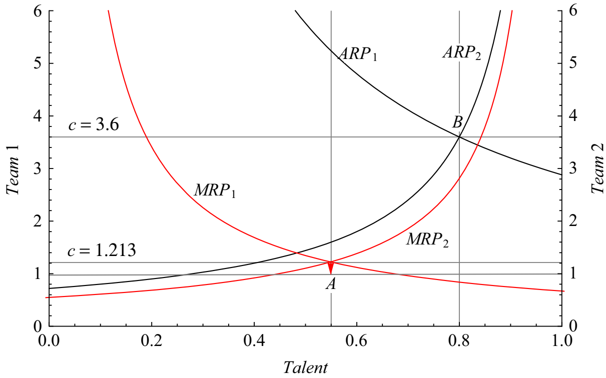

Figure 7. Sportsman win-max league.

Substitution of equation (30) into equation (28) gives the sportsman win--max wage rate:

$$c = \frac{3m_1m_2}{2(m_1+m_2)} \text{ for } \frac{m_1}{m_2} < 2 \tag{31}$$

Equation (31) implicitly defines the market size combinations $\frac{3m_1m_2}{2(m_1+m_2)} \geq 1$ necessary for all talent to be employed $t_1 + t_2 = 1$ in the vertical portion of the talent supply curve.

IX Revenue Sharing in Win-Max League

The question whether the *revenue-sharing paradox* holds in a win-max league can be answered by modifying (12), so that the revenue-sharing function for team 1 becomes:

$$R_1' = \frac{(1+\alpha)m_1\left(\frac{1}{2}t_1^2 + t_1t_2\right) + (1-\alpha)m_2\left(\frac{1}{2}t_2^2 + t_1t_2\right)}{2(t_1+t_2)^2} \tag{32}$$

This yields the average revenue product for talent in a revenue-sharing win-max league:

$$ARP_1' = \frac{(1+\alpha)m_1\left(\frac{1}{2}t_1 + t_2\right) + (1-\alpha)m_2\left(\frac{t_2^2}{2t_1} + t_2\right)}{2(t_1+t_2)^2} \tag{33}$$

Equation (33) yields the APR ratio for clubs 1 and 2 which is the implicit function of the relationship between revenue sharing α and the talent ratio t_1/t_2:

$$\sigma = \frac{(1+\alpha)\left(\frac{1}{2}t_2 + t_1\right) - (1-\alpha)\left(\frac{t_2^2}{2t_1} + t_2\right)}{(1+\alpha)\left(\frac{1}{2}t_1 + t_2\right) - (1-\alpha)\left(\frac{t_1^2}{2t_2} + t_1\right)} \tag{34}$$

This leads to a win-max revenue-sharing proposition:

Proposition 3: Sportsman win-max league equilibrium is characterized by:

(1) Increased dominance by the – market team with corner solution of $t_1 = 1$ and $t_2 = 0$ for $\frac{m_1}{m_2} \geq 2$ and interior solution of $\frac{t_1}{t_2} = \frac{(2\sigma-1)}{(2-\sigma)}$ for $\sigma < 2$ at A in Figure 8.

(2) A direct relationship between pooled revenue sharing and competitive balance and the league approaches perfect balance $t_1/t_2 \to 1$ as $\alpha \to 0$ at C in Figure 8.

(3) Competitive balance approaching the cartel profit maximum $t_1/t_2 \to m_1/m_2$ for $\alpha = (\sigma^2 + \sigma + 1)/(\sigma^2 + 3\sigma + 1)$ where $\sigma \equiv m_1/m_2$ at B in Figure 8

(4) Talent being paid its average revenue product which also reaches a max at cartel revenue max B in Figure 8.

The left frame of Figure 9 shows the inverse relationship between revenue sharing and competitive balance in a profit-max league for $\sigma = 1.5$. Progressive cartelization from duopsony talent distribution at $\sqrt{\sigma}$ for $\alpha = 1$, to pure syndicate at σ for $\alpha = 0$ maximizes total cartel revenue and optimizes the distribution of an inferior level of talent as $\alpha \to 0$.

The right frame of Figure 9 shows a positive relationship between revenue sharing and competitive balance in a sportsman win-max league. If the paradox does not hold in a sportsman league then revenue sharing would adjust the talent distribution toward perfect balance $t_1/t_2 = 1$ as $\alpha \to 0$. More importantly, revenue sharing could achieve the same revenue maximum as a cartel at B for $t_1/t_2 = m_1/m_2$. Equation (34) yields a simple sportsman-league revenue-sharing rule for optimizing league revenue:

$$\alpha^* = \frac{\sigma^2 + \sigma + 1}{\sigma^2 + 3\sigma + 1} \tag{35}$$

If $\sigma = 1.5$ then $\alpha = .613$ generates the revenue maximum solution shown at B in Figures 8 and 9. The revenue-sharing scheme that optimizes win-max revenue at $\sigma \equiv m_1/m_2$ also maximizes salaries of superior levels of talent. This compares to a profit-max cartel that optimizes talent distribution while minimizing salaries for an inferior level of talent. This leads to a summary proposition comparing profit-max and win-max leagues:

Proposition 4: The differences between profit-max and win-max leagues include:

(1) A profit-max league is more balanced at $\frac{t_1}{t_2} = \sqrt{\sigma}$ than a win-max league where $t_1 = 1$ and $t_2 = 0$ *if* $\sigma \geq 2$, *or* $\frac{t_1}{t_2} = \frac{2\sigma-1)}{(2-\sigma)}$ *for* $\sigma < 2$ in Figures 8 and 9 point A.

(2) Talent in a profit-max league is paid its reservation wage which is always less than MRP, while talent receives its ARP in a win-max sportsman league.

(3) The revenue-sharing paradox holds in a profit-max league where there is an inverse relationship between pooled revenue sharing and competitive balance: where $t_1/t_2 \to \sigma$ as $\alpha \to 0$. This is shown in Figure 9 left panel.

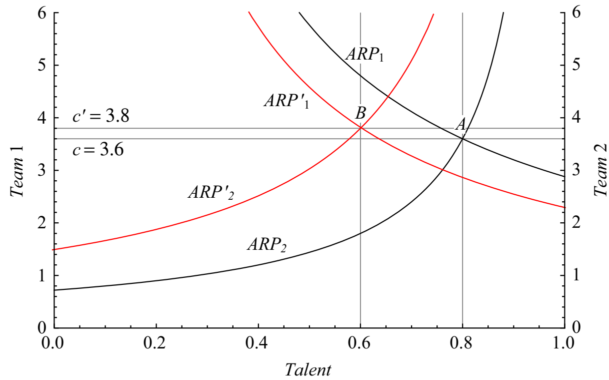

Figure 8. Optimum revenue sharing in win-max league.

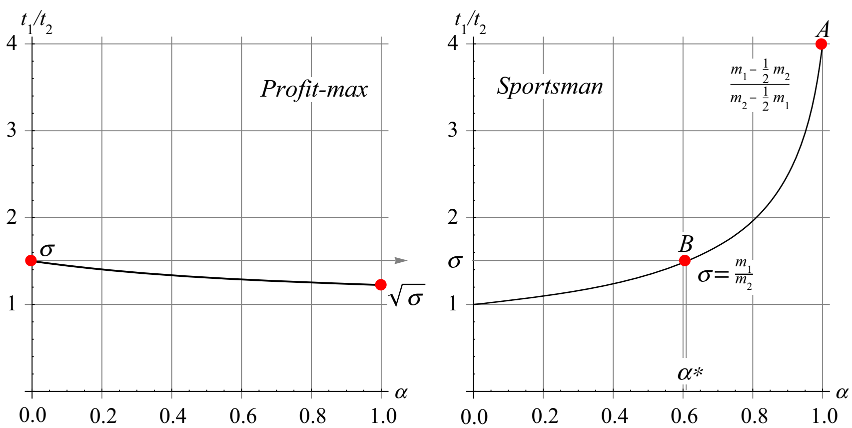

Figure 9. Revenue sharing and competitive balance.

(4) Revenue-sharing paradox does not hold in a sportsman win-max league where there is a direct relationship between revenue sharing and competitive balance: where $t_1/t_2 \rightarrow 1$ *as* $\alpha \rightarrow 0$. This is shown in right panel in Figure 9.
(5) Profit-max leagues maximize total revenue at $\alpha = 0$ but minimize payroll.
(6) Win-max leagues maximize total revenue at $\alpha^* = \frac{\sigma^2+\sigma+1}{\sigma^2+3\sigma+1}$ and maximize payroll.

X Optimal Competitive Balance

It is usually assumed that club owners behave as profit-maximizing monopolists in regionally distinct product markets for ticket pricing and local media rights. That leaves questions of league competition to be answered in duopsony talent markets shared by both profit-max and/or win-max sportsmen clubs. Dietl and Lang (2008) and Dietl *et al.* (2009) use a technique taken from Falconieri *et al.* (2004) to derive a product market demand function $d(m, p,q) = m(q\text{-}p)/q$, where q is the measure of match quality taken from equation (2) and the monopoly price is set $p^* = q/2$. Fan surplus is the integral of the demand function from p^* to the maximal price that a [0, 1] continuum of fans will pay and social welfare is simply $\frac{3}{8}(m_1q_1 + m_2q_2)$.

Social welfare is defined as the sum of fan surplus, aggregate profit and aggregate player salaries. Because total league quality is always proportional to total league revenue $(m_1q_1 + m_2q_2)$ it is not surprising to find that social welfare is maximized at the same talent ratio that maximizes league revenue. Optimal competitive balance is therefore, reached at the cartel solution $t_1/t_2 = \sigma$. This is the classic cartel solution in the presence of externalities. The optimality of the solution derives from the cartel's ability to jointly maximize league profit and internalize the Yankee paradox. So it is well known that the non-cooperative duopsony solution is *too* balanced for the internal profit maximum because it leaves potential revenue on the table, and that the sportsman win-max owner is *too* unbalanced because the single-minded pursuit of wins ignores the externality. The important result taken from this welfare analysis is that "a certain degree of imbalance is socially desirable" and that the duopsony profit-max win ratio $t_1/t_2 = \sqrt{\sigma}$ falls short of that optimum while the win-max solution $t_1/t_2 = \left(\sigma - \frac{1}{2}\right)/\left(1 - \frac{\sigma}{2}\right)$ goes beyond it. In this welfare model player salaries do not directly affect social welfare because they are simply a transfer from clubs owners to players.

As discovered above using the DV approach, players in a strategic duopsony game are always paid the salary minimum and all revenue goes to the owners at the league revenue maximum. Moreover, if the cartel solution is achieved through the taxation and subsidization of revenue sharing, then all talent in the league will vanish at the limit. It is easy to see that externalities are internalized and league revenues and fan welfare are maximized for the cartel, but it is difficult to accept a welfare criterion that optimizes the distribution of talent at the expense of the level of talent.

XI Fan Preference

Welfare criteria are of course ultimately a function of fan preference. The *Yankee paradox* is the empirical argument that fans prefer close wins instead of blow outs. Fan preference for competitive balance implies strictly concave revenue functions where $\varphi \in [0, 1]$ and $(1 - \varphi)$ reflects a general fan preference for competitive balance:

$$R_1 = m_1[\varphi w_1 + (1 - \varphi)w_1w_2] \; R_2 = m_2[\varphi w_2 + (1 - \varphi)w_2w_1] \tag{36}$$

The revenue function can be generalized in terms of talent and fan preference:

$$R_1 = m_1\left[\frac{\varphi t_1^2 + t_1 t_2}{(t_1+t_2)^2}\right] \; R_2 = m_2\left[\frac{\varphi t_2^2 + t_1 t_2}{(t_1+t_2)^2}\right] \tag{37}$$

And the MRP_1 function for both non-cooperative teams becomes:

$$MRP_1 = m_1\left[\frac{t_2^2 + (2\varphi-1)t_1 t_2}{(t_1+t_2)^3}\right] \; MRP_2 = m_2\left[\frac{t_1^2 + (2\varphi-1)t_1 t_2}{(t_1+t_2)^3}\right] \tag{38}$$

Divide both sides by t_1^2 and equation (38) becomes a quadratic that can be solved for $\frac{t_1}{t_2}$:

$$\frac{t_1}{t_2} = \frac{\sqrt{[(\sigma-1)(2\varphi-1)]^2 + 4\sigma} + (\sigma-1)(2\varphi-1)}{2} \tag{39}$$

General cooperative cartel revenue in terms of φ:

$$R^* = R_1 + R_2 = \frac{\sigma(\varphi t_1^2 + t_1 t_2) + (\varphi t_2^2 + t_1 t_2)}{(t_1+t_2)^2} \tag{40}$$

yields the cartel MRP_1 for team 1:

$$MRP_1^* = \frac{\sigma[t_2^2 + (2\varphi-1)t_1 t_2] - [(2\varphi-1)t_2^2 + t_1 t_2]}{(t_1+t_2)^3} \tag{41}$$

And the general cartel solution:

$$\frac{t_1}{t_2} = \frac{\sigma - (2\varphi-1)}{1-(2\varphi-1)\sigma} \tag{42}$$

The existence of an interior solution in equation (42) requires $\phi < (\sigma + 1)/2\sigma$. For example, if $\sigma = 1.5$, then an interior solution requires $\varphi < .833$. If $\varphi \geq .833$ then the large-market club would acquire all talent and the corner solution would become $t_1 = 1$ and $t_2 = 0$.

The general revenue-sharing function for team 1 is specified as:

$$R_1' = \frac{(1+\alpha)m_1(\varphi t_1^2 + t_1 t_2) + (1-\alpha)m_2(\varphi t_2^2 + t_1 t_2)}{2(t_1+t_2)^2} \tag{43}$$

This yields the revenue-sharing demand for talent in terms of fan preference:

$$\begin{aligned} MRP_1' &= \frac{(1+\alpha)m_1 t_2[(2\varphi-1)t_1 + t_2] - (1-\alpha)m_2 t_2[t_1 + (2\varphi-1)t_2]}{2(t_1+t_2)^3} \\ MRP_2' &= \frac{(1+\alpha)m_2 t_1[t_1 + (2\varphi-1)t_2] - (1-\alpha)m_1 t_1[(2\varphi-1)t_1 + t_2]}{2(t_1+t_2)^3} \end{aligned} \tag{44}$$

If $\alpha = 1$ then equation (44) becomes the non-cooperative game in equation (38), and if $\alpha = 0$ then equation (44) becomes a cooperative cartel in equation (41). An interior solution requires:

$$\varphi < \left[\frac{(1-\alpha)\sigma + (1+\alpha)}{2(1-\alpha)\sigma}\right] \tag{45}$$

The first term in equation (44) has two externalities that are internalized by non-cooperative duopsony teams. The Yankee paradox is reflected in the concavity of each team's revenue function, and a second externality $\frac{\partial w_1}{\partial t_1} = \frac{t_2}{(t_1+t_2)^2}$ is reflected in the concavity of each team's logistic *CSF*. As $\alpha \to 0$ the negative sign in the second term reveals another externality where the Yankee paradox interacts with an opponent's *CSF:* $\frac{\partial w_2}{\partial t_1} = \frac{t_2}{(t_1+t_2)^2}$. This interdependence is ignored in non-cooperative duopsony leagues but internalized in cooperative cartels. As $\alpha \to 0$ the demand for talent decreases for both clubs, but small-market team 2's demand decreases more than that of team 1 because the externality is larger for Team 2 ($m_1t_1 > m_2t_2$). The negative externality is mitigated when $\varphi \to 0$ because more team 2 talent also increases competitive balance and amplified when $\varphi \to 1$ because additional team 2 talent only decreases team 1 wins.

Setting $MRP_1' = MRP_2'$ and dividing by t_2^2 creates a quadratic in terms of $\frac{t_1}{t_2}$:

$$\begin{aligned}&\sigma\left\{(1+\alpha)\left[1+(2\varphi-1)\frac{t_1}{t_2}\right]+(1-\alpha)\left[(2\varphi-1)\frac{t_1^2}{t_2}+\frac{t_1}{t_2}\right]\right\}\\&=\left\{(1+\alpha)\left[\left(\frac{t_1^2}{t_2}+(2\varphi-1)\frac{t_1}{t_2}\right)\right]+(1-\alpha)\left[(2\varphi-1)+\frac{t_1}{t_2}\right]\right\}\end{aligned} \tag{46}$$

Implicit solutions of (46) are shown in the duopsony frame of Figure 10 for $\sigma = 1.5$. Duopsony solutions lie in the right axial plane where $\alpha = 1$ and cartel solutions lie in the left axial plane, where $\alpha = 0$. Infra-fan solutions are in the front axial plane $\varphi = 0$ and ultra-fans are toward the back in the $\varphi = 1$ plane. Duopsony revenue sharing $\alpha \to 0$ yields an optimal distribution $t_1/t_2 \to m_1/m_2$ of inferior talent $t_1 + t_2 \to 0$. The revenue-sharing paradox effect increases as $\phi \to 1$ because balance is irrelevant to ultra-fans and decreases as $\phi \to 0$ because balance is all-important to infra-fans.

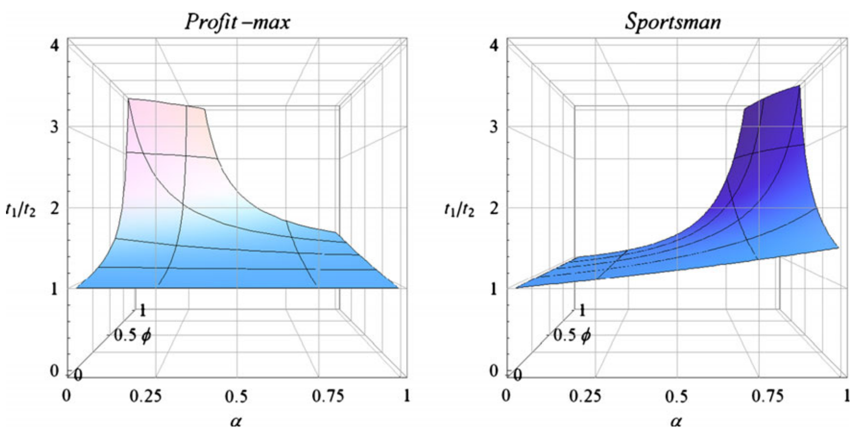

Figure 10. Revenue sharing, fan preference and talent distribution.

Finally sportsman objectives can be specified in terms of fan preference φ:

$$ARP_1 = ARP_2 = \frac{\sigma(\varphi t_1 + t_2)}{(t_1 + t_2)^2} = \frac{(\varphi t_2 + t_1)}{(t_1 + t_2)^2} \tag{47}$$

which yields:

$$\frac{t_1}{t_2} = \frac{\sigma - \varphi}{(1 - \varphi\sigma)} \tag{48}$$

An interior solution for a sportsman win-max league requires $\phi < 1/\sigma$. Substitution of equation (47) into equation (46) yields the sportsman wage rate:

$$c = \left(\frac{1 - \varphi^2}{1 - \varphi}\right)\left(\frac{\sigma}{\sigma + 1}\right) \tag{49}$$

The ARP'_1 function can also be generalized for revenue sharing and fan preference:

$$ARP'_1 = \frac{\sigma(1 + \alpha)(\varphi t_1 + t_2) + (1 - \alpha)\left(\varphi \frac{t_2^2}{t_1} + t_2\right)}{2(t_1 + t_2)^2} \tag{50}$$

Setting $ARP'_1 = ARP'_2$, $\sigma \equiv \frac{m_1}{m_2}$ defining and dividing by t_2 creates a quadratic for $\frac{t_1}{t_2}$:

$$\begin{aligned}\sigma\left[(1 + \alpha)\left(\varphi\frac{t_1}{t_2} + 1\right) - (1 - \alpha)\left(\varphi\frac{t_1^2}{t_2} + \frac{t_1}{t_2}\right)\right] \\ = \left[(1 + \alpha)\left(\varphi + \frac{t_1}{t_2}\right) - (1 - \alpha)\left(\varphi\frac{t_2}{t_1} + 1\right)\right]\end{aligned} \tag{51}$$

Implicit solutions of equation (51) are shown in Figure 10 for a sportsman for $\sigma = 1.5$. Infra-fans are on the front axial plane $\varphi = 0$ and ultra-fan solutions are toward the back axial plane $\varphi = 1$. Revenue sharing ($\alpha \to 0$) leads to absolutely balanced talent $t_1/t_2 = 1$ for all fan preferences in the limit. Substitution of the cartel optimum equation (42) in the profit-max frame into equation (51) in the sportsman frame yields revenue-sharing scheme necessary to achieve welfare optimal distribution of superior talent in a win-max league:

$$\alpha^* = \frac{\varphi(\sigma^2 + 2\sigma\varphi + 1)}{\sigma^2\varphi + 2\sigma(\varphi^2 - \varphi + 1) + \varphi} \tag{52}$$

XII General Solutions

The conventional assumption of $\varphi = .5$ for the *moderate fan* preference transforms equations in the previous section to their respective specifications and solutions derived earlier. Setting aside the mathematical convenience of $\varphi = .5$, it seems reasonable to suggest that preference for competitive balance in the real world is an empirical question. In the plausible absence of a Yankee paradox, certainty-seeking *ultra-fans* can be defined as those who prefer winning by any margin such that $\varphi = 1$. At the other extreme uncertainty-preferring *infra-fans* are defined as those who prefer perfect balance $\varphi = 0$. Table 1 compares moderate fan results found earlier to solutions for these extreme cases.

General solutions of relative talent t_1/t_2 for α and φ from equation (45) and equation (51) are shown in Figure 10 for profit-max (left frame) and sportsman leagues (right frame). The profit-max frame compares all of the non-cooperative duopsony solutions along the right vertical axis where $\alpha = 1$ to the perfect cartel solutions along the left vertical axis where $\alpha = 0$. (See the limiting case solutions in Table 1). It is clear that revenue sharing is essentially a tax and redistribution scheme to internalize the Yankee paradox. It is also obvious that the revenue-sharing paradox holds true for all profit-max solutions with the exception of the infra-fan ($\varphi = 0$), where the non-cooperative and cooperative duopsony solutions are the same (the *invariance proposition*). In all other cases, the cartel talent imbalance is greater than the non-cooperative imbalance (the revenue-sharing paradox) and the degree of imbalance increases with respect to fan preference for imbalance φ. The troubling aspect of the inference that each of the cartel schemes is also a welfare optimum is that as $\alpha \to 0$ in Figure 9, league revenues and fan welfare are maximized while player salaries are minimized and the league talent level goes to zero.

The sportsman right frame in Figure 11 shows the direct relationship between revenue sharing in a sportsman league and relative talent for selected fan preferences. The right axis where $\alpha = 1$ shows the win-max solutions from equation (48) for $\sigma = 1.5$. There are diminishing marginal returns on competitive balance as $\alpha \to 0$ and initial differences based on fan preference disappear in the limit where $t_1/t_2 \to 1$ as $\alpha \to 0$ regardless of fan preference. The degree of revenue sharing necessary to achieve the social optimum in a sportsman league described in equation (52) can be visualized by the substitution of the cartel solution equation (42) shown on the left vertical axis for $\alpha = 0$ into equation (51) in the sportsman frame for the same fan preference. For example, in the $\varphi = .5$ moderate fan case, the market ratio $\sigma = 1.5$ is the cartel

Table 1
Revenue sharing, fan preference and relative talent

	Profit max		Sports man	
Fan preferences	Duopsony $\alpha = 1$	Cartel $\alpha = 0$	Wine-max $\alpha = 1$	Optimum α^*
Ultra-fan $\varphi = 1$	σ	$1/0^{\dagger}$	$1/0$	1.0
Moderate fan $\varphi = .5$	$\sqrt{\sigma}$	σ	$\frac{\sigma-\frac{1}{2}}{1-\frac{\sigma}{2}}$	$\frac{\sigma^2+\sigma+1}{\sigma^2+3\sigma+1}$
Infra-fan $\varphi = 0$	$1/1$	$1/1$	σ	0.0

† If $\alpha > (\sigma - 1)/(\sigma + 1)$ then interior profit-max solution exists for ultra-fan.

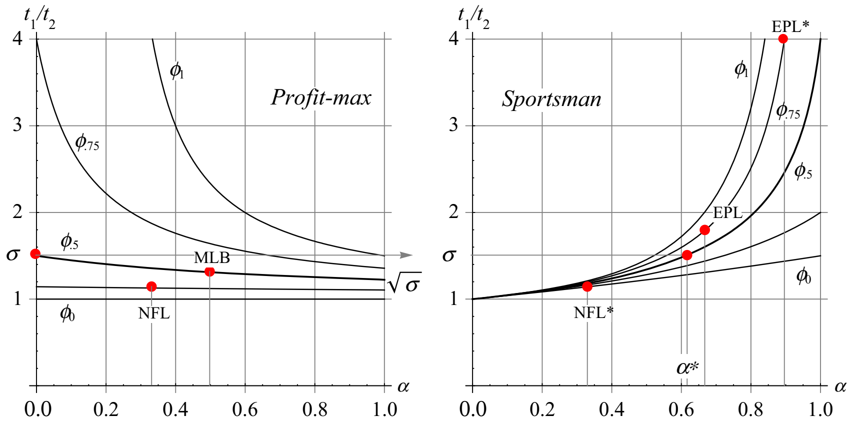

Figure 11. Revenue sharing, fan preference and talent distribution.

optimal talent distribution. Substitution of σ into the sportsman talent function yields $\alpha^* = .613$ on the α-axis of the sportsman talent function as shown in Figures 8–11.

The implicit function equation (51) can be solved for infinite optimizing triples. For example, the talent distribution of $\frac{t_1}{t_2} = 2$ that optimizes the fan preference φ =.625 for the cartel in the profit-max frame can be reached by $\alpha^* = .756$ in a sportsman league. The optimum talent distribution $\frac{t_1}{t_2} = 4$ for fan preference $\varphi = .75$ in a cartel can be reached with revenue sharing $\alpha^* = .896$ in a sportsman league. Optimizing win-max revenue sharing is preferred to payroll minimization schemes in duopsony leagues, because the win-max talent levels are superior. This suggests that optimal win-max revenue sharing is welfare superior to payroll minimization in cartel leagues because it leads to the optimal distribution of superior talent. A welfare proposition is immediate:

Proposition 5: Social welfare optimality in sports leagues is characterized by:

(1) League revenues, profits and fan welfare are maximized by sports league cartels, because they internalize the talent interdependence among clubs.

(2) Revenue sharing is optimal in profit leagues when revenues are maximized at $\alpha = 0$. Visitors share $(1 - \alpha)$ is an index of cartelization and welfare optimization.

(3) Conventional social welfare criteria are problematic because profit-max leagues also minimize player salaries and revenue sharing leads to an evacuation of talent.

(4) Non-cooperative duopsony leagues are more balanced than cartels with the same fan preference and the degree of imbalance increases with revenue sharing $\alpha \to 0$.

(5) Infra-fans $\varphi = 0$ are the exception where talent is always equal regardless of revenue sharing. (The invariance proposition only applies in the infra-fan case).

(6) Ultra-fan leagues have a corner solution optimum because they have the identical preference to win as the win-max sportsman
(7) A win-max sportsman league is ($\alpha = 1$) welfare inferior because of the linear objective to win beyond the optimum while ignoring the Yankee paradox externality.
(8) There is a direct relationship between revenue sharing and competitive balance in sportsman win-max leagues.
(9) There exists an optimal revenue sharing scheme that maximizes league revenues, fan surplus, and player salaries while optimizing the distribution (competitive balance) and level of superior talent: $\alpha^* = \frac{\varphi(\sigma^2+2\sigma\varphi+1)}{\sigma^2\varphi+2\sigma(\varphi^2-\varphi+1)+\varphi}$.

XIII Between the Lines

In professional sports leagues these limiting cases are not necessarily empirically extreme. The sustained revenue growth of extremely unbalanced leagues throughout Europe suggests that European football fans are perhaps more *ultra* than *moderate*.[13] The economic models of the Big 5 European clubs have been built on the belief that fans prefer dynasties over balanced competition.[14] Given the preferences of the ultra-fan the competitive imbalance in cartels and sportsman leagues may indeed be welfare superior. This is because the sportsman win-max owner and ultra-fan have identical preferences.

In contrast to European football, the economic model of the North American NFL seeks absolute parity among the clubs so that any given team can defeat any other. The financial success and popularity of an essentially random league, where mediocre clubs are defeating each other implies that NFL fans are more *infra-fan* than *moderate*.[15] By comparison, MLB fans are benchmark *moderate* fans where the highest rated matchups usually involve a dominant team facing an upstart challenger.

The dynamics of competitive balance in real world leagues representing these three hypothetical fan types can be observed through a simple autoregressive β-estimate of the continuity of winning percentages w_{ijt} for team i in league j from season $t-1$ to season t:

[13] Deloitte estimates total Euro football market of \$26.4 billion in 2012 with 48% Big 5 league-share of \$12.7 billion compared to \$14.1 billion 10 years ago (2003) with 53% Big 5 share of \$7.6 billion (€1 = \$1.36).

[14] In North America, the NBA has sought to increase national media revenues by marketing individual super-stars and promoting team dynasties. In the 30 years of salary cap era (1984), only 8 different clubs have won the NBA Championship. NBA: $\beta = .75$. NHL $\beta = .5$, (similar β to NBA before hard salary cap in 2005). European Big 5: Spanish La Liga and Italian Serie A, $\beta = 1$; German Bundesliga and French Ligue 1 $\beta = .5$.

[15] One possible explanation is the changing nature of the NFL fan base from rising popularity of fantasy football in the US. The Fantasy Sports Trade Association estimates that over 30 million NFL fantasy players spend more than the total revenue of the NFL (\$10 billion annually) directly on fantasy football. Fantasy teams are comprised of players drafted from existing NFL teams and scoring depends on individual rather than team performance. As a result, different combinations of players throughout the league have become the NFL media product rather than the performance of individual clubs.

$$w_{ijt} = \alpha + \beta w_{ijt-1} + \varepsilon_{ijt} \tag{53}$$

$\alpha \in [0, .5]$ and $\beta \in [0, 1]$. If α = .500 and β = 0, then w_{ijt} = .500, and each season is a random walk where teams have a fair and even chance to win. At the other extreme if α = 0 and β = 1 then $w_{ijt} = w_{ijt-1}$ and each season is a predictably deterministic repetition of the past.[16] Beta balance coefficients are shown in Figure 11 over the period 1970–2013/14 for the English Premier League (EPL), the National Football league (NFL) and Major League Baseball (MLB). This evidence suggests that the EPL is virtually predetermined with β = .75 → 1, the NFL is essentially random β = 0 → .25 and that MLB strikes a historical competitive balance β = .5 between the two extremes.

Actual competitive balance (β) is determined by ownership preference and revenue sharing (cartelization) schemes (α), and the optimality of that balance is a function of fan preference (φ). Although a variety of factors contribute to competitive balance in these three structurally different leagues these three parameters are sufficient to separate empirical differences between owner and fan preferences. These three leagues are hypothetically shown in Figure 12 based on the following structural comparison.

General: Revenue growth is strong in all three leagues. This suggests that owner and fan preferences are in synch and that competitive balance β can serve as an estimate of φ. Media coverage transforms dominant clubs into quasi-public goods and therefore an increased media share of revenues should increase fan preference for dominant clubs φ.[17]

There is a strong positive empirical relationship between revenue sharing and a balanced distribution of talent (α and β). This implies *ipso facto* that all leagues have a significant sportsman presence rather than profit-max ownership. This is not necessarily true, however, unless it is supported by evidence on the players' share of league revenues. Here is an optimal revenue-sharing simulation under the different league configurations:

EPL is a $4 billion league that shares 33% of total revenue. Media has grown from 12% of revenue in 1996 to over 50% in 2014. This explains the upward trend in beta balance.[18] Including overseas media, 67% is shared evenly and 33% is based on merit. EPL payrolls have grown from 50% of revenues in 1996 to 70% in 2014. *Conclusion: EPL is a sportsman league with*

[16] Beta estimates were first used to isolate the randomness of MLB clubs when player salaries doubled in the four years before the MLB strike of 1994–95 (see MLB betas in Figure 10). The 3-tiered labor MLB labor market had separated a player's performance from his pay and tier 3 free agent players (6+ years of experience) were overpaid by one-third and tier 1 (0–3 years) was underpaid by two-thirds. (Vrooman, 1996).

[17] SPORT+MRKT estimated that Barcelona FC had 5.5 million domestic fans (29% of La Liga base) and 57.8 million fans throughout Europe, Real Madrid has 6.8 million domestic fans (36% of La Liga base) and 31.3 million in Europe. Manchester United had 30.6 million fans in Europe including 4.7 million at home (18% of EPL fans); and Chelsea FC attracted 21 million fans in Europe including 1.6 million domestic fans (6% of the EPL fan base). Italian Serie A shares 25% of TV rights based on these fan support estimates.

[18] Another explanation is UEFA Champions League distortion of domestic balance (Vrooman, 2007).

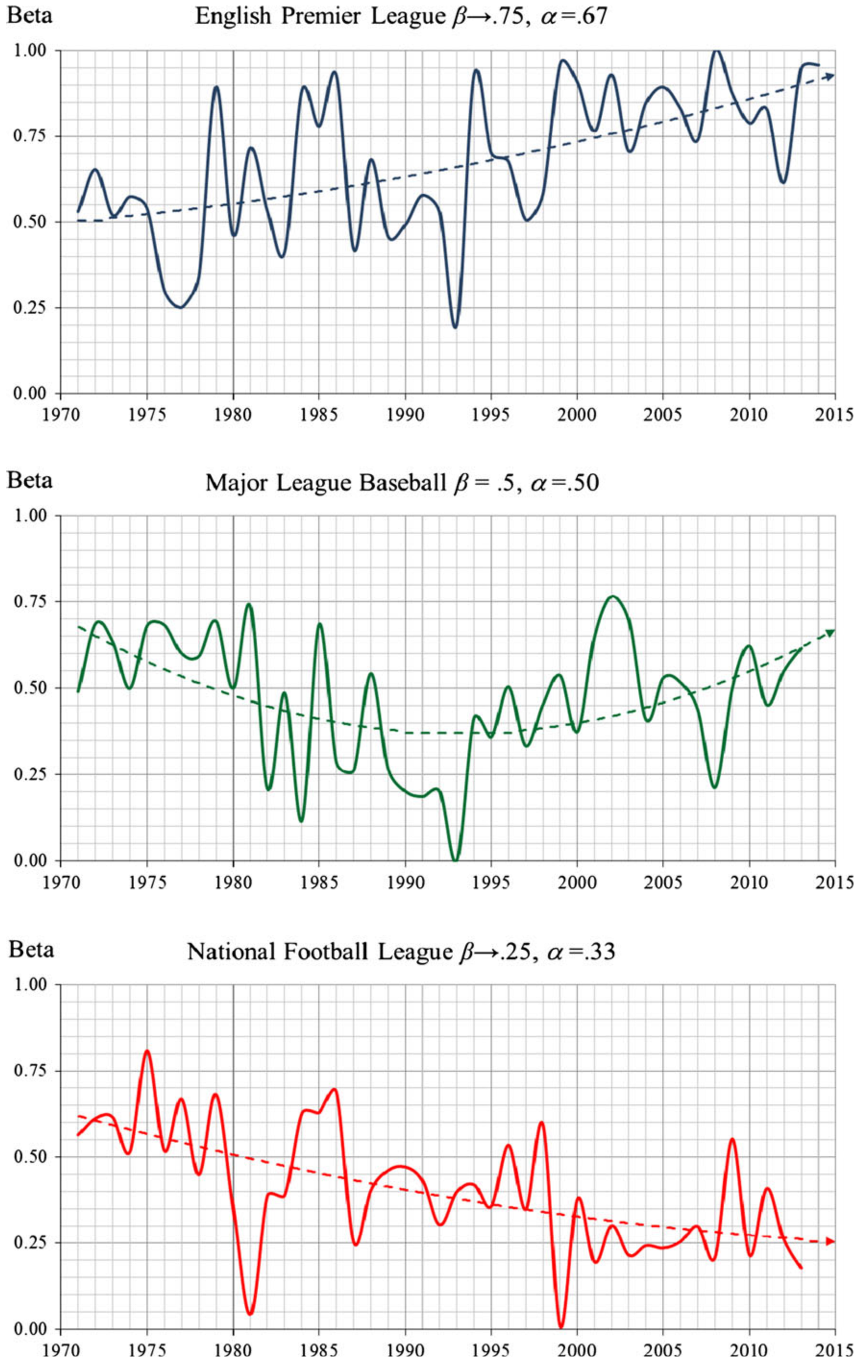

Figure 12. Autoregressive beta balance.

moderate/ultra-fans: α = .67; $\beta = \varphi \rightarrow .75$. EPL fan and player optimality could be reached with modest 10% revenue sharing at EPL in Figure 11.*

MLB is an \$8 billion league that shares 50% of its revenue. Media revenue is 50% of the total and 50% of media is national and shared evenly. 31% of all local revenue including media, gate and venue are shared. Players' share of revenue has fallen to 42% in 2014 from 63% in 2003. *Conclusion: MLB is a profit-max league with moderate fans $\alpha = 50$; $\beta = \varphi = .5$. Increased revenue sharing could move MLB relative talent toward the cartel max profits and fan welfare, but talent exploitation remains a significant problem.*[19]

NFL is a \$10 billion league that shares 67% of total revenue. National media revenue is 60% of total and shared evenly. Gate revenue is 20% of total and 34% is shared. Venue revenue is 20% and not shared. Player payrolls are capped at less than 50% of revenues.[20] *Conclusion: Revenue sharing and competitive balance suggest a sportsman league but 50% payroll cap implies profit-max cartel with infra/moderate fans: α = .33; $\beta = \varphi \rightarrow .25$. NFL 67% sharing optimizes league profits and fan welfare regardless if NFL is an infra-fan cartel at NFL or a sportsman league with any fan preference at NFL*. Inferior talent exploitation and asymmetric venue-revenue sharing among teams is a major problem.*[21]

XIV Conclusion

Since its modern origins in QFV theory, there has been major confusion in the game-theoretic modeling of profit-max sports leagues. The controversy centers on how to model duopsony in a two-team league with perfectly inelastic aggregate talent supply. SK (2004) attempted to reconcile the issue with a distinction between open talent markets with a perfectly elastic supply and closed markets with a perfectly inelastic supply. Open and closed cases lead to different conclusions about the invariance of competitive balance with respect to revenue sharing, but both confirm the revenue-sharing paradox.

In the course of the SK open league critique, there has also been confusion about the use of duopsony game theory in a two-team league. Club owners are seen as monopolists in the product market but then viewed as passive wage-takers in a labor market where they hold considerable power. There has also been a move toward using talent *expenditure* instead of talent as a strategy

[19] MLB owners were found guilty of collusion against free agents after 1985–87 and 2002–03 seasons.

[20] When payroll cap is used without revenue sharing it creates balance (NHL) in either profit or win-max leagues. When cap is used with revenue sharing (NFL) in a profit-max league then revenue sharing should dominate the cap in the limit and create cartel imbalance. If there is a minimum (90% of NFL payroll cap) then all teams would have equal talent at the minimum. About 20% of NFL teams function below the cap. When cap and sharing are combined win-max league teams are virtually cloned in the limit $\alpha = 0$ with equal talent, revenues, wages and profit (Vrooman, 2007, 2009). It is unlikely that a win-max league would pursue a cap.

[21] In the 2011-20 NFL CBA players; share is capped at 55% of TV, 45% NFL Properties and 40% of local revenue (48% overall). Rookie contracts are capped with 4-year max (5th year option for 1st round picks).

variable. It was argued that an inelastic wage is undefined (Madden, 2011), when it is really a solvable problem of infinite equilibria (DV, 2014). The game-theoretic remedy for infinite equilibria involves the solution of a duopsony limit game as total talent supply infinitesimally approaches perfect inelasticity.

Using talent as the strategic variable the DV approach yields the same competitive balance results as existing price-taking models (Vrooman, 2007, 2009) with one important exception. Wages are less than marginal revenue product, but they are always reduced to the reservation wage in the duopsony limit game. DV analysis finds that revenue sharing increases relative talent and cartelizes revenues and profit, but it also reduces total talent to zero. Vanishing talent results from any revenue function (including the widely used logistic *CSF*) where probability of winning is only a function of relative talent.

The win-max sportsman owner is constrained by zero profit rather than maximum profit, and the sportsman will maximize wins by spending all revenue on talent. This creates an increasingly unbalanced league dominated by the large-market clubs, but the players' share of revenue is maximized in a win-max league. The sportsman league is consistent with highly successful football leagues throughout Europe that are dominated by a very few aggressive clubs where the players' share exceeds 70 percent.

Sports leagues are natural cartels because they can cooperatively internalize the interdependencies inherent among sports teams. Profit-max cartel solutions are superior to non-cooperative duopsony solutions for precisely that reason. If the strategic variable is talent and the *CSF* depends on relative talent then sports team owners will always scale-down absolute talent to optimize talent *distribution*. Revenue sharing is progressive cartelization that internalizes inefficient externalities, but it also leads to optimal relative talent at the expense of absolute talent levels for the league. League social welfare criteria should obviously consider absolute talent as well as relative talent.

Non-cooperative duopsony leagues are too balanced for welfare optimality and win-max leagues are to unbalanced. Revenue sharing leads to less balance a profit-max league and more balance in a win-max league. So revenue sharing naturally emerges as a welfare optimization policy in any type of league. It is difficult to argue that a cartel is welfare optimal when it maximizes league revenue, fan welfare and profit, while it also minimizes payroll and league talent in the process. Although sportsmen are by nature disinterested in maximizing welfare, optimal revenue sharing does exist for a sports league commissioner to maximize league revenues and payrolls while achieving both an optimal allocation and level of talent. Sportsman leagues with optimal revenue sharing are welfare superior to league cartels because they can lead to the same welfare maxima in revenue and fan welfare, but they also optimize the distribution of superior talent.

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