WIDESPREAD EXTERNALITIES AND PERFECTLY COMPETITIVE MARKETS: EXAMPLES

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ABSTRACT

We consider four examples of economies with widespread externalities. These examples illustrate various forms of interactions between externalities and markets. We suggest that some phenomena usually regarded as incompatible with general equilibrium theory are in fact results of interactions of widespread externalities and perfect competition and that other phenomena usually regarded as economic problems separate from externalities may be manifestations of externalities. With our examples as a whole, we show that in many cases externalities are inseparable from the determination of economic activities, values and the market itself.

1 INTRODUCTION

Externalities can pose serious socio-economic problems. Some of these problems are directly related to economic activities and might not be separable from them. One such problem is the failure of the market mechanism to achieve an efficient allocation. Even when an efficient allocation is not achieved, the market mechanism may work in the sense that each individual economic agent optimizes and the market succeeds in balancing demands and supplies at common market prices; the market functions but the outcome is not efficient. In addition to causing inefficiency, the presence of externalities affects market outcomes in various ways, and, in particular, markets and externalities may have mutual interactions. This paper illustrates, via examples, various forms of interactions between markets and externalities. The examples as a whole are intended to convey some basic relationships between externalities and markets.

We consider externalities called "widespread". These are externalities created by and simultaneously affecting large numbers of members of the society. They include pollution of the environment (such as the effects of wastes, automobile emissions), over-exploitation of exhaustible resources (fishing banks or other common property) and others which are more psychological (fashion and neighborhood effects in housing). These examples have the common feature that they are problems of large societies–each individual, although affected by the externalities, is negligible relative to the externalities. Widespread externalities differ from ones created by and affecting only a few members of the society. Such local externalties are problems of small numbers of individual units and could be resolved by negotiations between these units. Those behaviors may, however, destroy the competitive functioning of the market. In contrast, widespread externalities are related to the entire society, and cannot be removed by negotiations between individual units in the society. When externalities are widespread, the market may function competitively, though the resulting competitive outcome may not be Pareto efficient.[1]

In principle, the emergence of competitive prices equating demands and supplies is a different issue than the Pareto efficiency of the outcomes of competitive markets. The first issue concerns whether or not markets themselves function to bring about competitive outcomes; the second concerns social welfare properties of ther resultant competitive allocation. We distinguish these two issues.

In small economies, externalties may cause market failure in both senses of efficiency and of market function. In such economies, however, it is difficult to separate these issues of market failure. The notion of competitive equilibrium is not necessarily adequate in small economies with or without externalities, as noted by Arrow-Hahn (1971, p. 136), since individuals can significantly affect both the externalities and also prices and these effects may induce non-market activities.

When externalities are widespread, they cannot be affected by the individual, who takes them as given — he has no effective alternative. The notion of widespread externalities is parallel to the classical assumption for perfectly competitive markets that each individual is negligible relative to aggregate economic variables, demands and supplies, and thereby prices. In such an

[1] Some authors have stressed the importance of widespread externalities (see the related discussion in Baumol-Oates (1988, p. 9, Section 3)).

economy, a *competitive equilibrium in an economy with widespread externalities* is defined so that

(i) each economic agent maximizes his utility or profit under given prices and given levels of externalities;

(ii) the induced demands and supplies balance and

(iii) the resulting levels of externalities coincide with initially given levels.

Here an individual agent treats widespread externalities as parametrically given in the same way as he treats prices as given. A large population justifies the parametric treatments of prices and externalities. Both widespread externalities and perfectly competitive markets are phenomena of large societies.

The existence of a (Pareto-inefficient) competitive equilibrium with externalities is discussed in McKenzie (1955), Arrow-Hahn (1971), Hammond-Kaneko-Wooders (1989), and Kaneko-Wooders (1989). McKenzie (1955) and Arrow-Hahn (1971) suggested the existence of a competitve equilibrium with externalities in a finite, convex economy. Hammond-Kaneko-Wooders (1989) and Kaneko-Wooders (1989) considered the existence of a competitive equilibrium in an economy with a continuum of traders and with widespread externalties. They also argued that the compatibility of the competitive equilibrium and widespread externalities is supported from the viewpoint of recontracting (the f-core); trade and recontracting occur within small groups of people, and the emergent exchange ratios are competitive price ratios. These papers dealt with theoretical issues, rather than the phenomena captured by widespread externalities. Here we consider what kinds of phenomena can be captured by the widespread externality framework.

This paper presents four examples of economies with widespread externalities. With these examples, we emphasize various forms of mutual interactions of markets and externalities. More specifically, we suggest that some phenomena usually regarded as incompatible with general equilibrium theory are in fact results of interactions of widespread externalities and perfect competition and that other phenomena usually regarded as economic problems separate from externalities may be manifestations of externalities. With our examples as a whole, we show that in many cases externalities are inseparable from the determination of economic activities, values and the market itself.

2 AN AUTOMOBILE MARKET WITH POLLUTION; A PURE PUBLIC BAD EXTERNALITY

The example of this section illustrates that externalities may be determined by markets and that the effects may be only in one direction — externalities do not necessarily affect market activities.[2] The second subsection considers government intervention to control the pollution and welfare effects of this intervention.

2.1 A Free Market for Automobiles

Each point in the interval $[0,1)$ represents a consumer. The utility function of consumer c is given by

$$u_c(\delta_c, m_c, \pi) = 2\delta_c + m_c - \alpha\pi,$$

where $\delta_c = 1$ if the consumer owns a car and $\delta_c = 0$ otherwise, m_c is a perfectly divisible composite commodity, π is the pollution level, and α is a constant. The pollution level π is given by $\pi = \int_{[0,1)} \delta_j \, d\mu$, where μ is Lebesgue measure. We assume that the income of the c-th consumer is I_c, $I_c > 1$, and that the consumers do not initially have automobiles.

The sellers or dealers of automobiles are the points in the interval $[1,3)$. Each seller owns one automobile for which he has a reservation price of 1 (instead of this, we could have producers of automobiles). For welfare consideration, the sellers, while they are participants in the market, are not treated as members of the society.[3]

The fact that the measure of sellers is greater than the measure of consumers ensures that the competitive price p of an autombile is 1. A possible competitive equilibrium in this economy is:

$$\begin{aligned}
&p = 1\\
&\delta_c^* = 1 \text{ for all } c \in [0,1)\\
&\delta_s^* = 0 \text{ for all } s \in [1,2) \text{ and } \delta_s^* = 1 \text{ for all } s \in [2,3).
\end{aligned}$$

[2] This example actually describes widespread public bads including environment-destroying wastes (PCBs), pesticides, and ozone-destroying carbon dioxide.

[3] We could modify this assumption and count sellers from the welfare viewpoint.

Each seller in the interval $[1,2)$ trades an automobile with a consumer for 1 unit of money. The utility of the competitive outcome for consumer c is

$$u_c^* = 2\delta_c^* + I_c - \delta_c^* p - \alpha \int_{[0,1)} \delta_j^* \, d\mu = I_c + 1 - \alpha.$$

In this example, each individual has one choice-whether or not to buy an automobile. When the price of an automobile is 1, buying an automobile gives higher utility than going without an automobile, given any level of the externalities. This gives us a total demand for automobiles of 1 (per capita). When $p = 1$, sellers are indifferent between selling and not selling. Thus, any supply (up to 2 cars per consumer) is possible at $p = 1$ and supply can be equated to demand at this price.

The efficiency of competitive outcomes depends on α. For $\alpha \leq 1$, the competitive outcome is efficient, while for $\alpha > 1$, it is inefficient. When $\alpha > 1$, the average damage of pollution is greater than the average benefit. The average damage, from the viewpoint of the individual, is fixed; he cannot significantly affect the pollution level. When $\alpha = 1$, all outcomes are efficient; average benefits to car buyers are exactly offset by average social costs imposed on non-car buyers. For $\alpha < 1$, the competitive outcome is efficient - average social benefits of car ownership exceed average social costs.

Here, the case where α is significantly large (i.e., $\alpha > 1$) is relevant to our consideration. In this case, the problem is not simply Pareto inefficiency but rather the extent of the inefficiency. If the problem were a small discrepancy from efficiency, then it would not be serious, but it is the case that the resulting discrepancy is large.

2.2 A Controlled Market for Automobiles

In the above example, without introducing some action of the society as a whole such as government intervention, nothing seems to prevent markets from bringing about the competitive outcome. If we want to change the resulting social outcomes, we need to think about some sort of social actions. The simplest (but possibly infeasible) way to prevent a very bad outcome is that the government prohibits the use of automobiles or levies severe taxes on automobiles, and, of course, there are many more practical policy recommendations for such an automobile problem.[4]

[4] Uzawa (1974) extensively discussed the social cost induced by automobiles, such as road-construction/maintainance costs and injury-death costs, other than pollution. Specifically,

policy, we need also to consider redistribution and equity effects. In the above example, the introduction of tickets increases every consumer's utility. This result requires that every consumer have enough income; if not, the result may change substantially.

Let the income distribution I_c be given as

$$I_c = 1 + \lambda/(1-c) \text{ for all } c \text{ in } [0,1),$$

where λ is "small" (more precisely, $\lambda < {}^3/_{16}$). There are some poor people and some very rich. We again assume that each consumer is assigned 3 tickets.

In this economy, the competitive price of an automobile is also 1. In equilibrium, each consumer c in $[{}^1/_4, 1)$ buys 1 ticket and an automobile, and each consumer c in $[0, {}^1/_4)$ sells 3 tickets. Here the marginal consumer, $c = {}^1/_4$, buys an automobile by applying all his endowment to buy an automobile and a ticket. Hence he pays $\frac{\lambda}{(1-{}^1/_4)} = {}^{4\lambda}/_3$ for a ticket. Thus the competitive price p_t of a ticket is

$$p_t = \frac{4\lambda}{3}.$$

Each consumer in $[0, {}^1/_4)$ cannot buy 1 ticket at price ${}^{4\lambda}/_3$, so each simply sells his 3 tickets at price ${}^{4\lambda}/_3$.

When λ is very small, the ticket price $p_t = {}^{4\lambda}/_3$ is also very small. In this case, the relatively rich people $[{}^1/_4, 1)$ benefit from using automobiles with the small payments for tickets, but the poor people $[0, {}^1/_4)$ are excluded from the use of automobiles with small compensations $3p_t = 4\lambda$. The utility configuration of the new competitive equilibrium is:

$$\begin{aligned} u_c^{**} &= I_c + 4\lambda - \tfrac{3\alpha}{4} && \text{for } c \in [0, {}^1/_4) \\ u_c^{**} &= I_c + 1 - \tfrac{4\lambda}{3} - \tfrac{3\alpha}{4} && \text{for } c \in [{}^1/_4, 1) \end{aligned}$$

Let $1 < \alpha < 4$ and $\lambda < {}^{(4-\alpha)}/_{16}$ (which implies $\lambda < {}^{3\alpha}/_{16}$), that is, α is assumed relatively large but not "very" large and λ is small relative to α. Then the utility level u_c^{**} for consumer c in $[{}^1/_4, 1)$ is higher than u_c^* in the original competitive equilibrium without tickets, but the converse holds for consumers in $[0, {}^1/_4)$, i.e.,

$$u_c^{**} = I_c + 1 - \frac{4\lambda}{3} - \frac{3\alpha}{4} > I_c + 1 - \alpha = u_c^* \text{ for } c \in [{}^1/_4, 1)$$

but

$$u_c^{**} = I_c + 4\lambda - \frac{3\alpha}{4} < I_c + 1 - \alpha = u_c^* \text{ for } c \in [0, {}^1/_4).$$

In the literature of economics, however, it is often regarded as a virtue to avoid direct intervention of a central authority. Along this line of economic thought, the internalization of externalities into the price system — making externalities into marketed commodities — have been often suggested (cf., Coase (1960), Arrow (1970), Newbery (1980), Laffont (1987)). An idea often discussed is that externality problems can be alleviated by creation of markets for externalities, thus making them tradable commodities. However, it is not always possible to introduce such new commodities so that externalities — new commodities — are traded in the same way as the standard private goods. In our example, while we cannot transform externalities into private goods, we can alleviate the extent of pollution by the introduction of a new market.

We suppose that a government endows each individual with 3 tickets and requires a car driver to own 4 tickets. Trade in tickets is permitted so that each person who obtains 4 tickets (an entire right) could drive a car.[5] Given these property rights, when the income I_c of each consumer is large enough ($I_c > 2$ for all c), we have a new competitive outcome with:

> $p_t = 1/4$ for a ticket and $p = 1$ for a car;
> $3/4$ of the population driving cars (each driver buys 1 ticket) and the remainder of the population selling their tickets.

The price of $1/4$ for a ticket makes every consumer indifferent between buying an additional ticket to have a whole right to drive a car and selling his 3 assigned tickets, i.e.,

$$I_c + 2 - 1 - 1/4 - 3\alpha/4 = I_c + 3/4 - 3\alpha/4.$$

Observe that (when $\alpha > 1$) this outcome is a Pareto improvement over the initial competitive outcome, i.e.,

$$I_c + 3/4 - 3\alpha/4 > I_c + 1 - \alpha \text{ for all } c \text{ in } [0, 1).$$

Pareto-optimality still requires that no individual be reallocated any positive amount of the right to drive a car.

Our problem was whether or not the restriction of rights to drive automobiles by assigning 3 tickets to each individual will bring about a Pareto improvement. However, when we evaluate the social welfare effect of a governmental

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he discussed the over-use of automobiles and inadequate separation of automobiles from pedestrians in Japan.

[5] This sort of market for pollution rights is currently being considered by the U.S. government to control the total pollution emission of industry. See "Giving Greed a Chance - Is the "right" to pollute an ecologically sound idea?" *Time*, February 12th, 1990, p. 67.

There is some common fishing waters, and a society consisting of both fishermen and consumers of fish. The fishermen are represented by the points i in the interval $[0, 1)$ with Lebesgue measure μ. The fishermen each have two possibilities: to fish or not to fish. The amount of fish caught depends on an individual decision and the total fishing activity: the catch of the i-th fisherman is given by $2\Delta\delta_1$ where

$\Delta = (1 + \epsilon - \int_{[0,1)} \delta_j d\mu)$, and $\epsilon > 0$ is assumed "small";

$\delta_j = 1$ if fisherman j fishes, and $\delta_j = 0$ otherwise.

When the market price of fish is p, the profit of fisherman i is $2\Delta\delta_i p - C\delta_i$, where $C = 1$ is the cost of the fishing activity.

The production coefficient $\Delta = (1 + \epsilon - \int_{[0,1)} \delta_j \, d\mu)$ is meant to describe a long-run average per period. When many fishermen have fished over many time periods, the fish resource is nearly depleted, and the production coefficient is very small.

The consumers are indexed by the points in $[1, 2)$ and consumer c has a reservation price of $^1/_{(2-c)}$ for 1 unit of fish. We assume, for simplicity, that each consumer desires either 0 or 1 units of fish.

The unique competitive outcome is that:

$\delta_i = 1$ for all i in $[0, 1)$ — every fisherman is active, each consumer in $[2 - 2\epsilon, 2)$ buys one unit of fish at the competitive price of $^1/_{2\epsilon}$.

The story behind the competitive outcome is as follows: With low levels of fishing activity, profits in the industry attract more fishermen. As more fishermen become active, the catch of each fisherman decreases, but fishing still remains profitable. When the total catch becomes very small, fish becomes a rare commodity and the price becomes very high. Finally the resulting outcome is the above competitive equilibrium.

We compare the competitive outcome with a situation where only a fraction ϵ of the fishermen go to sea. Observe that, when only a fraction ϵ of fishermen are active, the total catch is the same as when all are active, but the total cost

In this example, the introduction of tickets induces a new situation where only the relatively rich people benefit from the use of automobiles and the decrease of pollution is achieved by excluding the poor people from the use of automobiles with small compensations.

We considered the introduction of tickets for the right to drive an automobile. A similar consideration can be made for the introduction of taxes and subsidies. That is, the government levies a tax on a consumer if he buys a car and pays him a subsidy if he decides not to have a car. We require the total automobile tax revenue to coincide with the total subsidies. If the govenment wants to decrease the pollution level to $^3/_4$, then the level of tax should be $^{4\lambda}/_3$ and that of subsidy should be 4λ. Then a consumer c in $[0, {}^1/_4)$ cannot pay $^{4\lambda}/_3$ and receives the subsidy 4λ, while every consumer c in $[{}^1/_4, 1)$ buys an automobile and pays the small tax. The result is exactly the same as that of the introduction of tickets.

The above considerations are summarized as follows. In the automobile market with pollution, the free competition leads the society to a very bad state, without any governmental intervention. The resulting competitive equilibrium is very inefficient. We examined an internalization of pollution by the introduction of a ticket market. This decreased the pollution level and increased the total social surplus. However, this may, or may not, lead to a Pareto improvement; it is possible that only rich people benefit from the governmental intervention while the poor are harmed. Market effects may wipe out the intended fairness of the even distribution of tickets. As a whole, the internalization of widespread externalities is not necessarily a solution to the problem.

3 THE TRAGEDY OF THE COMMONS; OVER-EXPLOITATION CAUSED BY EXTERNALITY AND MARKET FEEDBACK

In the example of this section, interactions of the market and the externalities are in both directions; these interactions escalate the level of the externalities and simultaneously the price of goods obtained from the over-exploited resource.[6]

[6] We refer the reader to Gordon (1954) for extensive discussions of environmental problems related to this subject and to Baumol-Oates (1988, p. 28) for further references.

is ϵC, instead of C. In terms of ongoing long run averages, when only a few fishermen go to sea the resource is not depleted, so the same catch is obtained as when all fishermen go to sea. In this sense, the above competitive outcome causes over-exploitation of the resource and, of course, is Pareto-inefficient.

As in the pollution example, we can think of a market for "tickets," in this case (tradable) licenses to fish. By issuing some appropriate amount of licenses, a government can ensure more efficient outcomes.

The present example has a significant difference from the previous one. In the previous example, the externalities affect individual welfare but not individual economic activities. In the present example, however, the externalities and markets feedback on each other, and this feedback escalates the degree of exploitation and the fish price. Thus, externalities sometimes almost separately coexist with markets and sometimes have strong interactions with markets.

4 AN ECONOMY WITH CONFORMISTS AND NON-CONFORMISTS; A PURE PSYCHOLOGICAL EXTERNALITY

The externalities in the previous examples are environmental. In the example of this section, externalities come purely from individual tastes.

The consumers are again indexed by the points in the interval $[0, 1)$. A fraction of these consumers, say those in $[0, \alpha)$, are *conformists* and the remainder of the population consists of *non-conformists*. There are three goods, red sweaters, blue sweaters, and a composite commodity. The extent of an individual's conformity is expressed by how he feels about the colour of his sweater relative to the percentage of the population wearing the same colour sweater. Each individual needs one sweater (only). Let t_i be a vector listing the number of sweaters of each colour worn by the ith individual i.e. $t_i = (t_{iB}, t_{iR})$, where

$$\begin{aligned} t_{iB} &= 1 \text{ if } i \text{ wears a blue sweater} \\ &= 0 \text{ if } i \text{ does not wear a blue sweater;} \\ t_{iR} &= 1 \text{ if } i \text{ wears a red sweater} \\ &= 0 \text{ if } i \text{ does not wear a red sweater;} \end{aligned}$$

and $t_{iB} + t_{iR} = 1$. Each consumer has an endowment of the composite commodity, money, of more than 1 unit.

and $t_{cA} + t_{cB} = 1$. Let λ_A denote the average income of consumers (per landholding) who buy houses in area A and λ_B, the average income of consumers in area B:

$$\lambda_A = \frac{1}{3}\left(\int_{T_A} I_c d\mu\right), \text{ where } T_A = \{c \in [0,5) \mid t_{cA} = 1\},$$

and similarly for B. Let m_c denote the amount of the composite commodity consumed by c. The utility function of the cth consumer is given by

$$u_c(t_c, m_c, \lambda_A, \lambda_B) = \frac{t_{cA}\lambda_A + t_{cB}\lambda_B}{5} + (m_c)^\alpha$$

and, given prices p_A and p_B for housing in neighbourhoods A and B, his budget constraint is $I_c - p_A t_{cA} - p_B t_{cB} = m_c$.

The consumer is affected by the aggregate income per landholding of the neighbourhood in which he buys a house. We have taken λ_A and λ_B to be calculated on the basis of initital income; this may be explained by the maintenance of housing by consumers and also by sellers.

For simplicity, we assume $\alpha = 1$. We consider only equilibria which are limits of equilibria for α approaching one from below. This enables us to capture some aspects of income effects (for $\alpha < 1$) while keeping calculations simple, and restricts the set of competitive outcomes considered to two kinds.

The first kind of competitive equilibria are:

> $p_A = p_B = 1$ and the average income is the same in both areas, i.e., $\lambda_A = \lambda_B$.

These are unstable because movement by any small fraction of consumers disturbs the income balance. The other kind consists of two segregated competitive outcomes, which are symmetric with respect to the regions A and B. In one of them, the high income consumers all live in area A and the low income ones in B;

$$t^*_{cA} = 1 \text{ for all } c \in [2,5) \text{ and } t^*_{cB} = 1 \text{ for all } c \in [0,2).$$

In this case $p_A = 1.7$ and $p_B = 1$. (The other equilibrium of this kind has all the high income consumers filling area B.) Indeed, in the above allocation, since area B has a potential excess supply of housing, the competitive price p_B must be 1. The externality coefficients are calculated as $\lambda_A = {}^{11}/_2 = 5.5$ and

A point of this example is to illustrate a widespread externalities problem that arises purely from tastes. In addition to the difference between physiological and psychological externalities, this example is the opposite of the first example in that, in the first example, the preferences relevant to individual economic behavior are not affected by the externalities, while here the preferences relevant to market behavior are themselves externalities and without physiological effects.

5 A HOUSING MARKET WITH NEIGHBORHOOD EFFECTS; PSYCHOLOGICAL EXTERNALITIES CREATE ECONOMIC VALUE

In the preceding example, externalities are purely a matter of tastes, and the competitive outcome is determined by these tastes. However, externalities of the above kind do not have direct physiological effects on people. Thus markets with pure taste externalities may not be regarded as economically very important. This section presents an example of externalities in tastes in which the externalities significantly affect important economic variables. A phenomenon usually regarded as a typical economic problem appears through externalities.

We have a set of consumers consisting of the points in the interval [0,5). The income of c-th consumer is given by

$$I_c = 2 + c \text{ for all } c \in [0,5).$$

A set of landholdings is given by the interval $[5,11)$, divided into two neighbourhoods $A = [5,8)$ and $B = [8,11)$. We assume that each landholding is owned by one seller, and we index the set of sellers by their landholdings. Each seller has a reservation value of 1.

Each consumer wants one and only one house (landholding). Let $t_c = (t_{cA}, t_{cB})$ be a vector describing the housing consumption of consumer c, where

$$t_{cA} = \begin{cases} 1 & \text{if } c \text{ has a house in } A \\ 0 & \text{if } c \text{ does not have a house in } A; \end{cases},$$

$$t_{cB} = \begin{cases} 1 & \text{if } c \text{ has a house in } B \\ 0 & \text{if } c \text{ does not have a house in } B; \end{cases}$$

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each consumer c in the mixed equilibrium is higher than that in the segregated equilibrium, that is

$$\frac{^{15}/_4}{5} + I_c - 1 = {}^3/_4 + I_c - 1 > {}^2/_5 + I_c - 1 = {}^{5.5}/_5 + I_c - 1.7.$$

The external economy of high income people getting together is absorbed by the increase in the land price from competition of consumers; only the landholders in area A benefit and actually enjoy more than the economic surplus created.

The point of this example is that externalities in tastes significantly affect economic variables, more precisely, the externalities create the market value. It is an interesting additional feature that the market mechanism allocates this market value to the landholders. In a segregated equilibrium, the externalities make landholding "scarce" in the high income neighbourhood. This is the only source of scarcity (other than the scarcity of the composite commodity) in the example.

6 CONCLUSION

We considered four examples with various forms of interactions between widespread externalities and markets. We observed that the effects of externalities and markets on each other can be in either direction or in both directions and the interactions of markets and externalities can be fundamentals of economic phenomena. Also, certain phenomena usually regarded as incompatible with equilibrium theory are in fact results of interactions of widespread externalities and perfect competition, and other phenomena usually regarded as economic problems without externalities may manifest themselves through externalities. With our examples as a whole, we suggest that in many situations externalities are inseparable from the determination of economic activities, values and the market itself.

Besides the examples given in this paper, other economical and environmental situations fit into the widespread externalities framework. Hybrids of some of our examples may be interesting and relevant to the consideration of socio-economic problems. Also, as the third example includes some sociological aspects, there may be sociological phenomena which are well-captured by widespread externalities. We expect further explorations into widespread externalities.

$\lambda_B = 2$. The price $p_A = 1.7$ is calculated by making the marginal consumer $c = 2$ indifferent between A and B, i.e.

$$\frac{\lambda_A}{5} + 4 - p_A = \frac{\lambda_B}{5} + 4 - 1$$

In fact, every consumer is indifferent between areas A and B in the equilibrium.[7] Notice the contrast between this situation and the situation where there are no externality effects, in which case, the price of housing in both neighbourhoods would be 1. The economic value of housing in the wealthier area is created by the neighbourhood effects.

It can be shown that, for a segregated equilibrium, the total surplus generated by the externality is given by

$$\frac{3\lambda_A}{5} + \frac{2\lambda_B}{5} = \frac{3}{5} \times 5.5 + \frac{2}{5} \times 2 = 4.1$$

For the mixed equilibrium (with $\lambda_A^* = \lambda_B^* = {}^{15}/_{4}$), the total surplus generated by the externality is

$$\frac{2.5}{5}\lambda_A + \frac{2.5}{5}\lambda_B = 2(\frac{2.5}{5} \times \frac{15}{4}) = 3.75$$

Thus, the segregated equilibrium generates a larger surplus than the mixed equilibrium.

In fact, the segregated equilibrium outcome is Pareto-optimal. Indeed, consider a movement of a small measure of high income consumers to the low income area. There is a loss of surplus for the remaining consumers in the high income area because the average income per landholding decreases. This loss is not offset by gains in the low income area because the measure of the consumers affected is smaller.

This is the first example in which the reasonable competitive equilibria are Pareto optimal. However we have the surprising feature that in the segregated equilibrium, more than the newly generated surplus goes to landholders; the movement from a mixed equilibrium to a segregated equilibrium is not a Pareto-improvement for either rich or poor consumers.[8] The utility level of

[7] This depends crucially upon the linearly separable form of the utility function, which does not reflect income effects. If we take income effects into account, by setting $\alpha < 1$, then only the marginal consumer c is indifferent between A and B. In this case, the very rich may have a strong preference for the wealthier area.

[8] This feature is reminiscent of Ricardian theory, in which an increase in the amount of capital and/or population has the consequence that "only real gainers are the landlords". See Ricardo (1821, "on profit").

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