Imperfect Competition and Quality Signaling

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ABSTRACT

We examine the interplay of imperfect competition and incomplete information in the context of price competition among firms producing horizontally- and vertically-differentiated substitute products. We find that incomplete information about vertical quality (e.g., consumer satisfaction) that is signaled via price softens price competition. Low-quality firms always prefer playing the incomplete information game to the full-information analog: their prices are higher and so are their profits. Moreover, for "high-value" markets, if the proportion of high-quality firms is high enough, high-quality firms also prefer incomplete information to full information. We find conditions such that an increase in the loss to consumers associated with consuming the low-quality product may perversely benefit low-quality firms. We discuss the implications of our analysis for recent tort reform proposals and for professional licensing.

1. Introduction

In this paper we examine the interplay of imperfect competition and incomplete information in the context of a multi-firm industry producing horizontally-differentiated substitute products with an associated vertical quality measure, such as consumer satisfaction with a firm's product. We find that incomplete information about quality that is signaled via price softens price competition by firms. Further, we show that low-quality firms always prefer playing the incomplete information game to the full-information analog: their prices are higher and so are their profits. Moreover, for "high-value" markets (suitably defined), if the proportion of high-quality firms is high enough, high-quality firms also prefer incomplete information to full information. This is in contrast to the results for a monopolist, who would prefer full information so as to avoid the price distortion associated with signaling.

Other unexpected results of the interplay between imperfect competition and incomplete information also emerge; these results reflect both a firm's best-response behavior vis-a-vis its rivals and its incentive compatibility conditions vis-a-vis its own alter ego. For high-value markets, equilibrium prices, quantities, and profits for both types of firms are increasing in the proportion of high-quality firms; this parameter does not affect equilibrium play in the monopoly signaling model or in the full-information model. In equilibrium, low-quality firms produce greater output than do high-quality firms, a reversal of the result that obtains under full-information imperfect competition. Finally, an increase in the loss borne by consumers due to the low-quality product can (for portions of the parameter space) perversely increase the low-quality firm's price, quantity, and profits; this effect also does not arise in the monopoly signaling model or in the full-information model. This last result suggests that recent proposals for tort reform may actually increase the likelihood of harm and lawsuits and that licensing of professional services can result in increased competitiveness and lower prices.

Plan of the Paper

In Section 2 we provide a brief review of the literature. Section 3 provides the model and results. Section 4 discusses some implications and applications of the model. Section 5 provides a brief summary and conclusions. Supplementary material, including complex formulae and selected proofs, is contained in the Appendix.[1]

2. Related Literature

There are several strands of literature that are related to this work. One body of related work involves a monopolist using price to signal product quality. Bagwell and Riordan (1991) examine a two-type model in which a high-quality product is more costly to produce than is a low-quality product (we adopt this formulation below, but with multiple firms). In equilibrium, the low-quality firm chooses its full-information price, while the high-quality firm distorts its price upward relative to the full-information price for high quality.[2] Daughety and Reinganum (1995) provide a model with a continuum of types in which quality is viewed as product safety. When a product fails and harms a consumer, the liability system determines how the associated losses are allocated across the parties. In equilibrium, higher prices signal safer products when the consumer bears a sufficiently high share of the loss, while lower prices signal safer products when the firm bears a sufficiently high share of the loss. Daughety and Reinganum (2005) consider a model in which quality is a

[1] Proofs of Propositions 1, 3, and 4 are provided. Proofs of Propositions 2, 5, and 6 are straightforward but tedious, and are omitted (example calculations are provided for Proposition 6).

[2] Bagwell (1992) conducts a related analysis of a monopolist producing a product "line."

safety attribute and the firm may engage in confidential settlement of lawsuits. Following first-period production, the monopolist learns its product's safety through harmed consumers who seek compensation. The firm settles lawsuits confidentially, which (potentially) reduces the viability of suits and prevents future consumers from directly observing product safety. Although confidentiality lowers the firm's expected liability costs, it also depresses demand for its product. Daughety and Reinganum (2005) characterize when this trade-off induces the firm to prefer confidentiality versus a regime of openness (in which suits cannot be settled confidentially, and thus future consumers directly observe the product's safety).

There is a strand of the literature which considers price and advertising as joint signals of product quality. For example, Milgrom and Roberts (1986) provide a two-type monopoly model in which the cost of high quality may be higher or lower than that of low quality, and repeat sales are an important attribute of the model. They identify various conditions under which high quality may be signaled with a high price alone, a low price alone, or a combination of price and advertising expenditure.[3] Hertzendorf and Overgaard (2001b) and Fluet and Garella (2002) examine very similar duopoly models in which firms use price and advertising expenditure to signal their qualities. While consumers do not know either firm's quality, *both firms know both firms' qualities*.[4]

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[3] Hertzendorf (1993) argues that, if advertising is stochastically-observed, price and advertising expenditure will never be used in combination. Another interesting paper is Linnemer (1998), in which a firm uses price and advertising to signal to two different audiences: it signals its product quality to consumers and its marginal cost to a potential entrant.

[4] See also Hertzendorf and Overgaard (2001a) for a duopoly model in which firms can only use price to signal quality; again, both firms know both firms' qualities. Harrington (1987), Bagwell and Ramey (1991), and Orzach and Tauman (1996) consider limit pricing models in which two or more incumbent firms with *common* private information about production costs attempt to deter entry using price as a signal of cost. These papers are more closely-related to the models of Hertzendorf and Overgaard, and Fluet and Garella, than to ours.

Moreover, consumers do not have a preference between the two goods, provided they are of the same quality and charge the same price (i.e., there is no horizontal differentiation). In equilibrium, price alone can signal quality when vertical differentiation is substantial, but otherwise advertising is required as well. When advertising is not used, quality is signaled with upward-distorted prices, but when advertising is used, prices may be driven below their full-information levels.

Daughety and Reinganum (forthcoming) provide a duopoly model in which each firm uses its price to signal its product quality; this model differs from those above in three important respects. First, only price can be used to signal quality. Second, the products on offer are differentiated, both horizontally and, possibly, vertically. Third, each firm's quality is its private information. This information structure arises naturally in the context of quality as safety under a regime that permits confidential settlement of lawsuits (as described above). By settling confidentially with consumers harmed in the current period, the firm prevents future consumers and its rival from learning its quality. In the next period, each firm has private information about its own quality, which sets up the signaling game in which a firm's price may reveal its product quality. That paper employs a post-sale subgame involving tort liability, and therefore makes other simplifying assumptions to enhance tractability; in particular, a fixed number of consumers distributed along a line is assumed, with each consumer demanding a single unit. Moreover, it is assumed that the market is always fully covered; that is, every consumer buys a unit of the good. As a consequence, the market size is fixed exogenously.

A special case of the model described above is one wherein consumers bear the full loss associated with low quality; this can be interpreted as a "consumer satisfaction" model as introduced in Milgrom and Roberts (1986). In a consumer satisfaction model, the consumer simply receives lower utility from low-quality products than from high-quality products but, because utility is unverifiable, no compensation can be promised (e.g., a warranty cannot be used to insure the consumer against a loss of utility from low quality).[5]

The current paper also considers a consumer satisfaction model in which each firm has private information about its product quality, but employs a more general demand and market structure. A representative consumer has a linked system of demand functions for n differentiated products (each produced by a single firm), and each firm will non-cooperatively choose its price given its private information. Thus, the size of each firm's market, as well as the size of the total market for all n differentiated products, is determined endogenously. This means that we are able to examine the effect of changes in the number of firms, in the degree of substitutability of the products, and in the consumer's willingness-to-pay on equilibrium behavior.

Other (more tangentially-related) literatures include those involving quality-guaranteeing prices and those involving disclosure of quality. In the quality-guaranteeing price literature (dating back to the early 1980's; see, e.g., Klein and Leffler, 1981), firms choose their qualities, as well as their prices, while consumers observe only the prices. Equilibrium is characterized by a price premium that is sufficient to induce firms subsequently to provide high quality. Thus, unobservable quality relaxes price competition. A recent example is Bester (1998), who relates the magnitude of this effect to the degree of endogenous horizontal product differentiation. Levin, Peck and Ye (2005) provide a model in which two firms have private information about the quality of their

[5] If the consumer's loss were verifiable then a firm could offer a warranty contract specifying the consumer's extent of recovery in the event of this loss. Since the warranty contract is endogenous, warranties could serve as another route via which quality could be signaled (see Lutz, 1989, for such a model).

respective products, but can engage in costly disclosure. Consumers are located along a line between the two products, reflecting horizontal product differentiation. The cost of production is independent of quality and thus no signaling is possible. In equilibrium, firms engage in socially-excessive disclosure. The current paper differs from the quality-guaranteeing price literature because Nature chooses firm quality in our model, and differs from the disclosure literature because firms cannot credibly disclose quality and must instead resort to signaling.

Finally, there is also a small literature on non-cooperative signaling when each firm has private information about its cost of production. The most closely-related paper is Mailath (1989), which provides an n-firm oligopoly model with linear demand and constant marginal costs in which firms produce horizontally-differentiated products and engage in non-cooperative price competition across two periods.[6] A firm's first-period price can signal its (privately observed) marginal cost of production, which influences its rivals' pricing behavior in the second period.[7] Consumers have no inference problem, since they care only about prices, not marginal costs. Mailath finds that firms' prices are upward-distorted (in order to persuade rivals to price higher in the second period) relative to the "non-signaling benchmark," which retains incomplete information in the first period, but

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[6] Gal-Or (1988) also considers a two-period model in which firms learn about their costs, and those of their rivals, over time; her paper specifically omits consideration of the signaling problem. Mailath (1988) establishes conditions guaranteeing the existence of separating equilibria in abstract two-period games with simultaneous signaling.

[7] In a similar vein, Martin (1995) considers a two-period model in which two incumbent firms signal their privately known marginal costs, both to each other and to a potential entrant. When firms use price strategies in a horizontally-differentiated products model, an incumbent firm would like its incumbent rival to perceive it as high-cost, but this perception also invites entry. This can result in pooling equilibria. Das Varma (2003) considers a model in which the auction of a cost-reducing innovation precedes market competition. When firms compete in price strategies, each firm bids less aggressively for the innovation in order to persuade its rivals that, should it win the auction, it will be a softer competitor in the subsequent market.

assumes that the firms' types are exogenously revealed prior to the second period (so the signaling motive is removed).

Although we also use a horizontally-differentiated-products model with linear demand and constant marginal costs, our model differs from that of Mailath in other ways. First, we consider a one-shot (three-period) model wherein each firm signals its quality to consumers, rather than to its rivals. Second, a firm's product quality (type) affects both its constant marginal cost of production and the demand curve it faces, because product quality also reflects vertical differentiation. Finally, in our model the non-signaling benchmark is the full-information outcome in which both rivals and consumers observe product quality directly. Like Mailath, we find that equilibrium prices are upward-distorted relative to our (full-information) benchmark prices.

3. Model Set-Up and Results

Our model employs a representative consumer, who consumes some of each product, and n firms, each of whom produces one of the products, under conditions of constant marginal costs. The products are horizontally- and vertically-differentiated, where the quality of the product (the vertical attribute) takes on two possible levels (high and low). In period one, Nature independently draws a type for each firm from a common distribution and each firm observes its type. In period two, firms simultaneously choose prices. Finally, in period three, the representative consumer observes all prices and buys quantities of the products accordingly. In the incomplete-information model, firms do not observe the types of other firms, and consumers do not directly observe the type of any firm. In the full-information model, firms and consumers observe all the types in period two before firms choose prices. In all settings we restrict the analysis to interior equilibria.

Consumer Model

To keep things as simple as possible, we consider a single consumer[8] who consumes a variety of goods; products 1, 2, ..., n are differentiated substitute goods and good n + 1 is a numeraire good. Each product is made by a different firm, and we assume there are $n \geq 2$ products. Products 1, 2, ..., n may be of either high or low quality (signified by H or L, respectively). Let θ_i be an indicator function which takes on the value 1 when product i is of high quality and the value 0 when product i is of low quality. We assume that the consumer derives utility from the product, less a loss per unit consumed, which is zero for the high-quality good and $\delta > 0$ for the low-quality good.[9] The occurrence of this loss is unverifiable (e.g., an uncomfortable mattress, a lazy real estate agent, or a mediocre meal) and therefore cannot be covered by a warranty, or compensated via a lawsuit. Nature determines product quality independently for each firm, and Pr{H} is given by $\lambda \in (0, 1)$.[10] The consumer receives higher utility from a high-quality product i than a low-quality product i, but both versions of product i are worthwhile. In particular, we assume the consumer's utility function is quadratic in the n differentiated products, with the parameters $\alpha > 0$, $\beta > 0$, and $\gamma > 0$:

$$U(q_1, ..., q_n) = \sum_i [\alpha - (1 - \theta_i)\delta]q_i - \tfrac{1}{2}(\sum_i \beta q_i^2 + \sum_i \sum_{j \neq i} \gamma q_i q_j),$$

where γ is the degree of product substitution between any two products in the class of interest. We

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[8] Given constant returns to scale, the same results apply for multiple identical consumers.

[9] One could generalize the model to allow θ_i to take on values between zero and one, and then interpret θ_H as the probability that a product i of high quality does not result in a loss of δ, and θ_L ($< \theta_H$) as the probability that a product of low quality does not result in a loss of δ. In this case, a higher-quality product is less likely to result in a loss of δ, but both qualities of products can fail to provide complete satisfaction to the customer.

[10] For certain results we will consider limits where λ goes to zero or one.

take γ to lie in the interval $(0, \beta)$.[11] Product quality enters through the linear coefficient on q_i; this coefficient is α if product i is of high quality but falls to $\alpha - \delta$ if product i is of low quality.

The consumer with income I chooses $(q_1, ..., q_n)$ so as to maximize her utility of consumption (the consumption of the numeraire good is found as the residual):

$$\max_{q_1, ..., q_n} \{U(q_1, ..., q_n) + I - \sum_i p_i q_i\}.$$

Thus, for positive demands, the inverse demand function for product i is:

$$p_i(q_1, ..., q_n) = \alpha - (1 - \theta_i)\delta - \beta q_i - \gamma \sum_{j \neq i} q_j.$$

Since we are interested in firms using price strategies to signal their product quality, we solve for the ordinary demand functions:

$$q_i(p_1, ..., p_n) = a - b(1 - \theta_i)\delta + g\sum_{j \neq i}(1 - \theta_j)\delta - bp_i + g\sum_{j \neq i} p_j, \tag{1}$$

where $a \equiv \alpha/(\beta + (n - 1)\gamma)$, $b \equiv (\beta + (n - 2)\gamma)/(\beta - \gamma)(\beta + (n - 1)\gamma)$, and $g \equiv \gamma/(\beta - \gamma)(\beta + (n - 1)\gamma)$.

These represent the consumer's demand functions when quality is observable. When quality is unobservable to the consumer, she will have perceptions of product quality, which we will denote by $\tilde{\theta}_j$, j = 1, 2, ..., n. Then equation (1), modified by substituting perceived for true qualities, still describes the consumer's demand functions. We will later discuss in greater detail how the consumer's perceptions are formed based on observed prices.

Firm and Industry Model

For simplicity, we assume that each firm has constant marginal costs which depend on the quality of its product. The cost of producing a unit of a low-quality product is normalized to zero,[12]

[11] Note that if $\gamma = 0$, then each product is independent of each other product, and each firm has a monopoly in its product market.

[12] We generalize this cost structure to allow positive costs for the low-quality product later in the paper in discussing one of our applications.

and the cost of producing a unit of a high-quality product is $k > 0$. We also assume that $\delta > k$, so that the additional utility generated by a unit of a high-quality product justifies its incremental production cost (i.e., a consumer would be willing to pay k to receive higher quality, thus avoiding the loss δ).

A firm's profits can be written as a function of its product's true quality, its product's perceived quality (from the consumer's point of view) and its price, given the perceived qualities and prices of its rivals. If quality were observable, then the perceived qualities would coincide with the true qualities. However, perceived quality may differ from true quality if quality is not observable. Then profits for firm i, when it charges price p_i, its true quality is θ_i and its perceived quality is $\tilde{\theta}_i$ (and the vector of other firms' prices is p_{-i} and the vector of other firms' perceived qualities is $\tilde{\theta}_{-i}$) can be written as:

$$\pi_i(p_i, \theta_i, \tilde{\theta}_i \mid p_{-i}, \tilde{\theta}_{-i}) = (p_i - k\theta_i)(a - b(1 - \tilde{\theta}_i)\delta + g\textstyle\sum_{j\neq i}(1 - \tilde{\theta}_j)\delta - bp_i + g\sum_{j\neq i}p_j).$$

Thus, firm i's profits are a product of the true price-cost margin and the consumer's demand for product i, which is based on prices and her perceptions of quality.

We want to characterize a symmetric separating perfect Bayesian equilibrium for this game, wherein each firm's product quality is its private information; that is, firm i's product quality is unknown both to the consumer and to firm i's rivals.[13] Suppose that all other firms employ the same separating pricing rule $p^*(\theta)$; that is, $p^*(1) \neq p^*(0)$. Then because this is a separating strategy, firm i predicts that the consumer's perception of all rival firms' product qualities will be correct. Moreover, firm i also predicts that each of its rivals will charge the price $p^*(1)$ with probability λ

[13] In fact, we show that the separating equilibrium we discuss is the only symmetric separating equilibrium to survive refinement under the Intuitive Criterion. Moreover, this same refinement eliminates all (pure) pooling equilibria.

and the price $p^*(0)$ with probability $1 - \lambda$. Thus, firm i's expected profits, when it charges price p_i, its true quality is θ_i and its perceived quality is $\tilde{\theta}_i$ (and all rival firms use the separating strategy $p^*(\bullet)$) can be written as:

$$E_{-i}\{\pi_i(p_i, \theta_i, \tilde{\theta}_i \mid p_{-i}, \theta_{-i})\} = (p_i - k\theta_i)E_{-i}\{a - b(1 - \tilde{\theta}_i)\delta + g\textstyle\sum_{j\neq i}(1 - \theta_j)\delta - bp_i + g\textstyle\sum_{j\neq i}p^*(\theta_j)\},$$

where E_{-i} denotes the expectation with respect to the vector of rivals' product quality θ_{-i}. Since the second term above is linear in the rivals' types and prices, and the types are identically and independently drawn, this term is simply $a - b(1 - \tilde{\theta}_i)\delta + g(n - 1)(1 - \lambda)\delta - bp_i + g(n - 1)E(p^*)$, where $E(p^*) \equiv \lambda p^*(1) + (1 - \lambda)p^*(0)$. Thus, firm i's expected profits can be written as:

$$\Pi_i(p_i, \theta_i, \tilde{\theta}_i \mid E(p^*)) \equiv (p_i - k\theta_i)(a - b(1 - \tilde{\theta}_i)\delta + g(n - 1)(1 - \lambda)\delta - bp_i + g(n - 1)E(p^*)).$$

Note that for any given price, it is always more profitable to be perceived as type H, regardless of true type; and for any given price, it is better to be type L, regardless of perceived type. Thus no type will incur a signaling distortion in order to be perceived as type L.

To formalize the notion of the consumer's perception of quality, we define a belief function which specifies the type (high- or low-quality) that the consumer assigns to a firm. The firms (of a given type) are completely symmetric, so it is natural to impose a symmetry assumption. Moreover, firm types are independently drawn and no firm has any information about its rivals' types when it chooses its price, so its price cannot vary with the rivals' types, and hence its price cannot convey any information about the rivals' types. Thus, it is also natural to assume that the consumer's beliefs about firm i's type do not vary with the price charged by firm j, $j \neq i$. Fudenberg and Tirole (1991, pp. 332-3) incorporate this restriction (which they refer to as "no signaling what you don't know") into their definition of perfect Bayesian equilibrium for a general class of abstract games of which ours is a special case. Let $B(p)$ be the belief function; thus, if firm i charges p_i, then it is inferred to be of type $B(p_i)$. This belief function depends only on the price chosen by firm i, and it is the same function for all firms, so it reflects the preceding discussion of symmetry and invariance with respect to rivals' prices.

In equilibrium, each firm maximizes its expected profits, given the pricing functions of its rivals and given the consumer's belief function, and the consumer's beliefs are consistent with the equilibrium pricing function used by the firms. This is formalized in the following definition.

Definition. A symmetric separating perfect Bayesian equilibrium consists of a pair of prices $(p^*(0), p^*(1)) \equiv (P_L, P_H)$ and beliefs $B^*(p)$ such that, for i = 1, 2, ..., n:

(i) $\Pi_i(P_L, 0, 0 \mid E(p^*)) \geq \max_p \Pi_i(p, 0, B^*(p) \mid E(p^*))$;

(ii) $\Pi_i(P_H, 1, 1 \mid E(p^*)) \geq \max_p \Pi_i(p, 1, B^*(p) \mid E(p^*))$;

(iii) $B^*(P_L) = 0$, $B^*(P_H) = 1$;

(iv) $E(p^*) = \lambda P_H + (1 - \lambda)P_L$.

Part (i) says that a firm that has a low-quality product, and is perceived to have a low-quality product, would prefer to charge P_L rather than its best alternative price, given the consumer's belief function and the expected rival price. Part (ii) says that a firm that has a high-quality product, and is perceived to have a high-quality product, would prefer to charge P_H rather than its best alternative price, given the consumer's belief function and the expected rival price. These conditions reflect both a firm's best-response behavior vis-a-vis its rivals and its incentive compatibility conditions vis-a-vis its own alter ego. Part (iii) says that the consumer's beliefs are correct in equilibrium; she believes the product is of low quality when the firm charges P_L and she believes the product is of high quality when the firm charges P_H. Finally, part (iv) says that the prior-weighted expectation of P_H and P_L equals the expected rival price; that is, it is a mutual best response for all firms to play the price strategy (P_L, P_H). Let (Q_L, Q_H) denote the associated quantities and let (Π_L, Π_H) denote the associated profit levels.

Results

In this section we provide the primary results for the model above, along with a discussion of the intuition for the results. The comparative statics results (which are explored in Propositions 2, 5, and 6) are summarized in Table 1 near the end of the section. Unless otherwise specified, all references to equilibrium entities (i.e., prices, quantities, and profits) are *interim* versions (that is, each firm knows its type, but not the type of any rival).

We need to restrict the parameters for the analysis so as to guarantee an interior equilibrium, both *ex ante* and *ex post*, because the derivations above implicitly assume this. First we consider *ex ante* restrictions. Define the firm's true-type-dependent marginal costs: $c_s \equiv k\theta_s$, where s = L,H. The firm's demand (based on its perceived type) is $(d_t - p)b$, where $d_t \equiv \{a - b(1 - \theta_t)\delta + g(n - 1)(1 - \lambda)\delta + g(n - 1)E(p^*)\}/b$, t = L, H. Thus we can use the short-hand notation $\Pi_{st} = (p - c_s)b(d_t - p)$, for s, t = L, H.

For some parameter combinations, one or both types might not be able to choose prices that yield positive expected profits; under such parameter combinations, agents would potentially conjecture that the prior probability of a firm being an H-type was not λ, and should be updated, something that is not of particular interest to the current analysis. To ensure that there is always a profitable price for a firm, regardless of the consumer's perceptions of quality and regardless of the rival firm's expected price, we need $d_t > c_s$ for all s, t. The tightest such constraint is $d_L > c_H$; that is, $\{a - b\delta + g(n - 1)(1 - \lambda)\delta + g(n - 1)E(p^*)\}/b > k$. Recognizing that λ may be arbitrarily close to 1, $E(p^*)$ may be arbitrarily close to zero, and k (though smaller) may be arbitrarily close to δ, we employ the following sufficient condition, which we maintain throughout the paper.

Assumption 1. $a > 2\delta b$; that is, $\alpha/\delta > 2 + 2(n - 1)[\gamma/(\beta - \gamma)]$.

Assumption 1 requires that the product be of sufficiently "high value" (indicated by the maximum willingness-to-pay, α) relative to the possible loss, δ; in the case of a monopoly, we would require $\alpha/\delta > 2$. The second term on the right-hand-side reflects the intensity of competition and combines both the number of firms and the degree of product substitution. Thus, for this assumption to hold, we require that competition must be "sufficiently imperfect" in the sense that the extent of substitutability γ (relative to β) or the number of firms n must be sufficiently small. Notice that Assumption 1 is a strong sufficient condition for $d_L > c_H$, not a necessary condition.

We also need the equilibrium realized demand to be positive, so a second assumption addresses *ex post* interiority. Below we provide a proposition concerning the separating equilibrium prices (P_L, P_H) which are specified in the Appendix. Using those defined values note that $P_H > P_L + \delta$. As a consequence, when firms use the prices (P_L, P_H), a firm's realized demand is lowest when it is of type H and all of its rivals are of type L. As shown in the Appendix, this lowest realized demand is positive if α is sufficiently high. In the sequel, we maintain this assumption. Unfortunately, due to the complexity of the algebra involved, it is unclear whether this required level is greater or less than that implied by Assumption 1 (in our later computations, we make sure that the results respect both assumptions).

Assumption 2. α is sufficiently large as to ensure that equilibrium demand for firm i is positive for all realizations of (θ_i, θ_{-i}).

The following proposition indicates that (under Assumptions 1 and 2) there always exists a symmetric perfect Bayesian equilibrium (PBE) in which the two types separate. Moreover, by employing a refinement (the Intuitive Criterion; see the Appendix) we select one separating equilibrium and show that this is the only symmetric equilibrium (separating or pooling) which survives refinement. Due to the complexity of the algebra of expressing the equilibrium prices, quantities and profits, we leave these details to the Appendix.

Proposition 1 (Existence of a Unique Refined Equilibrium). There is a unique (refined) symmetric separating perfect Bayesian equilibrium consisting of a pair of prices (P_L, P_H), with $P_H > P_L$, and supporting beliefs $B^*(p)$, with $B^*(p) = 0$ when $p < P_H$, and $B^*(p) = 1$ when $p \geq P_H$. Moreover, $Q_L > Q_H$; that is, a low-quality firm produces a higher equilibrium output than a high-quality firm.

In this equilibrium, P_H is the lowest price that an H-type firm can charge and still deter mimicry by an L-type firm; a higher price also signals high quality, but is less profitable, while a lower price provides a profitable deviation for an L-type firm, if consumers were to infer that the firm that is charging that price is an H-type firm. As is discussed in Proposition 3 below, these separating-equilibrium prices exceed their full-information counterparts (see the Appendix for the relevant expressions). Moreover, unlike the full-information equilibrium, wherein a high-quality firm would produce more than a low-quality firm, in the refined asymmetric-information equilibrium, a low-quality firm produces more output than a high-quality firm: $Q_L - Q_H > 0$. This reflects the distortion in prices due to signaling quality: the high-quality firm's price is so much higher than the price charged by the low-quality firm that consumers are redistributed toward the L-type firm.[14]

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[14] That this effect arises due to signaling is seen by examining the Bayesian-Nash equilibrium, in which firms must choose prices under uncertainty about rival quality, but consumers can observe all firms' quality levels before buying. In this case (as in the case of full information), a high-quality firm will sell more output than a low-quality firm. We thank a referee for this observation.

It is straightforward to show that an L-type firm's *interim* profits exceed the *interim* profits of an H-type firm; this is a common finding in models wherein high quality is signaled by a high price (see, e.g., Bagwell and Riordan, 1991, and Daughety and Reinganum, 2005). This is not problematical in this model, since firms are unable to choose their quality levels. However, if one were to contemplate a more general model that allows firms to influence their product quality, this suggests that they would all prefer to produce low-quality products. We believe that this inference ignores some other important considerations, and we provide a detailed discussion of this issue at the end of this section.

Some of the results to follow are global, that is, independent of the parameter values (given that they satisfy Assumptions 1 and 2), while others can be demonstrated via sufficient conditions concerning one or more of the parameters. In particular, a number of our results employ a sufficient condition on the maximum willingness-to-pay, α. For convenience, we use the terminology "high value" to denote markets wherein α satisfies a given sufficiency condition (that is, it is "large enough"), which will be made more precise in each result. In particular, we will employ the following convention: a lower bound on a parameter that is specific to a result in a proposition will be subscripted by that proposition number (and sub-part, if needed). Thus, for example, there will be an α_2 asserted in Proposition 2, but no α_1, since Proposition 1 does not require any other lower bound than is implied by Assumptions 1 and 2.

<u>Definition 2</u>. A "high-value" market indicates the presence of a finite lower bound for α.

Interestingly, a perusal of the formulae (in the Appendix) for P_H and P_L reveals that both prices are functions of λ, the prior probability that a firm is of type H. Thus, unlike separating equilibria in most other models, where only the support of the prior affects the equilibrium,[15] here the prior probabilities themselves influence the separating equilibrium through the expected prices for the rival firms. As the following proposition indicates, increasing λ raises both prices, the gap between the prices, the gap between the quantities, the L-type quantities and profits, and (when dealing with a high-value market) the quantities and profits for the H-type firms.

Proposition 2 (Effect of Prior Probability Distribution on Prices, Quantities and Profits).

(i) An increase in λ increases:

1) the equilibrium prices P_L and P_H;

2) the difference between the equilibrium prices $P_H - P_L$;

3) the difference between the equilibrium output levels $Q_L - Q_H$; and

4) the L-type's quantity, Q_L, and profits, Π_L.

(ii) In high-value markets, an increase in λ results in an increase in the H-type's quantity, Q_H, and profits, Π_H. More formally, $\exists \alpha_2 < \infty$ such that $\forall \alpha > \alpha_2$, $\lambda\uparrow \Rightarrow Q_H\uparrow$ and $\Pi_H\uparrow$.[16]

The source of this effect can be understood by reconsidering the definition of equilibrium provided earlier. The incentive compatibility conditions (items (i) and (ii) in the Definition) depend upon the expected price of the competitors, $E(p^*)$. Since the equilibrium involves the H-type firm posting a higher price than the L-type firm, an increase in the proportion of firms that are likely to be of type

[15] Some exceptions in the literature considering signaling by one firm do exist; see Matthews and Mirman (1983) and Daughety and Reinganum (1995, 2005); see also Daughety and Reinganum (forthcoming) for a duopoly example with a simpler demand structure which yields results similar to Propositions 2 and 3 below.

[16] Note that α_2 is a function of the parameters of the model. This will be similarly true for other results below, which require a restriction that α be sufficiently large. We use α as the critical variable here because it can be increased without threatening to violate Assumptions 1 and 2.

H shifts the incentive compatibility constraints. Moreover, since firms' prices are strategic complements ($\gamma > 0$), best response functions are upward-sloping, so that an increase in the expected price of a firm's competitors encourages each firm to increase its price.

This effect turns out to be parameter-independent for the L-type's price, quantity and profits, and for the H-type's price; moreover, the difference between the high and low price (and between the low and high quantity) also increases as λ increases, again, for all portions of the parameter space. While we cannot provide the same global result for the H-type's quantity and profit, if the market is high-value, then increasing λ also increases the H-type's quantities and profits.

The above results suggest that incomplete information about the quality of the good may act to soften competition; since the firms are choosing prices in a non-cooperative manner, incomplete information may allow them to achieve higher prices. This leads us to the next proposition, which provides a comparison of equilibrium prices under incomplete versus full information. Let $P^F(\theta_i, \theta_{-i})$ denote the full information price for firm i if its true quality is θ_i and the vector of its rivals' true qualities is θ_{-i}; the formula for the full-information price function is provided in the Appendix. Likewise, let $Q^F(\theta_i, \theta_{-i})$ and $\Pi^F(\theta_i, \theta_{-i})$ denote the corresponding full-information outputs and profits. It is straightforward to show that a firm's full-information price, output and profits are highest when all of its rivals have low-quality products. We will sometimes use the notation 0_{-i} to denote an (n-1)-dimensional vector of zeros, and 1_{-i} to denote an (n -1)-dimensional vector of ones.

> <u>Proposition 3 (Prices Under Alternative Information Structures).</u> The equilibrium price under incomplete information is higher, for both firm types, than the corresponding price under full information, regardless of the rivals' realized qualities; that is, for all $\lambda \in (0, 1)$, $P_L > P^F(0, \theta_{-i})$ and $P_H > P^F(1, \theta_{-i})$ for all θ_{-i}.

Thus, there is an upward price distortion for both types of firm, since the equilibrium price for each type under incomplete information exceeds the highest price that type of firm would charge (independent of the realizations of the firm's rivals' types) if quality were observable.

Next, the following proposition (also proved in the Appendix) provides a comparison of profits under incomplete information with those under alternative information structures. In particular, we find that an L-type firm always benefits from incomplete information. We obtain a weaker result for an H-type firm by comparing the profits of an H-type firm under incomplete information with a natural analog: the *ex ante* expected value of an H-type firm's profits under full information (that is, computed just before period 2, so that the firm knows it is of type H and takes the expected value of its full-information profits with respect to the rivals' types).

Proposition 4 (Profits Under Alternative Information Structures).

(i) The L-type's profits under incomplete information are strictly higher than its full-information profits for any realization of the rivals' types; that is, for all $\lambda \in (0, 1)$, $\Pi_L > \Pi^F(0, \theta_{-i})$ for all θ_{-i}.

(ii) As the proportion of H-type firms becomes arbitrarily close to one, the H-type price converges to a value that is higher than the full-information equilibrium price, but (for high-value markets) lower than the full-information price, denoted P^F_C, that would be set by a cartel in an industry comprised only of H-type firms. More formally, $\lim_{\lambda \to 1} P_H > P^F(1, 1_{-i})$ and

$$\exists \alpha_{4ii} < \infty \text{ such that } \forall \alpha > \alpha_{4ii}, \lim_{\lambda \to 1} P_H < P^F_C \equiv (\alpha + k)/2.$$

Thus, for high-value markets, an H-type firm's profits under incomplete information exceed the *ex ante* expected value of an H-type firm's profits under full information when the proportion of H-types is sufficiently high.

The fact that H-type firms (at least for some parameter configurations) price below the full-information cartel price is yet more significant when one considers the case of a monopolist (n = 1) with private information, which would involve pricing above the full-information monopoly price (due to the signaling distortion). Thus, in this sense, the presence of competitors moderates the distortionary effects of the presence of incomplete information. Moreover, the proposition implies that there are parameter configurations (high-value markets with a sufficiently high proportion of H-types) for which both types of firm prefer playing an incomplete information game involving product quality to the full-information counterpart. This provides another contrast with the monopoly version of the model, in which an L-type firm would charge its full-information price and obtain its full-information profits, while an H-type firm would distort its price upward and necessarily receive lower profits than under full information.

The degree of "competitiveness" and the consumer's maximum willingness-to-pay also affect the equilibrium, in an intuitively expected manner, as described in the following proposition.

Proposition 5 (Effects of Product Value, Product Substitutability and the Number of Firms).

(i) An increase in the consumer's willingness-to-pay (α) yields an increase in both L-type and H-type equilibrium prices, quantities, and profits, and an increase in the difference in equilibrium prices and the difference in equilibrium quantities; that is,

$$\alpha\uparrow \Rightarrow P_L\uparrow,\ Q_L\uparrow,\ \Pi_L\uparrow,\ P_H\uparrow,\ Q_H\uparrow,\ \Pi_H\uparrow,\ P_H - P_L\uparrow, \text{ and } Q_L - Q_H\uparrow.$$

(ii) In high-value markets, an increase in the degree of product substitutability (γ) yields a decrease in both L-type and H-type equilibrium prices, quantities, and profits, and a decrease in the differences both in equilibrium prices and in equilibrium quantities; that is,

$\exists\alpha_{5ii} < \infty$ such that $\forall\alpha > \alpha_{5ii}$, $\gamma\uparrow \Rightarrow P_L\downarrow, Q_L\downarrow, \Pi_L\downarrow, P_H\downarrow, Q_H\downarrow, \Pi_H\downarrow, P_H - P_L\downarrow$, and $Q_L - Q_H\downarrow$.

(iii) In high-value markets, an increase in the number of firms (n) yields a decrease in both L-type and H-type equilibrium prices, quantities, and profits, and a decrease in the differences both in equilibrium prices and in equilibrium quantities; that is,

$\exists\alpha_{5iii} < \infty$ such that $\forall\alpha > \alpha_{5iii}$, $n\uparrow \Rightarrow P_L\downarrow, Q_L\downarrow, \Pi_L\downarrow, P_H\downarrow, Q_H\downarrow, \Pi_H\downarrow, P_H - P_L\downarrow$, and $Q_L - Q_H\downarrow$.

Thus, higher-value markets have higher prices, greater sales volumes, and higher profits, while increases in competition (brought about either via less product heterogeneity or via the presence of more competitors in the market) reduce prices, sales volumes, and profits when the markets are high-value.

Finally, the loss (δ) suffered by a consumer of a low quality product affects the equilibrium, but in a surprising manner. In the full information model, an increase in δ reduces the equilibrium price, output, and profits of the low-quality firm and increases the equilibrium price, output, and profits of the high-quality firm. However, in the model with incomplete information, an increase in the loss δ may increase the equilibrium price, output, and profits of the L-type firm. This is because the increase in the consumer's loss increases the incentive for L-type firms to mimic H-type firms; to deter mimicry, the gap between the prices must increase with any increase in the amount of the loss that consumers of L-type products must bear. This could lead to increased demand for the L-type good, and therefore higher price and profits for the L-type firm. In this event, since the prices are strategic complements, the H-type's price would also rise, though this need not mean that the H-type's quantity or profit would increase. The following proposition characterizes the results of an exogenous increase in the consumer loss parameter, δ.

Proposition 6 (Effects of Low-Quality Loss).

(i) The difference in prices is always increasing in the loss associated with low quality, δ; that is, $\delta\uparrow \Rightarrow P_H - P_L\uparrow$. For high-value markets, the difference in quantities is always increasing in the loss associated with low quality, δ; that is, $\delta\uparrow \Rightarrow Q_L - Q_H\uparrow$.

(ii) As the proportion of H-types becomes arbitrarily small, an increase in the loss δ results in a decrease in the L-type's price, quantity and profits. More formally:

$\lim_{\lambda\to 0} \partial P_L/\partial\delta < 0$;

$\lim_{\lambda\to 0} \partial Q_L/\partial\delta < 0$;

$\lim_{\lambda\to 0} \partial \Pi_L/\partial\delta < 0$.

(iii) For high-value markets, there are values of the proportion of H-types such that an increase in the loss δ results in an increase in the L-type's price, quantity and profits, and an increase in the H-type's price as well; that is,

$$\exists\alpha_{6iii} < \infty \text{ and } \lambda_{6iii} \in (0, 1) \text{ such that } \forall\lambda > \lambda_{6iii},\ \delta\uparrow \Rightarrow P_L\uparrow,\ Q_L\uparrow,\ \Pi_L\uparrow, \text{ and } P_H\uparrow.$$

The above proposition paints a surprising picture: for an industry that is comprised of a sufficiently large proportion of H-type firms, an increase in the loss actually can make the L-type firms better off. Examples of this are illustrated in Figure 1 below.[17] The Figure illustrates five computations of the curve $\partial P_L/\partial\delta$, for different values of the relevant parameters (without loss of generality we set $\beta = \delta = 1$). The heaviest line illustrates the case wherein $n = 3$, $\alpha/\delta = 15$ and $\gamma/\beta = 0.5$. Variations on this case include increasing n, decreasing α/δ or modifying γ/β; the underlined number in the triple indicates which parameter was changed to obtain the curve. We have included

[17] Of course, only parameter combinations that also satisfied Assumptions 1 and 2 were considered.

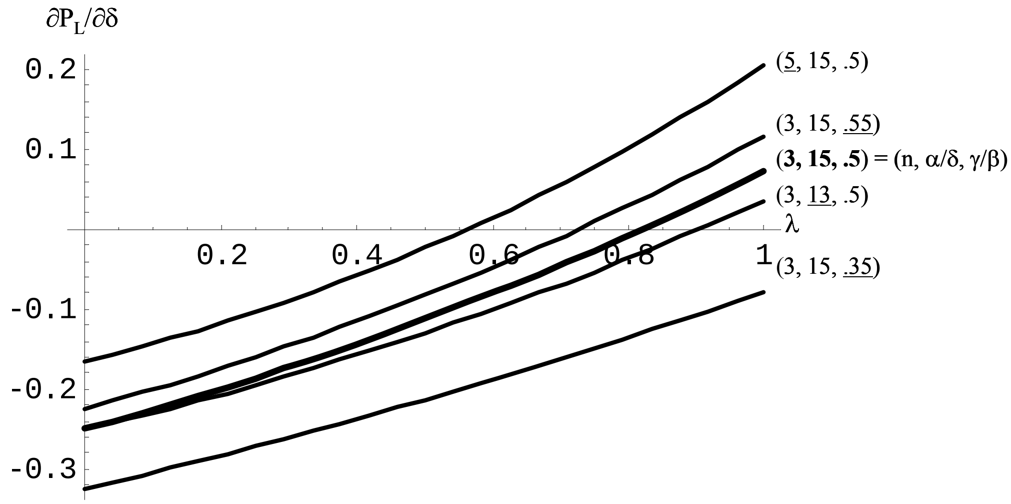

Figure 1. The Effect of an Increase in the Loss on the L-type Price

one parameter set, (n, α/δ, γ/β) = (3, 15, 0.35) to illustrate that for some portions of the parameter space, $\partial P_L/\partial\delta$ is always negative. However, the primary point of the Figure is that there are large portions of the parameter space where, for sufficiently high (but still fractional) values of λ, $\partial P_L/\partial\delta$ is strictly positive. In these portions of the parameter space, Q_L and Π_L increase as well.

We have illustrated this effect in the computations for the Figure for a triopoly, but there are duopoly configurations that also provide this effect. Moreover, as one might conjecture from Proposition 6, higher values of n (consistent with Assumptions 1 and 2) also produce such curves. Thus, both smaller and larger industries could satisfy the conditions of the proposition. We again emphasize that this result is due to the interplay between imperfect competition and incomplete information; absent either, we would not see such a result.[18] In Section 4 we discuss the implications of this finding in a number of contexts.

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[18] In particular, for a monopoly (n = 1), P_L, Q_L, and Π_L are all declining in δ.

Summary of Comparative Statics Results

The Table below summarizes the results provided in Propositions 2, 5, and 6. The following notation is used: 1) a global (i.e., parameter-independent) positive sign on a derivative is denoted as ⊕; 2) a positive (or negative) sign for a derivative which requires that α be sufficiently large will be denoted as **+** (or **-**), while an unknown sign is denoted by a question mark (**?**); 3) the positive sign for the influence of δ, which is dependent upon both the size of α and of λ, is denoted $+_{\lambda}$.

Table 1: Comparative Statics Results

	α	γ	n	λ	δ
P_L	⊕	-	-	⊕	$+_{\lambda}$
Q_L	⊕	-	-	⊕	$+_{\lambda}$
Π_L	⊕	-	-	⊕	$+_{\lambda}$
P_H	⊕	-	-	⊕	+
Q_H	⊕	-	-	+	-
Π_H	⊕	-	-	+	?
P_H - P_L	⊕	-	-	⊕	⊕
Q_L - Q_H	⊕	-	-	⊕	+

Discussion of Interim Profits, Quality, and Entry

In this analysis, we have assumed that quality is determined exogenously by Nature, and we have taken the number of firms as given. In a more comprehensive model, one might allow both of these assumptions to be relaxed. While a full analysis of the implications of relaxing these assumptions is beyond the scope of this paper, we briefly discuss each of these issues in turn.

As mentioned above, it is common in models wherein high quality is signaled by a high price to find that the *interim* profits for an H-type firm are lower than those for an L-type firm. This would seem to provide perverse incentives regarding the provision of quality if we allowed firms to influence the quality of their products. However, if this *interim* stage is placed in a larger context, there are several effects that can reverse this inference.

First, although a firm producing a high-quality product may have a higher marginal cost, it may have lower fixed costs. For instance, if high quality is due to having a better-educated and more reliable work force, then wages may be higher but training and turnover costs may be lower for a high-quality firm. The nature of the workforce available to the firm may be a function of attributes of the firm's geographic location, including the extent of funding for public education and the workers' possibilities for alternative employment. The firm's location is most likely a longer-run decision than either public support for education or the strength of the local economy (indeed, changes in these attributes could result in quality shocks; see below). Thus, a firm that wanted to affect its quality (in either direction) might do so by relocating, but this is costly and it does not guarantee that local conditions will remain the same.

Second, one could posit that quality is a firm-wide attribute and that firms produce other products that benefit from high quality. This is the approach taken by Daughety and Reinganum (2005), who suggest that, for example, a technology could be used to produce both branded therapeutic drugs and generic multi-vitamins which are sold in bulk to private-label distributors. A "better" technology would improve safety when used to produce therapeutic drugs, and would yield higher output when used to produce generic multi-vitamins. In this case, while high quality is disadvantageous in the market for the product with the safety attribute, it can still be advantageous for the firm overall.

Third, the model we are exploring might concern a new product whose quality is initially uncertain but will eventually become common knowledge. Thus, one can view this model as pertaining to an "introductory period," and augment the model with a "long-run" period in which quality is common knowledge. This introductory period need not be very brief, if consumers buy relatively infrequently, arrive sequentially, and do not observe past prices; this is the approach used by Bagwell and Riordan (1991) to explain the phenomenon of high and declining prices over a product's life-cycle. While high quality is disadvantageous in the introductory period, it is advantageous in the long run.

Fourth, one could view this model as part of a stationary equilibrium, in which firms draw their technologies (or labor forces) anew each period. There is a common basic observable quality level (which is reflected in the demand curve through, for example, the maximum willingness-to-pay, α). In addition, there is a random (uncontrollable) shock each period, which affects both quality and costs. Thus, in each period each firm has a chance λ of being an H-type firm and a chance 1 - λ of being an L-type firm. By Proposition 4, in high-value markets with a sufficiently high proportion of H-type firms, both H-type firms and L-type firms make higher profits with this stochastic technology with private information about quality as compared to the profits that would arise if all firms were known to have low quality. Thus the firms enjoy a collective benefit from this stochastic technology (though a firm that draws an H-type technology in a given period suffers a temporary disadvantage). Even though there might be an individual temptation to invest in R&D (or relocate) to lower one's own frequency of being the H-type, if this kind of investment (or its consequence) is detectable by rivals, then it might be deterred by the anticipation that rival firms would follow suit and all firms would end up (worse off) in the equilibrium with known low quality.

As mentioned above, our analysis takes the number of firms as given. If the firm is a multi-product firm, or if entry is limited by rent-generating attributes such as "know-how" or patented technologies, then the number of firms operating in this particular industry is not determined by a free-entry condition, and may be taken as exogenous. Alternatively, suppose that firms can enter upon payment of an entry cost, denoted F, before learning the quality of their product. If all firms in the industry obtain positive operating profits regardless of their realized types and their perceived types (as is ensured by Assumptions 1 and 2), then the *ex post* distribution of firms will still be anticipated to involve a fraction λ of H-type firms and a fraction $1 - \lambda$ of L-type firms. *Ex ante* expected profits are given by $\lambda\Pi_H(\alpha, \beta, \gamma, \delta, \lambda, k, n) + (1 - \lambda)\Pi_L(\alpha, \beta, \gamma, \delta, \lambda, n)$, where all of the parameters on which the equilibrium profits depend are indicated. One question is whether the number of firms can be determined in equilibrium, and still be consistent with Assumptions 1 and 2. To see that it can, we suggest a three-step procedure (this admittedly awkward argument is necessitated by the complex dependence of the equilibrium profits on the various parameters). The steps are: 1) choose a target industry size $\hat{n}$. Recall that for high-demand markets (that is, for sufficiently high values of α), both $\Pi_H(\alpha, \beta, \gamma, \delta, \lambda, k, n)$ and $\Pi_L(\alpha, \beta, \gamma, \delta, \lambda, n)$ are decreasing in n; 2) choose a value of λ in (0, 1), values of $(\alpha, \beta, \gamma, \delta)$ that are consistent with Assumptions 1 and 2 (for n in a neighborhood of $\hat{n}$) and involve α sufficiently large to obtain *interim* profits decreasing in n for both firm types (for n in a neighborhood of $\hat{n}$)[19], and a value of $k < \delta$; and 3) let the fixed cost of entry be $F \equiv \lambda\Pi_H(\alpha, \beta, \gamma, \delta, \lambda, k, \hat{n}) + (1 - \lambda)\Pi_L(\alpha, \beta, \gamma, \delta, \lambda, \hat{n})$. Then $\hat{n}$ is an equilibrium with unrestricted entry. Each firm that enters obtains zero expected profits (net of the entry cost), which

[19] This can be done by simply raising α, if necessary, because α can be raised without risk of violating Assumptions 1 and 2.

is the same as it would earn by not entering; moreover, no additional firm would want to enter because *ex ante* expected profits are decreasing in n (by construction).

4. Applications to Tort Reform and Professional Licensing

In this section we use the model to address two issues. First, we show that the result concerning an increase in the loss δ may mean that recent proposals for tort reform may increase (not decrease) the number of consumers harmed and thus the number of lawsuits. Second, we argue that professional licensing regulations that serve to lower the loss due to low quality, δ, may provide an additional benefit to consumers by increasing competition, thereby reducing both prices.

Tort Reform

The foregoing model suggests that tort reforms currently being suggested, such as "capping" (limiting) damages awards or setting higher evidentiary standards for plaintiffs to win cases, may perversely help low-quality firms, possibly at the expense of high-quality producers. To see this, consider the following direct extension of our model. Assume that the consumer loss, δ, is really comprised of two parts, compensated damages (δ_C) and uncompensated damages (δ_U). An example of the former is medical costs for "economic losses" (such as hospitalization costs) and of the latter might be "non-economic losses" (such as emotional distress arising from the harm having occurred), so a cap on recovery of non-economic damages would mean that $\delta_U > 0$. Such caps have been in operation at the state level, and are currently under consideration in the U.S. Congress.[20] Damages that are compensated must be paid for by the firm, so this means that the low-quality firm's unit cost

[20] As of this writing, the most recent bill at the federal level is HR 2657, "Comprehensive Medical Malpractice Reform Act of 2005" (introduced May 26, 2005). The first section of the bill, Section 101, provides for limits on recovery of non-economic damages in medical malpractice suits.

of production is now positive: $c_L = \delta_C$. To maintain the usual incentives for revelation, assume that $\delta_C < c_H = k < \delta \equiv \delta_C + \delta_U$. Finally, for simplicity we ignore other losses that might arise due to the filing of lawsuits, settlement bargaining, or trial.[21]

As in the previous section, one can show that (for the modified model, holding δ constant) for high-value markets, the L-type's price, quantity and profit are increasing in δ_U for sufficiently high λ. Thus, the effect of a cap on non-economic damages due to torts arising from products provided in high-value markets, where there is a sufficiently high proportion of high-quality producers, is to enhance the prospects of the low-quality firms! It is probably not unreasonable to think of medical services as a high-value market, so such caps may lead to an increased number of malpractice lawsuits, as the price distortion associated with signaling shifts more consumers toward lower-quality medical providers (and more harms). While this discussion formally focuses on caps on damages awards, a simple variation would provide the same result for increases in evidentiary standards (that is, raising the evidentiary requirements that a plaintiff must meet in order to prove a defendant's liability for a tort), as this would increase the expected uncompensated loss.

Professional Licensing Requirements

In many industries, especially service industries, producers must meet certain licensing requirements. For instance, real estate brokers, health-care providers, lawyers, accountants, professional engineers, architects, public-school teachers, barbers, and restaurants are typically licensed by the state. One effect of licensing is (arguably) to provide a higher floor on consumer

[21] We assume a regime of strict liability, so a firm compensates a victim for "compensable" harms without reference to due care standards. While medical malpractice is technically under a regime of negligence, this difference is not particularly important in what follows, as a low-quality producer is negligent by assumption, while a high-quality producer is never negligent.

satisfaction with the product (in our model, a smaller value of δ) by providing specific training requirements for future practitioners, along with certification examinations.[22] Moreover, meeting the licensing requirement involves a fixed cost, leaving variable costs largely unaffected. In the full information model, the impact of a licensing requirement that lowers δ, leaving marginal costs unchanged, is to raise the low-quality firm's price, output, and profits and to lower the high-quality firm's price, output, and profits. However, in the model with incomplete information, such a licensing requirement can have a double benefit to consumers in high-value markets with a sufficiently high proportion of H-type firms. This is because, in addition to the direct benefit of reducing the loss due to low quality, there is also an indirect benefit since both H-type and L-type prices fall, and H-type output increases relative to L-type output.[23]

The conflicting effects that lead to these results are as follows. A reduction in δ would, in principle, allow an L-type firm to charge a higher price for its product. This would reduce the L-type firm's incentive to mimic, which would allow the H-type firm to lower its price, to which an L-type firm would respond by lowering its price as well. If the fraction of H-type firms is sufficiently high, then the second incentive outweighs the first, and the L-type firm's price falls in

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[22] An alternative view of professional licensing is that it converts some, but not all, L-type firms into H-type firms; thus, it essentially raises the proportion λ of high-quality firms in the market. From Proposition 2, this results in higher prices and profits (in high-value markets) for both types of firm. We thank a referee for suggesting this interpretation.

[23] Ronnen (1991) provides a full information model with endogenous quality in which minimum quality standards lower both firms' prices, which cannot occur in the full information version of our model. Ronnen's model is one of pure vertical differentiation with heterogeneous consumer willingness-to-pay for quality (akin to spatial models), in contrast to our symmetric differentiated products model. Crampes and Hollander (1995) argue that Ronnen's result is sensitive to the way variable costs depend on quality; in their version with a convex relationship, they find that a minimum quality standard benefits the low-quality firm and hurts the high-quality firm, which is consistent with the implications of our full-information model.

equilibrium, as does its quantity, Q_L, and its profits, Π_L.

5. Conclusions

In this paper, we combine two relatively well-known models from industrial organization and find new and unexpected results. We employ a signaling model in which the quality of a firm's product is its private information; the firm's choice of price may signal its quality to consumers. We integrate this signaling aspect into a model of imperfect competition in a product market with horizontally-differentiated substitute goods. Thus, in choosing its price, a firm must play a best response to its rivals' price strategies and, at the same time, deter mimicry by its own alter ego.

We generate a variety of results that do not occur in the separate portions of the model. For instance, we find that a low-quality firm produces more output than a high-quality firm under incomplete information; this does not occur under full information (though it does occur in a monopoly model with incomplete information).

We find that incomplete information always raises prices for both types of firm. Moreover, there are circumstances under which incomplete information also raises equilibrium profits in the case of imperfect competition, while incomplete information only lowers (or leaves unchanged) equilibrium profits in the case of a monopoly. Under imperfect competition, the need to signal high quality acts as a credible commitment to higher prices, which allows rival firms to price higher as well, and can raise equilibrium profits. Under monopoly, the need to signal high quality causes the monopolist to price higher than the full-information monopoly price, thereby reducing profits.

In our model, the parameter representing the proportion of high-quality firms is an important determinant of equilibrium prices, quantities and profits, all of which are increasing in this parameter. A higher proportion of high-quality rivals implies a higher expected rival price and, since prices are strategic complements, a higher own price. At the same time, a higher expected rival price shifts demand toward the firm so that it also produces a higher quantity of output. Combining these two effects clearly implies higher profits. This parameter does not matter in a monopoly version of the model, nor in the full-information version of the model.

We find that (for some parameter values) an increase in the loss associated with low quality can have the perverse effect of increasing the price, quantity and profits of a low-quality firm. A higher loss increases the incentive for a low-quality firm to mimic a high-quality firm, causing the high-quality firm to raise its price even further to signal its quality. This in turn shifts demand toward the low-quality firm and allows the low-quality firm to increase its price (due to strategic complementarity). This cannot occur in a monopoly version of the model with incomplete information, nor can it occur in the full-information version of the model.

It is reassuring that some cherished results do carry over in this model (at least for high-value markets). In particular, the incomplete-information imperfect-competition model behaves as expected with respect to the variables that measure market size, product substitutability and the number of firms. A higher-value market corresponds to higher prices, outputs and profits for both quality levels, while an increase in the substitutability of the products or an increase in the number of firms causes prices, output (per firm) and profits to fall.

We employ the model to address two applications. In the case of tort reform, our results on the effect of the increase in the consumer loss on low-quality firm prices, quantities and profits suggest that such reforms as caps on damages or increased evidentiary standards may backfire, possibly leading to more harmed consumers and more lawsuits. In the case of professional licensing, we find that such requirements may actually enhance competitiveness and lead to reduced prices, in addition to their straightforward effect on consumer satisfaction.

Appendix

Derivation of the Symmetric Separating Equilibrium Price Function

Derivation of Best Response Functions

Recall the function describing firm i's profit as a function of its price, p_i, its actual type, θ_i, and the type the consumer believes it to be, $\tilde{\theta}$:

$$\Pi_i(p_i, \theta_i, \tilde{\theta}_i \mid E(p^*)) \equiv (p_i - k\theta_i)(a - b(1 - \tilde{\theta}_i)\delta + g(n-1)(1 - \lambda)\delta - bp_i + g(n - 1)E(p^*)).$$

Note that for any given price, it is always more profitable to be perceived as type H, regardless of true type; and for any given price, it is better to be type L, regardless of perceived type. If there were no signaling considerations, then $\Pi_{st} \equiv (p - c_s)b(d_t - p)$ would be maximized by $\rho_{st} = (c_s + d_t)/2$, and the resulting profits would be $\Pi_{st} = b(d_t - c_s)^2/4$. These prices (actually, "best responses" to $E(p^*)$) are ordered as follows: $\rho_{HH} > \rho_{LH} > \rho_{HL} > \rho_{LL}$. The only non-obvious case is $\rho_{LH} > \rho_{HL}$; this holds if and only if $d_H - d_L > c_H - c_L$, which is ensured by the assumption that $\delta > k$.

Our method of deriving the separating equilibrium prices is to first derive a best response function for firm i that reflects the need to signal its type. This will consist of a pair of prices $(\rho_L(E(p^*)), \rho_H(E(p^*)))$. We will then impose the equilibrium condition that $E(p^*) = \lambda\rho_H(E(p^*)) + (1 - \lambda)\rho_L(E(p^*))$ and solve for a fixed point. Finally, the resulting solution is substituted into $(\rho_L(E(p^*)), \rho_H(E(p^*)))$ to obtain the equilibrium *interim* prices.

No firm is willing to distort its price away from its best response (were its type known) in order to be perceived as type L (since this is the worst type to be perceived to be). Thus, if a firm of type L is perceived as such, its best response is ρ_{LL}, which yields profits of $b(d_L - c_L)^2/4$. If a firm of type H is perceived as being of type L, its best response is ρ_{HL}, which yields profits of $b(d_L - c_H)^2/4$.

However, either firm would be willing to distort its price away from its best response (were its type known) in order to be perceived as type H. Thus, a candidate for a revealing equilibrium must involve a best response for type H that satisfies two conditions. First, it must deter mimicry by the type L firm (who thus reverts to ρ_{LL}); and second, it must be worthwhile for the type H firm to use this price rather than to allow itself to be perceived as a type L firm (and thus revert to ρ_{HL}). Formally, a separating best response for the type H firm is a member of the following set:

$$\{p \mid (p - c_L)b(d_H - p) \leq b(d_L - c_L)^2/4 \text{ and } (p - c_H)b(d_H - p) \geq b(d_L - c_H)^2/4\}.$$

The first inequality says that the type L firm prefers to price at ρ_{LL} (and be perceived as type L) than to price at p (and be perceived as type H). The second inequality says that the type H firm prefers to price at p (and be perceived as type H) than to price at ρ_{HL} (and be perceived as type L). Solving these two inequalities implies that the H-type firm's best response belongs to the interval:

$$[.5\{d_H + c_L + ((d_H - c_L)^2 - (d_L - c_L)^2)^{1/2}\}, .5\{d_H + c_H + ((d_H - c_H)^2 - (d_L - c_H)^2)^{1/2}\}].$$

Since $\delta > k$, this entire interval involves prices in excess of $\rho_{HH} = (d_H + c_H)/2$; thus, the type H firm distorts its price upwards from the best response function it would follow if it were known to be of type H.[24]

Refinement of Best Response Functions

We have identified an interval of candidates for the type H firm's best response. We now apply the Intuitive Criterion (see Mas-Colell, Whinston and Green, 1995, pp. 470-471, for a discussion of equilibrium domination and the Intuitive Criterion of Cho and Kreps, 1987). It is appropriate to apply this refinement at this stage in the game because, conditional on any common conjecture (common to firm i and the consumer) about the strategy being employed by all other firms (including the equilibrium strategy), what remains is simply a signaling game between firm i's two types and the consumer. The Intuitive Criterion says that the consumer should infer type H from firm i's price p so long as type H would be willing to charge p, yet mimicry by type L would be deterred, even under this most-favorable inference. Thus, the firm of type H distorts its best response <u>to the minimum extent necessary to deter mimicry</u> by its alter ego (type L). Formally, this means that firm i can convince the consumer that it is of type H by playing the separating best response $\rho_H(E(p^*)) = .5\{d_H + c_L + ((d_H - c_L)^2 - (d_L - c_L)^2)^{1/2}\}$. As argued above, type L's best response is $\rho_L(E(p^*)) = (d_L + c_L)/2$. Note that $E(p^*)$ enters these functions through the terms d_H and d_L. Recalling that $c_L = 0$, we can simplify to obtain: $\rho_H(E(p^*)) = .5\{d_H + ((d_H - d_L)(d_H + d_L))^{1/2}\}$ and $\rho_L(E(p^*)) = d_L/2$.

Derivation of Refined Equilibrium Prices

Each type of firm i plays a best response to the common rival separating strategy (which is summarized, for firm i's purposes, by its expected value). Then in a symmetric equilibrium, the equilibrium expected price, $E(p^*)$, is a solution to the equation:

$$X = \lambda\rho_H(X) + (1 - \lambda)\rho_L(X) = \lambda d_H/2 + (1 - \lambda)d_L/2 + (\lambda/2)((d_H - d_L)(d_H + d_L))^{1/2}. \tag{A1}$$

Upon substitution, we obtain:

$$X = [U + g(n-1)X]/2b + \lambda\delta/2 + \lambda\{\delta[(U + g(n-1)X)/2b] + \delta^2/4\}^{1/2}, \tag{A2}$$

where $U \equiv a - b\delta + g(n - 1)(1 - \lambda)\delta > 0$ by Assumption 1. Let $Y \equiv [U + g(n-1)X]/2b$; then $X = (2bY - U)/g(n-1)$, and (A2) becomes:

$$Y[2b - g(n - 1)]/g(n - 1) = U/g(n - 1) + \lambda\delta/2 + \lambda\{\delta(Y + \delta/4)\}^{1/2}. \tag{A3}$$

For later purposes, note that $\rho_L(E(p^*)) = d_L/2 = Y$.

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[24] The type H firm could also deter mimicry by the type L firm by using a downward-distorted price but type H would prefer to give up and be taken as a type L firm rather than use such a low price to distinguish itself.

Finally, let $W \equiv \{\delta(Y + \delta/4)\}^{½}$; then $Y = W^2/\delta - \delta/4$. For later purposes, note that $\rho_H(E(p^*)) = d_H/2 + W$ and $\rho_H(E(p^*)) - \rho_L(E(p^*)) = d_H/2 + W - d_L/2 = \delta/2 + W$. Substitution of Yin terms of W into (A3) and simplification yields the following quadratic in W:

$$\eta_1 W^2 + \eta_2 W + \eta_3 = 0, \tag{A4}$$

where $\eta_1 = [2b -(n - 1)g]/\delta$, $\eta_2 = - \lambda g(n-1)$ and $\eta_3 = - U - \lambda g(n-1)\delta/2 - (\delta/4)[2b -(n - 1)g]$. The coefficient η_1 is positive, while η_2 and η_3 are negative. Thus, equation (A4) has one positive and one negative root; given than W is defined as a square root, the solution we seek is the positive root of equation (A4). Solving and substituting back the original parameters, W* can be written as:

$$W^* = \left[\lambda\delta\gamma(n - 1) + \{(\lambda\delta\gamma(n - 1))^2 + 4\delta(2\beta + (n - 3)\gamma)\eta_4\}^{.5}\right]/2(2\beta + (n - 3)\gamma), \tag{A5}$$

where $\eta_4 \equiv \alpha(\beta - \gamma) - \delta(\beta + (n - 2)\gamma) + \gamma\delta(n - 1)(1 - \lambda) + \lambda\delta\gamma(n - 1)/2 + (\delta/4)(2\beta + (n - 3)\gamma)$. Using the fact that $W = \{\delta(Y + \delta/4)\}^{½}$ and (A3), we obtain:

$$Y^* = [U + \lambda\delta g(n - 1)/2 + \lambda g(n - 1)W^*]/[2b - (n - 1)g]. \tag{A6}$$

Using the fact that $X = (2bY - U)/g(n-1)$, we obtain $X^* = [U + b\lambda\delta + 2b\lambda W^*]/[2b - (n -1)g]$.

We noted above that $\rho_L(E(p^*)) = Y$ and $\rho_H(E(p^*)) - \rho_L(E(p^*)) = \delta/2 + W$. Thus the equilibrium *interim* prices P_L and P_H are simply $P_L = \rho_L(X^*) = Y^*$ and $P_H = \rho_H(X^*) = Y^* + \delta/2 + W^*$. The equilibrium *interim* quantities are $Q_L = bY^*$ and $Q_H = b(Y^* + \delta/2 - W^*)$. Finally, the equilibrium *interim* profits are $\Pi_L = b(Y^*)^2$ and $\Pi_H = (Y^* + \delta/2 + W^* - k)b(Y^* + \delta/2 - W^*)$.

Claim regarding positive realized equilibrium demand. In the text before the statement of Proposition 1, it was claimed that (i) $P_H > P_L + \delta$, and thus, when firms use the prices (P_L, P_H), a firm's realized demand is lowest when it is of type H and all of its rivals are of type L. Moreover, it was claimed that (ii) there exists $\alpha_{A2} < \infty$ such that for all $\alpha > \alpha_{A2}$, this lowest realized demand is positive.

Proof. (i) $P_H - P_L = \delta/2 + W^*$, and it is straightforward to show that $W^* > \delta/2$. Realized demand for firm i is given by $q_i = a - b(1 - \theta_i)\delta + g\sum_{j\neq i}(1 - \theta_j)\delta - bp_i + g\sum_{j\neq i}p_j$. This is smallest when the coefficient of b (that is, $(1 - \theta_i)\delta + p_i$) is largest and the coefficient of g (that is, $\sum_{j\neq i}(1 - \theta_j)\delta + \sum_{j\neq i}p_j$) is smallest. Since $P_H > P_L + \delta$, this occurs when firm i is of type H and all other firms are of type L.

(ii) Thus, the lowest realized demand is given by $q = a + g(n - 1)\delta - bP_H + g(n - 1)P_L$.. Substitution of $P_L = Y^*$ and $P_H = Y^* + \delta/2 + W^*$ into this formula, followed by substitution of Y* in terms of W* from equation (A6), and collecting terms yields: $q = ab/(2b - (n - 1)g) + K_1 - K_2W^*(a)$, where K_1 is a positive constant independent of a and K_2 is a positive constant independent of a. Since equation (A5) provides W* as a function of α and $\alpha = a(\beta + (n - 1)\gamma$, we indicate the dependence of W* on a by W*(a). The first term in the expression for q increases linearly in a, while a appears only under a square root in W*(a). Thus, there is a sufficiently high value of a, call it a_{A2}, such that the sum of the first two terms will exceed the last term for all $a > a_{A2}$. Since $a = \alpha/(\beta + (n - 1)\gamma)$, the

corresponding required value of α is $\alpha_{A2} = a_{A2}(\beta + (n - 1)\gamma)$.
Q.E.D.

Results and Proofs of Selected Propositions

Proof of Proposition 1**.** We have identified a unique (refined) candidate for a symmetric separating equilibrium. To verify that the strategies and beliefs do provide a separating equilibrium, suppose that all firms but firm i play the strategy (P_L, P_H) given above, with expected value X*, and that the consumer maintains the beliefs: $B^*(p) = 0$ when $p < P_H$, and $B^*(p) = 1$ when $p \geq P_H$. Then, by construction, the type L firm i would be unwilling to charge a price at or above P_H (which is equal to $\rho_H(X^*)$) in order to be taken for type H. Rather, it will prefer to be taken for type L and to charge the price P_L (which is equal to $\rho_L(X^*)$). On the other hand, the type H firm i would be willing to charge a price at or somewhat above P_H (which is equal to $\rho_H(X^*)$) in order to be taken for type H, but among these it prefers the lowest price; that is, P_H. The consumer's beliefs are correct in equilibrium, and $X^* = \lambda P_H + (1 - \lambda)P_L$. Finally, note that $P_H - P_L = \delta/2 + W^* > 0$, and $Q_L - Q_H = b(W^* - \delta/2)$, which is easily shown to be positive. QED.

Elimination of (Pure) Pooling Equilibria

Consider possible pooling equilibria. Firms will pool at a price P^P if the following incentive compatibility constraints hold:

$$\text{i) } \Pi_i(P^P, 0, \lambda | P^P) \geq \max_p \Pi_i(p, 0, 0| P^P) \text{ \underline{and} ii) } \Pi_i(P^P, 1, \lambda| P^P) \geq \max_p \Pi_i(p, 1, 0| P^P), \quad \text{(A7)}$$

with beliefs $B(P) = 0$ for $P \neq P^P$. The above incentive constraints express the profits of firm i at the pooling price, when the consumer believes it is an H-type firm with probability λ, supported by beliefs that treat any out-of-equilibrium price as indicating that the firm is L-type for sure. The pair of inequalities in (A7) generate the following associated inequalities in terms of the parameters of the problem:

$$\text{i) } P^P b(d^P - P^P) \geq b(d_L/2)^2 \text{ \underline{and} ii) } (P^P - k)b(d^P - P^P) \geq b((d_L - k)/2)^2,$$

where d^P is the "pooled" version of d_H and d_L, and can be shown to be $d^P = d_L + \lambda\delta$.

We need not actually construct a pooling equilibrium, as we need only show that if one exists, then there is a price to which the H-type firm could profitably defect (that would be unprofitable for an L-type firm) if the consumer were to update her beliefs and infer that the signal came from an H-type firm. Thus, P^P fails the Intuitive Criterion if there exists P* such that:

$$\text{i) } P^*b(d_H - P^*) \leq P^P b(d^P - P^P) \text{ \underline{and} ii) } (P^* - k)b(d_H - P^*) \geq (P^P - k)b(d^P - P^P). \quad \text{(A8)}$$

In (A8) the left-hand-side of each inequality are the profits that would be obtained by (respectively) the L-type and H-type firms by defecting (and being taken to be an H-type firm after the consumer has updated her beliefs), while the profits from the pooling equilibrium appear on the right-hand-side.

Let us denote the roots to (A8i) as P_L^- and $P_L^+ > P_L^-$, and the roots to (A8ii) as P_H^- and $P_H^+ > P_H^-$. Then (A8) is equivalent to asking if there exists P* such that $P^* \in [P_H^-, P_H^+]$ and $P^* \notin [P_L^-, P_L^+]$; if so, then P^P fails the Intuitive Criterion. With a little algebra one can show that $P_H^+ > P_L^+$, so such a P* exists for any P^P. Thus no pooling equilibrium survives refinement.

Full-Information Price and Profit Formula

Let firm i's type be θ_i and firm i's rivals' types be summarized by the vector of types θ_{-i}. Then the full-information equilibrium price for firm i is:

$$P^F(\theta_i, \theta_{-i}) = \Big[a - (b - (n-1)g)\delta + b(k+\delta)\theta_i - g\delta\textstyle\sum_{j\neq i}\theta_j$$
$$+ g\{n[a - (b - (n-1)g)\delta] + [(b - (n-1)g)\delta + bk]\textstyle\sum_j\theta_j\}/(2b - (n-1)g)\Big]/(2b+g).$$

The full-information price for an industry comprised only of L-type firms is $P_L^F \equiv P^F(0, 0_{-i})$ and the full-information price for an industry comprised only of H-type firms is $P_H^F \equiv P^F(1, 1_{-i})$, where 0_{-i} denotes an n - 1 vector of 0s and 1_{-i} denotes an n - 1 vector of 1s. Finally, substitution and simplification yields $\Pi^F(0, \theta_{-i}) = b(P^F(0, \theta_{-i}))^2$ and $\Pi^F(1, \theta_{-i}) = b(P^F(1, \theta_{-i}) - k)^2$.

Proof of Proposition 3. In each argument below, the first inequality follows from Proposition 2, and the remaining results follow from evaluating and comparing the pricing equations. First consider the L-type firm: for all $\lambda \in (0, 1)$, $P_L > \lim_{\lambda\to 0} P_L = (\alpha - \delta)(\beta - \gamma)/[2\beta + (n-3)\gamma] = P^F(0, 0_{-i}) \geq P^F(0, \theta_{-i})$ for all θ_{-i}. Now consider the H-type firm: for all $\lambda \in (0, 1)$, $P_H > \lim_{\lambda\to 0} P_H > (\alpha - \delta)(\beta - \gamma)/[2\beta + (n-3)\gamma] + \delta > P^F(1, 0_{-i}) \geq P^F(1, \theta_{-i})$ for all θ_{-i}. QED.

Proof of Proposition 4. (i) $\Pi_L = b(P_L)^2$ and $\Pi^F(0, \theta_{-i}) = b(P^F(0, \theta_{-i}))^2$, so the claim follows immediately from Proposition 3. (ii) Note that $\lim_{\lambda\to 1} P_H = [\alpha(\beta - \gamma) + 2(\beta + (n-2)\gamma)W^*]/[2\beta + (n-3)\gamma]$, while $P^F(1, 1_{-i}) = [\alpha(\beta - \gamma) + k(\beta + (n-2)\gamma)]/[2\beta + (n-3)\gamma]$. Thus, $\lim_{\lambda\to 1} P_H > P^F(1, 1_{-i})$ if and only if $2W^* > k$, which is easily verified. The full-information price for a collusive industry comprised only of H-type firms is $P_C^F = (\alpha + k)/2$. Then $\lim_{\lambda\to 1} P_H < (\alpha + k)/2$ if and only if $\alpha(n-1)\gamma + k(2\beta + (n-3)\gamma) > 4W^*(\beta + (n-2)\gamma)$. This is certainly true if α is large enough, since the left-hand-side is increasing linearly in α, while the right-hand-side increases with the square root of α. When $\lim_{\lambda\to 1} P_H$ is less than the collusive price, then we can conclude that $\lim_{\lambda\to 1} \Pi_H > \Pi^F(1, 1_{-i})$. This is because, when all firms charge the same price (whether it is the noncooperative full-information price, the collusive full-information price, or the *interim* price), each firm's profits are given by (p - k)(a - (b - (n - 1)g)p), which is a quadratic function that reaches its maximum at the collusive price. Thus, for high-value markets, $\lim_{\lambda\to 1} \Pi_H > \Pi^F(1, 1_{-i}) = \lim_{\lambda\to 1} E_{-i}\{\Pi^F(1, \theta_{-i})\}$, where the equality follows since in the limit only the term $\Pi^F(1, 1_{-i})$ has positive weight. Consequently, $\Pi_H > E_{-i}\{\Pi^F(1, \theta_{-i})\}$ when λ is sufficiently close to 1. QED.

Partial Proof of Proposition 6. Here we provide a partial proof of Proposition 6 as an illustration of how the other comparative statics results can be proved. Recall that $P_L = Y^*$ from equation (A6) and $P_H = Y^* + \delta/2 + W^*$ from equation (A5).

(i) First consider the effect of δ on W*.

$$\text{sgn}\{\partial W^*/\partial\delta\} = \text{sgn}\Big\{\lambda\gamma(n-1) + .5[2\delta(\lambda\gamma(n-1))^2 + 4(2\beta + (n-3)\gamma)(\partial(\delta\eta_4)/\partial\delta)]\big/[(\lambda\delta\gamma(n-1))^2 + 4\delta(2\beta + (n-3)\gamma)\eta_4]^{.5}\Big\}. \quad \text{(A9)}$$

Note that $\partial W^*/\partial\delta > 0$ for all $\lambda \in (0, 1)$ if $\partial(\delta\eta_4)/\partial\delta > 0$ for all $\lambda \in (0, 1)$. But

$$\partial(\delta\eta_4)/\partial\delta = \alpha(\beta - \gamma) - 2\delta(\beta + (n-2)\gamma) + 2\gamma\delta(n-1)(1-\lambda) + 2\lambda\delta\gamma(n-1)/2 + 2(\delta/4)(2\beta + (n-3)\gamma).$$

The first line is positive by Assumption 1, and the remaining terms are positive for all λ. Thus, we conclude that $\partial W^*/\partial\delta > 0$ for all $\lambda \in (0, 1)$. This implies that $P_H - P_L = \delta/2 + W^*$ is strictly increasing in δ for all $\lambda \in (0, 1)$, as claimed in part (i).
(ii) Recall that $P_L = Y^*$, $Q_L = bY^*$ and $\Pi_L = b(Y^*)^2$. Thus, to determine the effect of a change in δ on each of these expressions, it is sufficient to determine the effect of a change in δ on Y^*.

$$\text{sgn}\{\partial Y^*/\partial\delta\} = \text{sgn}\Big\{-(\beta - \gamma) - \gamma\lambda(n-1)/2 + \gamma\lambda(n-1)(\partial W^*/\partial\delta)\Big\}.$$

The sum of the first two terms is negative for all $\lambda \in (0, 1)$. Since $\partial W^*/\partial\delta > 0$ for all $\lambda \in (0, 1)$, the last term goes to zero as $\lambda \to 0$, but it is strictly positive for $\lambda \in (0, 1)$. Thus, $\lim_{\lambda\to 0} \partial Y^*/\partial\delta < 0$, which implies the claims made in part (ii).
(iii) The expression $\partial W^*/\partial\delta$ involves a ratio whose numerator is positive and increases linearly in α and a denominator that is positive and increases with the square root of α, while the negative terms are independent of α. Thus, for any fixed $\lambda \in (0, 1)$, there exists a value $\alpha^*(\lambda) < \infty$ that is sufficiently large to ensure that the positive term $\gamma\lambda(n-1)(\partial W^*/\partial\delta)$ balances the negative constant terms and, thus, that $\partial Y^*/\partial\delta = 0$. Let $\alpha^*(1) \equiv \lim_{\lambda\to 1} \alpha^*(\lambda)$; by construction, at $\alpha^*(1)$, $\lim_{\lambda\to 1} \partial Y^*/\partial\delta = 0$. Now choose any $\alpha_{6iii} > \alpha^*(1)$; at α_{6iii}, $\lim_{\lambda\to 1} \partial Y^*/\partial\delta > 0$. Consequently, there is a neighborhood of $\lambda = 1$, denoted $(\lambda_{6iii}, 1)$, such that $\partial Y^*/\partial\delta > 0$ for all $\lambda \in (\lambda_{6iii}, 1)$, as claimed in part (iii). QED

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