Technical Appendix for
"Cumulative Harm and Resilient Liability Rules for Product Markets"

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Technical Appendix, part A: Monopoly Results for the Quadratic Harm Model

In what follows we assume that h(•) and c(•) are twice continuously differentiable functions. This implies that the welfare function and all profit functions are twice continuously differentiable. In the text we assumed that $h(x)q^2$ is convex in (x, q); this implies that $h''(x)h(x) - 2(h'(x))^2 \geq 0$ for all x.

Welfare Maximization. The social welfare maximizing planner chooses q and x to maximize:

$$W(x, q) = \alpha q - (\beta/2)q^2 - h(x)q^2 - c(x)q.$$

While W(x, q) is not globally-concave, it is still very well-behaved. We first prove that any welfare-maximizing solution is interior; that is, it is socially optimal to exert some care and produce some output. To see this, first notice that W(x, 0) = 0 for all x. Moreover (see equation A.2 below), $W_q(x, q) \leq 0$ for all $q \geq 0$ if $x \geq \bar{x}$ such that $\alpha = c(\bar{x})$. Thus, $W(x, q) \leq 0$ for all (x, q) such that $x \geq \bar{x}$ and $q \geq 0$. However, since $W_q(x, 0) > 0$ for $x \in [0, \bar{x})$, there exist values of (x, q) such that $x \in [0, \bar{x})$ and q > 0 for which W(x, q) > 0. Thus, any welfare-maximizing solution must be one of these (x, q) combinations. To see that x = 0 cannot be part of a welfare-maximizing solution, note that (see equation A.1 below) $W_x(0, q) > 0$ for all q > 0 since $h'(0) < 0$ and $c'(0) = 0$. Thus, any welfare-maximizing solution must involve (x, q) such that $x \in (0, \bar{x})$ and q > 0.

Let $(\hat{x}^W, \hat{q}^W)$ denote a welfare-maximizing solution. Then $(\hat{x}^W, \hat{q}^W)$ must satisfy the first-order conditions:

$$W_x = -h'(x)q^2 - c'(x)q = 0; \tag{A.1}$$
$$W_q = \alpha - \beta q - 2h(x)q - c(x) = 0. \tag{A.2}$$

We now argue that <u>any</u> interior solution to the first-order conditions (A.1)-(A.2) must be a strict local maximum. This follows from the fact that the matrix of second derivatives is negative definite at any interior solution to (A.1)-(A.2). To see this, we verify that $W_{xx} < 0$, $W_{qq} < 0$, and $W_{xx}W_{qq} - (W_{xq})^2 > 0$ at any interior solution to (A.1)-(A.2). The relevant second derivatives are:

$$W_{xx} = -h''(x)q^2 - c''(x)q < 0;$$
$$W_{xq} = -2h'(x)q - c'(x);$$
$$W_{qq} = -\beta - 2h(x) < 0.$$

Notice that (for q > 0) equation (A.1) implies that $W_{xq} = -h'(x)q + (1/q)W_x = -h'(x)q > 0$ everywhere along the solution to (A.1). Evaluating the expression $W_{xx}W_{qq} - (W_{xq})^2$ yields:

$$(h''(x)q^2 + c''(x)q)(\beta + 2h(x)) - (h'(x)q)^2 > [2h''(x)h(x) - (h'(x))^2]q^2 > 0,$$

where the first inequality follows from the fact that $h''(x)$, $c''(x)$, β, and q are positive, and the second inequality follows from the convexity of expected harm in (x, q). Thus, any interior solution to (A.1)-(A.2) is a strict local maximum.

Equation (A.1) defines implicitly the socially-optimal care level for any given level of output, denoted $x^W(q)$; it is clear that $x^W(q) > 0$ for all $q > 0$, and $\lim_{q\to 0} x^W(q) = 0$. Differentiating equation (A.1) with respect to x and q and collecting terms yields $dx^W(q)/dq = W_{xq}/(-W_{xx})$. The denominator is positive, while the numerator is also positive along $x^W(q)$ because equation (A.1) implies that $W_{xq} = -h'(x)q + (1/q)W_x = -h'(x)q > 0$, since with q being treated as a parameter, $W_x = 0$ for each value of q. Thus, $x^W(q)$ is strictly increasing in q.

Equation (A.2) defines the socially-optimal output level for any given level of output, denoted $q^W(x)$, with $q^W(x) = (\alpha - c(x))/(\beta + 2h(x)) > 0$ for $x < \bar{x}$ and $q^W(\bar{x}) = 0$. Differentiating equation (A.2) with respect to q and x and collecting terms implies that $dq^W(x)/dx = W_{qx}/(-W_{qq})$. While we don't know the sign of this in general, we know that $W_{qx} > 0$ at $(\hat{x}^W, \hat{q}^W)$ (by the previous argument using equation (A.1)). Thus, $q^W(x)$ is strictly increasing in a neighborhood of $(\hat{x}^W, \hat{q}^W)$, where equations (A.1) and (A.2) are satisfied simultaneously.

In (x, q) space, we are graphing the functions $q^W(x)$ and the <u>inverse</u> of $x^W(q)$, denoted $\tilde{q}^W(x)$. Both of these are continuous functions. To see that $\tilde{q}^W(x)$ crosses $q^W(x)$ "from below," first notice that $\tilde{q}^W(0) = 0 < q^W(0)$, but $\tilde{q}^W(\bar{x}) > 0 = q^W(\bar{x})$; since both functions are continuous, they must cross at least once. To see that any crossing is "from below," note that $d\tilde{q}^W(x)/dx = 1/dx^W(q)/dq = (-W_{xx})/W_{xq} > W_{xq}/(-W_{qq}) = dq^W(x)/dx$, where the inequality follows from the fact that $W_{qq}W_{xx} - (W_{xq})^2 > 0$ at $(\hat{x}^W, \hat{q}^W)$.

Finally, we can conclude that $(\hat{x}^W, \hat{q}^W)$ is the unique interior solution to (A.1)-(A.2). For if there were another interior solution, then it would also have to be a strict local maximum; that is, the function $\tilde{q}^W(x)$ would have to cross the function $q^W(x)$ again "from below" at, say, $\hat{x}'$. But then there would have to be yet another value of x (between $\hat{x}^W$ and $\hat{x}'$) at which the function $\tilde{q}^W(x)$ would have to cross the function $q^W(x)$ "from above." But such a location involves an (x, q)-pair that would also satisfy (A.1)-(A.2) and yet it could not be a strict local maximum, which is a contradiction. Thus, the welfare-maximizing solution $(\hat{x}^W, \hat{q}^W)$ is the unique interior solution to the first-order conditions (A.1)-(A.2).

Profit-Maximization under Strict Liability. Under strict liability, the firm's profit is given by:

$$\Pi^{SL}(x, q) = (\alpha - \beta q)q - h(x)q^2 - c(x)q.$$

While $\Pi^{SL}(x, q)$ is not globally-concave, it is still very well-behaved. It can be shown (using an argument analogous to that used in the case of welfare-maximization) that any profit-maximizing solution is interior; that is, it must involve (x, q) such that $x \in (0, \bar{x})$ and $q > 0$. Let $(\hat{x}^{SL}, \hat{q}^{SL})$ denote a profit-maximizing solution under strict liability. Then $(\hat{x}^{SL}, \hat{q}^{SL})$ must satisfy the first-order conditions:

$$\Pi^{SL}_x = -h'(x)q^2 - c'(x)q = 0; \tag{A.3}$$
$$\Pi^{SL}_q = \alpha - 2\beta q - 2h(x)q - c(x) = 0. \tag{A.4}$$

To see that <u>any</u> interior solution to the first-order conditions (A.3)-(A.4) must be a strict local maximum, we need only verify that the matrix of second derivatives is negative definite at any interior solution to (A.3)-(A.4). The relevant second derivatives are:

$\Pi^{SL}_{xx} = -h''(x)q^2 - c''(x)q < 0;$
$\Pi^{SL}_{xq} = -2h'(x)q - c'(x);$
$\Pi^{SL}_{qq} = -2\beta - 2h(x) < 0.$

Notice that (for $q > 0$) equation (A.3) implies that $\Pi^{SL}_{xq} = -h'(x)q + (1/q)\Pi^{SL}_{x} = -h'(x)q > 0$ everywhere along the solution to (A.3). Evaluating the expression $\Pi^{SL}_{xx}\Pi^{SL}_{qq} - (\Pi^{SL}_{xq})^2$ yields:

$$(h''(x)q^2 + c''(x)q)(2\beta + 2h(x)) - (h'(x)q)^2 > [2h''(x)h(x) - (h'(x))^2]q^2 > 0,$$

where the first inequality follows from the fact that $h''(x)$, $c''(x)$, β, and q are positive, and the second inequality follows from the convexity of expected harm in (x, q). Thus, any interior solution to (A.3)-(A.4) is a strict local maximum.

Equation (A.3) defines implicitly the profit-maximizing care level for any given level of output, denoted $x^{SL}(q)$; it is clear that $x^{SL}(q) > 0$ for all $q > 0$, and $\lim_{q\to 0} x^{SL}(q) = 0$. Comparing equations (A.1) and (A.3), it is clear that $x^{SL}(q) = x^{W}(q)$ for all q. Differentiating equation (A.3) with respect to x and q and collecting terms implies that, along $x^{SL}(q)$, $dx^{SL}(q)/dq = \Pi^{SL}_{xq}/(-\Pi^{SL}_{xx})$. The denominator is positive, while the numerator is also positive along $x^{SL}(q)$, since equation (A.3) implies that $\Pi^{SL}_{xq} = -h'(x)q + (1/q)\Pi^{SL}_{x} = -h'(x)q > 0$, since with q being treated as a parameter, $\Pi^{SL}_{x} = 0$ for each value of q. Thus, $x^{SL}(q)$ is strictly increasing in q.

Solving equation (A.4) for the profit-maximizing output level for any given level of care, denoted $q^{SL}(x)$, we find that $q^{SL}(x) = (\alpha - c(x))/2(\beta + h(x)) > 0$ for $x < \overline{x}$ and $q^{SL}(\overline{x}) = 0$; moreover, for $x < \overline{x}$, $q^{SL}(x) < q^{W}(x) = (\alpha - c(x))/(\beta + 2h(x))$. Differentiating equation (A.4) with respect to q and x and collecting terms implies that $dq^{SL}(x)/dx = \Pi^{SL}_{qx}/(-\Pi^{SL}_{qq})$. While we don't know the sign of this in general, we know that $\Pi^{SL}_{qx} > 0$ at $(\hat{x}^{SL}, \hat{q}^{SL})$ (by the previous argument using equation (A.3)). Thus, $q^{SL}(x)$ is strictly increasing in a neighborhood of $(\hat{x}^{SL}, \hat{q}^{SL})$, where equations (A.3) and (A.4) are satisfied simultaneously.

In (x, q) space, we are graphing the functions $q^{SL}(x)$ and the <u>inverse</u> of $x^{SL}(q)$, denoted $\tilde{q}^{SL}(x)$. These are both continuous functions. To see that $\tilde{q}^{SL}(x)$ crosses $q^{SL}(x)$ "from below," first notice that $\tilde{q}^{SL}(0) = 0 < q^{SL}(0)$, but $\tilde{q}^{SL}(\overline{x}) > 0 = q^{SL}(\overline{x})$; since both functions are continuous, they must cross at least once. To see that any crossing is "from below," note that $d\tilde{q}^{SL}(x)/dx = 1/dx^{SL}(q)/dq = (-\Pi^{SL}_{xx})/\Pi^{SL}_{xq} > \Pi^{SL}_{xq}/(-\Pi^{SL}_{qq}) = dq^{SL}(x)/dx$, where the inequality follows from the fact that $\Pi^{SL}_{xx}\Pi^{SL}_{qq} - (\Pi^{SL}_{xq})^2 > 0$ at $(\hat{x}^{SL}, \hat{q}^{SL})$.

We can conclude that $(\hat{x}^{SL}, \hat{q}^{SL})$ is the unique interior solution to (A.3)-(A.4) using an argument similar to that used to establish the uniqueness of $(\hat{x}^{W}, \hat{q}^{W})$. Finally, since $q^{SL}(x) > q^{W}(x)$ for all x, while $x^{SL}(q) = x^{W}(q)$ for all q, it is immediate that $\hat{x}^{SL} < \hat{x}^{W}$ and $\hat{q}^{SL} < \hat{q}^{W}$.

Profit-Maximization under No Liability. Finally, under no liability, the firm's profit is given by:

$$\Pi^{NL}(x, q) = (\alpha - \beta q - 2h(x)q)q - c(x)q.$$

While $\Pi^{NL}(x, q)$ is not globally-concave, it is still very well-behaved. It can be shown (using an argument analogous to that used in the case of welfare-maximization) that any profit-maximizing solution is interior; that is, it must involve (x, q) such that $x \in (0, \overline{x})$ and $q > 0$. Let $(\hat{x}^{NL}, \hat{q}^{NL})$ denote a profit-maximizing solution under no liability. Then $(\hat{x}^{NL}, \hat{q}^{NL})$ must satisfy the first-order conditions:

$$\Pi^{NL}_{x} = -2h'(x)q^2 - c'(x)q = 0; \qquad (A.5)$$

$$\Pi^{NL}_{q} = \alpha - 2\beta q - 4h(x)q - c(x) = 0. \quad (A.6)$$

To see that <u>any</u> interior solution to the first-order conditions (A.5)-(A.6) must be a strict local maximum, we need only verify that the matrix of second derivatives is negative definite at any interior solution to (A.5)-(A.6). The relevant second derivatives are:

$$\Pi^{NL}_{xx} = -2h''(x)q^2 - c''(x)q < 0;$$
$$\Pi^{NL}_{xq} = -4h'(x)q - c'(x);$$
$$\Pi^{NL}_{qq} = -2\beta - 4h(x) < 0.$$

Notice that (for $q > 0$) equation (A.5) implies that $\Pi^{NL}_{xq} = -2h'(x)q + (1/q)\Pi^{NL}_{x} = -2h'(x)q > 0$ everywhere along the solution to (A.5). Evaluating the expression $\Pi^{NL}_{qq}\Pi^{NL}_{xx} - (\Pi^{NL}_{xq})^2$ yields:

$$(2h''(x)q^2 + c''(x)q)(2\beta + 4h(x)) - (2h'(x)q)^2 > [8h''(x)h(x) - 4(h'(x))^2]q^2 > 0,$$

where the first inequality follows from the fact that $h''(x)$, $c''(x)$, β, and q are positive, and the second inequality follows from the convexity of expected harm in (x, q). Thus, any interior solution to (A.5)-(A.6) is a strict local maximum.

Equation (A.5) defines implicitly the profit-maximizing care level for any given level of output, denoted $x^{NL}(q)$; it is clear that $x^{NL}(q) > 0$ for all $q > 0$, and $\lim_{q\to 0} x^{NL}(q) = 0$. Differentiating equation (A.5) with respect to x and q and collecting terms implies that, along $x^{NL}(q)$, $dx^{NL}(q)/dq = \Pi^{NL}_{xq}/(-\Pi^{NL}_{xx})$. The denominator is positive, while the numerator is also positive along $x^{NL}(q)$ because equation (A5) implies that $\Pi^{NL}_{xq} = -2h'(x)q + (1/q)\Pi^{NL}_{x} = -2h'(x)q > 0$, since with q being treated as a parameter, $\Pi^{NL}_{x} = 0$ for each value of q. Thus, $x^{NL}(q)$ is strictly increasing in q. Comparing equations (A1) and (A5) implies that $x^{NL}(q) = x^{W}(2q)$ for all q.

Solving equation (A.6) for the profit-maximizing output level for any given level of care, denoted $q^{NL}(x)$, we find that $q^{NL}(x) = (\alpha - c(x))/2(\beta + 2h(x)) > 0$ for $x < \bar{x}$ and $q^{NL}(\bar{x}) = 0$; moreover, $q^{NL}(x) = (1/2)q^{W}(x) < q^{SL}(x)$. Differentiating equation (A.6) with respect to q and x and collecting terms implies that $dq^{NL}(x)/dx = \Pi^{NL}_{qx}/(-\Pi^{NL}_{qq})$. While we don't know the sign of this in general, we know that $\Pi^{NL}_{qx} > 0$ at $(\hat{x}^{SL}, \hat{q}^{SL})$ (by the previous argument using equation (A.5)). Thus, $q^{NL}(x)$ is strictly increasing in a neighborhood of $(\hat{x}^{NL}, \hat{q}^{NL})$, where equations (A.5) and (A.6) are satisfied simultaneously.

In (x, q) space, we are graphing the functions $q^{NL}(x)$ and the <u>inverse</u> of $x^{NL}(q)$, denoted $\tilde{q}^{NL}(x)$. Both of these are continuous functions. To see that $\tilde{q}^{NL}(x)$ crosses $q^{NL}(x)$ "from below," first notice that $\tilde{q}^{NL}(0) = 0 < q^{NL}(0)$, but $\tilde{q}^{NL}(\bar{x}) > 0 = q^{NL}(\bar{x})$; since both functions are continuous, they must cross at least once. To see that any crossing is "from below," note that $d\tilde{q}^{NL}(x)/dx = 1/dx^{NL}(q)/dq = (-\Pi^{NL}_{xx})/\Pi^{NL}_{xq} > \Pi^{NL}_{xq}/(-\Pi^{NL}_{qq}) = dq^{NL}(x)/dx$, where the inequality follows from the fact that $\Pi^{NL}_{qq}\Pi^{NL}_{xx} - (\Pi^{NL}_{xq})^2 > 0$ at $(\hat{x}^{NL}, \hat{q}^{NL})$.

We can conclude that $(\hat{x}^{NL}, \hat{q}^{NL})$ is the unique interior solution to (A.5)-(A.6) using an argument similar to that used to establish the uniqueness of $(\hat{x}^{W}, \hat{q}^{W})$. Finally, the facts that $q^{NL}(x) = (1/2)q^{W}(x)$ and $x^{NL}(q) = x^{W}(2q)$ for all q imply that $\hat{x}^{NL} = \hat{x}^{W}$ and $\hat{q}^{NL} = (1/2)\hat{q}^{W}$.

Profit Maximization under Parametrized Liablity. The parametrized profit function is given by:

$$\Pi(x, q; \gamma) = (\alpha - \beta q)q - \gamma h(x)q^2 - c(x)q,$$

where $\gamma = 1$ corresponds to strict liability and $\gamma = 2$ corresponds to no liability. While $\Pi(x, q; \gamma)$ is not globally-concave, it is still very well-behaved. It can be shown (using an argument analogous to that used in the case of welfare-maximization) that any profit-maximizing solution is interior; that is, it must involve (x, q) such that $x \in (0, \bar{x})$ and $q > 0$. Let $(\hat{x}(\gamma), \hat{q}(\gamma))$ denote a profit-maximizing solution. Then $(\hat{x}(\gamma), \hat{q}(\gamma))$ must satisfy the first-order conditions:

$$\Pi_x = -\gamma h'(x)q^2 - c'(x)q = 0; \tag{A.7}$$
$$\Pi_q = \alpha - 2\beta q - 2\gamma h(x)q - c(x) = 0. \tag{A.8}$$

To see that <u>any</u> interior solution to the first-order conditions (A.7)-(A.8) must be a strict local maximum, we need only verify that the matrix of second derivatives is negative definite at any interior solution to (A.7)-(A.8). The relevant second derivatives are:

$$\Pi_{xx} = -\gamma h''(x)q^2 - c''(x)q < 0;$$
$$\Pi_{xq} = -2\gamma h'(x)q - c'(x);$$
$$\Pi_{qq} = -2\beta - 2\gamma h(x) < 0.$$

Notice that (for $q > 0$) equation (A.7) implies that $\Pi_{xq} = -\gamma h'(x)q + (1/q)\Pi_x = -\gamma h'(x)q > 0$ everywhere along the solution to (A.7). Evaluating the expression $\Pi_{qq}\Pi_{xx} - (\Pi_{xq})^2$ yields:

$$(\gamma h''(x)q^2 + c''(x)q)(2\beta + 2\gamma h(x)) - (\gamma h'(x)q)^2 > \gamma^2 [2h''(x)h(x) - (h'(x))^2]q^2 > 0,$$

where the first inequality follows from the fact that $h''(x)$, $c''(x)$, β, and q are positive, and the second inequality follows from the convexity of expected harm in (x, q). Thus, any interior solution to (A.7)-(A.8) is a strict local maximum. We can conclude that there is a unique interior solution $(\hat{x}(\gamma), \hat{q}(\gamma))$ to (A.7)-(A.8) using arguments similar to those used to establish the existence and uniqueness of $(\hat{x}^W, \hat{q}^W)$.

Differentiating the system (A.7)-(A.8) in terms of x, q, and γ, and solving for $d\hat{x}(\gamma)/d\gamma$ and $d\hat{q}(\gamma)/d\gamma$ yields:

$$d\hat{x}(\gamma)/d\gamma = \{\Pi_{qq}h'(x)q^2 - \Pi_{xq}2h(x)q\}/\{\Pi_{xx}\Pi_{qq} - (\Pi_{xq})^2\}$$

and

$$d\hat{q}(\gamma)/d\gamma = \{-\Pi_{xq}h'(x)q^2 + \Pi_{xx}2h(x)q\}/\{\Pi_{xx}\Pi_{qq} - (\Pi_{xq})^2\},$$

where all expressions are evaluated at the solution $(\hat{x}(\gamma), \hat{q}(\gamma))$. The denominator is positive in both cases. Consider the numerator in the expression for $d\hat{x}(\gamma)/d\gamma$. Upon substituting for Π_{qq} and Π_{xq}, and noting that (A.7) implies that $\Pi_{xq} = -\gamma h'(x)q$, the numerator becomes $\{\gamma h'(x)q2h(x)q - 2(\beta + \gamma h(x))h'(x)q^2\} = -2\beta h'(x)q^2 > 0$. Thus, $d\hat{x}(\gamma)/d\gamma > 0$, which implies that $\hat{x}^{NL} > \hat{x}^{SL}$. Now consider the numerator for the expression $d\hat{q}(\gamma)/d\gamma$. Upon substituting for Π_{xx} and $\Pi_{xq} = -\gamma h'(x)q$, the numerator becomes $q^2\{\gamma q[(h'(x))^2 - 2h(x)h''(x)] - 2h(x)c''(x)\}$. Since $c''(x) > 0$, a sufficient condition for this to be negative is that $[(h'(x))^2 - 2h(x)h''(x)] < 0$. But this inequality is implied by the convexity of expected harm in (x, q). Thus, $d\hat{q}(\gamma)/d\gamma < 0$, which implies that $\hat{q}^{NL} < \hat{q}^{SL}$.

Proof that welfare, evaluated at the profit-maximizing (x, q), is decreasing in γ. Recall that $W(\hat{x}(\gamma), \hat{q}(\gamma)) = \alpha\hat{q}(\gamma) - (\beta/2)(\hat{q}(\gamma))^2 - h(\hat{x}(\gamma))(\hat{q}(\gamma))^2 - c(\hat{x}(\gamma))\hat{q}(\gamma)$. Thus, $dW/d\gamma = W_x(d\hat{x}(\gamma)/d\gamma) + W_q(d\hat{q}(\gamma)/d\gamma)$, where W_q and W_x are evaluated at $(\hat{x}(\gamma), \hat{q}(\gamma))$. Evaluating W_x (equation (1) in the main text) at the solution to equation (A.1) implies that $W_x(\hat{x}(\gamma), \hat{q}(\gamma)) = (\gamma - 1)h'(\hat{x}(\gamma))(\hat{q}(\gamma))^2 < 0$ ($= 0$ at $\gamma = 1$). Since $d\hat{x}(\gamma)/d\gamma > 0$, it is clear that

$W_x(d\hat{x}(\gamma)/d\gamma) < 0$ (= 0 at $\gamma = 1$). Evaluating W_q (equation (2) in the main text) at the solution to equation (A.2) implies that $W_q(\hat{x}(\gamma), \hat{q}(\gamma)) = \beta\hat{q}(\gamma) + 2(\gamma - 1)h(\hat{x}(\gamma))\hat{q}(\gamma) > 0$. Since $d\hat{q}(\gamma)/d\gamma < 0$, it is clear that $W_q(d\hat{q}(\gamma)/d\gamma) < 0$. Combining the two terms implies that $dW/d\gamma < 0$; that is, strict liability is socially-preferred to no liability.

Technical Appendix, part B: Results for the Generalized Expected Harm Formulation Under Monopoly[1]

Initial considerations.

In this Appendix we re-formulate the monopoly model considered in the main text by allowing for both a more general model of expected harm and a more general model of the representative consumer's utility (which leads to a more general model of demand for the product). We first make some assumptions on the elements of the model and then some further assumptions on the payoff functions so as to ensure that the first order conditions for each optimization problem properly characterize the respective optimal solutions, and that those solutions are in the interior (i.e., all relevant variables remain positive). We then employ the resulting models to show which results of the main text carry over to the more general environment and which may require further restrictions. As before, unless specified otherwise, x and q are assumed to be non-negative, and $\gamma = 1$ if the liability regime is SL (strict liability) while $\gamma = 2$ if the liability regime is NL (no liability); NEG (negligence) will be handled as a composite of the two regimes and will be more precisely defined when needed. For readability we will separately analyze the SL and NL settings, though we will return to the parametrized version for the discussion of firm and social preferences over regimes. In what follows, the expected harm is denoted as $H(x, q)$.

Assumptions

B1) The representative consumer's utility function, $U(q, z; x, \gamma)$ is quasilinear in form:

$$U(q, z; x, \gamma) = u(q) + z - (\gamma - 1)H(x, q)$$

where z is the consumer's numeraire good and $u(q)$ is the consumer's direct utility for the good of interest. The consumer faces the budget constraint $pq + z \leq I$, where $I > 0$ is income and p is the (positive) price of the q-good; I is assumed sufficient to guarantee that the consumer always consumes positive amounts of the consumer's decision variable, q and z, so that the demand for q is independent of I. Assumptions on u are that:

i) $u(q)$ is twice continuously differentiable for all $q \geq 0$;
ii) $u(q) \geq 0$, $u'(q) > 0$, and $u''(q) < 0$ for all $q \geq 0$.

B2) Expected Harm is modeled by the function $H(x, q)$ which is thrice continuously differentiable with the following properties:

i) $H(x, 0) = 0$ for all $x \geq 0$; $H(x, q) > 0$ for all $x \geq 0$, and all $q > 0$;
ii) $H_x(x, q) < 0$ and $H_q(x, q) > 0$ for all $(x, q) > 0$; $H_x(x, 0) = 0$ for all $x > 0$;
iii) $H(x, q)$ is convex in (x, q) with strictly positive own-second partials: $H_{xx}(x, q) > 0$; $H_{qq}(x, q) > 0$ for all $(x, q) > 0$;
iv) $H_{xq}(q, x) < 0$ for all $(x, q) > 0$;
v) $\partial H_{qq}(x, q)/\partial x < 0$ for all $(x, q) > 0$.

B3) Production cost is modeled as in the main text: $C(x, q) = c(x)q$, with $c(\bullet)$ twice continuously differentiable, $c(x) > 0$, $c'(x) > 0$, and $c''(x) > 0$ for all $x \geq 0$.

B4) Optimization. All optimization problems have a unique interior maximum and all the respective

[1] See Marino (1988a) and Spulber (1989) for derivations overlapping some portions of this Appendix.

payoff functions (U, W, Π^{SL}, and Π^{NL}) have second-order matrices that are negative definite in a sufficiently large neighborhood of these optima.

Assumption B1 concerns the representative consumer's optimization problem. The utility function, u(q), is strictly increasing and concave, so the consumer's (inverse) demand function for the q-good can be written as:

$$p(q; x, \gamma) = \max [u'(q) - (\gamma - 1)H_q(x, q), 0],$$

with derivative $dp(q; x, \gamma)/dq = \min [u''(q) - (\gamma - 1)H_{qq}(x, q), 0]$. When $\gamma = 1$ this simplifies to:

$$p^{SL}(q) = u'(q) > 0, \text{ for all } q \geq 0,$$

and

$$dp^{SL}(q)/dq = u''(q) < 0 \text{ for all } q \geq 0,$$

where both properties come from Assumption B1. When $\gamma = 2$, under Assumption B1, the inverse demand function and derivative simplify to:

$$p^{NL}(q; x) = \max [u'(q) - H_q(x, q), 0] \geq 0,$$

and

$$dp^{NL}(q; x)/dq = u''(q) - H_{qq}(x, q) < 0 \text{ when } p^{NL}(q; x) \text{ is positive.}$$

The derivative $dp^{NL}(q; x)/dq$ is negative when $p^{NL}(q; x)$ is positive because of Assumptions B1(ii) and B2(iii). In order to avoid the kink in the NL case (in the inverse demand function), and assure that the willingness to pay is everywhere positive, we assume that $u'(q) - H_q(x, q) > 0$ over the relevant ranges of the x- and q-variables. This amounts to asserting that there are (x, q) combinations such that the product is socially valuable, and that all of the optima satisfy this requirement. Thus we take the inverse demand functions for the SL and NL cases to be the following:

$$p^{SL}(q) = u'(q) > 0, \text{ for all } q \geq 0,$$

and

$$p^{NL}(q; x) = u'(q) - H_q(x, q) > 0 \text{ over the relevant range of } (x, q).$$

Assumption B2 provides basic properties of the expected harm function; we have assumed that third derivatives are continuous due to the need for property B2(v), which we discuss in more detail below. Note that this is an expected harm function, so it is reflects both the probability of harm taking on specific possible values as well as those possible values. Assumption B2(i) states that when the quantity consumed is zero then the expected harm is zero, independent of the level of care taken; positive levels of use generate a positive level of expected harm, for any given level of care. Assumption B2(ii) states that the level of expected harm is decreasing in the level of care taken but increasing in the amount consumed; it further asserts that when usage (q) is zero, then as with total expected harm, the marginal effect of increasing x does not change the level of expected harm. Assumption B2(iii) states that expected harm is a convex function of x and q, and that the own second partials are strictly positive. Thus, increases in care have diminishing returns with respect to care, but increases in usage increase marginal (with respect to usage) expected harm. Note that B2(iii) rules out expected harm models of the traditional form (h(x)q) but allows for the one considered in the main text ($h(x)q^2$) which focuses around cumulative expected harm.[2] Assumption B2(iv) states that an increase in the level of use (q) raises the marginal benefit ($- H_x(x, q)$) of care; alternatively put, the cross derivative, H_{xq}, is negative; this, together with the fact that convexity implies that $H_{xx}(x, q)H_{qq}(x, q) \geq (H_{xq}(x, q))^2$ means that H_{xq}, while negative by assumption, is also restricted in magnitude. Lastly, Assumption

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[2] Assumption B2(iii), when applied to $H(x, q) = h(x)q^2$, means that h(x) satisfies the side requirement that $h''(x)h(x) \geq 2(h'(x))^2$, which in turn implies the condition in the discussion of the sign of $d\hat{q}/d\gamma$ in the main text.

B2(v) reflects an intuitive understanding that an increase in the level of care ameliorates the acceleration effect that usage has on expected harm (since H_{qq} is positive).

The convexity of H (Assumption B2(iii)) also implies that, for positive levels of output, the marginal expected harm exceeds the average expected harm:

$$H_q(x, q) > H(x, q)/q \text{ for all } (x, q) > 0.$$

To see this, observe that $H(x, q) = \int_{[0, q]} H_q(x, t)dt$, since by Assumption B2(i), $H_x(x, 0) = 0$. Since $H_{qq} > 0$, $H_q(x, t)$ is increasing in t. Thus, the average of the area under the function $H_q(x, t)$ as t ranges from 0 to q, $(1/q)\int_{[0, q]} H_q(x, t)dt$, must be less than the upper end of that curve, $H_q(x, q)$, yielding the desired result. As will be seen below, this plays an important role in the discussion of the firm's preference for strict liability over no liability.

Finally, Assumption B2(v) implies that:

$$H_{xq}(x, q) < H_x(x, q)/q \text{ for all } (x, q) > 0.$$

To see this (keep in mind that, by assumption, both sides of the above inequality are negative), observe that $H_x(x, q) = \int_{[0, q]} H_{xq}(x, t)dt$, since by Assumption B2(ii), $H_x(x, 0) = 0$. Thus, $H_x(x, q) - H_{xq}(x, q)q \gtrless 0$ as $\partial H_{xq}(x, q)/\partial q \lessgtr 0$. While this last derivative is not particularly obvious, $\partial H_{xq}(x, q)/\partial q = \partial H_{qq}(x, q)/\partial x$, which (by Assumption B2(v)) is negative. Thus, we know that $H_x(q, x) - H_{xq}(x, q)q > 0$, as claimed. We shall make use of this property later, too.[3]

Assumption B3 uses the same production cost function, c(x)q, as in the main text. We maintain the same assumption here so as to highlight the effect of the expected harm function on social optimality and on firm optimal choices of x and q.

Assumption B4 provides sufficient conditions that allow us to employ the first order conditions for all of our maximization models to characterize an (interior) optimum. Furthermore, the Assumption requires that at the respective (unique) optima, the sufficient conditions hold for a large enough portion of the space to allow for comparisons of the alternative equilibria and optimal outcomes.

Comparison of SL and NL regimes with the Efficient Outcome.

First, consider the (x, q) combination that maximizes social welfare. Denote social welfare as W(x, q), so that:

$$W(x, q) = u(q) - H(x, q) - c(x)q.$$

Consider the result of solving the following single-variable problem, where we treat q as a parameter:

$$\max_x W(x, q).$$

The optimal solution function for this problem, x(q), satisfies the first order condition $- H_x(x, q) - c'(x)q =$

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[3] Marino (1988a) instead considers the case wherein $H_{xq}(x, q) > H_x(x, q)/q$ for all $(x, q) > 0$. This is not consistent with our Assumption B2(v) which assumes that increasing care ameliorates the acceleration of harm arising from increased use.

0; second order conditions are clearly satisfied, namely $W_{xx}(x(q), q) < 0$. Totally differentiating the first order condition yields the derivative:

$$dx(q)/dq = W_{xq}/(-W_{xx}), \text{ where } W_{xq}(x(q), q) = - H_{xq}(x(q), q) - c'(x(q)).$$

Using the first-order condition to re-express $W_{xq}(x(q), q)$, it is straightforward to find that:

$$W_{xq}(x(q), q) = - H_{xq}(x(q), q) + H_x(x(q), q)/q > 0$$

from the earlier discussion employing Assumption B2(v). Thus, $dx(q)/dq > 0$ for the single-variable problem.

Now let us turn to the full optimization problem, namely:

$$\max_{(x,q)} W(x, q).$$

Under Assumption B4, the first-order-conditions characterize the social optimum in (x, q):

$$W_x = \quad - H_x(x, q) - c'(x)q = 0; \tag{B.1}$$
and
$$W_q = u'(q) - H_q(x, q) - c(x) = 0. \tag{B.2}$$

Solving the equation (B.1), $W_x = 0$, yields (for arbitrary q) the socially-optimal level of care as a function of the level of output, denoted again as $x^W(q)$. However, this is exactly the same as that derived in the single-variable problem analysis above, so we immediately know that $x^W(q)$ is increasing in q: $dx^W(q)/dq = W_{xq}(x, q)/(-W_{xx}(x, q)) > 0$. This, in turn, means that $W_{xq}(x, q) > 0$ along the entire path $x^W(q)$. Notice also that $W_{xq} > 0$ means that q and x are complements along the path $x^W(q)$ in the sense that increasing q raises the marginal contribution of x to welfare.

Solving the second first-order condition ($W_q = 0$) yields, for arbitrary x, the level of socially-optimal output as a function of the level of care, denoted as $q^W(x)$, and in a similar manner one finds that $dq^W(x)/dx = W_{xq}/(-W_{qq})$. Unfortunately, the same trick as used for the x-variable cannot be used for the q-variable: in general, $W_{xq}(x, q)$ can change sign along the path $q^W(x)$. However, since negative definiteness (and the assumption of interiority of the solution) means that the solution occurs where the x^W- and q^W-functions cross, denoted as $(\hat{x}^W, \hat{q}^W)$, $W_{xq}(x, q)$ must be positive at least in a neighborhood of the optimum, so that the slope of $q^W(x)$, $dq^W(x)/dx = W_{xq}(x, q)/(-W_{qq}(x, q))$, is positive (at least in a neighborhood of the social optimum).

Thus, when graphed in (x, q)-space, the slope of the curve $x^W(q)$ is given by $1/(dx^W(q)/dq)$, so that the issue of the relative position of this curve and that of the curve $q^W(x)$ is given by whether $1/(dx^W(q)/dq) \gtrless dq^W(x)/dx$; that is, by whether $(-W_{xx})/W_{xq} \gtrless W_{xq}/(-W_{qq})$. Since at the optimum the determinant of the second-order terms of W is positive, then $W_{xx}W_{qq} > (W_{xq})^2$, independent of the sign of W_{xq}. However since (locally) $W_{xq} > 0$ then both curves are upward-sloping and the curve $x^W(q)$ cuts the curve $q^W(x)$ from below, as was illustrated in the main text.

Next, consider the strict-liability regime. The firm's profit function under strict liability is:

$$\Pi^{SL}(x, q) = p^{SL}(q)q - H(x, q) - c(x)q.$$

Assumption B4 guarantees a unique (interior) optimum for the firm's decision problem in (x, q) which is characterized by the first-order conditions for the firm:

$$\Pi^{SL}_x = \qquad - H_x(x, q) - c'(x)q \quad = 0; \tag{B.3}$$

and

$$\Pi^{SL}_q = u'(q) + u''(q)q - H_q(x, q) - c(x) \quad = 0. \tag{B.4}$$

Note that the first-order condition for care, equation (B.3), is the same as that for maximizing welfare, equation (B.1), so that it is immediate that the optimal level-of-care function under strict liability (for an arbitrary level of q), $x^{SL}(q)$, equals that for maximizing welfare; that is, $x^{SL}(q) = x^W(q)$ for all q. Moreover, applying Assumption B2(v) again, equation (B.3) yields that $\Pi^{SL}_{xq}(x, q) > 0$ along $x^{SL}(q)$. The optimal quantity function, $q^{SL}(x)$, however is not the same as $q^W(x)$, as becomes clear since the second term in the first-order condition for quantity, equation (B.4), does not appear in the first-order condition for quantity in the welfare conditions, equation (B.2). Evaluating $\Pi^{SL}_q(x, q)$ at $q^W(x)$ shows that $\Pi^{SL}_q(x, q^W(x)) = u''(q^W(x))q^W(x) < 0$ from Assumption B1(ii); that is, the SL-firm would want to reduce output at any given level of x. Thus, $q^{SL}(x) < q^W(x)$ for all x: in (x, q)-space, the curve for $q^{SL}(x)$ is everywhere below that for $q^W(x)$. This immediately tells us that the solution to the first-order conditions for the firm under strict liability is "south-and-west" of that for welfare maximization: $(\hat{x}^{SL}, \hat{q}^{SL}) < (\hat{x}^W, \hat{q}^W)$, and (as illustrated in the main text) any policy that worked to increase the quantity provided by the firm would result (under strict liability) in a higher level of care provided. Moreover, if the quantity was increased to force $\hat{q}^{SL}$ to equal $\hat{q}^W$, then the firm would provide the associated socially-optimal level of care, $\hat{x}^W$, as movements will be along $x^W(q)$. As with $q^W(x)$, the slope of $q^{SL}(x)$ in a neighborhood of $(\hat{x}^{SL}, \hat{q}^{SL})$ must be positive, but this need not hold over the entire space.

The slope of $x^{SL}(q)$, $dx^{SL}(q)/dq$, is readily seen to be $\Pi^{SL}_{xq}(x, q)/(-\Pi_{xx}(x, q))$; since $x^{SL}(q) = x^W(q)$ and $\Pi^{SL}_{xx}(x, q) < 0$, then this means that $\Pi^{SL}_{xq}(x, q) > 0$ everywhere along $x^{SL}(q)$. In (x, q) space, the slope of the curve $x^{SL}(q)$ slope is given $1/\, dx^{SL}(q)/dq = (-\,\Pi^{SL}_{xx}(x, q))/\,\Pi^{SL}_{xq}(x, q)$. On the other hand, while the slope of the curve $q^{SL}(x)$, $dq^{SL}(x)/dx$, is readily found to be $\Pi^{SL}_{xq}(x, q)/(-\,\Pi^{SL}_{qq}(x, q))$, the sign of $\Pi^{SL}_{xq}(x, q)$ is only guaranteed to be positive in a neighborhood of where the two curves cross. Again, employing Assumption B4 we know that $\Pi^{SL}_{xx}\,\Pi^{SL}_{qq} > (\Pi^{SL}_{xq})^2$, so the curve $x^{SL}(q)$ cuts the curve $q^{SL}(x)$ from below.

Finally, we consider the no-liability regime. Here the consumer does not ignore the cost of expected harm since she will not be compensated and, as shown earlier, demand is now conditioned on the level of care chosen, so the inverse demand function is $p^{NL}(q; x)$ as derived earlier.
Therefore the firm's payoff function is:

$$\Pi^{NL}(x, q) = (u'(q) - H_q(x, q))q - c(x)q.$$

Again, Assumption B4 guarantees a unique (interior) optimum for the firm's decision problem in (x, q), which is characterized by the first-order conditions for the firm:

$$\Pi^{NL}_x = \qquad - H_{xq}(x, q)q \qquad - c'(x)q \quad = 0; \tag{B.5}$$

and

$$\Pi^{NL}_q = u'(q) + u''(q)q - H_{qq}(x, q)q - H_q(x, q) - c(x) \quad = 0. \tag{B.6}$$

Comparing equation (B.5) with equation (B.1) allows us to locate the firms's choice-of-care function under no liability, $x^{NL}(q)$, relative to the socially optimal level, $x^W(q)$. Once again, recalling the result that $H_{xq}(x, q) < H_x(x, q)/q$ for all $(x, q) > 0$, and evaluating $\Pi^{NL}_x(x, q)$ at $x^W(q)$, one finds that $\Pi^{NL}_x(x^W(q), q) > 0$ for each value of q. Thus, $x^{NL}(q) > x^W(q)$ for each q: the NL firm over-supplies (in comparison with the socially-efficient level) care when q is given. Note, as we will see below, this does not address the level of care provided in equilibrium, as the quantities of output may differ. Thus we know that:

$$x^{NL}(q) > x^W(q) = x^{SL}(q) \text{ for all q.}$$

Turning to the issue of the NL-firm's choice of output given an arbitrary level of care, x, compare equation (B.6) with equation (B.4); we make this comparison because both the SL-firm and the NL-firm exploit their market power in choosing the level of output to produce. Evaluating $\Pi_q^{NL}(x, q)$ at the function that solves equation (B.4), $q^{SL}(x)$, yields $\Pi_q^{NL}(x, q^{SL}(x)) = - H_{qq}(x, q^{SL}(x))q^{SL}(x) < 0$ by Assumption B2(iii). Thus, given any level x, the NL-firm would reduce output from that which the SL-firm would choose; that is, $q^{NL}(x) < q^{SL}(x)$. Bringing this together with the welfare-maximization results yields:

$$q^W(x) > q^{SL}(x) > q^{NL}(x) \text{ for all x.}$$

Again, we consider how the sets of curves are related, at least at the equilibrium solutions. Differentiating $\Pi_x^{NL}(x, q)$ yields $\Pi_{xq}^{NL}(x, q) = - H_{xq}(x, q) - H_{xqq}(x,q)q - c'(x)$. Substituting from $\Pi_x^{NL}(x, q) = 0$ yields $\Pi_{xq}^{NL}(x, q) = - H_{xqq}(x,q)q$, so employing Assumption B2(v) means that $\Pi_{xq}^{NL}(x, q) > 0$ for all (x, q) along $x^{NL}(q)$. This implies that:

$$dx^{NL}(q)/dq = \Pi_{xq}^{NL}(x, q)/(-\Pi_{xx}^{NL}(x, q)) > 0.$$

As with the other q-functions, it is not possible to sign $\Pi_{xq}^{NL}(x, q)$ along $q^{NL}(x)$ except in a neighborhood of the optimal solution for the NL-firm, in which case it must be positive, yielding:

$$dq^{NL}(x)/dx = \Pi_{xq}^{NL}(x, q)/(-\Pi_{qq}^{NL}(x, q)) > 0 \text{ in a neighborhood of the maximum, } (\hat{x}^{NL}, \hat{q}^{NL}).$$

Thus, both these curves are upward-sloping near the maximum profit for the NL-firm. As earlier, when viewed in (x, q)-space we need to invert the derivatives for the x-curves, when comparing them with the q-curves, and second order conditions for the profit function under no liability guarantee that $x^{NL}(q)$ crosses $q^{NL}(x)$ from below.

Given the earlier results about the relationship between the q-curves and the result that $x^{NL}(q) > x^W(q)$, the NL-equilibrium is not directly comparable to the SL-equilibrium, nor is it comparable to the welfare-optimum; that is; in general $(\hat{x}^{NL}, \hat{q}^{NL}) \neq (\hat{x}^W, \hat{q}^W)$. As was shown in the main text, when $H(x, q) = h(x)q^2$, then $\hat{x}^{NL} = \hat{x}^W$ (but $\hat{q}^{NL} < \hat{q}^W$). However, this result (that under no liability the firm provides the socially optimal level of care) is due to employing a specific functional form; to see this simply modify the expected harm function again. We can (in general) show that $\hat{q}^{NL} < \hat{q}^W$ whenever $\hat{x}^{NL} \leq \hat{x}^W$, simply by reference to the earlier results on $q^{NL}(x)$ versus $q^W(x)$. Also, evaluating $\Pi_x^{NL}(x, q)$ at $(\hat{x}^W, \hat{q}^W)$ yields $\Pi_x^{NL}(\hat{x}^W, \hat{q}^W) > 0$, so at $q = \hat{q}^W$, the NL-firm would wish to expand the level of care <u>above</u> $\hat{x}^W$: the NL-firm is inefficient in its choice of care if forced (or induced) to choose output at the socially-efficient level.

Firm Preferences over Strict vs No Liability

For convenience, recall that the profit function for a firm under strict liability is:

$$\Pi^{SL}(x, q) = p^{SL}(q)q - H(x, q) - c(x)q,$$

while that for a firm under no liability is:

$$\Pi^{NL}(x, q) = p^{NL}(q; x)q - c(x)q = (p^{SL}(q) - H_q(x, q))q - c(x)q.$$

In a parallel manner to the main text, we now introduce a parametrized version of the profit function, $\Pi(x, q; \gamma)$, where $\gamma = 1$ corresponds to SL and $\gamma = 2$ corresponds to NL:

$$\Pi(x, q; \gamma) = (p^{SL}(q) - (\gamma - 1)H_q(x, q))q - (2 - \gamma)H(x, q) - c(x)q.$$

Under Assumption B4, for all γ in the interval [1, 2], the first order conditions:

$$\Pi_x(x, q; \gamma) = \qquad - (\gamma - 1)H_{xq}(x, q)q \qquad - (2 - \gamma)H_x(x, q) - c'(x)q = 0 \tag{B.7}$$
and
$$\Pi_q(x, q; \gamma) = u'(q) + u''(q)q - (\gamma - 1)\{H_{qq}(x, q)q + H_q(x, q)\} - (2 - \gamma)H_q(x, q) - c(x) = 0 \tag{B.8}$$

yield the (continuous and continuously differentiable) solution functions denoted as $(\hat{x}(\gamma), \hat{q}(\gamma))$. Then using the envelope theorem,

$$d\Pi(\hat{x}(\gamma), \hat{q}(\gamma); \gamma)/d\gamma = - H_q(\hat{x}(\gamma), \hat{q}(\gamma))\hat{q}(\gamma) + H(\hat{x}(\gamma), \hat{q}(\gamma)).$$

From the discussion concerning the implications of the convexity of H earlier in this Appendix, we know that the marginal expected harm exceeds the average expected harm, so this means that the above derivative is negative: the firm prefers SL to NL, since increasing γ lowers optimized profits.

As in the main text and Appendix A, we can totally differentiate the first order conditions for $\Pi(x, q; \gamma)$ and obtain the following results for the signs of $d\hat{x}(\gamma)/d\gamma$ and $d\hat{q}(\gamma)/d\gamma$:

$$\text{sign}(d\hat{x}(\gamma)/d\gamma) = \text{sign}(\Pi_{qq}(H_{xq}q - H_x) - \Pi_{xq}H_{qq}q);$$

$$\text{sign}(d\hat{q}(\gamma)/d\gamma) = \text{sign}(\Pi_{xq}(H_x - H_{xq}q) + \Pi_{xx}H_{qq}q).$$

These sign conditions are due to the negative definiteness of Π^{SL} and Π^{NL} (and thus of $\Pi(x, q; \gamma)$) with respect to (x, q) from assumption B4. With the more general expected harm function employed in this Appendix, while elements of the right-hand-sides of the above equations can be signed, the entire expression cannot be signed for all possible values. However, one can show that it is not possible for $\hat{q}$ to be increasing in γ simultaneously with $\hat{x}$ decreasing in γ; such a sign pattern is mutually exclusive. In other words, in moving from SL to NL, the firm would not choose to reduce the level of care and simultaneously try to sell more output. This is because as the firm is moving from SL to NL demand is falling since consumers expect to have to cover their own expected harm, forcing a lower price; increasing output simply forces a yet further lower price. All other sign patterns are possible and we summarize the results as follows:

$d\hat{q}(\gamma)/d\gamma > 0$ implies $d\hat{x}(\gamma)/d\gamma > 0$;
$d\hat{x}(\gamma)/d\gamma < 0$ implies $d\hat{q}(\gamma)/d\gamma < 0$;
$d\hat{x}(\gamma)/d\gamma > 0$ and $d\hat{q}(\gamma)/d\gamma < 0$ can occur;
$d\hat{x}(\gamma)/d\gamma < 0$ and $d\hat{q}(\gamma)/d\gamma > 0$ cannot occur.

Recall from the main text that when $H(x, q) = h(x)q^2$ is a convex function, then we can show that $d\hat{x}(\gamma)/d\gamma > 0$ and $d\hat{q}(\gamma)/d\gamma < 0$ for all γ in the interval [1, 2]: the shift from SL to NL results in an increase in care provided but a fall in output provided to the market. In the more general expected harm model, at $\gamma = 1$, $d\hat{q}(\gamma)/d\gamma < 0$ and $d\hat{x}(\gamma)/d\gamma > 0$, but the result is limited to $\gamma = 1$ (i.e, is local).

On the Instability of Negligence as a Policy

Again, the implication for negligence, wherein the firm is not liable if $x \geq \hat{x}^W$, but is otherwise fully liable, is that since the consumer can observe the chosen level of care, the firm will "signal" the consumer that the firm will be fully liable for harm by choosing $x = \hat{x}^{SL}$. This holds since if $\hat{x}^{NL} \geq \hat{x}^W$, then if compliant with negligence, the firm meets (or more than meets) the standard, but since the firm prefers SL to NL, it will choose to violate the standard and instead produce the outcome $(\hat{x}^{SL}, \hat{q}^{SL})$. On the other hand, should it be that $\hat{x}^{NL} < \hat{x}^W$, the firm will again wish to switch regimes to SL, and it again will choose $(\hat{x}^{SL}, \hat{q}^{SL})$. Since in either possibility, upon observing $\hat{x}^{SL}$ the consumer knows that perfect compensation will obtain, the consumer's demand function becomes that under strict liability, $p^{SL}(q)$, which is what the firm desires to have happen. Thus, once again, negligence is unstable as a policy.

The discussion in the main text about unobservable care levels carries over to the more general expected harm function as well. Under strict liability the consumer expects to be fully compensated and so need not observe the level of care, while under no liability the consumer must conjecture about the level of care taken, which in equilibrium results in x = 0 (so as to minimize production cost).

Welfare Preferences over Strict vs No Liability

Since $W(\hat{x}(\gamma), \hat{q}(\gamma)) = u(\hat{x}(\gamma), \hat{q}(\gamma)) - H(\hat{x}(\gamma), \hat{q}(\gamma)) - c(\hat{x}(\gamma))\hat{q}(\gamma)$, then:

$$dW(\hat{x}(\gamma), \hat{q}(\gamma))/d\gamma = (W_x(\hat{x}(\gamma), \hat{q}(\gamma)))(d\hat{x}(\gamma)/d\gamma) + (W_q(\hat{x}(\gamma), \hat{q}(\gamma)))(d\hat{q}(\gamma)/d\gamma).$$

Evaluating $\Pi_x(x, q; \gamma) = 0$ (equation B.7) at $(\hat{x}(\gamma), \hat{q}(\gamma))$ and substituting into equation (B.1) via the $c'(x)q$ term yields:

$$W_x(\hat{x}(\gamma), \hat{q}(\gamma)) = (\gamma - 1)\{H_{xq}(\hat{x}(\gamma), \hat{q}(\gamma))\hat{q}(\gamma) - H_x(\hat{x}(\gamma), \hat{q}(\gamma))\} < 0 \text{ for } \gamma > 1.$$

Notice that at $\gamma = 1$, $W_x(\hat{x}(\gamma), \hat{q}(\gamma)) = 0$ (as it should, since the SL-firm's first-order care-choice condition is the same as that for welfare maximization, equation B.1). Evaluating $\Pi_q(x, q; \gamma) = 0$ (equation B.8) at $(\hat{x}(\gamma), \hat{q}(\gamma))$ and substituting into equation (B.2) via the term $c(x) - u'(q)$, we obtain:

$$W_q(\hat{x}(\gamma), \hat{q}(\gamma)) = - u''(\hat{q}(\gamma))\hat{q}(\gamma) + (\gamma - 1)H_{qq}(\hat{x}(\gamma), \hat{q}(\gamma)) > 0 \text{ for all } \gamma.$$

Thus, there are two "simple" settings wherein the effect of shifting regime on welfare can be predicted:

1) for $\gamma = 1$, since $W_x(\hat{x}(\gamma), \hat{q}(\gamma)) = 0$ and $d\hat{q}(\gamma)/d\gamma < 0$, then $dW(\hat{x}(\gamma), \hat{q}(\gamma))/d\gamma < 0$;

and 2) for $\gamma > 1$, if for all γ $d\hat{x}(\gamma)/d\gamma > 0$ occurs jointly with $d\hat{q}(\gamma)/d\gamma < 0$, then $dW(\hat{x}(\gamma), \hat{q}(\gamma))/d\gamma < 0$.

For the two other possible combinations ($d\hat{q}(\gamma)/d\gamma > 0$, which implies $d\hat{x}(\gamma)/d\gamma > 0$, and $d\hat{x}(\gamma)/d\gamma < 0$ which implies that $d\hat{q}(\gamma)/d\gamma < 0$ for $\gamma > 1$), then the effect on welfare is determined by the relative magnitudes of the two terms in $dW(\hat{x}(\gamma), \hat{q}(\gamma))/d\gamma$ and cannot be signed *ex ante*.

Summary of Results from this Appendix

Conditional on the restrictions placed on the expected harm function and the need for concavity of welfare and profit functions, many of the primary results obtained in the main text for a specific expected harm function extend to a more general version:

1) $x^{SL}(q) = x^{W}(q) < x^{NL}(q)$; all are upward sloping functions;

2) $q^{W}(x) > q^{SL}(x) > q^{NL}(x)$; all are upward sloping at the respective W, SL, NL optima, all are cut from below by the respective x-functions at the respective optima;

3) $(\hat{x}^{SL}, \hat{q}^{SL}) < (\hat{x}^{W}, \hat{q}^{W})$; $(\hat{x}^{NL}, \hat{q}^{NL}) \neq (\hat{x}^{W}, \hat{q}^{W})$;

4) if the NL firm is forced to produce $q = \hat{q}^{W}$ then $x^{NL}(\hat{q}^{W}) > \hat{x}^{W}$;

5) firms prefer SL to NL;

6) negligence without employing extra penalties for not employing the socially-efficient level of care is unstable: if care is observable, the firm will choose to provide $(\hat{x}^{SL}, \hat{q}^{SL})$ instead.

7) Society's preferences are more difficult to characterize in the more general expected harm case. If $d\hat{x}(\gamma)/d\gamma > 0$ occurs jointly with $d\hat{q}(\gamma)/d\gamma < 0$, then $dW(\hat{x}(\gamma), \hat{q}(\gamma))/d\gamma < 0$ for all γ; at $\gamma = 1$, $dW(\hat{x}(\gamma), \hat{q}(\gamma))/d\gamma < 0$, but this latter result is very local. However, once again, even if, from an initial position, NL was more efficient than SL, an exogenous increase in output level (e.g., due to a regulator of market performance) would increase the level of care so that the SL-outcome would eventually be more efficient than the corresponding NL-outcome.

8) SL provides a resilient policy regime in that:
   a) increases in q will lead to increases in x converging in the direction of $(\hat{x}^W, \hat{q}^W)$;
   b) this means that eventually SL is more efficient than NL, even if $(\hat{x}^{SL}, \hat{q}^{SL}) < (\hat{x}^W, \hat{q}^W)$, since the $x^{NL}(q)$-path does not lead to the social optimum;
   c) firms will endeavor to undermine negligence;
   d) unobservability of the level of care does not undermine SL while it does undermine NL.

Technical Appendix, part C: Material for Oligopoly Analysis of Quadratic Cumulative Harm Model

In the interests of brevity, we derive only the equations necessary to characterize the symmetric equilibria. Our maintained assumptions (e.g., the convexity of expected harm in (x, q)) are sufficient to imply that the matrix of second partial derivatives is negative definite at the associated solution.

Independent Cumulative Harm

Strict Liability. Firm i's profits are given by $\Pi_i^{SL}(x_i, q_i; n) = (\alpha - \beta Q)q_i - h(x_i)(q_i)^2 - c(x_i)q_i$. The first-order conditions are:

$$\Pi_{i\,x}^{SL} = -h'(x_i)(q_i)^2 - c'(x_i)q_i = 0;$$
$$\Pi_{i\,q}^{LS} = \alpha - 2\beta q_i - \beta Q_{-i} - 2h(x_i)q_i - c(x_i) = 0,$$

where $Q_{-i} \equiv Q - q_i$ is the aggregate output of all other firms. At a symmetric equilibrium, all firms will choose the same output and care levels; incorporating this symmetry yields equations (9)-(10) from the text.

No Liability. Firm i's profits under no liability are given by $\Pi_i^{NL}(x_i, q_i; n) = (\alpha - \beta Q - 2h(x_i)q_i)q_i - c(x_i)q_i$. The first-order conditions are:

$$\Pi_{i\,x}^{NL} = -2h'(x_i)(q_i)^2 - c'(x_i)q_i = 0;$$
$$\Pi_{i\,q}^{NL} = \alpha - 2\beta q_i - \beta Q_{-i} - 4h(x_i)q_i - c(x_i) = 0.$$

At a symmetric equilibrium, all firms will choose the same output and care levels; incorporating this symmetry yields equations (11)-(12) from the text.

Joint Cumulative Harm

Strict Liability. Using the pure market share approach (wherein firm i is responsible for the share $\mu(x_i)q_i/M$ of the expected harm), firm i's profits are given by $\pi_i^{JSL}(x_i, q_i; n) = (\alpha - \beta Q)q_i - \lambda\mu(x_i)q_iM - c(x_i)q_i$. The use of the lower-case π distinguishes this from the "adjusted" profit function that will be used in the main text. It will be convenient to define the total contribution to exposure for all firms except firm i: $M_{-i} = M - \mu(x_i)q_i$. The first-order conditions are:

$$\pi_{ix}^{JSL} = -\lambda[2\mu(x_i)\mu'(x_i)(q_i)^2 + \mu'(x_i)q_iM_{-i}] - c'(x_i)q_i = 0;$$
$$\pi_{iq}^{JSL} = \alpha - 2\beta q_i - \beta Q_{-i} - \lambda[2(\mu(x_i))^2q_i + \mu(x_i)M_{-i}] - c(x_i) = 0.$$

Using symmetry, we can find a system of equations that characterizes the symmetric equilibrium care and output levels:

$$-\lambda(n + 1)\mu(x)\mu'(x)q^2 - c'(x)q = 0; \tag{C.1}$$
$$\alpha - (n + 1)\beta q - \lambda(n + 1)(\mu(x))^2q - c(x) = 0. \tag{C.2}$$

Notice that equations (13) and (C.1) are not the same; that is, for $n \geq 2$, noncooperative firms will provide less care than would be socially-optimal, for a given level of output, q, and number of firms, n.

Next, consider the "adjusted" profit function, wherein firm i's liability for harm is given by the share $(2n/(n + 1))\mu(x_i)q_i/M$. In this case, profit is $\Pi_i^{JSL}(x_i, q_i; n) = (\alpha - \beta Q)q_i - (2n/(n + 1))\lambda\mu(x_i)q_iM - c(x_i)q_i$. The first-order conditions are:

$$\Pi_{ix}^{JSL} = -\lambda(2n/(n + 1))[2\mu(x_i)\mu'(x_i)(q_i)^2 + \mu'(x_i)q_i M_{-i}] - c'(x_i)q_i = 0;$$
$$\Pi_i^{JL} = \alpha - 2\beta q_i - \beta Q_{-i} - \lambda(2n/(n + 1))[2(\mu(x_i))^2q_i + \mu(x_i)M_{-i}] - c(x_i) = 0.$$

At a symmetric equilibrium, all firms will choose the same output and care levels; incorporating this symmetry yields equations (15)-(16) from the text.

<u>No Liability</u>. Firm i's profits are given by $\Pi_i^{JNL}(x_i, q_i; n) = (\alpha - \beta Q - 2\lambda M\mu(x_i))q_i - c(x_i)q_i$. The first-order conditions are:

$$\Pi_{ix}^{JNL} = -2\lambda[2\mu(x_i)\mu'(x_i)(q_i)^2 + \mu'(x_i)q_i M_{-i}] - c'(x_i)q_i = 0;$$
$$\Pi_{iq}^{JNL} = \alpha - 2\beta q_i - \beta Q_{-i} - 2\lambda[2(\mu(x_i))^2q_i + \mu(x_i)M_{-i}] - c(x_i) = 0.$$

At a symmetric equilibrium, all firms will choose the same output and care levels, yielding equations (17)-(18) from the text.

Remark. It was claimed in the text that under joint cumulative harm the optimal level of care and the equilibrium level of care under no liability are (1) invariant to the number of firms; and (2) equal to each other.

Proof of (1). To see that the functions $q^{JW}(x; n)$ and $x^{JW}(q; n)$ cross at the same x value for all n, first solve equation (13) for $\tilde{q}^{JW}(x; n) = c'(x)/(-2\lambda n\mu(x)\mu'(x))$ and solve equation (14) for $q^{JW}(x; n) = (\alpha - c(x))/n[\beta + 2\lambda(\mu(x))^2]$. Then, setting these two functions equal to each other yields an equation for x that is independent of n: $c'(x)/(-2\lambda\mu(x)\mu'(x)) = (\alpha - c(x))/[\beta + 2\lambda(\mu(x))^2]$.

To see that the functions $q^{JNL}(x; n)$ and $x^{JNL}(q; n)$ cross at the same value for all n, first solve equation (17) for $\tilde{q}^{JNL}(x; n) = c'(x)/(-2\lambda(n + 1)\mu(x)\mu'(x))$ and solve equation (18) for $q^{JW}(x; n) = (\alpha - c(x))/(n + 1)[\beta + 2\lambda(\mu(x))^2]$. Then, setting these two functions equal to each other yields an equation for x that is independent of n: $c'(x)/(-2\lambda\mu(x)\mu'(x)) = (\alpha - c(x))/[\beta + 2\lambda(\mu(x))^2]$.

Proof of (2). Simply note that the equations defining x are the same for case JW and for case JNL.