Technical Appendix to
"Search, Bargaining, and Signaling in the Market for Legal Services"

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In this Technical Appendix, we provide: (1) the discussion of how a small cost of conducting the auction renders the continuation equilibrium wherein C accepts L2's demand Pareto superior to the one in which C conducts the auction, and how L2 can make C an offer that she strictly prefers to accept; (2) the detailed analysis of equilibrium beliefs and behavior in the continuation equilibrium between L2 and C, for the case of unconstrained F and for the case wherein F is constrained to be zero; (3) the equilibrium refinement arguments using D1 to select the equilibrium that involves the smallest likelihood of a second search in the case wherein F is unconstrained; (4) the equilibrium refinement arguments using D1 to justify the use of skeptical beliefs when F = 0; and (5) the equilibrium refinement arguments using D1 to eliminate a separating equilibrium at the lower root of equation (6) when F = 0.

1. Effects of a Small Cost of Conducting the Auction under PFI

When it is costless to conduct the auction and A is common knowledge, then it is always a best response for C to conduct the auction after having visited L2. Anticipating this, it is an optimal strategy for L2 to make a demand that leaves C indifferent between accepting and conducting the auction. Note that L2 cannot break C's indifference: (1) in the case of F = 0, he cannot make a lump-sum transfer and any other α would be worse from C's point of view; and (2) in the case of unrestricted transfers, L2 is already bidding away the full case value. Thus, there is also a continuation equilibrium wherein C conducts the auction. We now argue that this equilibrium is not robust to a small cost, k, of conducting the auction.

First, consider the case of unrestricted F. When C visits L2, if there is a small cost k to conduct the auction, then L2 can demand $(1,\ \Pi^L(1, A) - k)$, which leaves C indifferent between accepting this demand and conducting the auction and leaves L2 with profit of k if accepted, and profit of 0 if rejected. So it is Pareto superior for C to accept, rather than reject, this demand. Moreover, if C were to reject this demand when indifferent (with positive probability), then L2 could offer $(1,\ \Pi^L(1, A) - k + \delta)$, for some $\delta \in (0, k)$, and induce C to accept for sure. But this δ can be arbitrarily small, so there cannot be an equilibrium wherein C rejects the demand $(1,\ \Pi^L(1, A) - k)$ with positive probability. On the other hand, if C were to accept this demand when indifferent, then L2 would find this to be the optimal demand. So there is an equilibrium wherein C accepts the demand $(1,\ \Pi^L(1, A) - k)$. Tracing the effects of this cost backward, it follows that L1 will demand $(1, \Pi^L(1, A) - s - k)$, which C will accept in equilibrium. The equilibrium payoffs are: $\Pi^L(1, A) - 2s - k$ for C; s + k for L1; and 0 for L2. The equilibrium in the text corresponds to k = 0.

Next, consider the case wherein F is restricted to be zero. When C visits L2, if there is a small cost k to conduct the auction, then L2 can demand the contingent fee $\tilde{\alpha}^C(A; k)$, which is defined implicitly by $\Pi^C(\tilde{\alpha}^C(A; k), A) = \Pi^C(\alpha^C(A), A) - k$. Notice that $\tilde{\alpha}^C(A; k) > \alpha^C(A)$ for $k > 0$, with $\tilde{\alpha}^C(A; 0) = \alpha^C(A)$. C is indifferent about accepting this demand, whereas L2 strictly prefers that C accept it; thus, it is Pareto superior for C to accept, rather than reject, this demand. Moreover, if C were to reject this demand (with positive probability) when indifferent, then L2 could offer the contingent fee $\tilde{\alpha}^C(A; k - \delta)$, for some $\delta \in (0, k)$, and induce C to accept for sure (since this demand is closer to C's ideal share, $\alpha^C(A)$). But this δ can be arbitrarily small, so there cannot be an equilibrium wherein C rejects the demand $\tilde{\alpha}^C(A; k)$ with positive probability. On the other hand, if C were to accept the demand $\tilde{\alpha}^C(A; k)$ when indifferent, then L2 would find this to be the optimal demand. So there is an equilibrium wherein C accepts the demand $\tilde{\alpha}^C(A; k)$. Tracing the effects of this cost backward, it follows that L1 will demand $\tilde{\alpha}^L(A; k)$ such that $\Pi^C(\tilde{\alpha}^L(A; k), A) = \Pi^C(\tilde{\alpha}^C(A; k), A) - s = \Pi^C(\alpha^C(A), A) - s - k$, which C accepts in equilibrium. The equilibrium payoffs are: $\Pi^C(\alpha^C(A), A) - 2s - k$ for C; $\Pi^L(\tilde{\alpha}^C(A; k), A)$ for L1; and 0 for L2. The equilibrium in the text corresponds to k = 0.

These arguments are modified below for the cases of PAI with F unconstrained, and with F = 0. The relevant continuation games between C and L2 are described first for the case wherein it is costless to conduct the auction; subsequently we indicate how one could incorporate a small cost of conducting the auction and how this results in the equilibrium outcome wherein C accepts L2's demand rather than conducting the auction.

2. Equilibrium Beliefs and Behavior in the Continuation Game Between C and L2

Analysis of the Continuation Game Between C and L2 when F is Unconstrained

Recall that $B_2(\alpha_2, F_2 \mid B(\alpha_1, F_1))$ denotes C's posterior belief, after having arrived at L2 with beliefs $B_1(\alpha_1, F_1)$ and having received the demand (α_2, F_2) from L2, where:

$$B_2(\alpha_2, F_2 \mid \underline{A}) = \underline{A} \text{ for } (\alpha_2, F_2) \in u(\underline{A}); \text{ for all other } (\alpha_2, F_2),\ B_2(\alpha_2, F_2 \mid \underline{A}) \in \{\underline{A}, \overline{A}\}.$$

$$B_2(\alpha_2, F_2 \mid \overline{A}) = \overline{A} \text{ for } (\alpha_2, F_2) \in \text{epi}(u(\underline{A})) \cap \text{hypo}(u(\overline{A})); \text{ for all other } (\alpha_2, F_2),\ B_2(\alpha_2, F_2 \mid \overline{A}) \in \{\underline{A}, \overline{A}\}.$$

We can now characterize the client's optimal behavior in response to L2's demand (α_2, F_2). Suppose that $B_1(\alpha_1, F_1) = \underline{A}$. If L2's demand (α_2, F_2) is on $u(\underline{A})$, then C's beliefs are confirmed and she is willing to accept (or reject) any such demand. This follows since she is indifferent between every point on the curve $u(\underline{A})$ and the point $(1, \Pi^L(1, \underline{A}))$, which is what she expects to obtain from the auction. If L2's demand (α_2, F_2) is above $u(\underline{A})$ but below $u(\overline{A})$, then C will accept it if she continues to believe $\underline{A}$ and she will conduct the auction if she revises her belief upward to $\overline{A}$. If L2's demand (α_2, F_2) is below $u(\underline{A})$, then C will initiate the auction regardless of her beliefs. If L2's demand (α_2, F_2) is on the curve $u(\overline{A})$, then she is indifferent between accepting it and conducting the auction if she revises her belief upward to $\overline{A}$, and she will accept this demand if she continues to believe $\underline{A}$. Finally, if L2's demand is above $u(\overline{A})$, then C will accept regardless of her beliefs.

Alternatively, suppose that $B_1(\alpha_1, F_1) = \overline{A}$. If L2's demand (α_2, F_2) is on $u(\overline{A})$, then C's beliefs are confirmed and she is willing to accept (or reject) any such demand. This follows since she is indifferent between every point on the curve $u(\overline{A})$ and the point $(1, \Pi^L(1, \overline{A}))$, which is what she expects to obtain from the auction (and the most she could ever hope to obtain). If L2's demand (α_2, F_2) is on or above $u(\underline{A})$ but below $u(\overline{A})$, then C continues to believe $\overline{A}$ and thus she will conduct the auction. Again, if L2's demand (α_2, F_2) is below $u(\underline{A})$, C will initiate the auction regardless of her beliefs and if L2's demand (α_2, F_2) is above $u(\overline{A})$, C will accept the demand regardless of her beliefs.

We now characterize L2's optimal demand (α_2, F_2), given C's beliefs upon approaching L2, $B_1(\alpha_1, F_1)$, and given L2's own observation of the expected case value. In principle, there are four possibilities (though only the first one will occur in the overall equilibrium). First, suppose that L2 observes $\underline{A}$ and C believes $B_1(\alpha_1, F_1) = \underline{A}$. Then L2 can obtain a payoff of zero by demanding $(\alpha_2, F_2) = (1, \Pi^L(1, \underline{A}))$, which the client accepts, or by offering (α_2, F_2) below $u(\underline{A})$, which the client rejects in favor of an auction. An offer of (α_2, F_2) on $u(\underline{A})$, but different from $(1, \Pi^L(1, \underline{A}))$, is acceptable to C but yields a negative payoff for L2; this is also true for offers (α_2, F_2) on or above $u(\overline{A})$. An offer of (α_2, F_2) above $u(\underline{A})$ but below $u(\overline{A})$ yields a payoff of zero to L2 if C believes $\overline{A}$ and therefore rejects it, or a negative payoff if C believes $\underline{A}$ and therefore accepts it. Thus, it is clear that $(\alpha_2, F_2) = (1, \Pi^L(1, \underline{A}))$ is an optimal demand by L2 when the expected value of the case is $\underline{A}$, C believes $B_1(\alpha_1, F_1) = \underline{A}$ and C accepts this demand; C obtains a payoff of $\Pi^L(1, \underline{A})$.

Second, suppose that L2 observes $\overline{A}$ but C believes $B_1(\alpha_1, F_1) = \underline{A}$. Then L2 can obtain a payoff of $\Pi^L(1, \overline{A}) - \Pi^L(1, \underline{A})$ by demanding $(\alpha_2, F_2) = (1, \Pi^L(1, \underline{A}))$, which the client accepts (because it confirms her beliefs). Although C would also be willing to accept any other offer on $u(\underline{A})$, these would yield lower profits for L2. Similarly, any offer above $u(\underline{A})$ and below $u(\overline{A})$ would either provoke an auction (from which L2 expects to obtain a payoff of zero) if C were to revise her beliefs upward, or be accepted (yielding lower profits to L2) if C were to maintain her beliefs. Offers above $u(\overline{A})$ would be accepted but yield lower profits to L2, while offers below $u(\underline{A})$ would result in an auction, yielding profits to L2 of zero. Thus, it is clear that $(\alpha_2, F_2) = (1, \Pi^L(1, \underline{A}))$ is the optimal demand by L2 when the expected case value is $\overline{A}$ but C believes $B_1(\alpha_1, F_1) = \underline{A}$, and C accepts this demand; C obtains a payoff of $\Pi^L(1, \underline{A})$. **To summarize:** when the client approaches L2 believing $B_1(\alpha_1, F_1) = \underline{A}$, then an optimal demand for L2 is to confirm C's beliefs by offering $(\alpha_2, F_2) = (1, \Pi^L(1, \underline{A}))$, prompting C to accept L2's demand.

Third, suppose that L2 observes $\underline{A}$ but C believes $B_1(\alpha_1, F_1) = \overline{A}$. Then L2 can do no better than to provoke an auction (from which L2 expects a payoff of zero). This is because any offer on $u(\overline{A})$ would yield a negative payoff to L2, and C's beliefs are skeptical in that she does not revise her beliefs downward when L2 makes a demand below $u(\overline{A})$. Notice that even if C did revise her beliefs down to $\underline{A}$ for, say, some (α_2, F_2) below $u(\overline{A})$ but on or above $u(\underline{A})$, and were to accept such a demand (with positive probability), L2 would still strictly prefer the auction with the single exception of the demand $(\alpha_2, F_2) = (1, \Pi^L(1, \underline{A}))$, at which L2 is indifferent between acceptance and the auction.

Fourth, suppose that L2 observes $\overline{A}$ and C believes $B_1(\alpha_1, F_1) = \overline{A}$. Then L2 can obtain a payoff of zero by demanding $(\alpha_2, F_2) = (1, \Pi^L(1, \overline{A}))$, which confirms C's beliefs and which she accepts; or by offering (α_2, F_2) below $u(\overline{A})$, which the client rejects in favor of an auction (since she continues to believe that the expected value of the case is $\overline{A}$). An offer of (α_2, F_2) on $u(\overline{A})$, but different from $(1, \Pi^L(1, \overline{A}))$, is acceptable to C but yields a negative payoff for L2; this is also true for offers (α_2, F_2) above $u(\overline{A})$. Thus, it is clear that $(\alpha_2, F_2) = (1, \Pi^L(1, \overline{A}))$ is an optimal demand by L2 when the expected value of the case is $\overline{A}$, C believes $B_1(\alpha_1, F_1) = \overline{A}$, and C accepts this demand; C obtains a payoff of $\Pi^L(1, \overline{A})$.

Notice that this last situation is where C's skeptical beliefs come into play. To see why these are the "right" out-of-equilibrium beliefs, consider what would happen if C were to revise her beliefs downward following a demand (α_2, F_2) below $u(\overline{A})$ but on or above $u(\underline{A})$, and thus accept such a demand (with positive probability). The result would be that L2 could make a positive profit by deviating to such a demand and falsely persuading C that her case has a low expected value when it actually has a high expected value. Moreover, only an L2 of type $\overline{A}$ could profit from such a deviation, since we showed immediately above that if L2 observed $\underline{A}$ but C believed $B_1(\alpha_1, F_1) = \overline{A}$, then L2 could never make a positive profit from such a deviation, even if she were able to convince C that the expected case value was $\underline{A}$. Thus, C should rationally attribute such a deviation to an L2 of type $\overline{A}$.

Adding a small cost to conduct the auction. From the discussion above, it is clear that if the auction is costless, then it is a best response for C to conduct the auction, regardless of her beliefs, after having visited two lawyers. Thus there is another equilibrium wherein L2's demand confirms C's beliefs and, though she is indifferent, C conducts the auction. We now argue that this equilibrium is not robust to a small cost, k, of conducting the auction. Define the curves $\tilde{u}(A; k)$ as follows:

$$\tilde{u}(A; k) = \{(\alpha_2, F_2) \mid F_2 = \Pi^L(1, A) - \Pi^C(\alpha_2, A) - k\}, \text{ for } A \in \{\underline{A}, \overline{A}\}.$$

This is the set of demands for L2 that would make C indifferent between accepting the demand and conducting the auction under the belief that the case has expected value A (whereas L2 prefers that C accept as this yields profit to L2 of k). This curve lies below the curve u(A) for $k > 0$, with $\tilde{u}(A; 0) = u(A)$. Furthermore, we modify C's beliefs so that if she visits L2 believing that $A = \underline{A}$, then she continues to believe that $A = \underline{A}$ for any demand (α_2, F_2) in an arbitrarily small open neighborhood of the curve $\tilde{u}(\underline{A}; k)$. (Note that heretofore, any belief was allowed off the curve $u(\underline{A})$, so this modification is not inconsistent with any of the previous analysis for $k = 0$; that is, it does not change anything that the previous analysis relied upon.) C's beliefs are otherwise unchanged from those specified above. The arguments above can be modified, substituting the curves $\tilde{u}(A; k)$ for the curves u(A) (and modifying the demands to reflect k in the obvious ways), but now when C visits L2 with the belief $B_1(\alpha_1, F_1) = \underline{A}$, then L2 can make a demand above (but arbitrarily close to) $\tilde{u}(\underline{A}; k)$ and C will strictly prefer to accept this. Thus, there cannot be an equilibrium wherein C rejects L2's demand along $\tilde{u}(\underline{A}; k)$ when $B_1(\alpha_1, F_1) = \underline{A}$. A similar argument implies that there cannot be an equilibrium wherein C rejects L2's demand along $\tilde{u}(\overline{A}; k)$ when $B_1(\alpha_1, F_1) = \overline{A}$, since C would always want to accept any demand above this curve. Tracing the effects of this cost backward, the rest of the analysis in Section 4.2 of the text can be repeated replacing the term s by the term $\sigma \equiv s + k$. The equilibrium contract for $A = \underline{A}$, $(\underline{\alpha}^*, \varphi(\underline{\alpha}^*, \underline{A}))$, and the probability with which it is rejected, $r(\underline{\alpha}^*, \varphi(\underline{\alpha}^*, \underline{A}))$, will now depend on k. The equilibrium payoffs are: $\Pi^L(1, A) - 2s - k$ for C, $A \in \{\underline{A}, \overline{A}\}$; if $A = \overline{A}$, then L1 obtains $s + k$ and L2 obtains 0; if $A = \underline{A}$, then L1 obtains $(1 - r(\underline{\alpha}^*, \varphi(\underline{\alpha}^*, \underline{A})))(\Pi^L(\underline{\alpha}^*, \underline{A}) + \Pi^C(\underline{\alpha}^*, \underline{A}) - \Pi^L(1, \underline{A}) + s + k)$, and L2's payoff is $r(\underline{\alpha}^*, \varphi(\underline{\alpha}^*, \underline{A}))k$. The equilibrium in the text corresponds to $k = 0$.

Analysis of the Continuation Game Between C and L2 when F = 0

Recall that $B_2(\alpha_2 \mid B(\alpha_1))$ denotes C's posterior belief, after having arrived at L2 with beliefs $B_1(\alpha_1)$ and having received the demand α_2 from L2, where:

$$B_2(\alpha_2 \mid \underline{A}) = \underline{A} \text{ for } \alpha_2 = \alpha^C(\underline{A}); \text{ for all other } \alpha_2,\ B_2(\alpha_2 \mid \underline{A}) \in \{\underline{A}, \overline{A}\}.$$

$$B_2(\alpha_2 \mid \overline{A}) = \overline{A} \text{ for all } \alpha_2.$$

We can now characterize the client's optimal behavior in response to L2's demand α_2. First, suppose that $B_1(\alpha_1) = \underline{A}$. If L2's demand is $\alpha_2 = \alpha^C(\underline{A})$, then C's beliefs are confirmed and she is being offered her most-preferred contingent fee, so she is willing to accept (or reject) this demand. If L2's demand α_2 is anything except $\alpha^C(\underline{A})$ and C continues to believe that $B_2(\alpha_2 \mid \underline{A}) = \underline{A}$, then C rejects the demand α_2 in favor of an auction, from which she expects a payoff of $\Pi^C(\alpha^C(\underline{A}), \underline{A})$. If L2's demand α_2 is anything except $\alpha^C(\underline{A})$

and C revises her belief upward to $B_2(\alpha_2 \mid \underline{A}) = \overline{A}$, then C is willing to accept the demand $\alpha_2 = \alpha^C(\overline{A})$, but she rejects all other demands α_2 in favor of an auction, from which she expects a payoff of $\Pi^C(\alpha^C(\overline{A}), \overline{A})$. Now suppose that $B_1(\alpha_1) = \overline{A}$. Since C does not revise her beliefs, the only demand that C is willing to accept is $\alpha^C(\overline{A})$, which makes her indifferent; all others will be rejected in favor of an auction, from which she expects a payoff of $\Pi^C(\alpha^C(\overline{A}), \overline{A})$.

We now characterize L2's optimal demand α_2, given C's beliefs upon approaching L2, $B_1(\alpha_1)$, and given L2's own observation of the expected case value. In principle, there are four possibilities (though only the first one will occur in the overall equilibrium). First, suppose that L2 observes $\underline{A}$ and C believes $B_1(\alpha_1) = \underline{A}$. Then L2 can obtain a payoff of $\Pi^L(\alpha^C(\underline{A}), \underline{A})$ by demanding $\alpha^C(\underline{A})$, which confirms the client's beliefs, and which she accepts. This is strictly better than his other options and their resulting payoffs: 1) he can obtain a payoff of $\Pi^L(\alpha^C(\overline{A}), \underline{A})$ by demanding $\alpha^C(\overline{A})$ if C is thereby persuaded to revise her beliefs upward to $\overline{A}$ and to accept this demand, but $\alpha^C(\overline{A}) < \alpha^C(\underline{A})$ and the lawyer prefers the higher contingent fee $\alpha^C(\underline{A})$; 2) he can obtain a payoff of $.5\Pi^L(\alpha^C(\underline{A}), \underline{A})$ by provoking an auction using a demand that C associates with $\underline{A}$; or 3) he can obtain a payoff of $.5\Pi^L(\alpha^C(\overline{A}), \underline{A})$ by provoking an auction using a demand that C associates with $\overline{A}$.

Second, suppose that L2 observes $\overline{A}$ but C believes $B_1(\alpha_1) = \underline{A}$. Then L2 can obtain a payoff of $\Pi^L(\alpha^C(\underline{A}), \overline{A})$ by demanding $\alpha_2 = \alpha^C(\underline{A})$, which confirms C's beliefs and which she accepts. Again, this is strictly better than L2's other options: 1) he can obtain a payoff of $\Pi^L(\alpha^C(\overline{A}), \overline{A})$ by demanding $\alpha^C(\overline{A})$ if C is thereby persuaded to revise her beliefs upward to $\overline{A}$ and to accept this demand, but $\alpha^C(\overline{A}) < \alpha^C(\underline{A})$ and the lawyer prefers the higher contingent fee $\alpha^C(\underline{A})$; 2) he can obtain a payoff of $.5\Pi^L(\alpha^C(\underline{A}), \overline{A})$ by provoking an auction using a demand that C associates with $\underline{A}$; or 3) he can obtain a payoff of $.5\Pi^L(\alpha^C(\overline{A}), \overline{A})$ by provoking an auction using a demand that C associates with $\overline{A}$. **To summarize:** when the client approaches L2 believing $B_1(\alpha_1) = \underline{A}$, then L2's unique optimal strategy is to confirm C's beliefs by offering $\alpha_2 = \alpha^C(\underline{A})$, prompting C to accept L2's demand.

Third, suppose that L2 observes $\underline{A}$ but C believes $B_1(\alpha_1) = \overline{A}$. Since there is nothing L2 can do to change C's beliefs, L2's best demand is $\alpha_2 = \alpha^C(\overline{A})$, which confirms C's beliefs and which is accepted, yielding a payoff of $\Pi^L(\alpha^C(\overline{A}), \underline{A})$ for L2 and a payoff of $\Pi^C(\alpha^C(\overline{A}), \underline{A})$ for C. Any other demand would be rejected but C would continue to believe $\overline{A}$, so L2 would expect a payoff of $.5\Pi^L(\alpha^C(\overline{A}), \underline{A})$ from the auction. Fourth, suppose that L2 observes $\overline{A}$ and C believes $B_1(\alpha_1) = \overline{A}$. Again, there is nothing L2 can do to change C's beliefs, so L2's best demand is $\alpha_2 = \alpha^C(\overline{A})$, which confirms C's beliefs and which is accepted, yielding a payoff of $\Pi^L(\alpha^C(\overline{A}), \overline{A})$ for L2 and a payoff of $\Pi^C(\alpha^C(\overline{A}), \overline{A})$ for C. Any other demand would be rejected but C would continue to believe $\overline{A}$, so L2 would expect a payoff of $.5\Pi^L(\alpha^C(\overline{A}), \overline{A})$ from the auction.

To see why C's skeptical beliefs are the "right" out-of-equilibrium beliefs when $B_1(\alpha_1) = \overline{A}$, consider what would happen if C were to revise her beliefs downward to $\underline{A}$ following an (out-of-equilibrium) demand of $\alpha_2 = \alpha^C(\underline{A})$ and accept this demand. The result would be that either type of L2 could make a positive profit by deviating to $\alpha_2 = \alpha^C(\underline{A})$, but type $\overline{A}$ would benefit more than type $\underline{A}$, since $\Pi^L(\alpha^C(\underline{A}), \overline{A})$ - $\Pi^L(\alpha^C(\overline{A}), \overline{A})$ - $[\ \Pi^L(\alpha^C(\underline{A}), \underline{A})$ - $\Pi^L(\alpha^C(\overline{A}), \underline{A})]$ has the same sign as $\Pi^L_{12} > 0$. Thus, using a D1-type argument (essentially, type $\overline{A}$ would be willing to defect for a higher rejection probability than $\underline{A}$) implies that C should rationally attribute a deviation to the out-of-equilibrium demand $\alpha_2 = \alpha^C(\underline{A})$ to an L2 of type $\overline{A}$.[1]

Adding a small cost to conduct the auction. From the discussion above, it is clear that whenever C's beliefs are confirmed by an offer from L2 of $\alpha^C(B_1(\alpha_1))$, she is indifferent between accepting L2's offer and conducting the auction. Although L2 strictly prefers that C accept, L2 cannot break C's indifference; he is forbidden to make a lump-sum payment and any other demand would be rejected by C in favor of the auction. So there is a second equilibrium wherein C conducts the auction. We now argue that this alternative equilibrium is not robust to a small cost, k, of conducting the auction. This requires a minor modification of C's beliefs so that, if she visits L2 believing that $A = \underline{A}$, then she continues to believe that $A = \underline{A}$ for any demand α_2 in an arbitrarily small open neighborhood of $\tilde{\alpha}^C(\underline{A}; k)$. (Note that heretofore, any belief was allowed off the equilibrium demand for type $\underline{A}$, so this modification is not inconsistent with any of the previous analysis for k = 0; that is, it does not change anything that the previous analysis relied upon.) C's beliefs are otherwise unchanged from those specified above. When C visits L2 with beliefs $B_1(\alpha_1) = \underline{A}$, then C is indifferent between accepting a demand of $\tilde{\alpha}^C(\underline{A}; k) > \alpha^C(\underline{A})$ and conducting the auction, whereas L2 prefers acceptance. But if C were to reject this demand (with positive probability), then L2 could make a demand that is less than, but arbitrarily close to, $\tilde{\alpha}^C(\underline{A}; k)$, and induce C to accept for sure (since C continues to believe that $A = \underline{A}$ and this is closer to her ideal demand $\alpha^C(\underline{A})$). So there cannot be an equilibrium wherein C rejects the demand $\tilde{\alpha}^C(\underline{A}; k)$ with positive probability when she arrives with beliefs $B_1(\alpha_1) = \underline{A}$. Similarly, when C visits L2 with beliefs $B_1(\alpha_1) = \overline{A}$, then there is a demand that is less than, but arbitrarily close to, $\tilde{\alpha}^C(\overline{A}; k)$, that C would strictly prefer to conducting the auction (and that L2 would prefer to make in order to induce acceptance). So there cannot be an equilibrium wherein C rejects the demand $\tilde{\alpha}^C(\overline{A}; k)$ when she arrives with beliefs $B_1(\alpha_1) = \overline{A}$. Tracing the effects of this cost backward, the rest of the analysis in Section 5.2 of the text can be repeated to find that L1 will still demand $\tilde{\alpha}^L(A; k)$ such that $\Pi^C(\tilde{\alpha}^L(A; k), A) = \Pi^C(\tilde{\alpha}^C(A; k), A) - s$. If $A = \overline{A}$, then C will accept for sure, but if $A = \underline{A}$, then C will reject this demand with positive probability r* (which now depends on k). The equilibrium payoffs are: $\Pi^C(\tilde{\alpha}^C(A; k), A) - 2s = \Pi^C(\alpha^C(A), A) - 2s - k$ for C, $A \in \{\underline{A}, \overline{A}\}$; if $A = \overline{A}$, then L1 obtains $\Pi^L(\tilde{\alpha}^L(\overline{A}; k), \overline{A})$ and L2 obtains 0; if A =

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[1] One could also imagine L2 using a demand α_2 other than $\alpha^C(\underline{A})$ simply to persuade C to revise her beliefs downward to $\underline{A}$, even though this demand provokes an auction in which L2 expects to obtain a payoff of $.5\Pi^L(\alpha^C(\underline{A}), A)$, where A is the expected value of the case observed by L2. This would not provide a profitable deviation for either $\underline{A}$ or $\overline{A}$ as long as $\Pi^L(\alpha^C(\overline{A}), A) \geq .5\Pi^L(\alpha^C(\underline{A}), A)$ for $A = \underline{A}, \overline{A}$. This inequality surely holds when $\alpha^C(\underline{A})$ and $\alpha^C(\overline{A})$ are not too far apart (recall that they are equal for our power-function example $p(x) = \lambda x^\theta$, and they will be sufficiently close provided that $\underline{A}$ and $\overline{A}$ are sufficiently close).

$\underline{A}$, then L1 obtains $(1 - r^*)\Pi^L(\tilde{\alpha}^L(\underline{A}; k), \underline{A})$ and L2 obtains $r^*\Pi^L(\tilde{\alpha}^C(\underline{A}; k), \underline{A})$. The equilibrium in the text corresponds to $k = 0$.

3. Equilibrium Refinement using D1 to Select the Least-cost Separating Equilibrium when F is Unconstrained

We argued in the text that the lawyer with an $\bar{A}$-type case must use the equilibrium strategy $(1, \varphi(1, \bar{A}))$, and this demand will be accepted for sure. We also argued that any separating equilibrium contingent fee for the lawyer with an $\underline{A}$-type case – say, $\hat{\alpha}$, which must be accompanied by the transfer $\varphi(\hat{\alpha}, \underline{A})$ – must belong to the interval $[\alpha^k, 1]$, where α^k is defined implicitly by $s = \Pi^L(\alpha^k, \bar{A}) - \varphi(\alpha^k, \underline{A})$. Recall that the minimal acceptance probability (to deter mimicry by $\bar{A}$) at α^k is 1; for all $\alpha \in (\alpha^k, 1]$, the minimal acceptance probability (to deter mimicry by $\bar{A}$) is strictly less than 1 (see Figure 1). In the text we denoted the separating equilibrium α-value for the $\underline{A}$-type as $\underline{\alpha}^*$; in what follows we suppress the * so as to simplify the notational overhead.

In the text, we identify a particular separating equilibrium given by $\{(\underline{\alpha}, \varphi(\underline{\alpha}, \underline{A})), (1, \varphi(1, \bar{A}))\}$, with $1 - r(1, \varphi(1, \bar{A})) = 1$ and $1 - r(\underline{\alpha}, \varphi(\underline{\alpha}, \underline{A})) = s/[\Pi^L(\underline{\alpha}, \bar{A}) - \varphi(\underline{\alpha}, \underline{A})]$, where $\underline{\alpha}$ maximizes $s[\Pi^L(\alpha, \underline{A}) - \varphi(\alpha, \underline{A})]/[\Pi^L(\alpha, \bar{A}) - \varphi(\alpha, \underline{A})]$. Although (as indicated above) the lawyer with an $\bar{A}$-type case must use the second component of the equilibrium strategy described above (and will be accepted for sure), there are many possible alternative equilibrium strategies of the form $(\alpha, \varphi(\alpha, \underline{A}))$ for the lawyer with an $\underline{A}$-type case, each one supported by an associated acceptance probability and out-of-equilibrium beliefs. Under the additional assumption that there is a unique global maximizer $\underline{\alpha}$, we claim that the selected equilibrium is the unique separating equilibrium outcome surviving refinement using D1. We state this additional assumption below and maintain it thereafter.

<u>Assumption T1</u>. Assume that $\underline{\alpha} = \text{argmax}\ \{ s[\Pi^L(\alpha, \underline{A}) - \varphi(\alpha, \underline{A})]/[\Pi^L(\alpha, \bar{A}) - \varphi(\alpha, \underline{A})] \mid \alpha \in [\alpha^k, 1]\}$ is unique.[2]

First, we will show that $IC(\bar{A})$ must hold with equality in any separating equilibrium surviving D1. This implies that any D1 separating equilibrium must involve an acceptance probability on the downward-sloping portion of the curve labeled "minimal rejection curve" in Figure 1, where the $\bar{A}$-type is just-deterred from mimicry. Second, we will argue that no separating equilibrium involving a strategy $(\hat{\alpha}, \varphi(\hat{\alpha}, \underline{A}))$ for which $\hat{\alpha} \neq \underline{\alpha}$ will survive D1. Finally, we will prove that the sole remaining candidate, $(\underline{\alpha}, \varphi(\underline{\alpha}, \underline{A}))$, does survive D1.

<u>Claim 1</u>. $IC(\bar{A})$ must hold with equality in any separating equilibrium surviving D1.

<u>Proof</u>. Suppose there exists a separating equilibrium with strategies $\{(\hat{\alpha}, \varphi(\hat{\alpha}, \underline{A})), (1, \varphi(1, \bar{A}))\}$ with associated acceptance probabilities of $1 - r(1, \varphi(1, \bar{A})) = 1$ and $1 - r(\hat{\alpha}, \varphi(\hat{\alpha}, \underline{A}))$. Moreover, suppose that

[2] We conjecture that if there were multiple optima, then the following arguments would still apply, and only the separating equilibria involving these values of the contingent fee would survive refinement using D1.

IC($\overline{A}$) is slack. Then 1 - r($\hat{\alpha}$, $\varphi(\hat{\alpha}, \underline{A})$) < 1 (since it must be strictly less than the minimal acceptance probability to deter mimicry by $\overline{A}$). Then the $\overline{A}$- type's equilibrium payoff is:

$$s > [1 - r(\hat{\alpha}, \varphi(\hat{\alpha}, \underline{A}))][\Pi^L(\hat{\alpha}, \overline{A}) - \varphi(\hat{\alpha}, \underline{A})],$$

while the $\underline{A}$-type's equilibrium payoff is:

$$[1 - r(\hat{\alpha}, \varphi(\hat{\alpha}, \underline{A}))][\Pi^L(\hat{\alpha}, \underline{A}) - \varphi(\hat{\alpha}, \underline{A})].$$

Now consider a defection to ($\hat{\alpha}$, $\varphi(\hat{\alpha}, \underline{A}) + \varepsilon$), where ε is a small positive number. This defection must be met with a sufficiently high rejection probability in order to support the hypothesized separating equilibrium (which means the out-of-equilibrium beliefs must place a sufficiently high probability on type $\overline{A}$; in the text, we assumed the beliefs assigned a probability of 1 to type $\overline{A}$, leading to rejection). We will argue that the equilibrium refinement D1 requires that – starting from this putative equilibrium wherein IC($\overline{A}$) is slack – beliefs at this out-of-equilibrium demand be $\underline{A}$, leading to acceptance by the client. This, in turn, would induce the lawyer of type $\underline{A}$ to pursue the (profitable) defection and upset the original separating equilibrium at ($\hat{\alpha}$, $\varphi(\hat{\alpha}, \underline{A})$).

To see this, notice that the $\underline{A}$-type would be willing to defect to ($\hat{\alpha}$, $\varphi(\hat{\alpha}, \underline{A}) + \varepsilon$) for any acceptance probability 1 - r such that:

$$(1 - r)[\Pi^L(\hat{\alpha}, \underline{A}) - \varphi(\hat{\alpha}, \underline{A}) - \varepsilon] \geq [1 - r(\hat{\alpha}, \varphi(\hat{\alpha}, \underline{A}))][\Pi^L(\hat{\alpha}, \underline{A}) - \varphi(\hat{\alpha}, \underline{A})];$$

that is, for any probability:

$$1 - r \geq 1 - \tilde{r}(\underline{A}) \equiv [1 - r(\hat{\alpha}, \varphi(\hat{\alpha}, \underline{A}))][\Pi^L(\hat{\alpha}, \underline{A}) - \varphi(\hat{\alpha}, \underline{A})]/[\Pi^L(\hat{\alpha}, \underline{A}) - \varphi(\hat{\alpha}, \underline{A}) - \varepsilon].$$

However, for sufficiently small ε, the $\overline{A}$-type would not be willing to defect to ($\hat{\alpha}$, $\varphi(\hat{\alpha}, \underline{A}) + \varepsilon$), since:

$$\lim_{\varepsilon \to 0} [1 - \tilde{r}(\underline{A})][\Pi^L(\hat{\alpha}, \overline{A}) - \varphi(\hat{\alpha}, \underline{A}) - \varepsilon] = [1 - r(\hat{\alpha}, \varphi(\hat{\alpha}, \underline{A}))][\Pi^L(\hat{\alpha}, \overline{A}) - \varphi(\hat{\alpha}, \underline{A})] < s.$$

Thus, for sufficiently small ε, an observed demand of ($\hat{\alpha}$, $\varphi(\hat{\alpha}, \underline{A}) + \varepsilon$) must (under D1) be inferred to have come from an $\underline{A}$-type. Since $\Pi^C(\hat{\alpha}, \underline{A}) + \varphi(\hat{\alpha}, \underline{A}) + \varepsilon > \Pi^L(1, \underline{A}) - s$, the client strictly prefers to accept the out-of-equilibrium demand under the belief $\underline{A}$. Finally, since 1 - r($\hat{\alpha}$, $\varphi(\hat{\alpha}, \underline{A})$) < 1, it follows that:

$$\Pi^L(\hat{\alpha}, \underline{A}) - \varphi(\hat{\alpha}, \underline{A}) - \varepsilon > [1 - r(\hat{\alpha}, \varphi(\hat{\alpha}, \underline{A}))][\Pi^L(\hat{\alpha}, \underline{A}) - \varphi(\hat{\alpha}, \underline{A})]$$

for sufficiently small ε and thus ($\hat{\alpha}$, $\varphi(\hat{\alpha}, \underline{A}) + \varepsilon$) provides a profitable deviation for type $\underline{A}$. QED

Claim 2. Any separating equilibrium involving ($\hat{\alpha}$, $\varphi(\hat{\alpha}, \underline{A})$), with $\hat{\alpha} \neq \underline{\alpha}$, does not survive D1.

Proof. Suppose, to the contrary, that there exists a separating equilibrium with strategies {($\hat{\alpha}$, $\varphi(\hat{\alpha}, \underline{A})$), (1, $\varphi(1, \overline{A})$)} and with associated acceptance probabilities of 1 - r(1, $\varphi(1, \overline{A})$) = 1 and 1 - r($\hat{\alpha}$, $\varphi(\hat{\alpha}, \underline{A})$), where $\hat{\alpha} \neq \underline{\alpha}$. This equilibrium is supported by out-of-equilibrium beliefs that assign a sufficiently high probability to the $\overline{A}$-type. We will argue that the $\underline{A}$-type can deviate to the demand ($\underline{\alpha}$, $\varphi(\underline{\alpha}, \underline{A}) + \varepsilon$) for sufficiently small positive ε, be identified under D1 and accepted, and thereby profit from the deviation, upsetting the hypothesized separating equilibrium.

To see this, first note that since IC($\bar{A}$) is tight, the $\bar{A}$-type's equilibrium payoff is:

$$s = [1 - r(\hat{\alpha}, \varphi(\hat{\alpha}, \underline{A}))][\Pi^L(\hat{\alpha}, \bar{A}) - \varphi(\hat{\alpha}, \underline{A})],$$

while the $\underline{A}$-type's equilibrium payoff is:

$$[1 - r(\hat{\alpha}, \varphi(\hat{\alpha}, \underline{A}))][\Pi^L(\hat{\alpha}, \underline{A}) - \varphi(\hat{\alpha}, \underline{A})].$$

Then the $\bar{A}$-type will be willing to defect to $(\underline{\alpha}, \varphi(\underline{\alpha}, \underline{A}) + \varepsilon)$ if the client's response, denoted in terms of the acceptance probability $1 - r$, is such that:

$$(1 - r)[\Pi^L(\underline{\alpha}, \bar{A}) - \varphi(\underline{\alpha}, \underline{A}) - \varepsilon] \geq [1 - r(\hat{\alpha}, \varphi(\hat{\alpha}, \underline{A}))][\Pi^L(\hat{\alpha}, \bar{A}) - \varphi(\hat{\alpha}, \underline{A})];$$

that is, if:

$$(1 - r) \geq [1 - r(\hat{\alpha}, \varphi(\hat{\alpha}, \underline{A}))][\Pi^L(\hat{\alpha}, \bar{A}) - \varphi(\hat{\alpha}, \underline{A})]/[\Pi^L(\underline{\alpha}, \bar{A}) - \varphi(\underline{\alpha}, \underline{A}) - \varepsilon] \equiv 1 - \tilde{r}(\bar{A}).$$

Similarly, the $\underline{A}$- type will be willing to defect to $(\underline{\alpha}, \varphi(\underline{\alpha}, \underline{A}) + \varepsilon)$ if the client's response, denoted in terms of the acceptance probability $1 - r$, is such that:

$$(1 - r)[\Pi^L(\underline{\alpha}, \underline{A}) - \varphi(\underline{\alpha}, \underline{A}) - \varepsilon] \geq [1 - r(\hat{\alpha}, \varphi(\hat{\alpha}, \underline{A}))][\Pi^L(\hat{\alpha}, \underline{A}) - \varphi(\hat{\alpha}, \underline{A})];$$

that is, if:

$$(1 - r) \geq [1 - r(\hat{\alpha}, \varphi(\hat{\alpha}, \underline{A}))][\Pi^L(\hat{\alpha}, \underline{A}) - \varphi(\hat{\alpha}, \underline{A})]/[\Pi^L(\underline{\alpha}, \underline{A}) - \varphi(\underline{\alpha}, \underline{A}) - \varepsilon] \equiv 1 - \tilde{r}(\underline{A}).$$

If $1 - \tilde{r}(\bar{A}) > 1 - \tilde{r}(\underline{A})$, then (according to D1), the out-of-equilibrium demand $(\underline{\alpha}, \varphi(\underline{\alpha}, \underline{A}) + \varepsilon)$ must be inferred to have been made by the $\underline{A}$ -type, since this type is willing to defect for the largest range of acceptance probabilities. The inequality $1 - \tilde{r}(\bar{A}) > 1 - \tilde{r}(\underline{A})$ holds if and only if:

$$[\Pi^L(\hat{\alpha}, \bar{A}) - \varphi(\hat{\alpha}, \underline{A})]/[\Pi^L(\underline{\alpha}, \bar{A}) - \varphi(\underline{\alpha}, \underline{A}) - \varepsilon] > [\Pi^L(\hat{\alpha}, \underline{A}) - \varphi(\hat{\alpha}, \underline{A})]/[\Pi^L(\underline{\alpha}, \underline{A}) - \varphi(\underline{\alpha}, \underline{A}) - \varepsilon];$$

that is, if and only if:

$$[\Pi^L(\underline{\alpha}, \underline{A}) - \varphi(\underline{\alpha}, \underline{A}) - \varepsilon]/[\Pi^L(\underline{\alpha}, \bar{A}) - \varphi(\underline{\alpha}, \underline{A}) - \varepsilon] > [\Pi^L(\hat{\alpha}, \underline{A}) - \varphi(\hat{\alpha}, \underline{A})]/[\Pi^L(\hat{\alpha}, \bar{A}) - \varphi(\hat{\alpha}, \underline{A})].$$

Notice that the inequality is true for $\varepsilon = 0$ since $\underline{\alpha}$ (uniquely) maximizes the expression $[\Pi^L(\alpha, \underline{A}) - \varphi(\alpha, \underline{A})]/[\Pi^L(\alpha\ \bar{A}) - \varphi(\alpha, \underline{A})]$. Thus, there is a sufficiently small (but still positive) ε for which the inequality still holds. Thus, such an out-of-equilibrium demand will be believed by the client to have come from the $\underline{A}$-type. Since $\Pi^C(\underline{\alpha}, \underline{A}) + \varphi(\underline{\alpha}, \underline{A}) + \varepsilon > \Pi^L(1, \underline{A}) - s$, the client will accept this out-of-equilibrium demand with probability 1. Finally, we claim that this will induce the $\underline{A}$-type to make this defection, thus upsetting the hypothesized separating equilibrium. To verify this final claim, recall that since we need only consider

α-values in $[\alpha^k, 1]$, it follows that $s/[\Pi^L(\alpha, \overline{A}) - \varphi(\alpha, \underline{A})] \leq 1$. Note that:

$$\Pi^L(\underline{\alpha}, \underline{A}) - \varphi(\underline{\alpha}, \underline{A}) \geq s[\Pi^L(\underline{\alpha}, \underline{A}) - \varphi(\underline{\alpha}, \underline{A})]/ [\Pi^L(\underline{\alpha}, \overline{A}) - \varphi(\underline{\alpha}, \underline{A})]$$

$$> s[\Pi^L(\hat{\alpha}, \underline{A}) - \varphi(\hat{\alpha}, \underline{A})]/[\Pi^L(\hat{\alpha}, \overline{A}) - \varphi(\hat{\alpha}, \underline{A})] = [1 - r(\hat{\alpha}, \varphi(\hat{\alpha}, \underline{A}))][\Pi^L(\hat{\alpha}, \underline{A}) - \varphi(\hat{\alpha}, \underline{A})].$$

The first inequality follows since $s/[\Pi^L(\underline{\alpha}, \overline{A}) - \varphi(\underline{\alpha}, \underline{A})] \leq 1$ and the second (strict) inequality follows since $\underline{\alpha}$ uniquely maximizes the ratio of the terms in brackets. By defecting, the $\underline{A}$-type will obtain $\Pi^L(\underline{\alpha}, \underline{A}) - \varphi(\underline{\alpha}, \underline{A}) - \varepsilon$. Since $\Pi^L(\underline{\alpha}, \underline{A}) - \varphi(\underline{\alpha}, \underline{A}) > [1 - r(\hat{\alpha}, \varphi(\hat{\alpha}, \underline{A}))][\Pi^L(\hat{\alpha}, \underline{A}) - \varphi(\hat{\alpha}, \underline{A})]$, there is a sufficiently small ε for which $\Pi^L(\underline{\alpha}, \underline{A}) - \varphi(\underline{\alpha}, \underline{A}) - \varepsilon > [1 - r(\hat{\alpha}, \varphi(\hat{\alpha}, \underline{A}))][\Pi^L(\hat{\alpha}, \underline{A}) - \varphi(\hat{\alpha}, \underline{A})]$. QED

Claim 3. The separating equilibrium outcome given by $\{(\underline{\alpha}, \varphi(\underline{\alpha}, \underline{A})), (1, \varphi(1, \overline{A}))\}$, with $1 - r(1, \varphi(1, \overline{A})) = 1$ and $1 - r(\underline{\alpha}, \varphi(\underline{\alpha}, \underline{A})) = s/[\Pi^L(\underline{\alpha}, \overline{A}) - \varphi(\underline{\alpha}, \underline{A})]$, where $\underline{\alpha}$ maximizes

$$s[\Pi^L(\alpha, \underline{A}) - \varphi(\alpha, \underline{A})]/[\Pi^L(\alpha, \overline{A}) - \varphi(\alpha, \underline{A})],$$

survives refinement using D1.

Proof. In the specified equilibrium, the $\overline{A}$-type's equilibrium payoff is:

$$s = [1 - r(\underline{\alpha}, \varphi(\underline{\alpha}, \underline{A}))][\Pi^L(\underline{\alpha}, \overline{A}) - \varphi(\underline{\alpha}, \underline{A})],$$

while the $\underline{A}$-type's equilibrium payoff is:

$$[1 - r(\underline{\alpha}, \varphi(\underline{\alpha}, \underline{A}))][\Pi^L(\underline{\alpha}, \underline{A}) - \varphi(\underline{\alpha}, \underline{A})].$$

Consider any out-of-equilibrium demand $(\alpha, \varphi(\alpha, \underline{A}) + \varepsilon)$, where $\alpha \neq \underline{\alpha}$ and $\varepsilon \geq 0$ or where $\alpha = \underline{\alpha}$ and $\varepsilon > 0$. Any out-of-equilibrium demand along $U(\underline{A})$ or between the loci $U(\overline{A})$ and $U(\underline{A})$ can be represented this way. Demands on or above $U(\overline{A})$ are accepted regardless of beliefs and demands below $U(\underline{A})$ are rejected regardless of beliefs, and neither type is tempted to deviate to these out-of-equilibrium demands. So it is only the out-of-equilibrium demands between the loci (and along $U(\underline{A})$) that must be considered. Some of these demands are also immune to defection (that is, there is no response by the client that would tempt either type to defect); what we need to show is that if the $\underline{A}$-type is willing to defect to a particular out-of-equilibrium demand for some responses by the client, then the $\overline{A}$-type is willing to defect to that demand for a strictly greater range of client responses. D1 then requires that the beliefs assign the $\overline{A}$-type to the defection, which will lead to certain rejection by the client, which will, in turn, deter both types from defecting from the separating equilibrium involving $\underline{\alpha}$.

The $\overline{A}$-type will be willing to defect to $(\alpha, \varphi(\alpha, \underline{A}) + \varepsilon)$ if the client's response, denoted in terms of the acceptance probability $1 - r$, is such that:

$$(1 - r)[\Pi^L(\alpha, \overline{A}) - \varphi(\alpha, \underline{A}) - \varepsilon] \geq [1 - r(\underline{\alpha}, \varphi(\underline{\alpha}, \underline{A}))][\Pi^L(\underline{\alpha}, \overline{A}) - \varphi(\underline{\alpha}, \underline{A})];$$

that is, if:

$$(1 - r) \geq [1 - r(\underline{\alpha}, \varphi(\underline{\alpha}, \underline{A}))][\Pi^L(\underline{\alpha}, \overline{A}) - \varphi(\underline{\alpha}, \underline{A})]/[\Pi^L(\alpha, \overline{A}) - \varphi(\alpha, \underline{A}) - \varepsilon] \equiv 1 - \tilde{r}(\overline{A}).$$

Similarly, the $\underline{A}$- type will be willing to defect to $(\alpha, \varphi(\alpha, \underline{A}) + \varepsilon)$ if the client's response, denoted in terms of the acceptance probability $1 - r$, is such that:

$$(1 - r)[\Pi^L(\alpha, \underline{A}) - \varphi(\alpha, \underline{A}) - \varepsilon] \geq [1 - r(\underline{\alpha}, \varphi(\underline{\alpha}, \underline{A}))][\Pi^L(\underline{\alpha}, \underline{A}) - \varphi(\underline{\alpha}, \underline{A})];$$

that is, if:

$$(1 - r) \geq [1 - r(\underline{\alpha}, \varphi(\underline{\alpha}, \underline{A}))][\Pi^L(\underline{\alpha}, \underline{A}) - \varphi(\underline{\alpha}, \underline{A})]/[\Pi^L(\alpha, \underline{A}) - \varphi(\alpha, \underline{A}) - \varepsilon] \equiv 1 - \tilde{r}(\underline{A}).$$

If $1 - \tilde{r}(\underline{A}) > 1 - \tilde{r}(\overline{A})$, then (according to D1), the out-of-equilibrium demand $(\alpha, \varphi(\alpha, \underline{A}) + \varepsilon)$ must be inferred to have been made by the $\overline{A}$-type, since this type is willing to defect for the largest range of acceptance probabilities.

The inequality $1 - \tilde{r}(\underline{A}) > 1 - \tilde{r}(\overline{A})$ holds if and only if:

$$[\Pi^L(\underline{\alpha}, \underline{A}) - \varphi(\underline{\alpha}, \underline{A})]/[\Pi^L(\alpha, \underline{A}) - \varphi(\alpha, \underline{A}) - \varepsilon] > [\Pi^L(\underline{\alpha}, \overline{A}) - \varphi(\underline{\alpha}, \underline{A})]/[\Pi^L(\alpha, \overline{A}) - \varphi(\alpha, \underline{A}) - \varepsilon];$$

that is, if and only if:

$$[\Pi^L(\underline{\alpha}, \underline{A}) - \varphi(\underline{\alpha}, \underline{A})]/[\Pi^L(\underline{\alpha}, \overline{A}) - \varphi(\underline{\alpha}, \underline{A})] > [\Pi^L(\alpha, \underline{A}) - \varphi(\alpha, \underline{A}) - \varepsilon]/[\Pi^L(\alpha, \overline{A}) - \varphi(\alpha, \underline{A}) - \varepsilon].$$

First, consider $\alpha \neq \underline{\alpha}$ and $\varepsilon = 0$; this is a deviation along the locus $U(\underline{A})$. Then the inequality holds since $\underline{\alpha}$ uniquely maximizes the ratio of the terms in brackets. Since the right-hand-side decreases as ε increases, the inequality also holds when $\alpha = \underline{\alpha}$ and $\varepsilon > 0$ and when $\alpha \neq \underline{\alpha}$ and $\varepsilon > 0$. QED

Finally, we note that in the text we specified the beliefs to be $B(\alpha, F) = \underline{A}$ for all (α, F) along the locus $U(\underline{A})$, even those involving $\alpha \neq \underline{\alpha}$. This was helpful for the purpose of exposition, it seems intuitively reasonable, and it suffices to support the separating equilibrium outcome involving $\underline{\alpha}$ (as specified above in Claim 3). However, as we have seen in the foregoing analysis, the out-of-equilibrium beliefs implied by D1 are somewhat harsher, requiring $B(\alpha, F) = \overline{A}$ (leading to rejection) for out-of-equilibrium values of α (i.e., for $\alpha \neq \underline{\alpha}$) along the locus $U(\underline{A})$. These harsher beliefs support the same separating equilibrium outcome.

4. Verification that Skeptical Beliefs Survive Refinement using D1 (uniquely) when Assumption 6 Holds (with a strict inequality) for the case of $F = 0$.

By "skeptical beliefs,"we mean that $B_1(\alpha) = \overline{A}$ for $\alpha \in (\alpha^L(\overline{A}), \alpha^L(\underline{A}))$. Recall that type $\overline{A}$'s equilibrium payoff is $\Pi^L(\alpha^L(\overline{A}), \overline{A})$ and type $\underline{A}$'s equilibrium payoff is $(1 - r^*)\Pi^L(\alpha^L(\underline{A}), \underline{A})$, where $1 - r^* = \Pi^L(\alpha^L(\overline{A}), \overline{A})/\Pi^L(\alpha^L(\underline{A}), \overline{A})$. Now consider an out-of-equilibrium demand $\alpha' \in (\alpha^L(\overline{A}), \alpha^L(\underline{A}))$. C will reject such a demand if she believes it comes from type $\overline{A}$ and she will accept it if she believes it comes from $\underline{A}$ (as long as $\alpha' \geq \alpha^C(\underline{A})$). Let ρ denote the probability that C believes the demand α' comes from type $\overline{A}$; then C will accept the demand α' with probability $1 - \rho$. Type $\overline{A}$ would be willing to defect from his equilibrium demand to the demand α' if $(1 - \rho)\Pi^L(\alpha', \overline{A}) \geq \Pi^L(\alpha^L(\overline{A}), \overline{A})$. Type $\underline{A}$ would be willing to defect to the demand α' if $(1 - \rho)\Pi^L(\alpha', \underline{A}) \geq (1 - r^*)\Pi^L(\alpha^L(\underline{A}), \underline{A})$. The minimum acceptance threshold for type $\overline{A}$ is $(1 - \rho(\overline{A})) \equiv [\Pi^L(\alpha^L(\overline{A}), \overline{A})/\Pi^L(\alpha', \overline{A})]$, while the minimum acceptance threshold for type $\underline{A}$ is $(1 - \rho(\underline{A})) \equiv (1 - r^*)[\Pi^L(\alpha^L(\underline{A}), \underline{A})/\Pi^L(\alpha', \underline{A})] = [\Pi^L(\alpha^L(\overline{A}), \overline{A})/\Pi^L(\alpha^L(\underline{A}), \overline{A})][\Pi^L(\alpha^L(\underline{A}), \underline{A})/\Pi^L(\alpha', \underline{A})]$, upon substituting for $1 - r^*$. According to D1, if the minimum acceptance threshold is strictly lower for $\overline{A}$ than for $\underline{A}$, then the out-of-equilibrium beliefs must associate the demand α' with type $\overline{A}$ (if the two thresholds are equal, it is allowable to associate the demand α' with type $\overline{A}$, but not required). After some algebraic manipulation, it can be shown that: $(1 - \rho(\overline{A})) < (=) (1 - \rho(\underline{A}))$ as $\Pi^L(\alpha', \underline{A})/\Pi^L(\alpha^L(\underline{A}), \underline{A}) > (=) \Pi^L(\alpha', \overline{A})/\Pi^L(\alpha^L(\underline{A}), \overline{A})$. Since $\alpha' < \alpha^L(\underline{A})$ and $\overline{A} > \underline{A}$, it follows from Assumption 6 that the inequality holds. Thus, the skeptical beliefs survive refinement using D1; when the inequality in Assumption 6 is strict, then such out-of-equilibrium demands α' must be assigned $B_1(\alpha') = \overline{A}$.

5. Equilibrium Refinement using D1 to Eliminate a Separating Equilibrium at the Lower Root of Equation (6).

Recall that equation (6) has two roots (where C is indifferent between accepting and visiting L2); the larger root is what we refer to as $\alpha^L(A)$ and this is L1's preferred solution under precontracting full information. Could there be another separating equilibrium (under asymmetric information) in which the lower root is used by one or both types? Consider type $\overline{A}$; in any separating equilibrium the lawyer of type $\overline{A}$ (the weak type) plays his full-information strategy, $\alpha^L(\overline{A})$, which is accepted for sure (so that he makes his full-information payoff). Now consider the lawyer of type $\underline{A}$, and let the lower root to equation (5) be denoted as $\alpha^\#$; can there be a separating equilibrium wherein type $\underline{A}$ demands $\alpha^\#$ and type $\overline{A}$ demands $\alpha^L(\overline{A})$? First, if $\alpha^\# < \alpha^L(\overline{A})$, then type $\underline{A}$ would prefer to defect to $\alpha^L(\overline{A})$, which is higher and is accepted for sure (this configuration occurs in the power-function example, so there cannot be a second separating equilibrium at the lower root for the power-function example). Second, if $\alpha^\# = \alpha^L(\overline{A})$, then this pair of demands does not separate types. Finally, if $\alpha^\# > \alpha^L(\overline{A})$, then C must reject $\alpha^\#$ with positive probability to deter type $\overline{A}$ from defecting to $\alpha^\#$. The corresponding IC constraints are (where r is the probability that the demand $\alpha^\#$ is rejected):

$$IC(\overline{A}):\quad \Pi^L(\alpha^L(\overline{A}), \overline{A}) \geq (1 - r)\Pi^L(\alpha^\#, \overline{A});$$

$$IC(\underline{A}):\quad (1 - r)\Pi^L(\alpha^\#, \underline{A}) \geq \Pi^L(\alpha^L(\overline{A}), \underline{A}).$$

Taken together, these imply that $1 - r \in [\Pi^L(\alpha^L(\overline{A}), \underline{A})/\Pi^L(\alpha^\#, \underline{A}), \Pi^L(\alpha^L(\overline{A}), \overline{A})/\Pi^L(\alpha^\#, \overline{A})]$. Assuming that C uses the lowest rejection probability consistent with deterring mimicry by $\overline{A}$, then the equilibrium rejection probability, denoted $r^\#$, is given by the upper endpoint of the interval: $1 - r^\# = \Pi^L(\alpha^L(\overline{A}), \overline{A})/(\Pi^L(\alpha^\#, \overline{A})$. Assume for the moment that $\alpha^\#$ does provide a second separating equilibrium (rather strange out-of-equilibrium beliefs are needed to support it, but that will be discussed later). We can compare the equilibrium payoffs to the L1 of type $\underline{A}$ at the two separating equilibria (C and the L1 of type $\underline{A}$ are indifferent). In the equilibrium wherein L1 plays $\alpha^L(\underline{A})$, his expected payoff is $(1 - r^*)\Pi^L(\alpha^L(\underline{A}), \underline{A})$; using r* as defined in the text, this becomes: $\pi(\alpha^L(\underline{A}) \equiv [\Pi^L(\alpha^L(\bar{A}), \bar{A})/\Pi^L(\alpha^L(\underline{A}), \bar{A})]\Pi^L(\alpha^L(\underline{A}), \underline{A})$. In the equilibrium wherein L1 plays $\alpha^\#$, his payoff is $\pi(\alpha^\#) \equiv (1 - r^{\#\prime})\Pi^L(\alpha^\#, \underline{A}) = [\Pi^L(\alpha^L(\bar{A}), \bar{A})/\Pi^L(\alpha^\#, \bar{A})]\Pi^L(\alpha^\#, \underline{A})$. Notice that (after simplification) $\pi(\alpha^L(\underline{A}))$ (>, =, <) $\pi(\alpha^\#)$ as $\Pi^L(\alpha^\#, \bar{A})/\Pi^L(\alpha^L(\underline{A}), \bar{A})$ (>, =, <) $\Pi^L(\alpha^\#, \underline{A})/\Pi^L(\alpha^L(\underline{A}), \underline{A})$. By Assumption 6, it follows that $\pi(\alpha^L(\underline{A})) \geq \pi(\alpha^\#)$ (with a strict inequality if the ratio in Assumption 6 is strictly increasing). Thus, we argue that C should expect the L1 of type $\underline{A}$ to demand $\alpha^L(\underline{A})$, and she should accord this the belief that $B(\alpha^L(\underline{A})) = \underline{A}$, and reject this demand with probability 1 - r* (this still deters mimicry by type $\bar{A}$). Alternatively, the L1 of type $\underline{A}$ can make a speech to C to this effect: "I am demanding $\alpha^L(\underline{A})$, and you should accord this the belief that $B(\alpha^L(\underline{A})) = \underline{A}$, and reject this demand with probability 1 - r* (this still deters mimicry by type $\bar{A}$)."

An alternative way to select the equilibrium involving $\alpha^L(\underline{A})$ is to notice that, in order to support a separating equilibrium at $\alpha^\#$, C needs to reject demands in the interval $(\alpha^\#, \alpha^L(\underline{A}))$ with a sufficiently high probability. Since C would want to accept (reject) such a demand for sure if she believed it came from an L1 of type $\underline{A}$ (of type $\bar{A}$), she would have to assign a sufficiently high probability to such a demand coming from an L1 of type $\bar{A}$ in order to be willing to reject it with the requisite probability. We argue that, if the ratio in Assumption 6 is strictly increasing in A, then the beliefs required to support the separating equilibrium at $\alpha^\#$ do not survive refinement using D1. (Note that for the power function example, the ratio in Assumption 6 is constant in A; however, the "second" equilibrium at the lower root is directly eliminated in this case; see the discussion above). Therefore, in what follows, we assume that the ratio in Assumption 6 is strictly increasing in A.

Consider an out-of-equilibrium demand $\alpha' \in (\alpha^\#, \alpha^L(\underline{A}))$, and let ρ denote the probability that C believes the demand α' comes from type $\bar{A}$; then she will accept such a demand with probability $1 - \rho$. Type $\bar{A}$ would be willing to defect from his equilibrium demand $\alpha^L(\bar{A})$ to the demand α' if $(1 - \rho)\Pi^L(\alpha', \bar{A}) \geq \Pi^L(\alpha^L(\bar{A}), \bar{A})$. Type $\underline{A}$ would be willing to defect from his equilibrium demand $\alpha^\#$ to the demand α' if $(1 - \rho)\Pi^L(\alpha', \underline{A}) \geq (1 - r^\#)\Pi^L(\alpha^\#, \underline{A})$. The minimum acceptance threshold for type $\bar{A}$ is $(1 - \rho(\bar{A})) \equiv [\Pi^L(\alpha^L(\bar{A}), \bar{A})/\Pi^L(\alpha', \bar{A})]$, while the minimum acceptance threshold for type $\underline{A}$ is $(1 - \rho(\underline{A})) \equiv (1 - r^\#)[\Pi^L(\alpha^\#, \underline{A})/\Pi^L(\alpha', \underline{A})] = [\Pi^L(\alpha^L(\bar{A}), \bar{A})/(\Pi^L(\alpha^\#, \bar{A})][\Pi^L(\alpha^\#, \underline{A})/\Pi^L(\alpha', \underline{A})]$.

According to D1, if the minimum acceptance threshold is strictly higher for $\bar{A}$, then the out-of-equilibrium demand α' should be associated with type $\underline{A}$ (and thus accepted by C for sure, which would in turn induce defection from their equilibrium demands by both types, since $\alpha' > \alpha^\# > \alpha^L(\bar{A})$ and lawyers always prefer a higher contingent fee). After some algebraic manipulation, it can be shown that: $(1 - \rho(\bar{A})) > (1 - \rho(\underline{A}))$ if $\Pi^L(\alpha^\#, \bar{A})/\Pi^L(\alpha', \bar{A}) > \Pi^L(\alpha^\#, \underline{A})/\Pi^L(\alpha', \underline{A})$. Assumption 6 (with strictly increasing ratio) implies that this inequality holds.