Technical Appendix for "Products Liability, Signaling and Disclosure"
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Derivation of a unique (refined) separating signaling equilibrium when the disclosure cost is prohibitively high

When the disclosure cost is prohibitively high, then neither firm will engage in disclosure; any information transmission will occur through signaling. As shown in the text, the payoff function for a firm charging a price p, whose true safety is θ_i and whose perceived safety is $\tilde{\theta}$, is given by $\pi(p, \theta_i, \tilde{\theta}) = (p - c_i)(\alpha - (1 - \tilde{\theta})\delta - p)/\beta$.

When neither firm type discloses its safety, consumers will look to the price for information about safety. We first note that no firm type would distort its price away from its full-information price in order to be taken as an L-type firm. Hence, in a separating equilibrium, the L-type firm will charge $P_L^s = P_L^f = (\alpha_L + c_L)/2$ and receive its full-information profits $\Pi_L^s = \Pi_L^f = (\alpha_L - c_L)^2/4\beta$. However, the H-type firm is willing to distort its price (at least to some extent) in order to be taken as an H-type firm. If an H-type firm is to use its price to signal high safety, it must charge a price P_H^s such that:

(1) the H-type firm prefers to charge P_H^s and be taken as an H-type firm rather than to charge any other price (and perhaps to be taken as an L-type firm); and

(2) the L-type firm would not find it profitable to mimic this price, even if by doing so it would be taken as an H-type firm.

First, consider condition (1). If the H-type firm allows itself to be taken as an L-type firm, its best price is the one that maximizes $\pi(p, \theta_H, \theta_L)$. That is, the firm will charge $\rho_{HL} \equiv (\alpha_L + c_H)/2$ and receive profits of $\Pi_{HL} \equiv (\alpha_L - c_H)^2/4\beta$. Thus, the incentive compatibility condition for the H-type implies:

$$(P_H^s - c_H)(\alpha_H - P_H^s)/\beta \geq (\alpha_L - c_H)^2/4\beta,$$

which is satisfied for all

$$P_H^s \in A \equiv [a_1, a_2] = [.5(\alpha_H + c_H - \{(\alpha_H - c_H)^2 - (\alpha_L - c_H)^2\}^{1/2}), .5(\alpha_H + c_H + \{(\alpha_H - c_H)^2 - (\alpha_L - c_H)^2\}^{1/2})].$$

Note that the H-type firm's full information price is given by $P_H^f \equiv (\alpha_H + c_H)/2$ and that $P_H^f \in [a_1, a_2]$.

Next, consider condition (2). If the L-type firm were to charge the price P_H^s and it was therefore inferred to be of type H, it would obtain profits of $\Pi_{LH} \equiv (P_H^s - c_L)(\alpha_H - P_H^s)/\beta$. Thus, the incentive compatibility condition for the L-type implies:

$$(\alpha - (1 - \theta_L)\delta - c_L)^2/4\beta \geq (P_H^s - c_L)(\alpha - (1 - \theta_H)\delta - P_H^s)/\beta,$$

which is satisfied for all

$$P_H^s \notin B \equiv (b_1, b_2) = [.5(\alpha_H + c_L - \{(\alpha_H - c_L)^2 - (\alpha_L - c_L)^2\}^{1/2}), .5(\alpha_H + c_L + \{(\alpha_H - c_L)^2 - (\alpha_L - c_L)^2\}^{1/2})].$$

Thus, any price belonging to the set A but <u>not</u> the set B provides a separating equilibrium price for the H-type firm. Each of these equilibria is supported by consumer beliefs that interpret any other price as coming from an L-type firm. The Intuitive Criterion (Cho and Kreps, 1987) argues that such beliefs are unreasonable, since no L-type would choose such a price even if it would be inferred to be an H-type; rather, consumers should infer that such prices are associated with an H-type firm. Under these reasonable beliefs, the H-type firm would select the price that sacrifices the least profit relative to the full-information benchmark.

The following functions and their properties will be used in the proofs of two lemmas. Let $g(c) \equiv .5(\alpha_H + c - \{(\alpha_H - c)^2 - (\alpha_L - c)^2\}^{1/2})$. Since $\alpha_H > \alpha_L > 0$, it follows that $g'(c) > 0$. Let $h(c) \equiv .5(\alpha_H + c + \{(\alpha_H - c)^2 - (\alpha_L - c)^2\}^{1/2})$. Under the maintained assumption that $\alpha_L > \max\{c_H, c_L\}$ (Assumption 2), it follows that $h'(c) > 0$.

<u>Lemma 1</u>. If $c_H > c_L$, then (i) $b_1 < a_1$ and (ii) $a_2 > b_2 > P_H^f$.

<u>Proof of Lemma 1</u>. Assume that $c_H > c_L$. (i) Note that $b_1 = g(c_L)$ and $a_1 = g(c_H)$. Since $g'(c) > 0$, it follows that $b_1 < a_1$. (ii) To see that the first inequality holds, note that $b_2 = h(c_L)$ and $a_2 = h(c_H)$. Since $h'(c) > 0$, it follows that $a_2 > b_2$. The second inequality also holds, under the maintained assumptions that $\alpha_H - c_H > \alpha_L - c_L$ and $\alpha_L > \max\{c_H, c_L\}$ (Assumptions 1 and 2, resp.). QED

Lemma 1 (along with the fact that $P_H^f \in [a_1, a_2]$) provides a complete ordering of the five prices, demonstrating that the set of prices in A but <u>not</u> in B $= [b_2, a_2]$ when $c_H > c_L$. The implication of Lemma 1 is that, although a price $P_H^s < b_1$ would deter mimicry, the H-type would prefer to allow itself to be taken as an L-type rather than use such a P_H^s to signal its true type (H). On the other hand, there is an interval of prices $[b_2, a_2]$, all of which involve pricing above the H-type's full-information price, that both deter mimicry by type L and are preferable for H to allowing itself to be taken for an L-type. Thus, when $c_H > c_L$, the H-type will signal high safety by distorting its price upward (relative to full information). The price that conveys this signal with the least distortion (and thus sacrifices the least profit) is $P_H^s = b_2$.

<u>Lemma 2</u>. If $c_L > c_H$, then (i) $b_2 > a_2$; (ii) $a_1 < b_1$; and (iii) $b_1 (<=>) P_H^f$ as $(c_L - c_H)^2 (<=>) (\alpha_H - \alpha_L)(\alpha_H + \alpha_L - 2c_L)$.

<u>Proof of Lemma 2</u>. Assume that $c_L > c_H$. (i) Note that $b_2 = h(c_L)$, $a_2 = h(c_H)$, and recall that $h'(c) > 0$. Since $c_L > c_H$, it follows that $b_2 > a_2$. (ii) Note that $b_1 = g(c_L)$, $a_1 = g(c_H)$, and recall that $g'(c) > 0$. Since $c_L > c_H$, it follows that $b_1 > a_1$. (iii) This follows directly from comparing the two expressions. QED

Lemma 2 (along with the fact that $P^f_H \in [a_1, a_2]$) provides a complete ordering of the five prices, demonstrating that the set of prices in A but <u>not</u> in B = $[a_1, b_1]$ when $c_H > c_L$. The implication of Lemma 1 is that, although a price $P^s_H > b_2$ would deter mimicry, the H-type would prefer to allow itself to be taken as an L-type rather than use such a P^s_H to signal its true type (H). On the other hand, there is an interval of prices $[a_1, b_1]$, that both deter mimicry by type L and are preferable for H to allowing itself to be taken for an L-type.

As can be seen from part (iii), when $c_L - c_H$ is "moderate," then $b_1 < P^f_H$ and hence P^f_H cannot be used to signal high safety; the H-type firm must signal high safety by distorting its price <u>downward</u> (relative to full information). To get an idea of what it means for $c_L - c_H$ to be "moderate," note that if the H-type firm's full-information price is higher than that of the L-type firm – that is, if $P^f_H = (\alpha_H + c_H)/2 > (\alpha_L + c_L)/2 = P^f_L$ – then $c_L - c_H < \alpha_H - \alpha_L$ and $c_L - c_H < \alpha_H + \alpha_L - 2c_L$, which ensures that $b_1 < P^f_H$. In order to have $b_1 > P^f_H$, so that the H-type firm can signal with its full-information price, c_L must be "large" relative to c_H. Again, the Intuitive Criterion selects from the interval $[a_1, b_1]$ the price that sacrifices the least profit (relative to full information), and hence $P^s_H = \min \{b_1, P^f_H\}$.

Finally, it is straightforward to verify that $P^s_H \in (c_H, \alpha_H)$ under the maintained assumptions that $\alpha_H - \alpha_L > c_H - c_L$ and $\alpha_L > \max\{c_H, c_L\}$ (Assumptions1 and 2, resp.); thus, both the equilibrium price-cost margin and output are positive for the H-type firm. Since the L-type firm uses its full-information price, its equilibrium price-cost margin and output are also positive.

Characterization of pooling equilibria and elimination via refinement

In this section we first characterize a pooling equilibrium, in which both firms charge the same price, denoted P^P. We then argue that no such equilibrium survives refinement using the Intuitive Criterion.

If both firms charge the same price P^P, then consumers believe that the firm is of type H with probability λ, in which case they demand $(\alpha_H - P^P)/\beta$ units, and of type L with probability1 - λ, in which case they demand $(\alpha_L - P^P)/\beta$ units. Let $\bar{\alpha} \equiv \lambda\alpha_H + (1 - \lambda)\alpha_L$. Then both firms expect to sell $(\bar{\alpha} - P^P)/\beta$ units if they charge the price P^P. Both firms will pool at a price P^P if the following incentive compatibility constraints hold:

i) $(P^P - c_H)(\bar{\alpha} - P^P)/\beta \geq \max_p (p - c_H)(\alpha_L - p)/\beta$ <u>and</u> ii) $(P^P - c_L)(\bar{\alpha} - P^P)/\beta \geq \max_p (p - c_L)(\alpha_L - p)/\beta$.

These inequalities indicate that both types would prefer to charge the pooling price and sell the average quantity demanded at that price, rather than deviating to any other price and being taken as an L-type firm. That is, the pooling price is supported by beliefs that assign the worst type (type L) to any price other than the pooling price. Any price P^P satisfying these two inequalities provides a pooling equilibrium.

We need not actually construct a pooling equilibrium, as we need only show that if one exists, then there is a price to which the H-type firm could profitably defect and that would be unprofitable for an L-type firm, even if the consumer were to update her beliefs and infer that the signal came from an H-type firm. Thus, P^P fails the Intuitive Criterion if there exists P* such that:

(iii) $(P^* - c_H)(\alpha_H - P^*)/\beta \geq (P^P - c_H)(\bar{\alpha} - P^P)/\beta$ and iv) $(P^* - c_L)(\alpha_H - P^*)/\beta \leq (P^P - c_L)(\bar{\alpha} - P^P)/\beta$.

In the inequalities (iii)-(iv), the left-hand-side of each inequality is the profit that would be obtained by (respectively) the H-type and L-type firms by defecting (and being taken to be an H-type firm after the consumer has updated her beliefs), while the profits from the pooling equilibrium appear on the right-hand-side.

Let us denote the roots to the equality version of inequality (iii) as P_H^- and $P_H^+ > P_H^-$, and the roots to the equality version of inequality (iv) as P_L^- and $P_L^+ > P_L^-$. Then satisfaction of the inequalities (iii)-(iv) is equivalent to asking if there exists P* such that $P^* \in [P_H^-, P_H^+]$ and $P^* \notin [P_L^-, P_L^+]$; if so, then P^P fails the Intuitive Criterion.

The roots for the equality versions of inequalities (iii)-(iv) are given by:

$$P_i^+ = .5(\alpha_H + c_i + \{(\alpha_H - c_i)^2 - 4(P^P - c_i)(\bar{\alpha} - P^P)\}^{1/2})$$

and

$$P_i^- = .5(\alpha_H + c_i - \{(\alpha_H - c_i)^2 - 4(P^P - c_i)(\bar{\alpha} - P^P)\}^{1/2}), \text{ for } i = H, L.$$

It is straightforward to show that P_H^+ (> = <) P_L^+ and P_H^- (> = <) P_L^- as c_H (> = <) c_L. Thus, if $c_H > c_L$, then there is a non-empty interval of prices $[P_L^+, P_H^+]$ satisfying (iii)-(iv), and any P* in this interval upsets the pooling equilibrium. On the other hand, if $c_L > c_H$, then there is a non-empty interval of prices $[P_H^-, P_L^-]$ satisfying (iii)-(iv), and any P* in this interval upsets the pooling equilibrium (recall that we explicitly eliminate the knife-edge case of $c_H > c_L$, or, equivalently, k = γ; see footnote 13). Thus no pooling equilibrium survives refinement.

Comparison of equilibrium signaling profits

Claim. (a) $\Pi_H^s < \Pi_L^s$ when $c_H > c_L$; (b) $\Pi_H^s > \Pi_L^s$ when $c_H < c_L$.

Proof. (a) $\Pi_L^s = (P_L^s - c_L)(\alpha_L - P_L^s)/\beta \geq (P_H^s - c_L)(\alpha_H - P_H^s)/\beta > (P_H^s - c_H)(\alpha_H - P_H^s)/\beta = \Pi_H^s$, where the first inequality follows from Incentive Compatibility and the second follows from $c_H > c_L$.

(b) $\Pi_H^s = (P_H^s - c_H)(\alpha_H - P_H^s)/\beta \geq (P_L^s - c_H)(\alpha_L - P_L^s)/\beta > (P_L^s - c_L)(\alpha_L - P_L^s)/\beta = \Pi_L^s$, where the first inequality follows from Incentive Compatibility and the second follows from $c_H < c_L$. QED