Please note the following two typos in the published version of the paper. On page 308, equation (5) should read as:

\[- Nq^n_i FMC_x(x_i) - t \} + \{Nq^n_i \gamma \sum_j [- \partial q^n_j / \partial x_j] \} = 0.

Notice the “j” subscript in the summation within the second set of braces; in the published version, this appears as an “i” subscript. Second, on page 309, line nine should read:

“[- \partial q^n_j / \partial x_j] = - \gamma FMC(x_j) / [(2\beta - \gamma)(2\beta + (n-1)\gamma)] > 0, so that ...”

Again, note the “j” subscript instead of the “i” subscript that appears in the published version. While both of these errors cropped up somewhere along the line in the galleys, we apologize for having missed them.

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Another useful benchmark is captured by considering a social planner who can set both safety effort and output. We refer to this as the unrestricted social planner (USP) and provide here results comparable to those provided in the main text for the restricted social planner (RSP).¹

**Social Optimality in the Two-Party Case**

In the USP case, the optimal levels will be denoted $X^n$ and $Q^n$. USP’s problem is:

\[
\text{max}_{X,Q} NU(Q_1, ..., Q_n) - \sum_i [NQ_i FMC(X_i) + tX_i]. \quad (WA.1)
\]

\[
Q^n = (\alpha - FMC(X^n)) / (\beta + (n-1)\gamma). \quad (WA.2)
\]

\[
H(X^n; A^n) = 0, \text{ where } A^n = (t/N)(\beta + (n-1)\gamma). \quad (WA.3)
\]

Note that $A^{ni} > A^n$ for each $(\gamma, n)$, which means that USP chooses a higher level of safety effort than does RSP: $X^n > X^{ni}$. Thus, RSP prefers a lower level of safety effort than does a “broader” social planner (USP), because RSP anticipates inefficiently low output. Moreover, since both $A^n$ and $A^{ni}$ are monotonically increasing in $\gamma$ and in $n$, both $X^n$ and $X^{ni}$ are monotonically decreasing in $\gamma$ and in $n$ (for $\gamma > 0$). Finally, from the definitions of $A^n$ and $A^{ni}$ it is evident that $X^n$ and $X^{ni}$ are both increasing in $N$ (and $\alpha$) and decreasing in $t$ and $\beta$. 
Comparisons Between USP’s Choices and the Equilibrium Oligopoly Outcomes

Let us compare $x^n$ and $q^n$ with $X^n$ and $Q^n$. Comparing (WA.2) and (6) in the main text, it is immediately clear that, for any fixed level of safety effort, $q^n(x) < Q^n(x)$. Comparing the solutions to (7) in the main text and (WA.3) amounts to comparing $a^n$ and $A^n$. Once again, from the properties of $H$ we know that $x^n \geq X^n$ if and only if $A^n > a^n$. Some tedious algebra shows that, when $n = 2$, $A^n < a^n$ for all values of $\gamma$, and thus that $X^n > x^n$ for all values of $\gamma$ when $n = 2$. However, for $n \geq 3$, $A^n > a^n$ as $\gamma > \Gamma_n(\beta, n)$, where $\Gamma_n(\beta, n)$ equates $A^n$ and $a^n$. $\Gamma_n(\beta, n)$ provides the value of $\gamma$ wherein the function describing $X^n$ just crosses that for $x^n$; at this point, the equilibrium safety effort produced by the $n$-firm oligopoly is the same as the unrestricted social planner would have chosen. It can be shown that there is a unique $\Gamma_n(\beta, n) \in (0, \beta)$ for all $n > 2$ and that $\partial \Gamma_n(\beta, n)/\partial n < 0$. Thus, a result similar to that under RSP holds: the set of $\gamma$-values wherein $x^n$ exceeds $X^n$ increases as $n$ increases. We formalize this in Proposition WA1 (which is the USP version of Proposition 2 in the main text) and illustrate this in Figure WA1 below.

**Proposition WA1.**

i) For any given $(\gamma, n)$, the restricted planner chooses a lower level of safety than the unrestricted planner: $X^n > x^n$. Moreover, $X^n$ and $x^n$ are both decreasing in $t, \beta, \gamma$ and $n$ (for $\gamma > 0$), and increasing in $N$ and $\alpha$.

ii) With respect to the unrestricted planner’s level of safety, firms in market equilibrium under-supply safety if and only if $\gamma < \Gamma_n(\beta, n)$; otherwise they over-supply safety. Moreover, when firms under-supply safety they under-supply output as well.

iii) $\Gamma_n(\beta, n) \in (0, \beta)$ for all $n > 2$ and $\partial \Gamma_n(\beta, n)/\partial n < 0$.

**Social Optimality with Third Parties and $t > 0$**

Next we consider the unrestricted planner’s choice of safety effort and output, now in the presence of third parties. We again assume that USP’s (reduced-form) problem is strictly concave in $X$. Thus, USP’s problem is $\max_{X,0} \{N[n(\alpha Q - \beta Q^2/2 - (n-1)\gamma Q^2/2) - nFMC^\theta(X)Q] - ntX\}$. Let $(X^n, Q^n)$ solve USP’s problem; then they satisfy the following conditions:

$$Q^n = (\alpha - FMC^\theta(X^n))/(\beta + (n-1)\gamma),$$

and
Since t > 0, $X^n < \tilde{x}$. To understand the relationship between $x^n$ and $X^n$, we re-write (A.6) as:

$$- (\alpha - \text{FMC}^S(x^n))\text{FMC}^S(x^n) - (t/N)(\beta + (n-1)\gamma) = 0.$$  \hspace{1cm} (WA.5)

Now recall (12) in the text, the firm’s first-order-condition for the choice of safety effort, which can be re-expressed as:

$$- (\alpha - \text{FMC}^f(x^n))\text{FMC}^f(x^n) - (t/N)((2\beta - \gamma)(2\beta + (n-1)\gamma)^2)/([2\beta - \gamma)(2\beta + (n-1)\gamma) + \gamma^2(n-1)] = 0.$$  \hspace{1cm} (WA.6)

From its definition in Section 4, at $\gamma = \Gamma^n(\beta, n)$, the terms multiplying $(t/N)$ in (WA.6) and (WA.7) are equal, so that at $\gamma = \Gamma^n(\beta, n)$, $x^n = X^n$ if $\tilde{u}(\bullet) = \tilde{u}_*(\bullet) = 0$. However, since $\tilde{u}(\bullet) > 0$ and $\tilde{u}_*(\bullet) < 0$, when $\varphi > 0$ and the term in square brackets in (WA.6) is positive when evaluated at $x^n$, then $x^n < X^n$ at $\gamma = \Gamma^n(\beta, n)$. This last result holds because the sign of the left-hand-side of (WA.6) at $x^n$ is positive; hence, $X$ needs to be increased to achieve optimality for USP.
Market Equilibrium and Social Optimality in the Three-Party Case when $t$ is Small

Since third-party harm depends upon both the level of safety effort and the level of output, but RSP can only adjust the safety level, this will mean that $X^{nq}$ is likely to differ from $X^n$. In particular, one should expect there to be circumstances such that $X^{nq}$ exceeds $X^n$. This is important because if RSP’s choices resemble the decisions of courts determining whether “sufficient” safety effort was employed then employment of tort law (which generally is mute on the question of whether a defendant produced an appropriate level of output) may lead to standard setting that is yet greater than that which USP would have chosen, something that will never occur in the two-party case.

When $t = 0$, it is immediate that $x^n = \bar{x}^f$ and that $X^n = \bar{x}^s$. Since $\bar{x}^f < \bar{x}^s$, this means that $x^n < X^n$. Hence, from the perspective of USP, the market under-supplies safety effort. Under the assumption that $t = 0$, RSP’s optimal safety effort level is implicitly defined by:

$$- (\alpha - \text{FMC}^f(X^{nq}))\text{FMC}^i(X^{nq})$$
$$+ \phi \left[ (2\beta + (n-1)\gamma)/(3\beta + (n-1)\gamma) \right] [u(X^{nq})\text{FMC}^i(x^n) - u_i(x^n)(\alpha - \text{FMC}^f(X^{nq})] = 0.$$  (WA.8)

If (WA.8) is evaluated at $x = \bar{x}^f$, then $\text{FMC}^i(\bar{x}^f) = 0$ and the left-hand-side is positive, meaning that $X^{nq} > \bar{x}^f$. Thus, when $t = 0$, the market always supplies too little safety from the perspective of RSP, independent of $\gamma$ and $n$. Moreover, this means that $\text{FMC}^i(x^{nq}) > 0$. That is, RSP would prefer the firm to operate on the upward-sloping portion of $\text{FMC}^i$.

More significantly, there are conditions under which $X^{nq} > X^n$. To see this, re-consider (WA.8) above, which can be re-written as:

$$- Q^{nq}(x)\text{FMC}^i(x) - (\beta Q^{nq}(x) - \varphi u(x))\text{FMC}^i(x)/(2\beta + (n-1)\gamma) = 0.$$  

At $x = \bar{x}^s$, $\text{FMC}^i(x) = 0$ and $\text{FMC}^i(x) > 0$. The above first-order-condition, evaluated at $\bar{x}^s$, is positive if $\beta Q^{nq}(\bar{x}^s) < \varphi u(\bar{x}^s)$. Thus, $X^{nq} > \bar{x}^s = X^n$ if and only if $\beta Q^{nq}(\bar{x}^s) < \varphi u(\bar{x}^s)$. This condition clearly fails when $\varphi = 0$. However, consider $\varphi$ positive and fixed. Notice that $Q^{nq}(\bar{x}^s)$ is a monotonically decreasing (to zero) function of $n$, while $\varphi u(\bar{x}^s)$ is independent of $n$. Thus, for any given $\varphi$, there always exists an $n$ such that $\beta Q^{nq}(\bar{x}^s) < \varphi u(\bar{x}^s)$. This result is summarized in Proposition WA2.

**Proposition WA2.** If $t = 0$ and $\varphi > 0$, then there exists an $n^*$ (dependent upon $\varphi$) such that:

$$n > n^* \Rightarrow X^{nq} > X^n.$$
In other words, if there are enough firms in the market equilibrium, RSP will set a standard for safety effort which is distorted upwards from what USP would set. This proposition once again reflects the interdependence of safety effort and quantity, in this case because of the presence of third parties. Since RSP cannot instruct firms to adjust their output (i.e., force them to adhere to Q^n instead of q^n), the RSP solution is to increase safety effort, certainly above what the firms would choose, and possibly above what USP would choose.

It seems somewhat odd that RSP would prefer a level of safety effort that exceeds x^S. However, this can be understood if we make the following observations. First, a non-cooperative firm has an incentive to produce less output than USP would prefer, as seen by comparing the denominators of q^n(x) and Q^n(x); this reflects the oligopoly incentive to restrict output. On the other hand, a non-cooperative firm has an incentive to produce more than USP would choose, since the firm faces FMC_f, not FMC_S; that is, the firm does not face the full social costs. Thus, just as it is possible for X^n to exceed X^s, it is possible for Q^n (or, equivalently, q^n) to exceed Q^s. Indeed, it can be shown that βQ^n(x^S) < φu~(x^S) (and hence, X^n > x^s = X^s) if and only if Q^n(x^S) > Q^n(x^S). In this case, non-cooperative firms would produce too much output, and thus RSP raises X^n beyond x^S in order to raise FMC_f (which is increasing in x at x^S) and thereby induce the non-cooperative firms to reduce their output, which reduces uncompensated losses to third parties.

Finally, it bears repeating that since these results hold for t = 0, they hold for sufficiently small t as well (or sufficiently large α or N). Thus, at least when markets for the products are sufficiently large, or when per-unit safety effort cost is small, there is a substantial divergence between what the firm would choose to do in equilibrium, what courts might wish to impose as a standard for safety effort, and what a social planner with the ability to adjust both x and q would choose.

Summary of Welfare Comparisons Between USP and RSP

We found that, in general, USP and RSP disagree about the appropriate safety standard to set. In the two-party case, RSP always sets the safety standard too low when compared with USP, while in the most significant third-party setting (high exposure rate, large third-party uncompensated losses or a large number of firms), RSP may set the safety standard too high when compared with USP. This is because RSP has direct control only over safety effort, which it then adjusts so as to
indirectly manipulate output. Furthermore, this limitation on instruments means that a court, which is analogous to RSP, will not be able to properly pursue standard-setting (for example, if the legal regime involved a negligence regime instead of strict liability) unless it performs a full analysis of the product market (the number of firms, the nature of the oligopolistic interaction, the degree of product substitutability, etc.) along with an analysis of the costs of safety design and product manufacture. In order to achieve the full social optimum, the court would need to have the authority to adjust output standards, too. This would mean a substantial change in our current institutional design of the legal system. An alternative to the use of litigation is _ex ante_ regulation of safety effort and output. Although agencies with these powers exist, they tend to have divided jurisdictions (some governing safety while others focus on output), while the problem calls for coordinated regulation.

**Social and Market Implications of Assuming Price Competition for the Market Subgame**

We have assumed that firms choose quantity strategies (rather than price strategies) for two reasons. First, and most important, we want the safety effort choice problem to be well-behaved for all degrees of product differentiation (that is, ranging from independence to perfect substitutes). This is not true of a price model, since as products become perfect substitutes the firms compete away all their profits. Since the market subgame cannot generate profits sufficient to cover safety investments, there cannot be a Nash equilibrium in pure (safety effort) strategies with positive safety effort in the limit as $\gamma - \beta$. At the same time, there cannot be a Nash equilibrium with zero safety effort (unless $t$ is prohibitively high). This holds since if all rival firms are choosing $x = 0$, a firm $i$ that deviates to safety effort $x_i > 0$ will be able to capture all sales to all consumers at a profit margin of $FMC(0) - FMC(x_i)$, thereby making a profit if $t$ is not too high. Thus, equilibrium will require the use of mixed strategies (in safety effort) when the products are sufficiently good substitutes. Second, the model is algebraically much simpler in the case of quantity strategies since consumers simply deduct anticipated uncompensated losses from their willingness to pay for the product (a simple shift of the inverse demand curve). This results in the firm facing the full marginal cost of its product. The analogous expressions are much more complex when the ordinary demand curves are used.

Nevertheless, we have explored the pricing subgame, and some of its implications for the
choice of safety effort. Firm j’s equilibrium price is a decreasing function of firm i’s safety effort level; that is, higher safety effort by firm i intensifies price competition from its rivals. Thus, firm i now faces an additional cost associated with higher safety effort due to the strategic response of rivals, which is to reduce their prices. This is the opposite of what occurs when firms choose outputs; in that case, firm i enjoys an additional benefit associated with higher safety effort due to the strategic response of rivals, which is to reduce their output levels. This suggests that the under-provision of safety effort (in equilibrium) may be even more likely when firms compete in price strategies.

Endnotes

1. Note, we do not consider the case of a social planner who can also manipulate the number of firms. A detailed analysis is beyond the scope of this paper, but allowing products which are imperfect substitutes suggests that social optimality may involve n > 1.

2. Firm i’s profit will be N[α - FMC(0)][FMC(0) - FMC(x_i)]/β - tx_i; unless t is very high, this will be maximized at a positive value of x_i.