

# Partisan strength and legislative bargaining

Journal of Theoretical Politics

2019, Vol. 31(1) 6–45

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DOI: 10.1177/0951629818809416

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## Abstract

We extend the canonical Baron–Ferejohn model of majoritarian legislative bargaining in order to analyze the effects of partisanship on bargaining outcomes. We consider three legislators, two of whom are party affiliated, with each partisan placing some value on the share of the dollar obtained by his copartisan in addition to his own share. We characterize the equilibrium of our model as a function of the strength of party affiliation and the degree to which the legislators have concern for the future; and we determine how the equilibrium varies in response to changes in these two parameters. We show how partisanship advantages the affiliated legislators relative to the nonpartisan and identify the circumstances in which a majority party legislator proposes a bipartisan outcome.

## Keywords

Baron–Ferejohn; distributive politics; legislative bargaining; partisanship

## 1. Introduction

Casual observers of American politics are constantly bombarded with headlines about how contemporary legislative politics have become increasingly partisan.

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Whether due to the natural geographic sorting of voters into more ideologically homogeneous constituencies (e.g., Chen and Rodden, 2013; Dodd and Oppenheimer, 2017), gerrymandering (e.g., La Raja, 2009), or the general empowerment of party leaders (e.g., Aldrich and Rohde, 2017), many congressional scholars have suggested that parties in the United States Congress are more ideologically divergent than they have been for over a century (e.g., Hare and Poole, 2014). Likewise, scholars of political psychology (e.g., Hetherington and Rudolph, 2015) have argued that this increase in partisanship has corresponded to a decrease in trust across the parties, which makes the policymaking process increasingly contentious in Washington.

These scholarly and journalistic accounts suggest that the rise of partisan conflicts in Congress has led to a rise in legislative gridlock or, at least, an increase in the incidence of one-sided policy outcomes being obtained, with the minority party being effectively shut out of the policymaking process. As legislators have come to place greater value on partisanship, majority party members have had less of an incentive to reach out to members of the minority party for their support even on relatively minor legislative issues (Lee, 2016); and the majority party leadership have had more of an incentive to make concessions to dissonant voices in their own party. In addition, members of the minority party are increasingly expected to do whatever they can to prevent a policy victory for the majority party, even if that victory might benefit the minority party's interests as well.

While such arguments seem sensible at face value, and comport nicely with recent journalistic accounts of contemporary legislative politics, other factors may limit their applicability. Of particular importance is the way that partisanship interacts with the value that legislators place on future outcomes, such as a party obtaining (or retaining) majority party status or successfully implementing its agenda in later sessions of the legislature. It is plausible to conjecture that in order to secure policy agreements, in *some* circumstances the leadership of a majority party would be either willing to reach out to members of the minority party or willing not to cater to extreme interests in its own party. It is also plausible to believe that there are circumstances in which members of a minority party are able to offer some members of the majority party *something* so as to secure policy outcomes that they find mutually advantageous.

To engage with these issues, we develop a model of legislative bargaining over a fixed divisible resource—a dollar—that builds on the distributive politics models of Baron and Ferejohn (1989) and Calvert and Dietz (2005). Our model accounts for both the strength of a legislator's party affiliation and the extent to which future outcomes matter for current negotiations. Specifically, we consider three legislators, two of whom (the partisans) belong to the same political party, who bargain over how to divide a dollar. In each period until majority agreement is reached, a legislator is recognized with equal probability to make a proposal. Future shares are discounted by a common discount factor. The nonpartisan's payoff is simply the present value of his share of the dollar. A partisan's payoff, however, consists of a fraction of what his copartisan obtains added to the amount that he receives

(discounted to the present). The weight that a partisan places on his copartisan's share of the dollar is our measure of the strength of partisanship (i.e., of party affiliation).

We allow both the party affiliation parameter and the discount factor to vary. Hence, our model allows us to assess how the outcome of legislative bargaining depends on both the strength of party affiliation and on the value that legislators place on future payoffs. The model of Baron and Ferejohn (1989) for the case when there are three legislators and the partisan symmetric baseline model of Calvert and Dietz (2005) are special cases of our own. Baron and Ferejohn permit the discount factor to vary, but assume that legislators only care about their own shares. Calvert and Dietz allow for any degree of partisan affiliation, but only consider the case in which the legislators are infinitely patient (i.e., the discount factor is 1).

The Baron–Ferejohn model is concerned with determining how particularistic goods—the shares of a fixed resource—are distributed when the interests of the legislators exhibit an extreme form of conflict: each legislator seeks to maximize his own share at the expense of the other legislators. By having a partisan care about what his copartisan receives, not just about his own share, as in Calvert and Dietz (2005), we are regarding partisanship as a form of preference similarity. This is a natural way to introduce preference similarity when there is no ideological component to a legislator's preferences, as is the case here when bargaining only concerns the distribution of a benefit. The concern for a copartisan's share could arise because legislators might have electoral (or within party) benefits from providing goods to a copartisan's district. For example, when a majority party's members have similar interests, the party is better able to deliver on the policies it has proposed, which enhances its members' chances of being re-elected.

As in Calvert and Dietz (2005), a partisan trades off his share with that of his copartisan at a fixed rate. In other words, the partisans do not exhibit any inequality aversion about how the resource is shared among themselves. Furthermore, the payoff externality appears symmetrically in the two partisans' utility functions. Both of these assumptions could be relaxed. Montero (2007) extends the  $n$ -person Baron–Ferejohn legislative bargaining model by having each legislator care about the equitableness of the division of the dollar, not just his own share. Montero (2008) relaxes the symmetry assumption in a three-legislator model by allowing the rate at which a legislator trades off shares to be legislator specific. However, she assumes that each legislator weights the shares received by each of the other legislators by a common factor, and so does not allow for partisanship.<sup>1</sup> Allowing for either inequality aversion or differential share weights when there is partisanship would significantly complicate the analysis, so we do not consider these possibilities here. Our results can be viewed as providing a benchmark for assessing the extent to which these factors play a role in determining the equilibrium shares.

The assumption that members of the same political party experience positive utility from shares of the dollar being allocated to their copartisans is consistent with recent scholarship on the 'particularistic president' (e.g., Kriner and Reeves, 2015; Rogowski, 2016), which has demonstrated how presidents have historically distributed federal dollars disproportionately to areas represented by their

copartisans. It is likewise consistent with the empirical findings of Levitt and Snyder (1995) and others, who have demonstrated how the distribution of federal outlays by Congress is also biased toward districts whose voters are mostly of the same party affiliation as the majority party.<sup>2</sup>

Baron (1989) regards a party as being a means by which a group of legislators can gain control of the legislature and thereby obtain legislative outcomes that benefit themselves. Krehbiel (1993), however, challenges this view because it cannot distinguish between the outcomes that are obtainable by a party from those obtainable by unaligned legislators acting independently when the degree of preference similarity is the same in both cases. Krehbiel's point applies to our model as well. While we interpret our model as one in which two legislators belong to the majority party and the third legislator constitutes the minority party, we could equally well view our model as being one in which there are no parties and outcomes are determined by the degree to which preferences are similar. The latter interpretation is the one adopted by Calvert and Dietz (2005), who regard their analysis as providing a baseline from which the influence of parties on legislative outcomes can be determined. For concreteness, we employ the party-based interpretation of our model in the rest of this article, but it should be borne in mind that it can also be interpreted in terms of independent legislators, two of whom exhibit some degree of preference similarity.

We restrict our attention to partisan symmetric stationary subgame perfect equilibria in this bargaining game. As in Baron and Ferejohn (1989) and Calvert and Dietz (2005), agreement is reached without delay. In our main proposition, we provide a complete description of the equilibrium of our model as a function of the strength of party affiliation and the discount factor. We show that the range of possible values for the discount factor can be partitioned into three intervals whose boundaries depend on the value of the party affiliation parameter, with the qualitative features of the equilibrium proposals differing between these intervals. When legislators place a high value on potential future interactions, a majority party proposer is willing to seek the support of either his copartisan or the nonpartisan by offering enough of the dollar to obtain a vote in favor of his proposal. Thus, bipartisan coalitions can be obtained in equilibrium with positive probability in this case.<sup>3</sup> For smaller values of the discount factor, however, the prospects for bipartisan agreements break down. For intermediate values, a majority party proposer is only willing to split the dollar between himself and his copartisan. When legislators are sufficiently impatient, a majority party proposer keeps the whole dollar for himself, relying on the strength of his copartisan's party affiliation to obtain his support. Only the first of these three possibilities is identified by Calvert and Dietz (2005) because they assume that the discount factor takes its highest possible value, 1.

In addition to characterizing the equilibria of our model, we also perform a number of comparative static exercises. In particular, we investigate how the features of the equilibrium proposals and how the shares of the dollar that each legislator can expect to receive, *ex ante*, respond to changes in the strength of party affiliation and in the discount factor.

Alternative ways of modeling legislative partisanship have been proposed. The models of negative agenda setting analyzed by Cox and McCubbins (2005), for example, account for the role of partisanship by identifying the majority party median as the agent that has particular procedural rights; specifically, the power to keep policy proposals off the agenda. Likewise, Krehbiel et al. (2015) account for legislative partisanship by identifying party agenda setters as the agents that have particular parliamentary rights or the ability to engage in vote buying. Krehbiel and Meirowitz (2002) assume that party affiliations determine which agents move first in advancing a sequence of policy proposals and amendments in the legislative process, with a majority party member moving first, followed by the minority party legislator. Diermeier and Feddersen (1998) show that institutional features of legislatures, particularly the parliamentary vote of confidence, can promote majority party cohesion that facilitates coalition building based on the expectations of future payoffs to this party's legislators. Baron (1993) describes the effects of legislative bargaining institutions on the electoral competition between parties in a proportional representation system; whereas Baron (1989) explores the relationships between the sizes of coalitions (which could be interpreted as being political parties) and legislative bargaining outcomes. However, none of these models (nor many others), parsimoniously captures the value of partisanship to rank-and-file members of the majority or minority parties independent of the spatial location of their ideal points (which are often assumed to be positively correlated with party affiliation) or their procedural rights.<sup>4</sup> Hence, analysis of our model reveals numerous insights that speak to ongoing debates about the impact of partisanship on legislative politics; the model is sufficiently tractable so as to facilitate further analytical extensions in future scholarship.

Our model of partisan legislative bargaining is presented in Section 2. The solutions for the special cases of our model considered by Baron and Ferejohn (1989) and Calvert and Dietz (2005) are described in Section 3. The equilibria of our model are characterized in Section 4. We investigate how the equilibrium proposals respond to variations in the party affiliation parameter and the discount factor in Section 5. In Section 6, we determine the equilibrium ex ante shares of the legislators and show how they vary with changes in these two parameters. Some concluding remarks are offered in Section 7. The proofs of our results may be found in the appendix.

## 2. The partisan legislative bargaining game

There are three legislators (indexed by  $i = 1, 2, 3$ ) whose objective is to determine how to distribute a unit of benefits, which we hereafter refer to as a *dollar*, among themselves. A *distribution* is a nonnegative vector  $x = (x_1, x_2, x_3)$  for which  $\sum_i x_i = 1$ . The set of possible distributions is  $X$ .

Legislators 1 and 2 are members of the same party (hereafter referred to as the *partisans*), each of whom cares not only for his own share of the dollar, but also for the share received by his *copartisan*. Each of the partisan legislators assigns the same weight to his copartisan's share, with this weight being strictly less than the

weight of 1 that he assigns to his own share. Legislator 3, the *nonpartisan*, only cares about his own share of the dollar. Formally, the three legislators' utilities obtained with the distribution  $x$  in the period in which  $x$  is adopted are

$$U^1(x) = x_1 + \alpha x_2, \quad (1)$$

$$U^2(x) = x_2 + \alpha x_1, \quad (2)$$

and

$$U^3(x) = x_3, \quad (3)$$

where  $\alpha \in [0, 1)$ . The parameter  $\alpha$  measures the extent to which either of the partisans takes account of his copartisan's share of the dollar. It can be interpreted as being the value of party affiliation. If  $\alpha = 0$ , each partisan legislator only cares about his own share, which is the case considered by Baron and Ferejohn (1989).

Legislators discount future payoffs using a common *discount factor* of  $\delta \in [0, 1]$ . Thus, if  $t$  periods in the future legislator  $i$  receives a utility payoff of  $u^i$ , this has a present value of  $\delta^t u^i$ . If  $\delta = 1$ , there is no discounting, which is the case considered by Calvert and Dietz (2005). When  $\delta < 1$ , the legislators exhibit impatience, preferring to receive benefits sooner rather than later.

As in Baron and Ferejohn (1989), in the initial period, a legislator is recognized with equal probability to make a proposal for dividing the dollar. We assume that the legislature uses a *closed rule*, in which the distribution  $x$  that has been proposed is voted on without amendment against the status quo. If a simple majority votes in favor of  $x$ , the bargaining ends and the dollar is distributed according to the agreed upon allocation. If  $x$  fails to secure a majority, however, this procedure is repeated with a one period delay. Bargaining continues until there is majoritarian agreement.

A *strategy* for a legislator consists of a proposal rule and a voting rule. A legislator's *proposal rule* specifies for each period what proposal he would make should he be recognized. His *voting rule* specifies for each period which distributions he would vote for. In general, the decisions in each period could depend on the past history of the bargaining. Following Baron and Ferejohn (1989) and Calvert and Dietz (2005), however, we restrict our attention to stationary strategies, in which the decisions made in any period are not contingent on past history. With a *stationary strategy*, a legislator makes the same proposal each time he is recognized and only considers the current distribution being voted on when deciding whether to support it. We exploit the symmetry of legislators 1 and 2 in the model by supposing that legislator 2's proposal is the same as that of legislator 1 *mutatis mutandis*.<sup>5</sup> We call such a strategy *partisan symmetric*. Our equilibrium concept is partisan symmetric stationary subgame perfect equilibrium. In a *partisan symmetric stationary subgame perfect equilibrium*, (i) each legislator uses a partisan symmetric stationary strategy and (ii) the profile of the three legislators' strategies is a Nash equilibrium when restricted to any subgame.

Because the proposer only needs the support of one of the other legislators to obtain a majority, he will only offer just enough of the dollar to one other legislator that will secure his support. For legislator 1, he proposes the distribution

$$x^{12} = (1 - x_P, x_P, 0) \quad (4)$$

if he wants to secure the support of his copartisan and the distribution

$$x^{13} = (1 - x_N, 0, x_N) \quad (5)$$

if he wants to secure the support of the nonpartisan. He chooses to seek the support of his copartisan with probability  $p$  and of the nonpartisan with probability  $1 - p$ , where  $p \in [0, 1]$ . In summary, a (*stationary*) *proposal for legislator 1* consists of (i) the two distributions  $x^{12}$  and  $x^{13}$  and (ii) the probability  $p$  with which the first distribution is offered.<sup>6</sup>

A (*stationary*) *proposal for legislator 2* consists of the distributions

$$x^{21} = (x_P, 1 - x_P, 0) \quad (6)$$

and

$$x^{23} = (0, 1 - x_N, x_N) \quad (7)$$

together with the probability  $p$  that the first distribution is offered.

Because legislator 3 wants to secure the support of one other legislator at the least cost to himself, he will offer the same amount  $x_S$  to secure the support of either of them. Because the other two legislators stand in exactly the same relationship to legislator 3, we suppose that he seeks their support with equal probability. Thus, a (*stationary*) *proposal for legislator 3* consists of the distributions

$$x^{31} = (x_S, 0, 1 - x_S) \quad (8)$$

and

$$x^{32} = (0, x_S, 1 - x_S), \quad (9)$$

with each offered with probability  $1/2$ .<sup>7</sup>

A legislator's *continuation value* is his expected utility at the beginning of the next period should agreement not be reached in the current period. With our stationarity assumption, this value is time invariant. The continuation value discounted by one period is the utility that a legislator must be provided in order to gain his support for the distribution being voted on.

The continuation value of legislator  $i$  is

$$V^i = \frac{1}{3} [pU^i(x^{12}) + (1 - p)U^i(x^{13})] + \frac{1}{3} [pU^i(x^{21}) + (1 - p)U^i(x^{23})] + \frac{1}{3} \left[ \frac{1}{2} U^i(x^{31}) + \frac{1}{2} U^i(x^{32}) \right]. \quad (10)$$

The first term in square brackets in this expression is the expected utility obtained with legislator 1's proposal. Similarly, the second and third terms in square brackets are his expected utilities obtained with the proposals of legislators 2 and 3, respectively. All three terms receive a weight of  $1/3$  because each legislator is recognized

with equal probability. Using equations (1) to (9), the continuation values in equation (10) can be rewritten as

$$V^1 = V^2 = \frac{1}{3}(1 + \alpha) \left[ 1 - (1 - p)x_N + \frac{1}{2}x_S \right] \quad (11)$$

and

$$V^3 = \frac{1}{3}[2(1 - p)x_N + 1 - x_S]. \quad (12)$$

Henceforth, we refer to the game introduced in this section as the *partisan legislative bargaining game*.

### 3. Benchmark cases

Before analyzing our general model of partisan bargaining, we begin by presenting the benchmark cases considered by Baron and Ferejohn (1989) and Calvert and Dietz (2005). The former is the special case of our model in which legislators are purely self-interested, but discounting of future payoffs is permitted. The latter is the special case in which partisans may care about each other's welfare, but there is no discounting.

#### 3.1. Baron–Ferejohn

Baron and Ferejohn (1989) consider the special case of our model in which  $\alpha = 0$ .<sup>8</sup> Because each legislator is only concerned with his own share of the benefit, party affiliation has no influence over the equilibrium allocations of the dollar.

For the case where  $n = 3$ , in equilibrium, any proposer offers with equal probability the fraction  $\delta/3$  of the dollar to one of the other two legislators, with the remaining  $1 - \delta/3$  allocated to himself. Each legislator has a continuation value of  $1/3$ , as can be confirmed by setting  $\alpha = 0$ ,  $p = 1/2$ , and  $x_N = x_S = \delta/3$  in equations (11) and (12). Thus,  $\delta/3$  is what needs to be offered to gain a legislator's support; so each legislator will vote for any offer of at least this amount. Thus, the distribution voted on in the first period is adopted, and each pair of legislators forms a winning coalition with equal probability.

The proposer's share is decreasing in  $\delta$ . If  $\delta = 1$ , the legislators are infinitely patient, with the consequence that in equilibrium the proposer receives  $2/3$ , whereas any coalition partner receives  $1/3$ . If  $\delta = 0$ , however, nobody cares about the future, thereby removing any threat value of withholding support for a proposal. Hence, the proposer can allocate all of the dollar to himself.

#### 3.2. Calvert–Dietz

Calvert and Dietz (2005) consider the special case of our model in which there is no discounting ( $\delta = 1$ ). As we do here, they allow for any amount of symmetric



altruism between the two partisans provided that a partisan assigns less weight to his copartisan than to himself.

The equilibrium values of the choice variables are functions of the value  $\alpha$  that partisans place on each other's share. A partisan proposer (i) seeks the support of his copartisan with probability  $p = (1 + \alpha)/2$  and (ii) he offers  $x_P = 1/(\alpha + 3)$  to his copartisan and  $x_N = (1 - \alpha)/(\alpha + 3)$  to the nonpartisan. The nonpartisan offers  $x_S = \alpha + x_N = (\alpha^2 + 2\alpha + 1)/(\alpha + 3)$  with equal probability to the other two legislators, if recognized. The continuation values are  $x_S$  for each of the partisans and  $x_N$  for the nonpartisan, with each legislator voting to accept any distribution that provides him with his continuation value. Thus, as in the Baron–Ferejohn equilibrium, agreement is reached without delay.

For the special case in which  $\alpha = 0$ , the equilibrium is identical to the Baron–Ferejohn equilibrium for the case in which there is no discounting. The probability that a partisan seeks the support of his copartisan is increasing in the parameter  $\alpha$ , attaining its minimum value of  $1/2$  when  $\alpha = 0$  and approaching a limiting maximum value of  $1$  as  $\alpha$  goes to  $1$ . A partisan proposer exploits his copartisan's party affiliation by decreasing his offer  $x_P$  to him as the value of  $\alpha$  is increased. The offer attains its maximum value of  $1/3$  when  $\alpha = 0$  and it is bounded below by the limiting value of  $1/4$  as  $\alpha$  goes to  $1$ . As  $\alpha$  increases, there is less need for a partisan to solicit the support of the nonpartisan. Consequently, the offer  $x_N$  made to the nonpartisan is decreasing in  $\alpha$ , attaining its maximum value of  $1/3$  when  $\alpha = 0$  and having a value of  $0$  in the limit when  $\alpha$  goes to  $1$ . The difference  $x_P - x_N$  in the offers made to a copartisan and to the nonpartisan is increasing in  $\alpha$ , ranging from  $0$  when  $\alpha = 0$  to a limiting value of  $1/4$  as  $\alpha$  approaches  $1$ . As  $\alpha$  increases, the nonpartisan must make an ever more generous offer  $x_S$  to gain the support of a partisan, ranging from an offer of  $1/3$  when  $\alpha = 0$  to a limiting value of  $1$  as  $\alpha$  approaches  $1$ . The probability that the partisans form a winning coalition is equal to  $2p/3 = (1 + \alpha)/3$ . This value is increasing in  $\alpha$ , attaining its minimum value of  $1/3$  when  $\alpha = 0$  and approaching a limiting maximum value of  $2/3$  as  $\alpha$  goes to  $1$ .

### 3.3. Comparison of the benchmarks

In comparing across these benchmark results, we see that regardless of the strength of party affiliation, with positive probability, a partisan proposer in the Calvert–Dietz model offers the nonpartisan a share of the dollar. In other words, when  $\delta = 1$ , there is always the prospect for a bipartisan outcome. Moreover, should a partisan offer a positive share to his copartisan, he retains a larger share of the dollar  $((2 + \alpha)/(3 + \alpha))$  than the share retained ( $2/3$ ) in the Baron–Ferejohn model when there is no discounting. Nevertheless, the copartisan supports receiving a smaller share himself because of the benefit he receives from the partisan proposer's share. Because  $x_N$  and  $1 - x_S$  are decreasing in  $\alpha$ , if the nonpartisan legislator is part of the winning coalition, he receives less of the dollar in the Calvert–Dietz equilibrium than he would in the Baron–Ferejohn equilibrium regardless of who makes the proposal. Of course, if he is not part of the winning coalition, he receives nothing in either case.

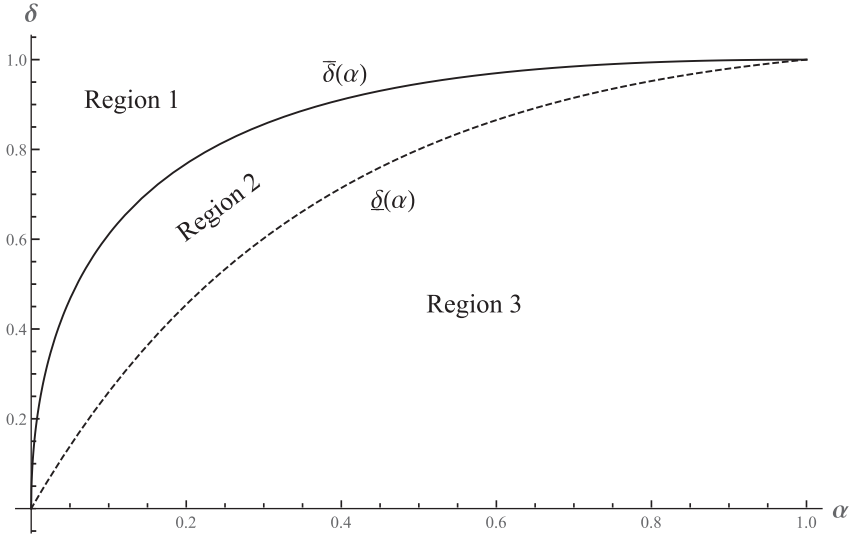


Figure 1. Partisan legislative bargaining game equilibrium regions.

#### 4. Equilibria of the partisan legislative bargaining game

The equilibrium of the partisan legislative bargaining game depends on the values of the preference parameter  $\alpha$  and the discount factor  $\delta$ . We show that the set of possible values of these two parameters can be partitioned into three regions whose equilibria exhibit qualitatively different features. For each value of  $\alpha$ , the boundaries of these regions are determined by specifying two threshold values of  $\delta$ . These thresholds are given by

$$\underline{\delta}(\alpha) = \frac{6\alpha}{(1 + \alpha)(2 + \alpha)} \tag{13}$$

and

$$\bar{\delta}(\alpha) = \frac{-3\alpha - \alpha^2 + \sqrt{24\alpha + 33\alpha^2 + 6\alpha^3 + \alpha^4}}{2(1 + \alpha)}. \tag{14}$$

The graphs of these two functions are shown in Figure 1.<sup>9</sup> Both of these functions are increasing in  $\alpha$ , with  $\underline{\delta}(0) = \bar{\delta}(0)$  and both  $\underline{\delta}(\alpha)$  and  $\bar{\delta}(\alpha)$  having limiting values of 1 as  $\alpha$  goes to 1. Furthermore,  $0 < \underline{\delta}(\alpha) < \bar{\delta}(\alpha) < 1$  when  $0 < \alpha < 1$ . For a given value of  $\alpha$ , the threshold values  $\underline{\delta}(\alpha)$  and  $\bar{\delta}(\alpha)$  partition the interval  $[0, 1]$  into three regions:  $[\bar{\delta}(\alpha), 1]$ ,  $[\underline{\delta}(\alpha), \bar{\delta}(\alpha))$ , and  $[0, \underline{\delta}(\alpha))$ , which we refer to as Regions 1, 2, and 3, respectively.<sup>10</sup> Note that if  $\alpha = 0$ , only the first region is nonempty, whereas for any  $\alpha \in (0, 1)$ , all three regions are nonempty.

For each  $\alpha \in [0, 1)$  and  $\delta \in [0, 1]$ , the equilibrium of the partisan legislative bargaining game is characterized in Proposition 1.

**Proposition 1.** *A set of strategies is a partisan symmetric stationary subgame perfect equilibrium if and only if:*

- (a) For any  $\delta \in [\underline{\delta}(\alpha), 1]$ ,
  - (i) when a partisan is the proposer,
    - i. with probability  $p = (6\alpha - 7\alpha\delta - \alpha^2\delta + \delta^2 + \alpha\delta^2)/(2\delta(\delta + \alpha\delta - 2\alpha))$ , he offers  $x_P = (-2\alpha + \alpha\delta + \delta)/((3 + \alpha)(1 - \alpha))$  to his copartisan, 0 to the nonpartisan, and proposes for himself to receive  $(3 - \alpha^2 - \delta - \alpha\delta)/((3 + \alpha)(1 - \alpha))$ ;
    - ii. with probability  $1 - p$ , he offers  $x_N = (-2\alpha + \alpha\delta + \delta)/(3 + \alpha)$  to the nonpartisan, 0 to his copartisan, and proposes for himself to receive  $((1 + \alpha)(3 - \delta))/(3 + \alpha)$ ;
  - (ii) when the nonpartisan is the proposer, with equal probability, he offers  $x_S = ((1 + \alpha)(\alpha + \delta))/(3 + \alpha)$  to one of the partisans and 0 to the other and proposes for himself to receive  $(3 - \alpha^2 - \delta - \alpha\delta)/(3 + \alpha)$ .

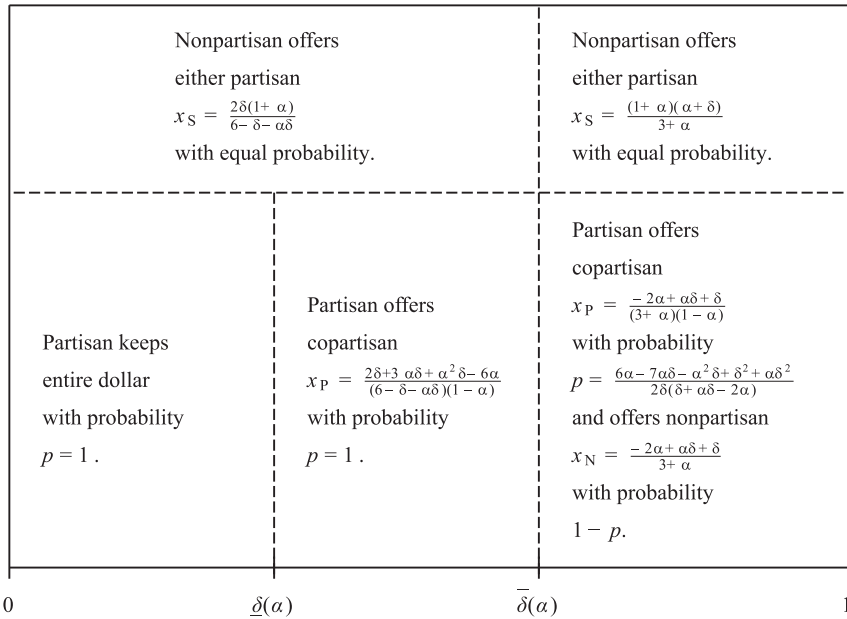
Regardless of who is the proposer, the nonpartisan votes for any distribution in which he receives at least  $(-2\alpha + \alpha\delta + \delta)/(3 + \alpha)$  and either partisan votes for any distribution that offers him utility at least  $((1 + \alpha)(\alpha + \delta))/(3 + \alpha)$ .

- (b) For any  $\delta \in [\underline{\delta}(\alpha), \bar{\delta}(\alpha))$ , when a partisan is the proposer, with probability 1, he offers  $x_P = (2\delta + 3\alpha\delta + \alpha^2\delta - 6\alpha)/((6 - \delta - \alpha\delta)(1 - \alpha))$  to his copartisan, 0 to the nonpartisan, and proposes for himself to receive  $(6 - 3\delta - 3\alpha\delta)/((6 - \delta - \alpha\delta)(1 - \alpha))$ .
- (c) For any  $\delta \in [0, \underline{\delta}(\alpha))$ , when a partisan is the proposer, with probability 1, he offers 0 to both of the other legislators and proposes for himself to receive 1.
- (d) For any  $\delta \in [0, \bar{\delta}(\alpha))$ , when the nonpartisan is the proposer, with equal probability, he offers  $x_S = (2\delta(1 + \alpha))/(6 - \delta - \alpha\delta)$  to one of the partisans and 0 to the other, and proposes for himself to receive  $(3(2 - \delta - \alpha\delta))/(6 - \delta - \alpha\delta)$ .
- (e) In (b), (c), and (d), regardless of who is the proposer, the nonpartisan votes for any distribution in which he receives at least  $(\delta(2 - \delta - \alpha\delta))/(6 - \delta - \alpha\delta)$  and either partisan votes for any distribution that offers him utility at least  $(2\delta(1 + \alpha))/(6 - \delta - \alpha\delta)$ .

Each of the distributions proposed receives the support of a majority.

The equilibrium proposals characterized in Proposition 1 are summarized in Figure 2. For both a partisan and a nonpartisan proposer, what is offered if recognized depends on the values of the party affiliation parameter  $\alpha$  and the discount factor  $\delta$ . Specifically, the proposal depends on the value of  $\delta$  relative to the thresholds  $\underline{\delta}(\alpha)$  and  $\bar{\delta}(\alpha)$ ; that is, it depends on which of the three regions in Figure 1  $\delta$  lies in.<sup>11</sup>

If a partisan proposer is recognized, working from right to left in Figure 2, for  $\delta \in (\bar{\delta}(\alpha), 1]$  (Region 1), he offers a positive share of the dollar to one of the other legislators, with the choice of whom is to receive the offer determined probabilistically. If instead a partisan proposer were to make an offer deterministically to either his copartisan or the nonpartisan, off the equilibrium path, the offer needed



**Figure 2.** Equilibrium proposals in the partisan legislative bargaining game.

to obtain the support of the legislator who was not in the coalition would leave the proposer better off than the offer made to the legislator in the coalition. To support the probabilistic offers in Region 1, the partisan proposer must be indifferent between his equilibrium offers to the other two legislators. For  $\delta \in (\underline{\delta}(\alpha), \bar{\delta}(\alpha)]$  (Region 2), a partisan proposer offers a positive share to his copartisan and excludes the nonpartisan from the bargaining process. Finally, for  $\delta \in [0, \underline{\delta}(\alpha)]$  (Region 3), a partisan proposer proposes to keep the whole dollar for himself. It is noteworthy that the finding by Calvert and Dietz (2005) that a partisan proposer offers a nonzero share of the dollar to a nonpartisan with positive probability when there is no discounting does not hold for all values of the discount factor. Rather, the bipartisanship that Calvert and Dietz identify *only* holds for values of  $\delta$  in Region 1; that is, only if the legislators have sufficient concern about the future.

Intuitively, when the legislators are very patient (Region 1), a copartisan's continuation value is high, with the consequence that it is quite costly to obtain his support. A partisan proposer can reduce this continuation value, and thereby retain more of the dollar, by seeking the support of the nonpartisan with positive probability. With intermediate degrees of patience (Region 2), it is less costly to secure a copartisan's support, so there is no need for a partisan to consider offering the nonpartisan a share. However, the copartisan is not so impatient that he is willing to support the partisan without receiving some of the dollar. When the legislators are very impatient (Region 3), a copartisan values the future so little that the

positive utility externality he obtains from a partisan proposer keeping the whole dollar for himself is sufficient to obtain his support.

In all three regions, the nonpartisan proposer makes a positive offer to one of the other legislators if recognized, provided that  $\delta \neq 0$ . Because of our symmetry assumption, each of them has an equal chance of receiving the offer, but the size of the offer does not depend on whom it is made to. The formula used to determine the magnitude of the nonpartisan's offer depends on whether  $\delta$  is in Regions 1 or 2, or in Region 3. When the strength of party affiliation nears 1, for almost all values of  $\delta$ , a partisan proposer keeps the whole dollar. When  $\delta = 0$ , no legislator cares about the future. In this case, the proposer keeps the whole dollar, regardless of the value of  $\alpha$ .

In interpreting these results, it is important to bear in mind that the boundaries between the three regions for  $\delta$  depend on the value of the party affiliation parameter  $\alpha$ , as illustrated in Figure 1. As  $\alpha$  increases, Region 1 shrinks, with the consequence that the threshold value  $\bar{\delta}(\alpha)$  above which bipartisanship can be sustained increases. Likewise, as  $\alpha$  increases, Region 3 expands, with the consequence that the threshold value  $\underline{\delta}(\alpha)$  below which a partisan proposer exploits his copartisan's strength of party affiliation in order to keep the whole dollar increases. That is, as party affiliation is strengthened, the likelihood that the nonpartisan is cut out of the process and that a partisan proposer keeps the whole dollar increases. When the strength of party affiliation nears 1, for almost all values of  $\delta$ , a partisan proposer keeps the whole dollar and bipartisanship cannot be sustained. In other words, when party strength is very strong, a partisan takes advantage of his proposal power to appropriate all of the current resources that are available unless the legislators exhibit substantial patience about when they receive any benefit. When the strength of party affiliation is close to 0, the possibility of bipartisanship exists for almost all values of  $\delta$ . In the limit when no value is placed on party affiliation ( $\alpha = 0$ ), a partisan proposer is willing to offer a positive share of the dollar to the nonpartisan for any value of  $\delta \neq 0$ . The size of the interval  $\delta \in (\underline{\delta}(\alpha), \bar{\delta}(\alpha)]$  in which a partisan proposer only makes a positive offer to his copartisan first increases and then decreases as  $\alpha$  increases.

Each of the proposals receives the support of a majority, with agreement reached without delay. The utility of a proposer, whoever he is, always exceeds his discounted continuation value, so there is a benefit to being recognized. Indeed, while a partisan proposer would accept for himself what he offers to his copartisan, he, in fact, is able to guarantee himself more than that amount. In all three regions, the offer made by the nonpartisan to one of the partisans makes the recipient indifferent between accepting the offer or rejecting it and moving to the next round of bargaining. In other words, the utility value of the share offered is equal to a partisan's continuation value, discounted by one period. In Region 1, any offer that a partisan makes to the nonpartisan is one that he is just willing to accept. In Regions 1 and 2, the same is true about any offer made by a partisan to his copartisan. However, in Region 3 (i.e., when the legislators are sufficiently impatient), the copartisan's utility exceeds his discounted continuation value even though he is offered none of the dollar. In this case, the positive value that the copartisan places

on the partisan proposer keeping all of the dollar is more than enough to win his support. The lower bound of 0 on an offer prevents a partisan proposer from extracting all of the surplus from his copartisan.<sup>12</sup>

## 5. Comparative statics of the equilibrium proposals

In many cases, it is not apparent from inspection of the formulas in Proposition 1 how the equilibrium values of the proposal variables respond to changes in the strength of party affiliation and the discount factor. Identifying the signs of these comparative static responses adds further insight into the nature of the legislative bargaining problem that we are considering. There are four endogenous variables in Proposition 1: (i) the probability  $p$  that a partisan proposer makes a positive offer to his copartisan, (ii) the share  $x_P$  that a partisan proposer offers to his copartisan, (iii) the share  $x_N$  that a partisan proposer offers to the nonpartisan, and (iv) the share  $x_S$  that a nonpartisan proposer offers to a partisan. We determine how the equilibrium values of each of these variables changes in response to changes in  $\alpha$  and  $\delta$ .

### 5.1. Comparative statics for the value of party affiliation

Proposition 2 presents our comparative static analysis for the strength of party affiliation parameter  $\alpha$ . In this proposition, for each of the endogenous variables, we restrict our attention to the region of the parameter space in which its value varies with  $\alpha$ . The value of the discount factor  $\delta$  is held fixed when determining these comparative static results.

**Proposition 2.** *In any partisan symmetric stationary subgame perfect equilibrium, an increase in the value of party affiliation  $\alpha$  results in:*

- (a) *an increase in the probability  $p$  that a partisan proposer makes a positive offer to his copartisan for  $\delta \in (\underline{\delta}(\alpha), 1]$ ;*
- (b) *a decrease in the share  $x_P$  that a partisan proposer offers to his copartisan for  $\delta \in (\underline{\delta}(\alpha), 1]$ ;*
- (c) *a decrease in the share  $x_N$  that a partisan proposer offers to the nonpartisan for  $\delta \in (\underline{\delta}(\alpha), 1]$ ;*
- (d) *an increase in the share  $x_S$  that a nonpartisan proposer offers to a partisan for  $\delta \in (0, 1]$ .*

A partisan proposer only makes an offer to the nonpartisan with positive probability if  $\delta \in (\underline{\delta}(\alpha), 1]$ ; that is, if  $\delta$  is in the interior of Region 1. In this region, we see from part (a) of Proposition 2 that the probability  $p$  with which the copartisan is made an offer is increasing in the strength of party affiliation and, hence, the probability with which the nonpartisan is made an offer is decreasing in this parameter. The intuition for this result is straightforward. As the value of party affiliation increases, a partisan proposer is able to offer less of the dollar to his copartisan and still obtain his support because the utility value to the copartisan of whatever

the partisan keeps for himself is increasing in this parameter. As a consequence, a partisan proposer benefits from proposing an increased share of the dollar for himself; he likewise benefits from any additional portion of the dollar that he allocates to his copartisan rather than to the nonpartisan (which would yield no positive copartisanship externalities).

Any offer  $x_P$  that a partisan proposer makes to his copartisan is only positive if  $\delta \in (\underline{\delta}(\alpha), 1]$ ; that is, if  $\delta$  is in Region 1 or in the interior of Region 2. When this is the case, from part (b) of Proposition 2, we see that the size of this offer is decreasing in the strength of party affiliation. Because an increase in the strength of party affiliation increases the utility value to the copartisan of the share of the dollar that goes to the partisan proposer, the latter can offer a smaller portion of the dollar to his copartisan as  $\alpha$  increases and still have his offer be accepted. A partisan proposer's offer to his copartisan can be thought of as being a form of vote buying. As the strength of party affiliation increases, the copartisan is willing to sacrifice more for his party, and so his support becomes cheaper to obtain.

As we have noted, a positive offer  $x_N$  is made by a partisan proposer to the nonpartisan only if  $\delta \in (\bar{\delta}(\alpha), 1]$ . By part (c) of Proposition 2, the size of this offer is also decreasing in the strength of party affiliation. As  $\alpha$  increases, it becomes more desirable for the partisans to form a winning coalition because of the increased value of the utility externalities, which reduces the bargaining power of the nonpartisan. As a consequence, the nonpartisan is willing to reduce the amount needed in order to obtain his support.<sup>13</sup>

No matter the value of  $\delta$ , the nonpartisan needs to obtain the support of one of the partisans in order to have his proposal accepted. By part (d) of Proposition 2, the nonpartisan needs to offer a larger share of the dollar  $x_S$  to one of the partisans in order to induce a partisan to accept as the strength of party affiliation increases whenever  $\delta \neq 0$ . This conclusion is an implication of his reduced bargaining power, as discussed previously. The nonpartisan prefers to have a minimally acceptable offer accepted in the current period than to have the offer turned down and move to the next period. In the next period, either (i) the nonpartisan is recognized as the proposer, in which case he would not only have to make a better offer in order to get it accepted but would also incur the cost of delaying agreement or (ii) one of the partisans is recognized as the proposer, in which case the nonpartisan would lose the added benefit that accrues to a proposer.

In the benchmark case considered by Baron and Ferejohn (1989), there is no party affiliation ( $\alpha = 0$ ). In their equilibrium, any proposer offers  $\delta/3$  with equal probability to either of the other legislators, keeping  $1 - \delta/3$  for himself. When there is party affiliation ( $\alpha > 0$ ), it is readily confirmed from part (a) of Proposition 1 that, in the case in which bipartisanship is possible ( $\delta > \bar{\delta}(\alpha)$ ), a partisan proposer favors his copartisan by making an offer to him instead of to the nonpartisan, with a probability that exceeds  $1/2$ . Moreover, as is shown in Proposition 2, regardless of whom he makes an offer to, the amount that a partisan proposer keeps is increasing in  $\alpha$ , provided that a positive offer is made. Hence, for any  $\delta > 0$  and any  $\alpha > 0$ , a partisan proposer keeps more for himself than in the Baron–Ferejohn benchmark case. We also know from Proposition 2 that a nonpartisan proposer's

own share is decreasing in  $\alpha$  for any value of  $\delta$ . Hence, he secures less of the dollar when the other legislators are affiliated than when they are not.

## 5.2. Comparative statics for the value of the discount factor

We now turn in Proposition 3 to our comparative static analysis for the discount factor  $\delta$ . For each of the endogenous variables, we restrict attention to the region of the parameter space in which its value varies with changes in  $\delta$ , holding  $\alpha$  fixed.

**Proposition 3.** *In any partisan symmetric stationary subgame perfect equilibrium, an increase in the discount factor  $\delta$  results in:*

- (a) *a decrease in the probability  $p$  that a partisan proposer makes a positive offer to his copartisan for  $\delta \in [\bar{\delta}(\alpha), 1)$  and  $\alpha \in (0, 1)$ ;*
- (b) *an increase in the share  $x_P$  that a partisan proposer offers to his copartisan for  $\delta \in [\underline{\delta}(\alpha), 1)$ ;*
- (c) *an increase in the share  $x_N$  that a partisan proposer offers to the nonpartisan for  $\delta \in [\bar{\delta}(\alpha), 1)$ ;*
- (d) *an increase in the share  $x_S$  that a nonpartisan proposer offers to a partisan for  $\delta \in [0, 1)$ .*

As  $\delta$  increases, the legislators become more patient, which implies that each of them is more willing to hold out for a better offer in the future, making them more expensive coalition partners. That is, each legislator's discounted continuation value is increasing in  $\delta$ . As a consequence, whenever a proposer (either partisan or nonpartisan) is willing to offer a positive share to a legislator (in which case, he offers the discounted continuation value), he must offer a larger share of the dollar to his coalition partner in order to induce acceptance in the current period. Thus, as shown in parts (b), (c), and (d) of Proposition 3,  $x_P$ ,  $x_N$ , and  $x_S$  are all increasing in  $\delta$  in the relevant regions.

When bipartisanship is viable ( $\delta > \bar{\delta}$ ), a partisan proposer must be equally well off making an offer either to his copartisan or to the nonpartisan. To gain some intuition as to why part (a) of Proposition 3 holds, we consider the utility implications for a partisan proposer if he does not adjust  $p$  when  $\delta$  increases. Without loss of generality, suppose that legislator 1 is the partisan proposer. With an offer of  $x_P$  to his copartisan, his utility would be  $U^1(x^{12}) = 1 - (1 - \alpha)x_P$ , and with an offer of  $x_N$  to the nonpartisan, his utility would be  $U^1(x^{13}) = 1 - x_N$ . Because we are considering out-of-equilibrium effects of a change in  $\delta$ , in our thought experiment, both  $x_N$  and  $x_P$  depend on  $p$ , not just on  $\alpha$  and  $\delta$ .<sup>14</sup> Because the nonpartisan and the copartisan are now more patient, each must be offered an additional amount to obtain his support. However, before discounting, the increase in  $\delta$  results in a decrease in the nonpartisan's continuation value and an increase in that of a partisan.<sup>15</sup> In both cases, there are two countervailing factors. In his role as a proposer, a legislator faces a more patient recipient, which tends to decrease the share he can keep; whereas, in his role as a recipient, he is now willing to hold out longer, which



tends to increase his share. Because of the payoff externality, the second factor is relatively more important than the first for a partisan than for a nonpartisan. Therefore,  $x_N$  increases by less than  $(1 - \alpha)x_P$  when  $\delta$  increases. As a consequence, a partisan proposer would be better off making an offer to the nonpartisan than to his copartisan. Hence, in order for the proposer to be equally well off with either coalition partner, he must adjust  $x_N$  up and  $x_P$  down by decreasing the probability  $p$  that a positive offer is made to his copartisan.

Consistent with conventional wisdom, our comparative static results suggest that when there is substantial partisanship (i.e., when  $\alpha$  is very high), bipartisan outcomes are not likely to be obtained. Holding the degree of partisanship constant, however, one is more likely to see bipartisanship when legislators are more patient. Given the longer time horizons of US Senators in comparison with members of the House, this result might help to explain why the Senate has traditionally cultivated a reputation for bipartisan cooperation. This result is also consistent with recent empirical findings that point to how majority party membership appears to be less important for facilitating legislative success in the Senate than in the House (Volden and Wiseman, 2018). When there is a single budget, which is the case in our model, this relationship suggests that budgets passed closer to hard deadlines (i.e., when legislators face impending consequences of inaction) will be more likely to be supported by partisan coalitions.

### 6. Expected shares and their comparative statics

The offers that are characterized in Proposition 1 are the equilibrium offers in the partisan legislative bargaining game that are made *conditional* on being recognized as the proposer. The shares of the dollar that a legislator expects to receive prior to commencing bargaining is a measure of his relative bargaining strength. It is of interest to determine what these *ex ante* shares are and how they respond to changes in the strength of party affiliation and the discount factor.<sup>16</sup>

From an *ex ante* perspective, the two partisans are in completely symmetric situations, and so have the same expected shares. Without loss of generality, we focus on the share of legislator 1. The *ex ante expected share of a partisan* is

$$\begin{aligned}
 E[x_1] &= \frac{1}{3} \left[ p(1 - x_P) + (1 - p)(1 - x_N) + px_P + \frac{1}{2}x_S \right] \\
 &= \frac{1}{3} \left[ p + (1 - p)(1 - x_N) + \frac{1}{2}x_S \right]
 \end{aligned}
 \tag{15}$$

for  $\delta \in [\bar{\delta}(\alpha), 1]$  and

$$E[x_1] = \frac{1}{3} \left[ 1 - x_P + x_P + \frac{1}{2}x_S \right] = \frac{1}{3} \left[ 1 + \frac{1}{2}x_S \right]
 \tag{16}$$

for  $\delta \in [0, \bar{\delta}(\alpha))$ . Because the expected shares sum to 1 and the two partisans have the same expected shares, the *ex ante expected share of a nonpartisan* is

$$E[x_3] = 1 - 2E[x_1]. \quad (17)$$

Because the equilibrium shares in Proposition 1 are all nonnegative, the ex ante expected shares are also nonnegative.

In Proposition 4, we use the equilibrium values of  $p$ ,  $x_P$ ,  $x_N$ , and  $x_S$  from Proposition 1 to express these expected shares in terms of the parameters of our model.

**Proposition 4.** *In the partisan legislative bargaining game:*

- (a) For any  $\delta \in [\bar{\delta}(\alpha), 1]$ , the ex ante expected share of a partisan is  $E[x_1] = (\alpha + \delta)/(3 + \alpha)\delta$  and the ex ante expected share of a nonpartisan is  $E[x_3] = (-2\alpha + \delta + \alpha\delta)/(3 + \alpha)\delta$ .
- (b) For any  $\delta \in [0, \bar{\delta}(\alpha)]$ , the ex ante expected share of a partisan is  $E[x_1] = 2/(6 - \delta - \alpha\delta)$  and the ex ante expected share of a nonpartisan is  $E[x_3] = (2 - \delta - \alpha\delta)/(6 - \delta - \alpha\delta)$ .

In the Baron and Ferejohn(1989) model,  $\alpha = 0$ . By Proposition 4, the expected share of any legislator is then 1/3 for any value of  $\delta$ . The three legislators are symmetrically situated, and thus have equal expected shares of the dollar. For any  $\alpha \neq 0$ , we see from part (b) of Proposition 4 that these expected shares are also 1/3 if  $\delta = 0$ . In this case, a legislator is recognized with probability 1/3 and keeps the whole dollar if recognized. We do not consider these special cases further.

For  $\delta \in [\bar{\delta}(\alpha), 1]$ , the expected shares may be rewritten as

$$E[x_1] = \frac{1}{3} + \frac{1}{3} \left[ \frac{(3 - \delta)\alpha}{(3 + \alpha)\delta} \right] \quad (18)$$

and

$$E[x_3] = \frac{1}{3} - \frac{1}{3} \left[ \frac{2(3 - \delta)\alpha}{(3 + \alpha)\delta} \right]. \quad (19)$$

Similarly, for  $\delta \in [0, \bar{\delta}(\alpha)]$ , they may be rewritten as<sup>17</sup>

$$E[x_1] = \frac{1}{3} + \frac{1}{3} \left[ \frac{(1 + \alpha)\delta}{[6 - (1 + \alpha)\delta]} \right] \quad (20)$$

and

$$E[x_3] = \frac{1}{3} - \frac{1}{3} \left[ \frac{2(1 + \alpha)\delta}{[6 - (1 + \alpha)\delta]} \right]. \quad (21)$$

The expressions in equations (18) to (21) show how the expected shares deviate from the Baron–Ferejohn benchmark case when  $\alpha > 0$ . When  $\alpha = 0$ , as we have seen, each

legislator has an expected share equal to  $1/3$ . For  $\alpha > 0$ , the second terms in equations (18) and (20) show how much is added to a partisan's expected share due to party affiliation, with the magnitude of this term depending on both  $\alpha$  and  $\delta$ . Similarly, the second terms in equations (19) and (21) show how much must be subtracted from a nonpartisan's expected share because of the partisans' party affiliation.<sup>18</sup>

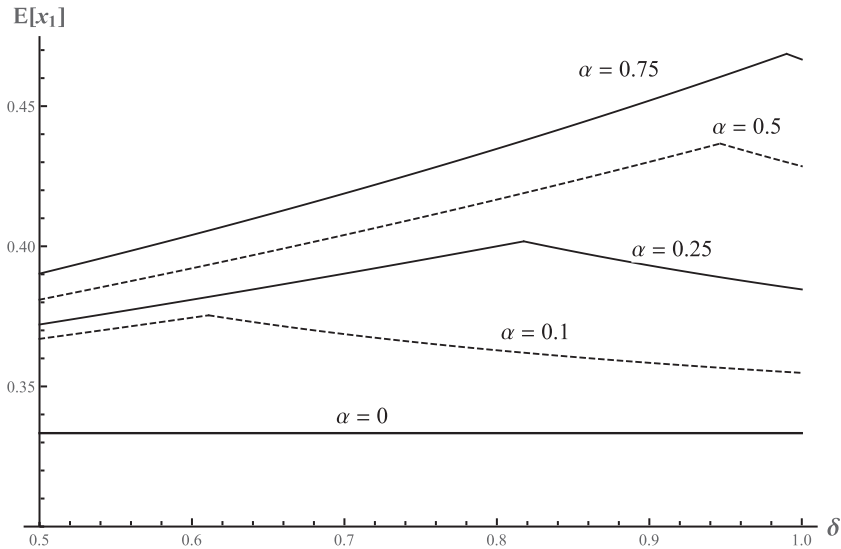
In Proposition 5, we determine how the expected shares respond to changes in the strength of party affiliation and the discount factor. Note that an immediate implication of equation (17) is that a partisan's expected share is increasing in response to a change in one of these parameters if and only if the nonpartisan's expected share is decreasing.

**Proposition 5.** *In any partisan symmetric stationary subgame perfect equilibrium:*

- (a) *If the discount factor  $\delta$  is positive, an increase in the value of party affiliation  $\alpha$  results in an increase in the expected share  $E[x_1]$  of a partisan legislator and a decrease in the expected share  $E[x_3]$  of the nonpartisan legislator.*
- (b) *If the value of party affiliation  $\alpha$  is positive, an increase in the discount factor  $\delta$  results in:*
  - (i) *a decrease in the expected share  $E[x_1]$  of a partisan legislator and an increase in the expected share  $E[x_3]$  of the nonpartisan legislator for  $\delta \in [\bar{\delta}(\alpha), 1]$ ;*
  - (ii) *an increase in the expected share  $E[x_1]$  of a partisan legislator and a decrease in the expected share  $E[x_3]$  of the nonpartisan legislator for  $\delta \in [0, \bar{\delta}(\alpha)]$ .*

The relationship between the ex ante share for legislator 1 and  $\delta$  is plotted in Figure 3 for several values of  $\alpha$ . In conformity with Proposition 5, we see that legislator 1's expected share of the dollar is (i) increasing in  $\alpha$  for fixed  $\delta$  and (ii) increasing and then decreasing in  $\delta$  for a fixed positive value of  $\alpha$ . For each  $\alpha > 0$ , the kink occurs at  $\bar{\delta}(\alpha)$ , the threshold beyond which the nonpartisan has a positive probability of being offered a positive share by a partisan. Because  $\bar{\delta}(\alpha)$  is increasing in  $\alpha$  (see Figure 1), this kink moves to the right as  $\alpha$  is increased.<sup>19</sup> From equation (16), it can be seen that there is no kink at  $\underline{\delta}(\alpha)$ , the threshold beyond which a partisan offers his copartisan a positive share. While there is a kink in the graph of  $x_p$  at this point, the symmetry of the two partisans' situations smooths this kink out when expected shares are computed.

Propositions 2 and 3 can be used to provide some intuition for Proposition 5. First, consider part (a). For  $\delta \leq \bar{\delta}(\alpha)$ , the two partisans share the dollar among themselves if one of them is recognized. Because they are symmetrically situated, conditional on one of them being recognized, they each have an expected share of  $1/2$ . This happens with probability  $2/3$ , so the expected share due to a partisan being the proposer is  $1/3$ , which is independent of  $\alpha$ . Thus, as shown in equation (16), a partisan legislator's expected share only varies with the amount  $x_S$  that a nonpartisan offers if recognized, which Proposition 2 shows is increasing in  $\alpha$ . For  $\delta > \bar{\delta}(\alpha)$ , by equation (15), a partisan legislator's expected share conditional on a partisan being recognized is  $[p + (1-p)(1-x_N)]/3 = [1 - (1-p)x_N]/3$ , which is increasing in  $\alpha$ , by Proposition 2. This effect reinforces the effect that an increase



**Figure 3.** Ex ante expected share for a partisan legislator.

in  $x_S$  has on a partisan’s expected share. Turning now to part (b), for  $\delta \leq \bar{\delta}(\alpha)$ , the preceding reasoning applies as before, except that appeal is now made to the fact that  $x_S$  is increasing in  $\delta$ , as shown in Proposition 3. Finally, for  $\delta > \bar{\delta}(\alpha)$ , the preceding argument is modified by observing that, by Proposition 3,  $[1 - (1 - p)x_N]/3$  is decreasing in  $\delta$ .<sup>20</sup>

### 7. Conclusion

While the contemporary political environment might lend itself to dire predictions about the likelihood of overcoming gridlock or the incidence of bipartisan policy outcomes, we know that political compromises do, in fact, occur, and bipartisan policies are, in fact, created. The relevant questions, then, are: When might bipartisan policy outcomes be obtained? What form will these bipartisan policies take, and why? We have provided a parsimonious model of legislative bargaining with partisan legislators that allows us to engage with these questions in a tractable way. Our analysis reveals that bipartisan outcomes are more likely to be obtained when legislators exhibit substantial concern for the future; but as the strength of party affiliation increases, it is increasingly difficult to forge bipartisan coalitions. Indeed, when the strength of party affiliation is sufficiently large, policy outcomes are quite one-sided, with a minority legislator being either entirely shut out of the policymaking process or only able to extract a small share of the benefits that are available.

Our results point to many possible extensions that are worthy of further study. From an empirical perspective, as alluded to, we find that bipartisan coalitions are more likely to be obtained in political environments in which legislators are

relatively more patient. Exploring the relationship between time-until-elections and the prevalence of bipartisan coalition formation would be of interest. Our results also suggest that bipartisanship is more likely to be realized when the value of party affiliation is relatively low. To the extent that one equates the value of party affiliation with the value of the party brand in the electoral arena (Kiewiet and McCubbins, 1991) or with the degree to which ideological polarization between parties conveys information about their ideological composition (Snyder and Ting, 2002), one might expect that bipartisanship would be more common when the parties are less ideologically distinct or when party affiliations carry less weight among voters. Moving beyond exploring the empirical implications of our model with observational data, the tractability of our model and the clarity of its predictions make it ripe for experimental exploration, perhaps in a laboratory setting.

From a theoretical standpoint, our model is obviously quite spartan in its representation of the policymaking process, and there are several ways that one might seek to enrich it. Obvious extensions would be to explore the impact of party affiliation on legislative bargaining outcomes when the chamber is choosing among policies that do not only have particularistic benefits, as is the case here, but also have collective components (Volden and Wiseman, 2007) or (spatial) ideological dimensions (Jackson and Moselle, 2002). Likewise, analyzing the model with more legislators would allow one to obtain more nuanced predictions regarding the likelihood and form of bipartisan policy outcomes, and how those outcomes might vary under different parliamentary arrangements (such as the presence of supermajoritarian voting requirements). Regardless of which directions are taken, we hope that the model employed here serves as a foundation for providing a greater appreciation of the factors that facilitate or inhibit the realization of partisan and bipartisan policy outcomes.

### **Acknowledgements**

We have benefited from the comments received when this article was presented at the Stanford Graduate School of Business Political Economy Theory Workshop, the Vanderbilt Political Economy Workshop, the 2017 North American Summer Meeting of the Econometric Society in St. Louis, the 2017 Millennium Institute for Market Imperfections Workshop on Political Economy and Political Science in Santiago, the 2018 Murat Sertel Workshop on Economic Design, Decisions, Institutions, and Organizations in Caen, and the 2018 Society for Social Choice and Welfare Conference in Seoul. We are particularly grateful for the comments received from Jim Snyder, Richard Van Weelden, and our referees.


### **Declaration of conflicting interests**

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

### **Funding**

The author(s) received no financial support for the research, authorship, and/or publication of this article.

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**Supplemental material**

Supplemental material for this article is available online.

**Appendix: Proofs<sup>21</sup>**

**Proof of Proposition 1.** We first establish the necessity part of the proof.

(a) Suppose that  $\delta \in [\bar{\delta}(\alpha), 1]$ . We first need to find expressions for the equilibrium values of  $x_N$ ,  $x_P$ ,  $x_S$ ,  $V^1$ ,  $V^2$ ,  $V^3$ , and  $p$  in terms of the model's two exogenous parameters,  $\alpha$  and  $\delta$ . We initially proceed on the assumption that the expressions we obtain for each of the three share variables and for a partisan's probability of making an offer to his copartisan are nonnegative and, in the latter case, do not exceed 1. We then show that this is indeed the case when  $\delta \in (\bar{\delta}(\alpha), 1]$ . By our symmetry assumption, the nonpartisan legislator makes the same offer to each partisan with equal probability.

For a partisan legislator to accept a proposal of  $x_S$ , he must weakly prefer it to his discounted continuation value. Using equation (11), we thus have

$$x_S \geq \frac{\delta}{3}(1 + \alpha) \left[ 1 - x_N + px_N + \frac{1}{2}x_S \right]. \quad (22)$$

Because a nonpartisan proposer would never offer a partisan legislator more than the minimum necessary to induce him to accept a proposal, this inequality binds in equilibrium. Combining terms involving  $x_S$  in the equality version of equation (22), we obtain

$$\left[ 3 - \delta(1 + \alpha)\frac{1}{2} \right] x_S = \delta(1 + \alpha)[1 - (1 - p)x_N],$$

from which it follows that

$$x_S = \frac{2\delta(1 + \alpha)[1 - (1 - p)x_N]}{6 - \delta(1 + \alpha)}. \quad (23)$$

Similarly, the share  $x_N$  offered to the nonpartisan legislator is equal to his discounted continuation value in equilibrium. Substituting the value of  $x_S$  from equation (23) into the expression in equation (12) for the nonpartisan's continuation value, we thus have that

$$x_N = \frac{\delta}{3} \left[ 2(1 - p)x_N + 1 - \frac{2\delta(1 + \alpha)[1 - (1 - p)x_N]}{6 - \delta(1 + \alpha)} \right]$$

or, equivalently,

$$x_N = \frac{\delta}{3} \left[ \frac{12(1-p)x_N + 6 - 3\delta(1+\alpha)}{6 - \delta(1+\alpha)} \right].$$

Combining terms involving  $x_N$  and expressing the coefficient of  $x_N$  in terms of a common denominator, it then follows that

$$\left[ \frac{18 - 3\delta(1+\alpha) - 12\delta(1-p)}{6 - \delta(1+\alpha)} \right] x_N = \delta \left[ \frac{6 - 3\delta(1+\alpha)}{6 - \delta(1+\alpha)} \right].$$

Dividing both sides of this equation by 3 and expanding the numerator on the left-hand side, we obtain

$$\left[ \frac{6 - 5\delta + 4\delta p - \alpha\delta}{6 - \delta(1+\alpha)} \right] x_N = \delta \left[ \frac{2 - \delta(1+\alpha)}{6 - \delta(1+\alpha)} \right].$$

Thus,

$$x_N = \frac{\delta[2 - \delta - \alpha\delta]}{6 - 5\delta + 4\delta p - \alpha\delta}. \quad (24)$$

We thus have an expression for  $x_N$  in terms of the model's parameters and  $p$  (whose value has yet to be determined).

To express  $x_S$  in terms of these parameters, we now substitute  $x_N$  from equation (24) into equation (23), thereby obtaining

$$x_S = \frac{2\delta(1+\alpha) \left[ 1 - (1-p) \frac{\delta[2 - \delta - \alpha\delta]}{6 - 5\delta + 4\delta p - \alpha\delta} \right]}{6 - \delta(1+\alpha)}$$

or, equivalently,

$$x_S = \frac{2\delta(1+\alpha) \left[ \frac{6 - 7\delta + 6\delta p - \alpha\delta + \delta^2 - \delta^2 p + \alpha\delta^2 - \alpha\delta^2 p}{6 - 5\delta + 4\delta p - \alpha\delta} \right]}{6 - \delta(1+\alpha)}.$$

Factoring the numerator in the term in square brackets yields

$$x_S = \frac{2\delta(1+\alpha) \left[ \frac{(6 - \delta(1+\alpha))(1 - (1-p)\delta)}{6 - 5\delta + 4\delta p - \alpha\delta} \right]}{6 - \delta(1+\alpha)},$$

from which it follows that

$$x_S = \frac{2\delta(1+\alpha)[1 - (1-p)\delta]}{6 - 5\delta + 4\delta p - \alpha\delta}. \quad (25)$$

Substituting the expressions for  $x_N$  and  $x_S$  in equations (24) and (25) into the expression in equation (11) for the partisan continuation value, after expanding terms we obtain

$$V^1 = V^2 = \frac{1}{3}(1 + \alpha) \times \left[ \frac{6 - 5\delta + 4\delta p - \alpha\delta - \delta(1-p)[2 - \delta - \alpha\delta] + \delta(1 + \alpha)[1 - (1-p)\delta]}{6 - 5\delta + 4\delta p - \alpha\delta} \right].$$

Simple algebra then shows that the partisan continuation value is

$$V^1 = V^2 = (1 + \alpha) \left[ \frac{2[1 - (1-p)\delta]}{6 - 5\delta + 4\delta p - \alpha\delta} \right]. \quad (26)$$

Similarly, substituting the expressions for  $x_N$  and  $x_S$  in equations (24) and (25) into the expression in equation (12) for the nonpartisan continuation value, after expressing every term using a common denominator, we obtain

$$V^3 = \frac{1}{3} \left[ \frac{2(1-p)\delta(2 - \delta - \alpha\delta) + 6 - 5\delta + 4\delta p - \alpha\delta - 2\delta(1 + \alpha)[1 - (1-p)\delta]}{6 - 5\delta + 4\delta p - \alpha\delta} \right].$$

Collecting terms and simplifying, the nonpartisan continuation value is

$$V^3 = \frac{2 - \delta - \alpha\delta}{6 - 5\delta + 4\delta p - \alpha\delta}. \quad (27)$$

We now determine the share  $x_P$  offered by a partisan to his copartisan. Without loss of generality, suppose that legislator 1 is the proposer and he offers the share  $x_P$  to legislator 2. This share would be accepted provided that the utility that legislator 2 receives from this proposal is at least as large as his discounted continuation value. That is, he will accept  $x_P$  if and only if

$$x_P + \alpha(1 - x_P) \geq \delta V^2$$

or, equivalently,

$$x_P \geq \frac{\delta V^2 - \alpha}{1 - \alpha}. \quad (28)$$

Because  $\alpha < 1$ ,  $x_P + \alpha(1 - x_P)$  is increasing in  $x_P$ . Thus, because  $x_P$  cannot be negative, the constraint in equation (28) binds if and only if the right-hand side of equation (28) is nonnegative. Rearranging equation (28), we see that it is optimal for legislator 1 to choose  $x_P$  so that equation (28) binds if and only if

$$\delta V^2 \geq \alpha. \quad (29)$$

Later, we shall show that equation (29) holds when  $\delta \in [\bar{\delta}(\alpha), 1]$ .

For now, we proceed on the assumption that equation (29) is satisfied. In this case, from equations (26) and (28)

$$x_P = \frac{1}{(1 - \alpha)} \left\{ \delta(1 + \alpha) \left[ \frac{2[1 - (1-p)\delta]}{6 - 5\delta + 4\delta p - \alpha\delta} \right] - \alpha \right\}.$$



Expressing the terms on the right-hand side of this equality in terms of a common denominator, we obtain

$$x_P = \frac{2\delta(1 + \alpha)[1 - \delta + \delta p] - \alpha(6 - 5\delta + 4\delta p - \alpha\delta)}{(6 - 5\delta + 4\delta p - \alpha\delta)(1 - \alpha)}$$

or, equivalently,

$$x_P = \frac{-6\alpha + 2\delta + 7\alpha\delta - 4\alpha\delta p + \alpha^2\delta - 2\delta^2 + 2\delta^2 p - 2\alpha\delta^2 + 2\alpha\delta^2 p}{(6 - 5\delta + 4\delta p - \alpha\delta)(1 - \alpha)}. \quad (30)$$

If a partisan proposer chooses  $p \in (0, 1)$ , he must be indifferent between offering the value of  $x_P$  in equation (30) to his partisan and the value of  $x_N$  in equation (24) to the nonpartisan. For now, we proceed on the assumption that the values of  $\alpha$  and  $\delta$  are such that this is the case and consider the possibility that  $p$  is either 0 or 1 later. Thus, we need to identify the value of  $p$  for which a partisan would be indifferent between offering a share to his copartisan and to the nonpartisan. As before, without loss of generality, suppose that the partisan proposer is legislator 1.

If he offers the value of  $x_N$  in equation (24) to the nonpartisan, legislator 1's utility would be

$$U^1(x^{13}) = 1 - x_N = 1 - \frac{\delta(2 - \delta - \alpha\delta)}{6 - 5\delta + 4\delta p - \alpha\delta}. \quad (31)$$

If, however, he offers the value of  $x_P$  in equation (30) to his copartisan, his utility would be

$$\begin{aligned} U^1(x^{12}) &= 1 - x_P + \alpha x_P \\ &= 1 - \frac{(-6\alpha + 2\delta + 7\alpha\delta - 4\alpha\delta p + \alpha^2\delta - 2\delta^2 + 2\delta^2 p - 2\alpha\delta^2 + 2\alpha\delta^2 p)}{(6 - 5\delta + 4\delta p - \alpha\delta)}. \end{aligned} \quad (32)$$

Setting the right-hand sides of equations (31) and (32) equal to each other and solving for  $p$ , we obtain

$$p = \frac{6\alpha - 7\alpha\delta - \alpha^2\delta + \delta^2 + \alpha\delta^2}{2\delta(\delta + \alpha\delta - 2\alpha)}, \quad (33)$$

which is the equilibrium value for  $p$ .

If  $x_P = 0$  in equation (32), the right-hand sides of equations (31) and (32) are equal if and only if

$$\delta(2 - \delta - \alpha\delta) = 0,$$

which is only satisfied if either (i)  $\delta = 0$  or (ii)  $2 - \delta - \alpha\delta = 0$ . Case (i) does not apply because 0 lies outside the range of values for  $\delta$  being considered. Case (ii) applies if and only if  $\delta = 2/(1 + \alpha)$ . Because  $\alpha < 1$ , this case is impossible because

$\delta$  cannot exceed 1. Thus, it must be the case that  $x_P > 0$  if  $p \in (0, 1)$  and, hence, equation (29) holds with a strict inequality.

Substituting the value of  $p$  from equation (33) into the expressions for the share offers  $x_P$ ,  $x_N$ ,  $x_S$  in equations (30), (24), and (25), respectively, we obtain

$$x_P = \frac{-2\alpha + \delta + \alpha\delta}{(3 + \alpha)(1 - \alpha)}, \quad (34)$$

$$x_N = \frac{-2\alpha + \delta + \alpha\delta}{3 + \alpha}, \quad (35)$$

and

$$x_S = \frac{(1 + \alpha)(\alpha + \delta)}{3 + \alpha}. \quad (36)$$

Because the nonpartisan offers a partisan a share equal to his discounted continuation value, equation (36) implies that

$$V^1 = V^2 = \frac{(1 + \alpha)(\alpha + \delta)}{\delta(3 + \alpha)}. \quad (37)$$

Similarly, because a partisan offers the nonpartisan a share equal to his discounted continuation value, equation (35) implies that

$$V^3 = \frac{-2\alpha + \delta + \alpha\delta}{\delta(3 + \alpha)}. \quad (38)$$

We have shown that the expressions in equations (33) to (38) are the equilibrium values of the endogenous variables expressed in terms of the two parameters  $\alpha$  and  $\delta$  provided that the three share values are nonnegative and that the partisan's offer probability  $p$  in equation (33) lies in  $(0, 1)$ . We proceed on the assumption that the same formulas for the share offers and the partisan's offer probability also apply when  $p = 0$  or  $p = 1$  and confirm that this is in fact the case later. Moreover, we shall show that these shares are nonnegative and that this probability is in  $[0, 1]$  when  $\delta \in [\bar{\delta}(\alpha), 1]$ .

We now use equations (34) to (36) to determine the shares a proposer would keep for himself in equilibrium. A partisan proposer's share when offering  $x_P$  is

$$1 - x_P = \frac{3 - \alpha^2 - \delta - \alpha\delta}{(3 + \alpha)(1 - \alpha)}. \quad (39)$$

A partisan proposer's share when offering  $x_N$  is

$$1 - x_N = \frac{(1 + \alpha)(3 - \delta)}{3 + \alpha}. \quad (40)$$

The nonpartisan proposer's share when offering  $x_S$  is

$$1 - x_S = \frac{3 - \alpha^2 - \delta - \alpha\delta}{3 + \alpha}. \quad (41)$$

A legislator will vote for any proposal in which he receives at least his discounted continuation value. Thus, using equation (37), a partisan will vote for any proposal in which he receives utility  $u$  for which

$$u \geq \frac{(1 + \alpha)(\alpha + \delta)}{(3 + \alpha)}. \quad (42)$$

Similarly, using equation (38), the nonpartisan will vote for any proposal in which he receives utility  $u$  for which

$$u \geq \frac{-2\alpha + \delta + \alpha\delta}{(3 + \alpha)}. \quad (43)$$

We have already seen that equation (29) must hold in order for  $x_P$  to be nonnegative. By equation (37), this holds if and only if

$$(1 + \alpha)(\alpha + \delta) \geq (3 + \alpha)\alpha$$

or, equivalently, if and only if

$$\delta \geq \frac{2\alpha}{1 + \alpha}. \quad (44)$$

We now show that equation (44) holds with a strict inequality when  $\delta = \bar{\delta}$ , which is the lowest value for  $\delta$  in the region being considered. First, note that

$$\sqrt{24\alpha + 33\alpha^2 + 6\alpha^3 + \alpha^4} > 7\alpha + \alpha^2$$

if and only if

$$24\alpha + 33\alpha^2 + 6\alpha^3 + \alpha^4 > 49\alpha^2 + 14\alpha^3 + \alpha^4$$

if and only if

$$3 > 2\alpha + \alpha^2.$$

Thus,

$$\bar{\delta} = \frac{-3\alpha - \alpha^2 + \sqrt{24\alpha + 33\alpha^2 + 6\alpha^3 + \alpha^4}}{2(1 + \alpha)} > \frac{2\alpha}{1 + \alpha} \quad (45)$$

because  $\alpha < 1$ . Hence, the expression for  $x_P$  in equation (34) is nonnegative.

We now determine the values for  $\alpha$  and  $\delta$  for which the value of  $p$  in equation (33) lies in  $[0, 1]$ . The upper bound on  $p$  is satisfied if and only if

$$p = \frac{6\alpha - 7\alpha\delta - \alpha^2\delta + \delta^2 + \alpha\delta^2}{2\delta(\delta + \alpha\delta - 2\alpha)} \leq 1. \quad (46)$$

When  $\alpha = 0$ ,  $p = 1/2$  in equation (46), and so this inequality is satisfied for all  $\delta$ . When  $\delta$  satisfies equation (44) and  $\alpha \in (0, 1)$ , equation (46) holds if and only if<sup>22</sup>

$$\delta \geq \frac{-3\alpha - \alpha^2 + \sqrt{24\alpha + 33\alpha^2 + 6\alpha^3 + \alpha^4}}{2(1 + \alpha)}. \quad (47)$$

This inequality holds with equality if and only if  $p = 1$ . Thus,  $p = 1$  if and only if  $\delta = \bar{\delta}$ .<sup>23</sup>

The lower bound on  $p$  is satisfied if and only if

$$p = \frac{6\alpha - 7\alpha\delta - \alpha^2\delta + \delta^2 + \alpha\delta^2}{2\delta(\delta + \alpha\delta - 2\alpha)} \geq 0. \quad (48)$$

For all  $\alpha \in [0, 1)$ , equation (48) holds if and only if  $\delta > 2\alpha/(1 + \alpha)$ . Moreover, there is no value of the parameters for which  $p = 0$ . Thus, for the range of values for  $\delta$  that we are considering, a partisan makes an offer to his copartisan with positive probability.

Because  $p$  is never 0 for the range of values for  $\delta$  being considered, we only need to confirm that the expression for  $p$  in equation (33) is valid when  $p = 1$ . That is, we need to show that the value of  $U^1(x^{12})$  in equation (32) is at least as large as the value of  $U^1(x^{13})$  in equation (31) when  $p = 1$ . The preceding argument shows that these two values are in fact the same when  $p = 1$  (i.e., when  $\delta = \bar{\delta}$  and  $\alpha \neq 0$ ).

We have already confirmed that the expression we have found for  $x_P$  is nonnegative when equation (47) is satisfied. It remains to be shown that the same is true for  $x_N$  and  $x_S$ .

From equations (35), (45), and (47), we have

$$\begin{aligned} x_N &= \frac{-2\alpha + \delta + \alpha\delta}{3 + \alpha} \geq \frac{-2\alpha + (1 + \alpha) \left[ \frac{-3\alpha - \alpha^2 + \sqrt{24\alpha + 33\alpha^2 + 6\alpha^3 + \alpha^4}}{2(1 + \alpha)} \right]}{3 + \alpha} \\ &> \frac{-2\alpha + (1 + \alpha) \left( \frac{2\alpha}{1 + \alpha} \right)}{3 + \alpha} = 0. \end{aligned}$$

Similarly, from equations (36), (45), and (47), we have

$$\begin{aligned} x_S &= \frac{(1 + \alpha)(\alpha + \delta)}{3 + \alpha} \geq \frac{(1 + \alpha) \left( \alpha + \left[ \frac{-3\alpha - \alpha^2 + \sqrt{24\alpha + 33\alpha^2 + 6\alpha^3 + \alpha^4}}{2(1 + \alpha)} \right] \right)}{3 + \alpha} \\ &> \frac{(1 + \alpha) \left( \alpha + \left[ \frac{2\alpha}{1 + \alpha} \right] \right)}{3 + \alpha} > 0. \end{aligned}$$

(b) Suppose that  $\delta \in [\underline{\delta}(\alpha), \bar{\delta}(\alpha))$  and that a partisan is the proposer. For these values of  $\delta$ , it is not possible to have  $\alpha = 0$ , so  $\alpha \in (0, 1)$ . We proceed on the assumption that it is optimal for a partisan to make his copartisan an offer that he

is just willing to accept. That is, we suppose that equation (29) is satisfied. Later, we shall confirm that this is in fact the case for the values of  $\delta$  being considered.

From equation (47), we know that it is not possible to have  $p < 1$  when  $\alpha \neq 0$  if  $\delta \leq (-3\alpha - \alpha^2 + \sqrt{24\alpha + 33\alpha^2 + 6\alpha^3 + \alpha^4}) / (2(1 + \alpha))$ . Thus,  $p = 1$ . Because a partisan proposer strictly prefers to offer his copartisan his continuation value rather than make a positive offer to the nonpartisan,  $x_N$  is not relevant. The formula for  $x_P$  in equation (30) continues to apply provided that it is nonnegative. Substituting  $p = 1$  into this equation, we obtain

$$x_P = \frac{2\delta + 3\alpha\delta + \alpha^2\delta - 6\alpha}{(6 - \delta - \alpha\delta)(1 - \alpha)}, \tag{49}$$

which is the equilibrium value of  $x_P$ . The partisan proposer’s share when offering  $x_P$  is

$$1 - x_P = 1 - \frac{2\delta + 3\alpha\delta + \alpha^2\delta - 6\alpha}{(6 - \delta - \alpha\delta)(1 - \alpha)} = \frac{6 - 3\delta - 3\alpha\delta}{(6 - \delta - \alpha\delta)(1 - \alpha)}, \tag{50}$$

Because  $\alpha < 1$  and  $\delta \leq 1$ , this share is positive.

From equation (49), we have  $x_P \geq 0$  if and only if

$$2\delta + 3\alpha\delta + \alpha^2\delta - 6\alpha \geq 0$$

or, equivalently,

$$\delta \geq \frac{6\alpha}{(1 + \alpha)(2 + \alpha)}, \tag{51}$$

The right-hand side of this inequality is  $\underline{\delta}(\alpha)$ , which is the smallest value of  $\delta$  in the interval being considered, so we have confirmed that the share offers are nonnegative.

It remains to confirm that it is optimal for a partisan to offer his copartisan exactly his discounted continuation value. That is, we need to confirm that equation (29) holds. Setting  $p = 1$  in equation (26), we find that

$$V^1 = V^2 = \frac{2(1 + \alpha)}{6 - \delta - \alpha\delta}. \tag{52}$$

Using equation (52), equation (29) holds if and only if

$$2\delta(1 + \alpha) \geq \alpha(6 - \delta - \alpha\delta).$$

Simple algebra shows that this inequality is equivalent to equation (51).

(c) Suppose that  $\delta \in [0, \underline{\delta}(\alpha))$  and that a partisan is the proposer. As in part (b), for values of  $\delta$  in this interval, it is not possible to have  $\alpha = 0$ , so  $\alpha \in (0, 1)$ . The argument in the proof of part (b) has also established that  $p = 1$ . Furthermore, because  $\delta < \underline{\delta}(\alpha)$ , the inequality in equation (51) is violated and, therefore, the nonnegativity constraint on  $x_P$  binds (i.e.,  $x_P = 0$ ). Thus, with probability 1, a

partisan proposes to keep all of the dollar for himself. Because  $p = 1$ , the continuation values for two partisans are given in equation (52).

(d) Suppose that  $\delta \in [0, \bar{\delta}(\alpha)]$  and that the nonpartisan is the proposer. From the proofs of parts (b) and (c), we know that when  $\delta$  is in this interval,  $p = 1$ . The formula for  $x_S$  in equation (25) continues to apply provided that it is nonnegative. Substituting  $p = 1$  into equation (25), we obtain

$$x_S = \frac{2\delta(1 + \alpha)}{6 - \delta - \alpha\delta}, \quad (53)$$

which is 0 if  $\delta = 0$ ; otherwise, it is positive. The nonpartisan proposer's share when offering  $x_S$  is

$$1 - x_S = 1 - \frac{2\delta(1 + \alpha)}{6 - \delta - \alpha\delta} = \frac{3(2 - \delta - \alpha\delta)}{6 - \delta - \alpha\delta}, \quad (54)$$

which is positive.

(e) Suppose that  $\delta \in [0, \bar{\delta}(\alpha)]$ . The continuation values for the partisans are given in equation (52). We have shown that  $p = 1$  when  $\delta < \bar{\delta}(\alpha)$ . By setting  $p = 1$  in equation (27), we find that the nonpartisan's continuation value is

$$V^3 = \frac{2 - \delta - \alpha\delta}{6 - \delta - \alpha\delta}. \quad (55)$$

A legislator will vote for any proposal in which he receives at least his discounted continuation value. Thus, using equation (52), a partisan will vote for any proposal in which he receives utility  $u$  for which

$$u \geq \frac{2\delta(1 + \alpha)}{6 - \delta - \alpha\delta}. \quad (56)$$

Similarly, using equation (38), the nonpartisan will vote for any proposal in which he receives utility  $u$  for which

$$u \geq \frac{\delta(2 - \delta - \alpha\delta)}{6 - \delta - \alpha\delta}. \quad (57)$$

It remains to be shown that each of the distributions proposed receives the support of a majority. Someone who is offered his discounted continuation value is indifferent between supporting or opposing the proposal. However, if he does not support it, the proposal is defeated and we do not have an equilibrium. There are three cases.

*Case 1:*  $\delta \in [\bar{\delta}(\alpha), 1]$ . From equations (34), (37), and (42), it follows that both partisans vote for  $x^{12} = (1 - x_P, x_P, 0)$  because

$$\begin{aligned} U^1(x^{12}) &= 1 - x_P + \alpha x_P = 1 - (1 - \alpha) \frac{[-2\alpha + \delta + \alpha\delta]}{(3 + \alpha)(1 - \alpha)} \\ &= \frac{(3 - \delta)(1 + \alpha)}{3 + \alpha} > \frac{(1 + \alpha)(\alpha + \delta)}{3 + \alpha} = \delta V^1 \end{aligned}$$

and

$$\begin{aligned} U^2(x^{12}) &= \alpha(1 - x_P) + x_P = \alpha + (1 - \alpha) \frac{[-2\alpha + \delta + \alpha\delta]}{(3 + \alpha)(1 - \alpha)} \\ &= \frac{(1 + \alpha)(\alpha + \delta)}{3 + \alpha} = \delta V^2. \end{aligned}$$

From equations (35), (37), (38), (42), and (43), it follows that the partisan proposer and the nonpartisan vote for  $x^{13} = (1 - x_N, 0, x_N)$  because

$$\begin{aligned} U^1(x^{13}) &= 1 - x_N = 1 - \frac{[-2\alpha + \delta + \alpha\delta]}{3 + \alpha} \\ &= \frac{(1 + \alpha)(3 - \delta)}{3 + \alpha} > \frac{(1 + \alpha)(\alpha + \delta)}{3 + \alpha} = \delta V^1 \end{aligned}$$

and

$$U^3(x^{13}) = x_N = \frac{-2\alpha + \delta + \alpha\delta}{3 + \alpha} = \delta V^3.$$

From equations (36), (37), (38), (42), and (43), it follows that the nonpartisan proposer and the partisan who is offered a positive share vote for  $x^{31} = (x_S, 0, 1 - x_S)$  because

$$U^3(x^{31}) = 1 - x_S = \frac{3 - \alpha^2 - \delta - \alpha\delta}{3 + \alpha} > \frac{-2\alpha + \delta + \alpha\delta}{3 + \alpha} = \delta V^3$$

and

$$U^1(x^{31}) = x_S = \frac{(1 + \alpha)(\alpha + \delta)}{3 + \alpha} = \delta V^1.$$

*Case 2:*  $\delta \in [\underline{\delta}(\alpha), \bar{\delta}(\alpha)]$ . From equations (49) and (52), it follows that both partisans vote for  $x^{12} = (1 - x_P, x_P, 0)$  because

$$\begin{aligned} U^1(x^{12}) &= 1 - x_P + \alpha x_P = 1 - (1 - \alpha) \frac{[2\delta + 3\alpha\delta + \alpha^2\delta - 6\alpha]}{(6 - \delta - \alpha\delta)(1 - \alpha)} \\ &= \frac{6 - 3\delta - 4\alpha\delta - \alpha^2\delta + 6\alpha}{6 - \delta - \alpha\delta} = \frac{(6 - 3\delta - \alpha\delta)(1 + \alpha)}{6 - \delta - \alpha\delta} \\ &> \frac{2\delta(1 + \alpha)}{6 - \delta - \alpha\delta} = \delta V^1 \end{aligned}$$

and

$$\begin{aligned}
 U^2(x^{12}) &= \alpha(1 - x_P) + x_P = \alpha + (1 - \alpha) \frac{[2\delta + 3\alpha\delta + \alpha^2\delta - 6\alpha]}{(6 - \delta - \alpha\delta)(1 - \alpha)} \\
 &= \frac{2\delta(1 + \alpha)}{6 - \delta - \alpha\delta} = \delta V^2.
 \end{aligned}$$

From equations (53) to (55), it follows that the nonpartisan proposer and the partisan who is offered a positive share vote for  $x^{31} = (x_S, 0, 1 - x_S)$  because

$$U^3(x^{31}) = 1 - x_S = \frac{3(2 - \delta - \alpha\delta)}{6 - \delta - \alpha\delta} > \frac{2 - \delta - \alpha\delta}{6 - \delta - \alpha\delta} = \delta V^3$$

and

$$U^1(x^{31}) = x_S = \frac{2\delta(1 + \alpha)}{6 - \delta - \alpha\delta} = \delta V^1.$$

*Case 3:*  $\delta \in [0, \underline{\delta}(\alpha)]$ . The proof in Case 2 for the nonpartisan's proposal applies in this case as well.

It remains to confirm that when a partisan proposes  $x^{12} = (1, 0, 0)$ , it will be approved. A partisan votes for  $x^{12} = (1, 0, 0)$  because

$$U^1(x^{12}) = 1 > \frac{2\alpha}{2 + \alpha} > \frac{\delta(1 + \alpha)}{3} \geq \frac{2\delta(1 + \alpha)}{6 - \delta - \alpha\delta} = \delta V^1,$$

where the first inequality holds because  $\alpha < 1$ , the second because  $\delta < 6\alpha/((1 + \alpha)(2 + \alpha))$ , and the third because  $\delta + \alpha\delta \geq 0$ .

The copartisan also supports  $x^{12} = (1, 0, 0)$  because

$$U^2(x^{12}) = \alpha \geq \frac{2\alpha}{2 + \alpha} > \frac{\delta(1 + \alpha)}{3} \geq \frac{2\delta(1 + \alpha)}{6 - \delta - \alpha\delta} = \delta V^2,$$

where the first inequality is strict if  $\alpha \neq 0$ .

This completes the necessity part of the proof.

For the sufficiency part of the proof, we need to show that the strategies described in the statement of the proposition are a partisan symmetric stationary subgame perfect equilibrium. In other words, we need to show that no legislator wants to deviate unilaterally from these strategies. To do this, we must show that: (i) no legislator in his role as a proposer wants to modify the share offered to one of the other legislators in order to receive his support, (ii) no legislator in his role as a proposer wants to modify the probabilities with which he makes offers to the other legislators, and (iii) no legislator wants to deviate from his voting strategy.

We have already shown that the specified shares are the minimal amounts needed to attain the support of the relevant legislator, so (i) holds. The last part of the necessity proof has established (iii). For a partisan proposer, the proofs of parts (a), (b), and (c) have shown that deviating from the specified probability  $p$  would require reducing a partisan proposer's share in favor of one of the other legislators. When the nonpartisan is the proposer, he receives the share  $1 - x_S$  regardless of



who ends up supporting him. Thus, the nonpartisan has no incentive to deviate from making offers to each partisan with probability 1/2. Hence, (ii) holds as well.

**Proof of Proposition 2.** Differentiation for all the comparative statics in this and the subsequent two proofs were carried out using *Mathematica*. The code for these derivations may be found in the supplementary online appendix. At a boundary of any of the three regions for  $\delta$ , the relevant one-sided derivative is used.<sup>24</sup>

(a) For  $\delta \in [\bar{\delta}(\alpha), 1]$ , by part (a) of Proposition 1

$$p = \frac{6\alpha - 7\alpha\delta - \alpha^2\delta + \delta^2 + \alpha\delta^2}{-4\alpha\delta + 2\delta^2 + 2\alpha\delta^2}. \quad (58)$$

Differentiating equation (58) with respect to  $\alpha$  for  $\delta \in (\bar{\delta}(\alpha), 1]$ , we obtain

$$\begin{aligned} \frac{\partial p}{\partial \alpha} &= \frac{6 + 2\alpha^2 - \alpha^2\delta - 5\delta - 2\alpha\delta}{2(2\alpha - \alpha\delta - \delta)^2} \\ &\geq \frac{6 + 2\alpha^2 - \alpha^2(1) - 5(1) - 2\alpha(1)}{2(2\alpha - \alpha\delta - \delta)^2} \\ &= \frac{1 - 2\alpha + \alpha^2}{2(2\alpha - \alpha\delta - \delta)^2} = \frac{(1 - \alpha)^2}{2(2\alpha - \alpha\delta - \delta)^2} > 0, \end{aligned} \quad (59)$$

where the first inequality follows because  $\delta \leq 1$  and the last because  $\alpha < 1$ .

(b) For  $\delta \in [\bar{\delta}(\alpha), 1]$ , by part (a) of Proposition 1

$$x_P = \frac{-2\alpha + \alpha\delta + \delta}{(3 + \alpha)(1 - \alpha)}. \quad (60)$$

Differentiating equation (60) with respect to  $\alpha$  for  $\delta \in (\bar{\delta}(\alpha), 1]$ , we obtain

$$\begin{aligned} \frac{\partial x_P}{\partial \alpha} &= \frac{-6 - 2\alpha^2 + \alpha^2\delta + 5\delta + 2\alpha\delta}{(3 + \alpha)^2(1 - \alpha)^2} \\ &\leq \frac{-6 - 2\alpha^2 + \alpha^2(1) + 5(1) + 2\alpha(1)}{(3 + \alpha)^2(1 - \alpha)^2} \\ &= \frac{-1 + 2\alpha - \alpha^2}{(3 + \alpha)^2(1 - \alpha)^2} \\ &= -\frac{(1 - \alpha)^2}{(3 + \alpha)^2(1 - \alpha)^2} = -\frac{1}{(3 + \alpha)^2} < 0, \end{aligned} \quad (61)$$

where the first inequality follows because  $\delta \leq 1$ .

For  $\delta \in [\underline{\delta}(\alpha), \bar{\delta}(\alpha)]$ , by part (b) of Proposition 1

$$x_P = \frac{2\delta + 3\alpha\delta + \alpha^2\delta - 6\alpha}{(6 - \delta - \alpha\delta)(1 - \alpha)}. \quad (62)$$

Differentiating equation (62) with respect to  $\alpha$  for  $\delta \in (\underline{\delta}(\alpha), \bar{\delta}(\alpha))$ , we obtain

$$\begin{aligned}
\frac{\partial x_P}{\partial \alpha} &= \frac{-3(12 - 12\delta - 4\alpha\delta + (1 + \alpha)^2\delta^2)}{(6 - \delta - \alpha\delta)^2(1 - \alpha)^2} \\
&\leq \frac{3(-12 + 12(1) + 4\alpha(1) - (1 + \alpha)^2(1)^2)}{(6 - \delta - \alpha\delta)^2(1 - \alpha)^2} \\
&= \frac{3(4\alpha - (1 + \alpha)^2)}{(6 - \delta - \alpha\delta)^2(1 - \alpha)^2} = \frac{-3(1 - 2\alpha + \alpha^2)}{(6 - \delta - \alpha\delta)^2(1 - \alpha)^2} \\
&= \frac{-3(1 - \alpha)^2}{(6 - \delta - \alpha\delta)^2(1 - \alpha)^2} = \frac{-3}{(6 - \delta - \alpha\delta)^2} < 0,
\end{aligned} \tag{63}$$

where the first inequality follows because  $\delta \leq 1$  and the last because neither  $\alpha$  nor  $\delta$  exceeds 1.

It then follows from the continuity of the equilibrium value of  $x_P$  in  $\alpha$  that  $x_P$  is also decreasing in  $\alpha$  when  $\delta = \bar{\delta}(\alpha)$ .

(c) For  $\delta \in [\bar{\delta}(\alpha), 1]$ , by part (a) of Proposition 1

$$x_N = \frac{-2\alpha + \alpha\delta + \delta}{3 + \alpha}. \tag{64}$$

Differentiating equation (64) with respect to  $\alpha$  for  $\delta \in (\bar{\delta}(\alpha), 1]$ , we obtain

$$\frac{\partial x_N}{\partial \alpha} = \frac{2(-3 + \delta)}{(3 + \alpha)^2} \leq \frac{2(-3 + 1)}{(3 + \alpha)^2} = \frac{-4}{(3 + \alpha)^2} < 0, \tag{65}$$

where the first inequality follows because  $\delta \leq 1$ .

(d) For  $\delta \in [\bar{\delta}(\alpha), 1]$ , by part (a) of Proposition 1

$$x_S = \frac{(1 + \alpha)(\alpha + \delta)}{3 + \alpha}. \tag{66}$$

Differentiating equation (66) with respect to  $\alpha$  for  $\delta \in (\bar{\delta}(\alpha), 1]$ , we obtain

$$\frac{\partial x_S}{\partial \alpha} = \frac{3 + 6\alpha + \alpha^2 + 2\delta}{(3 + \alpha)^2} \geq \frac{3 + 6(0) + 0^2 + 2(0)}{(3 + \alpha)^2} = \frac{3}{(3 + \alpha)^2} > 0, \tag{67}$$

where the first inequality follows because both  $\alpha$  and  $\delta$  are nonnegative.

For  $\delta \in [0, \bar{\delta}(\alpha))$ , by part (d) of Proposition 1

$$x_S = \frac{2\delta(1 + \alpha)}{6 - \delta - \alpha\delta}. \tag{68}$$

Differentiating equation (68) with respect to  $\alpha$  for  $\delta \in (0, \bar{\delta}(\alpha))$ , we obtain

$$\frac{\partial x_S}{\partial \alpha} = \frac{12\delta}{(6 - \delta - \alpha\delta)^2} > 0, \tag{69}$$

where the inequality holds because  $\delta > 0$ . Therefore,  $x_S$  is also increasing in  $\alpha$  in this interval for  $\delta$ .

It then follows from the continuity of the equilibrium value of  $x_S$  in  $\alpha$  that  $x_S$  is also increasing in  $\alpha$  when  $\delta = \bar{\delta}(\alpha) \neq 0$ .

**Proof of Proposition 3.** (a) For  $\delta \in [\bar{\delta}(\alpha), 1)$ , differentiating equation (58) with respect to  $\delta$ , we obtain

$$\frac{\partial p}{\partial \delta} = \frac{\alpha(\alpha^2\delta^2 - 12\delta + 5\delta^2 + 12\alpha - 12\alpha\delta + 6\alpha\delta^2)}{2\delta^2(2 - \alpha\delta - \delta)^2}. \quad (70)$$

For  $0 < \alpha < 1$ , the *Mathematica* derivations in the supplementary online appendix demonstrate that in order to establish that this derivative is negative, it is sufficient to show that

$$\delta > \frac{2\alpha}{1 + \alpha}. \quad (71)$$

Because  $0 < \alpha < 1$

$$\underline{\delta}(\alpha) = \frac{6\alpha}{(1 + \alpha)(2 + \alpha)} > \frac{2\alpha}{1 + \alpha}.$$

Therefore, because  $\bar{\delta}(\alpha) > \underline{\delta}(\alpha)$ , equation (71) holds. Because the derivative in equation (70) is negative for all  $\alpha > 0$ ,  $p$  is decreasing in  $\delta$  in this interval for  $\delta$ .

(b) For  $\delta \in [\bar{\delta}(\alpha), 1)$ , differentiating equation (60) with respect to  $\delta$ , we obtain

$$\frac{\partial x_P}{\partial \delta} = \frac{1 + \alpha}{(3 + \alpha)(1 - \alpha)} \geq \frac{1 + 0}{(3 + \alpha)(1 - \alpha)} > 0, \quad (72)$$

where the first inequality holds because  $\alpha \geq 0$  and the second because  $\alpha < 1$ .

For  $\delta \in [\underline{\delta}(\alpha), \bar{\delta}(\alpha))$ , differentiating equation (62) with respect to  $\delta$ , we obtain

$$\frac{\partial x_P}{\partial \delta} = \frac{12(1 + \alpha)}{(6 - \delta - \alpha\delta)^2(1 - \alpha)} \geq \frac{12(1 + 0)}{(6 - \delta - \alpha\delta)^2(1 - \alpha)} > 0, \quad (73)$$

where the first inequality holds because  $\alpha \geq 0$  and the second because  $\alpha < 1$ .

(c) For  $\delta \in [\bar{\delta}(\alpha), 1)$ , differentiating equation (64) with respect to  $\delta$ , we obtain

$$\frac{\partial x_N}{\partial \delta} = \frac{1 + \alpha}{3 + \alpha} \geq \frac{1 + 0}{3 + \alpha} > 0, \quad (74)$$

where both inequalities hold because  $\alpha \geq 0$ .

(d) For  $\delta \in [\bar{\delta}(\alpha), 1)$ , differentiating equation (66) with respect to  $\delta$ , we obtain

$$\frac{\partial x_S}{\partial \delta} = \frac{1 + \alpha}{3 + \alpha} > 0, \quad (75)$$

where the inequality follows as in the proof of part (c).

For  $\delta \in (0, \bar{\delta}(\alpha))$ , differentiating equation (68) with respect to  $\delta$ , we obtain

$$\frac{\partial x_S}{\partial \delta} = \frac{12(1 + \alpha)}{(6 - \alpha - \alpha\delta)^2} > 0, \quad (76)$$

where the inequality holds because the derivative in equation (76) is a positive multiple of the derivative in equation (73). The value of  $x_S$  is 0 if and only if  $\delta = 0$ , so  $x_S$  is also increasing in  $\delta$  at  $\delta = 0$ .

**Proof of Proposition 4.** We only show the derivations used to obtain the expressions for  $E[x_1]$  as it is straightforward to derive the expressions for  $E[x_3]$  from them using equation (17).

(a) Substituting the equilibrium values of  $p$ ,  $x_P$ ,  $x_N$ , and  $x_S$  in equation (15), we obtain

$$\begin{aligned} E[x_1] = & \frac{1}{3} \left[ \frac{6\alpha - 7\alpha\delta - \alpha^2\delta + \delta^2 + \alpha\delta^2}{-4\alpha\delta + 2\delta^2 + 2\alpha\delta^2} \right. \\ & + \left( 1 - \frac{6\alpha - 7\alpha\delta - \alpha^2\delta + \delta^2 + \alpha\delta^2}{-4\alpha\delta + 2\delta^2 + 2\alpha\delta^2} \right) \left( 1 - \frac{-2\alpha + \alpha\delta + \delta}{3 + \alpha} \right) \\ & \left. + \frac{1}{2} \left( \frac{(1 + \alpha)(\alpha + \delta)}{3 + \alpha} \right) \right]. \end{aligned}$$

Simplifying,

$$\begin{aligned} E[x_1] = & \frac{1}{3} \left[ \frac{-(\alpha + \delta)(6 - \delta - \alpha\delta)(2\alpha - \delta - \alpha\delta)}{-2\delta(2\alpha - \delta - \alpha\delta)(3 + \alpha)} + \frac{1}{2} \left( \frac{(1 + \alpha)(\alpha + \delta)}{3 + \alpha} \right) \right] \\ = & \frac{1}{3} \left[ \frac{(\alpha + \delta)(6 - \delta - \alpha\delta) + (\alpha + \delta)\delta(1 + \alpha)}{2\delta(3 + \alpha)} \right] \\ = & \frac{1}{3} \left[ \frac{6(\alpha + \delta)}{2\delta(3 + \alpha)} \right] = \frac{\alpha + \delta}{\delta(3 + \alpha)}. \end{aligned}$$

(b) Substituting the equilibrium values of  $p$ ,  $x_P$ ,  $x_N$ , and  $x_S$  in equation (15), we obtain

$$E[x_1] = \frac{1}{3} \left[ 1 + \frac{1}{2} \left( \frac{2\delta(1 + \alpha)}{6 - \delta - \alpha\delta} \right) \right].$$

Simplifying,

$$E[x_1] = \frac{1}{3} \left[ \frac{6 - \delta - \alpha\delta + \delta(1 + \alpha)}{6 - \delta - \alpha\delta} \right] = \frac{2}{6 - \delta - \alpha\delta}.$$

**Deriving equations (18) to (21).**

For  $\delta \in [\bar{\delta}(\alpha), 1]$ , we can rewrite the expression for  $E[x_1]$  in part (a) of Proposition 4 as

$$E[x_1] = \frac{1}{3} \left[ 1 + \frac{3(\alpha + \delta)}{(3 + \alpha)\delta} - \frac{(3 + \alpha)\delta}{(3 + \alpha)\delta} \right].$$

Simplifying,

$$E[x_1] = \frac{1}{3} \left[ 1 + \frac{(3\alpha + 3\delta - \alpha\delta - 3\delta)}{(3 + \alpha)\delta} \right] = \frac{1}{3} \left[ 1 + \frac{(3 - \delta)\alpha}{(3 + \alpha)\delta} \right],$$

which is equation (18). Similarly, for  $\delta \in [0, \bar{\delta}(\alpha)]$ , we can rewrite the expression for  $E[x_1]$  in part (b) of Proposition 4 as

$$E[x_1] = \frac{1}{3} \left[ 1 + \frac{6}{(6 - \delta - \alpha\delta)} - \frac{(6 - \delta - \alpha\delta)}{(6 - \delta - \alpha\delta)} \right].$$

Simplifying,

$$E[x_1] = \frac{1}{3} \left[ 1 + \frac{(\delta + \alpha\delta)}{[6 - (1 + \alpha)\delta]} \right] = \frac{1}{3} \left[ 1 + \frac{(1 + \alpha)\delta}{[6 - (1 + \alpha)\delta]} \right],$$

which is equation (20). The formulas in equations (19) and (21) are obtained by substituting equations (18) and (20) into equation (17).

**Proof of Proposition 5.** In light of equation (17), we only need to show how  $E[x_1]$  responds to increases in  $\alpha$  and  $\delta$ .

(a) By rewriting equation (18) as

$$E[x_1] = \frac{1}{3} \left[ 1 + \frac{(3 - \delta)}{(1 + \frac{\alpha}{\delta})\delta} \right],$$

it is clear that  $E[x_1]$  is increasing in  $\alpha$  when  $\delta \in [\bar{\delta}(\alpha), 1]$  and  $\delta \neq 0$ . Because an increase in  $\alpha$  decreases the denominator in the expression for  $E[x_1]$  in part (b) of Proposition 4,  $E[x_1]$  is also increasing in  $\alpha$  when  $\delta \in (0, \bar{\delta}(\alpha))$ .

(b) Because an increase in  $\delta$  decreases the numerator and increases the denominator in the fraction in equation (18) when  $\alpha > 0$ ,  $E[x_1]$  is decreasing in  $\delta$  when  $\delta \in [\bar{\delta}(\alpha), 1]$ . Because an increase in  $\delta$  decreases the denominator in the expression for  $E[x_1]$  in part (b) of Proposition 4,  $E[x_1]$  is increasing in  $\delta$  when  $\delta \in (0, \bar{\delta}(\alpha))$ .

## Notes

1. Calvert and Dietz (2005) also consider a model without partisanship in which payoff externalities are symmetric between pairs of legislators.
2. Kriner and Schickler (2016) illustrate this phenomenon with the case of President Reagan's EPA Administrator, Anne Gorsuch, who sought to manipulate 'Superfund' cleanup funds for political purposes so as to limit the benefits for Democrats from receiving the funds.
3. When the nonpartisan is the proposer, which happens with a one-third probability, he must form a bipartisan coalition in order to have his proposal accepted. Henceforth,

- when we speak of bipartisanship, we refer to the acceptance by the nonpartisan of a partisan's proposal, as this is the only kind of bipartisanship for which there is a viable partisan alternative.
4. A wide range of models analyze legislative bargaining in different political systems using agents who are political parties or factions. See, for example, Austen-Smith and Banks (1988), Baron and Diermeier (2001), and Sin and Lupia (2013).
  5. This assumption merely facilitates equilibrium selection rather than preventing any legislator from playing a best response.
  6. Because of the utility externality due to party affiliation, the utility that a copartisan obtains when a partisan makes an offer to the nonpartisan may be sufficient to obtain his support even if he is not offered any of the dollar himself. Nevertheless, it is never in the interest of a partisan to make an unacceptable offer to the nonpartisan so as to obtain his copartisan's support because if he simply kept the amount offered to the nonpartisan, the utilities of both partisans would be increased. Thus, a partisan only makes an offer to the nonpartisan if he wants his support.
  7. The symmetry assumptions we have made in defining the three proposals are the same as in Calvert and Dietz (2005) and are analogous to those made implicitly in Proposition 3 of Baron and Ferejohn (1989).
  8. Baron and Ferejohn (1989) allow for an arbitrary odd number of legislators. Our description of their equilibrium is for the three legislator case illustrated in their Figure 3.
  9. The *Mathematica* code for plotting the graphs in Figures 1 and 3 is provided in the supplementary online appendix.
  10. The equilibrium values of  $p$ ,  $x_P$ ,  $x_N$ , and  $x_S$  are continuous in the parameter values. In our discussion, it is sometimes more convenient to include  $\bar{\delta}(\alpha)$  in Region 2 and  $\underline{\delta}(\alpha)$  in Region 3.
  11. In the appendix, we show that, in equilibrium, each of the shares is nonnegative and that  $p$  is in the interval  $[0, 1]$ . In particular, we show that  $x_P > 0$  when  $\delta > \bar{\delta}$  because  $\bar{\delta} > 2\alpha/(1 + \alpha)$ . Thus, in order for a partisan to be offered a positive share by his copartisan, he must be sufficiently patient, with the degree of patience required increasing in the strength of party affiliation. See equations (44) and (45).
  12. The statements made in this paragraph are established formally in the proof of Proposition 1.
  13. The threshold values  $\underline{\delta}(\alpha)$  and  $\bar{\delta}(\alpha)$  that define the region boundaries are increasing in  $\alpha$ . When  $\delta$  is on the lower boundary of Region 1 ( $\delta = \bar{\delta}(\alpha)$ ), a marginal increase in  $\alpha$  results in this value of  $\delta$  being in Region 2. Before and after this change, a partisan only makes an offer to his copartisan if  $\delta \neq 0$ . Similarly, when  $\delta$  is on the lower boundary of Region 2 ( $\delta = \underline{\delta}(\alpha)$ ), a marginal increase in  $\alpha$  results in this value of  $\delta$  being in Region 3. No share of the dollar is offered by a partisan proposer to his copartisan either before or after this change.
  14. See equations (24) and (30). The following claims may be formally confirmed by differentiating equations (24), (26), (27), (30), (31), and (32) with respect to  $\delta$ .
  15. Both of their *discounted* continuation values increase.
  16. Alternatively, we could consider instead the expected utilities of the legislators. For the nonpartisan, this is equivalent to considering shares because his share is his utility. However, the partisan legislators' expected utilities include the externality from their copartisan's share. Therefore, the comparative statics for the expected utilities with respect to the partisan affiliation parameter  $\alpha$  involve making an interpersonal utility comparison between legislators with different degrees of partisanship and, thus, are difficult to interpret. Additional reasons for focusing on shares rather than utilities are

- that (i) shares are more easily measured empirically than are utilities and (ii) shares highlight the material incentives for the development and maintenance of partisan bonds.
17. The algebra used to derive equations (18) to (21) may be found in the appendix.
  18. Setting  $\delta = 1$  in Proposition 4 or in equations (18) to (21), we obtain formulas for the expected shares in the Calvert and Dietz (2005) model.
  19. Because  $\bar{\delta}(\alpha)$  is only smaller than 0.5 for values of  $\alpha$  significantly below 0.1, we do not include values of  $\delta$  smaller than 0.5 in Figure 3.
  20. Our discussion of Propositions 2 and 3 provides further intuition for Proposition 5 because it supplies intuition for the comparative static results that are appealed to here.
  21. Some of the expressions in this appendix were derived using *Mathematica*. The *Mathematica* code for these derivations may be found in the supplementary online appendix.
  22. The expression on the right-hand side of this inequality is obtained by replacing the inequality in equation (46) with an equality and then using the quadratic formula to solve the resulting equation. Only the positive root is relevant. The restriction on  $\delta$  implied by equation (48) is obtained similarly. See the supplementary online appendix for *Mathematica* code to implement these calculations.
  23. If  $p = 1$ , then part (a).(i).ii of Proposition 1 is trivially true.
  24. The equilibrium values of the variables are continuous in the parameters at a boundary between two regions, but they may not be differentiable.

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