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Predicting Navigational Error of Visual Binary Stars

ABSTRACT

For years, the Navy has been using a selection of bright stars for celestial navigation purposes that, by default, includes a number of stars in binary systems. Often these systems are resolved as a single star at the center-of-light of the system. However, from this list of navigational binary stars we have isolated fourteen such binary systems with at least 3.0 arcseconds apparent separation between the primary and secondary star. A separation of this magnitude is needed to resolve both stars in the pair at the magnification of most common navigational sextants. We estimate the maximum error associated with using each such binary star pair in a navigational fix when observed at maximum separation using a catalog position that corresponds to the center-of-light of the system. The change in the maximum navigational error over time appears to coincide with the period of the binary star system's orbit, as predicted. Additionally, it was found that the error varied with the observer's latitude, although this effect seems to be much less predictable. The system demonstrating the highest maximum navigational error came with HR 3579 (10 Ursae Majoris) at a predicted navigational error of 1169 m in 2009 using this method.

INTRODUCTION

The Navy mandates that each ship utilize two independent forms of navigation. The Navy relies on the GPS as its primary means of navigation. Since GPS signals are easily jammed or spoofed, the Navy has an ever-increasing need for alternate forms of navigation. Thus, celestial navigation has continued importance. That being said, many navigational stars are visual binary stars. Since instruments may have the ability to resolve the separate components of many binary systems, it is important to understand and characterize the possible navigational errors brought on by the apparent separation of binary system components in the sky. This paper examines the maximum error associated with some useful navigation stars.

Assumptions

Three stars are needed for a navigational fix using the altitude-intercept method of celestial navigation. Here, we assume that these three stars are optimally 120° apart from one another. Further, we assume that two of three stars in the fix have positions known with exact accuracy. The third star is assumed to be part of a wide binary system with a known orbit. The positions of its components can be calculated from the center-of-mass of the binary system at any point in time during their orbital period. The mass ratio of the two components in the system is needed in the calculation; when masses are not known, this is estimated from the spectral types of the stars. The maximum error translated to the navigational fix from a binary system is assumed to be the vertical separation of the components in the system relative to the observer's horizon. In the calculation of the offset in altitude due to the vertical orbital offset, refraction of Earth's atmosphere is not accounted for because it will be nearly the same for both component stars. Altitude observations of the binary system are assumed to be taken at 0° longitude and at all latitudes separated by a tenth of a degree.

Methodology

Orbital parameters for the binary systems of focus were taken from the *USNO Sixth Catalog of Orbits of Visual Binary Stars* (hereafter, the 6th orbit catalog). Our sample included the intersection between the 59 navigation stars from *The Nautical Almanac* (found in the "Index to Selected Stars") and the 6th orbit catalog. The 6th orbit catalog gave us all orbital parameters, except for the mass ratio of the two components and the center-of-mass position of the systems. Center-of-mass positions were taken from the *Basic Fifth Fundamental Catalogue* (FK5), where available, otherwise they were taken from the *Hipparcos Catalogue* (absolute accuracy in the positions was not required because we measured relative separation). These data were then run through a program that takes the orbital parameters and outputs the positions (in right ascension and declination) of both stars in the system at a given time, rendering an orbital position. Knowing both the right ascension and declination of each star in a binary star system easily translates to the offset in their observed altitudes. The *USNO Vector Astrometry Software* (NOVAS) function equ2hor takes right ascension and declination of a star during a certain epoch, as well as observer location on Earth, and returns the altitude and azimuth of that star. Running NOVAS on both stars in the binary system yields each star's altitude. Since the maximum error is assumed to be the distance between the altitudes of the two components, we simply subtract the two altitudes to yield the error in altitude.

In a navigational fix, any movement of a line of position (in meters) for an observed star correlates directly to an offset (Δa, in arcseconds) in the altitude at an average of about 30.87 m/arcsecond (simply the average radius of Earth multiplied by one arcsecond in radians). Since we are assuming the three navigation stars are optimally 120° degrees apart, the lines of position for the fix take the form of an equilateral triangle. Moving one line perpendicularly will change the height of the triangle by ΔH. Two thirds of the height of the triangle is the center of an equilateral triangle from the vertices, which is our assumed fix. Thus:

$$a_1 - a_2 = \Delta a$$

$$\Delta H = 30.87 \text{ m/arcsecond} * \Delta a \text{ arcseconds}$$

$$\Delta R = (2/3) * \Delta H \text{ m}$$

where ΔR is the change in the position of a navigational fix due to the offset in observed altitude from the binary motion from the center-of-mass for one navigational star in a three star navigational solution.

The results for ΔR were examined against location of the observer on Earth at latitudes at which the star was observable in the night sky. In this case, the observed longitude is assumed to be a constant at 0°. To determine the latitudes at which a star is visible, the declination was used. For positive declinations, the entire northern hemisphere can view the star. Additionally, any negative latitudes within 90° of the declination can view the star. For negative declinations, the entire southern hemisphere can view the star as well as positive latitudes within 90° of the declination. All visible latitudes could observe the star at some point during the day. In that range of visible latitudes, calculations at 0.1 degree intervals formed

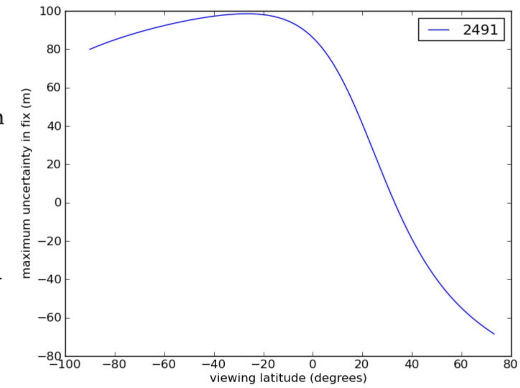

FIGURE 1. Maximum error in fix in meters vs. viewing latitude in degrees at a fixed time JD 2000.5 and longitude for HR 2491.

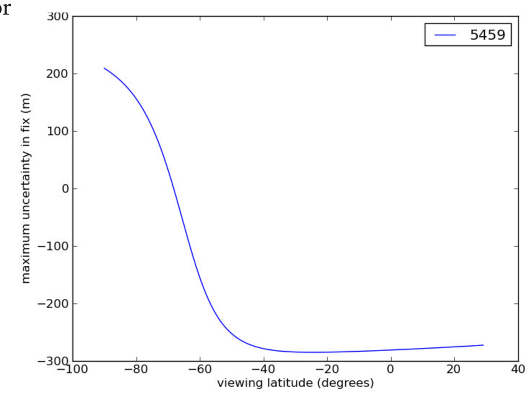

FIGURE 2. Maximum error in fix in meters vs. viewing latitude in degrees at a fixed time JD 2000.5 and longitude for HR 5459.

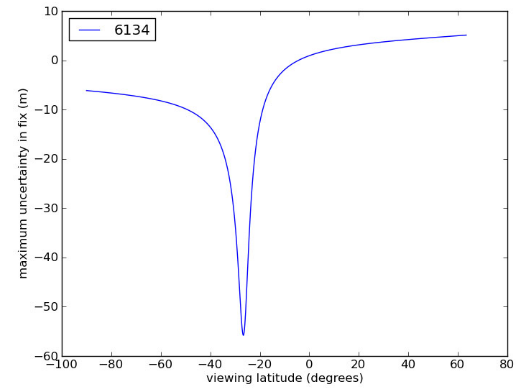

FIGURE 3. Maximum error in fix in meters vs. viewing latitude in degrees at a fixed time JD 2000.5 and longitude for HR 6134.

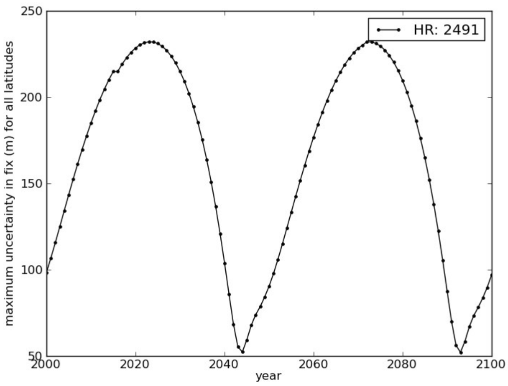

FIGURE 4. Maximum error over all latitudes vs. year for HR 2491.

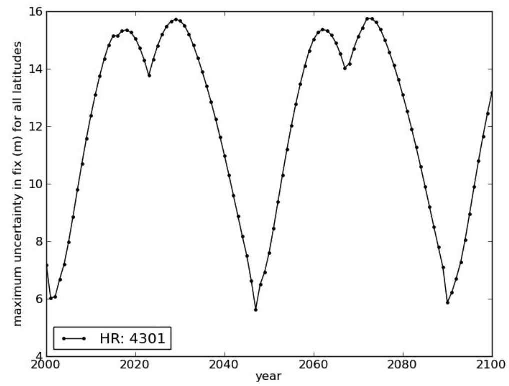

FIGURE 5. Maximum error over all latitudes vs. year for HR 4301.

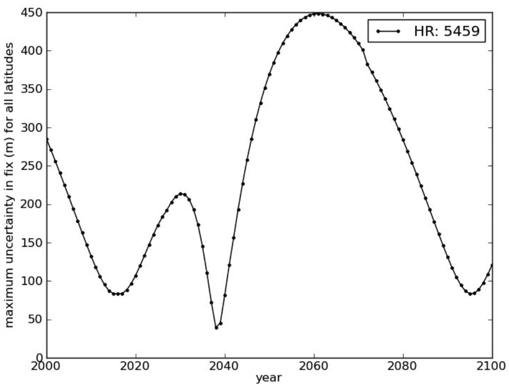

FIGURE 6. Maximum error over all latitudes vs. year for HR 5459.

a continuous curve of ΔR for each star (Figures 1-3) at epoch 2000.5 to illustrate the change in a navigational fix due to an observer's latitude. This data processing was repeated for each year over the course of a one-hundred-year period. For each year, the maximum absolute value error for any latitude was determined and used to characterize the maximum navigational error over the course of the orbital period of the system (Figures 4-6). Note that the maximum separation for a given year somewhere on Earth would just be the maximum separation between the stars times 30.87 meters/arcsecond. Thus, altitude and azimuth are unneeded for this analysis.

Additionally, some stars were chosen for analysis over varying longitudes as well as latitudes. This was done to simulate the change in maximum error throughout the day. This technique assumes that the star is visible at all times of the day, which is mostly not the case. However, this analysis was still useful in illustrating the change in maximum error as Earth rotates.

Results and Discussion

Figures 1-3 display the maximum error in a navigational fix in meters due to orbital offset for varying latitudes at a constant longitude of 0° to characterize the relationship between observer latitude and error of the fix.

As HR 2491 (a.k.a. Sirius) is a Southern Hemisphere star, Figure 1 illustrates that only latitudes of about 70° or lower can view this star. As the plot shows, the viewing angle of the star has changed the maximum error associated with the binary star system in a way that resembles a smooth curve.

HR 5459 (a.k.a. Alpha Centauri) in Figure 2 is also a Southern Hemisphere star, which is why only latitudes of about 30° degrees and below can view it. Much like Figure 1, it decreases in a smooth way. Interestingly, it seems to bottom out at a certain point.

HR 6134, shown in Figure 3, is a Southern Hemisphere star as well. This star is interesting as it shows a sharp decline in maximum error around -30° degrees latitude. It appears to approach some type of asymptote in the data. Since this is a stepwise analysis of 0.1 degrees latitude and not an actual smooth curve, a real asymptote is unlikely to appear in the data.

Figures 4-6 estimate the maximum navigational error in meters that binary systems can introduce over the course of their orbital period.

Figure 4 shows the periodic nature of the error using HR 2491 in a navigational fix at the latitude of maximum error at yearly intervals. This curve follows a periodic, almost sinusoidal path that corresponds to the 50-year period of the orbit of Sirius. As the Sirius system moves into different orbital positions, the apparent angular separations of their components in the night sky waxes and wanes.

Figure 5 displays the maximum error that HR 4301 (a.k.a. Alpha Ursae Majoris) experiences yearly at any latitude. Much like in Figure 4, the maximum error associated with HR 4301 waxes and wanes with the angular separation of the components of the system.

Figure 6 displays the maximum error that HR 5459 experiences yearly at any latitude.

Although this plot resembles a sine wave far less, the periodicity of the system's 80-year orbit can still be seen, beginning around 2018 and ending around 2098, peaking at a maximum error of 450 m in roughly the year 2060.

Additional analysis of this method using varying longitude was done stepwise over multiple plots, but the result would be too unwieldy for this paper format. It should be noted, however, that the plots smoothly flowed into one another when viewed in rapid succession in a way analogous to the plots of varying latitude.

Conclusions

The method described here, using the outlined assumptions, is able to characterize the error in a celestial navigation fix introduced by using a relatively wide binary star as one star in the three star navigational solution. As expected, the results of the fixed time and variable longitude analysis indicate variability of the maximum error in a fix depending on the observer's latitude on Earth. This variability occurs along a smooth curve, however, predicting the shape of such curves has generally proven challenging. It most likely has to do with the position angle at which the stars are separated.

Examining this maximum latitude over time yields the error in the navigational fix over the orbital period of the binary system. The periodic nature of the maximum error mirrors the orbital period of the system, however, it does not always vary smoothly. The most significant result is the magnitude of the maximum uncertainties, which could possibly translate into the error in a navigational fix over time such as 450 m for HR 5459 in 2060.

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