On-the-job-search, wage dispersion and trade liberalization

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Abstract. This paper builds a dynamic, general equilibrium, open economy model which allows for *ex-ante* homogeneous workers to experience different wage growth before and after trade liberalization. Key features of our model include a product market composed of heterogeneous firms and a labor market characterized by directed, on-the-job-search and dynamic contracts. We characterize the complementarity of these two sources of wage dispersion in an open economy setting where trade liberalization affects the endogenous equilibrium set of producers and the structure of wages. We find that worker job-to-job transitions, through on-the-job search, increases the impact of trade liberalization on wage dispersion by 4%.

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1 Introduction

Free trade is often supported by the argument that, in the long-run, trade liberalization will induce the reallocation of resources towards more productive uses. This, in turn, has the potential to generate substantial welfare gains both at home and abroad. In contrast, much policy discussion focusses on the impact that trade has on the unequal distribution of returns from trade across workers. This distinction has led to a clear disconnect between policy makers, who stress the impact of trade liberalization on employment and wage dispersion, and trade economists, who generally tend to emphasize the role trade liberalization has on equilibrium prices and resource allocation across countries.

This paper contributes to a growing literature that attempts to fill this gap. We build a dynamic, general equilibrium, open economy model with a frictional labour market. The model in this paper has several distinguishing features. First, we assume that single-worker firms offer dynamic wage contracts to attract workers, and the workers direct their search to a particular job offer. A single-worker firm setting allows us to derive clean characterization of the optimal dynamic contract even in the presence of on-the-job search and firm-heterogeneity. Second, the value of the contract induces the worker's effort, which positively affects the output of a match.[1] The value of the contract varies over time as the match continues. Therefore, by the design of the optimal contract, the single-worker firm with a fixed production technology can control its output level.[2] Third, we allow for on-the-job search of employed workers, and the worker job-to-job transition creates endogenous heterogeneity among ex-ante homogenous workers. In our model, firm productivity heterogeneity and dynamic contracts interact with the endogenous worker heterogeneity to generate novel insights into firm behavior over time in the face of international markets.

Our model builds upon two key papers in the literature of international trade and macro-labour search. The product market follows the Helpman et al. (2011) extension of Melitz (2003) which incorporates a frictional labour market. In particular, we consider a setting where firms with heterogeneous productivities produce horizontally differentiated goods using labour services acquired on a frictional labour market. While the Helpman et al. (2011) framework assumes a random search and matching process and that workers and firms are both heterogeneous, we, in contrast, consider a model based on a directed search process where workers are ex-ante homogenous. The labour market follows the Menzio and Shi (2010) model of on-the-job search and dynamic wage contracting. To embed this class of labour search models with single-worker firms into a monopolistically competitive product market, we apply the Tsuyuhara (2016) extension of Menzio and Shi (2010) which allows for variable worker effort.[3]

A large macro-labour search literature studies wage dispersion and employment dynamics in frictional labour markets. Following Hornstein et al. (2011), this literature recognizes that worker

[1] The intuition behind this structure follows that of the large body of efficiency wage models. See Davis and Harrigan (2011) or Amiti and Davis (2012) for examples of the application of efficiency wages to models of international trade. A theoretical basis in a search and matching model is provided by Tsuyuhara (2016).

[2] A number of recent papers explore the nature of trade in multi-worker setting including Helpman and Itskhoki (2010), Helpman et al. (2011), Fajgelbaum (2016), and Felbermayr et al. (2014). These papers apply a multi-worker firm framework to allow for variable firm-level output.

[3] For the firm problem to be consistent in a monopolistically competitive product market, the firm needs to be able to adjust its output level (or equivalently its price level) when facing a residual demand curve. Unlike Tsuyuhara (2016), however, we do not consider a setting where worker effort is subject to a moral hazard problem to simplify the exposition.

on-the-job search and the resulting backloading of wages play a crucial role in generating empirically reasonable measures of wage dispersion. In this sense, our model joins two classic sources of residual wage dispersion: firm productivity differences and on-the-job-search with dynamic contracts.[4]

Helpman et al. (2016) emphasize that firm-specific and firm-worker specific differences account for a large percentage of wage variation across firms and workers in an open economy. Our labour market directly captures this first element of wage variability in the sense that employers with higher productivity levels offer systematically higher wages than less productive employers. Further, conditional on productivity, our model will still generate variation in wages across workers due to wage-tenure contracts and on-the-job search. Incorporating this mechanism enables us to capture how dynamic wage contracts and worker mobility interact with endogenous dispersion in domestic revenues, export revenues and export participation in a environment without stochastic firm-level shocks.

International trade provides a natural setting to study this complementarity. The possibility of exporting affects the equilibrium set of profitable firms, the dynamic contracts offered to workers and the resulting wage dispersion. In our setting, more productive firms endogenously choose to export, offer higher wages and are characterized by lower turnover. The dynamic contracting environment in combination with endogenous employment dynamics implies that trade liberalization will affect both the set of exporters and the nature of export entry and output growth. A reduction in trade costs increases the profitability of exporting, which in turn influences the structure of wages offered to workers across the distribution of heterogeneous firms.

To illustrate the qualitative implications of our model, we first calibrate the model's parameters to match salient features of the US economy. In particular, we fit the production side of our model to match key features of firm-heterogeneity and wage-setting behavior among US manufacturers. The model's frictional labour market is likewise set to replicate observed features of the US labour market. The calibrated model shows that firm-heterogeneity, dynamic contracts and on-the-job-search are strong complements in generating larger wage dispersion among ex-ante homogenous workers. For instance, the ratio of the average wage and the minimum wage, "mean-min ratio," in our simulation is 2.63, which is significantly larger than that of previous studies and closer to its well-known empirical counterpart.[5]

We then use the quantitative model to investigate the impact of trade liberalization on equilibrium wage dispersion among US manufacturing workers. Trade liberalization increases the profitability of entering export markets for any given firm. While this induces previous non-exporters to start exporting, it also encourages firms to offer new, higher paying jobs to prospective workers. As workers move through the wage distribution they achieve higher wages through promotion or on-the-job-search than what was previously possible, even if the underlying productivity of their firm does not change. Across the distribution of ex-post heterogenous workers we find that this

[4]Instead of dynamic contracts as in our model and preceding papers such as Burdett and Coles (2003, 2010) and Shi (2009), Postel-Vinay and Robin (2002) and Cahuc et al. (2006) generate backloading of wages through wage bargaining with Bertrand competition between current and prospective employers. See Papp (2013) for a quantitative assessment of those models.

[5]Hornstein et al. (2011) demonstrate that the mean-min ratio is a useful measure for evaluating the model's capability to capture frictional wage dispersion in search models. They show that the canonical search model generates mean-min ratios of 1.05-1.1, while their estimates of empirical mean-min ratios, across a number of data sources, are between 1.5 and 2.

effect causes the dispersion of wages to increase by at least 3.2%.

Felbermayr et al. (2014) studies a monopolistic competition model of trade with heterogeneous firms and labour search frictions. The labour market in their model is based on Kaas and Kircher (2015). Our framework is similar: monopolistic competition and trade with heterogeneous firms and a labour market characterized by directed search. A key difference in our case is that we allow for on-the-job-search and for firms to offer dynamic wage contracts, instead of fixed-wage contracts. In addition, in our model, dynamic wage contracts generate endogenous productivity variation among ex-ante identical firms due to varying worker effort.

In a closely related model, Fajgelbaum (2016) also considers a monopolistic competition model of trade with heterogeneous firms and a labour market characterized by on-the-job-search. In that setting firm output grows when firms make fixed investments and accumulate workers over time. In contrast, firm output growth in our setting is driven by dynamic contracts and the resulting endogenous effort supplied by a worker. Moreover, wage dynamics and firm dynamics in Fajgelbaum (2016) are characterized by wage bargaining with Bertrand competition between current and prospective employers in a random search and matching framework, as in Cahuc et al. (2006) and Postel-Vinay and Robin (2002). In contrast, wage dynamics and resulting firm behaviour in our model are driven by dynamic contracting in an environment where worker search is a directed process. A directed search process, as opposed to random search, significantly increases tractability of the model and enables a clean characterization of the optimal dynamic contract.

Unlike Felbermayr et al. (2014) and Fajgelbaum (2016), where firm growth is captured by the number of employees, in our setting firm output growth is modeled in terms of the output of a single worker. This feature arises due to the technical difficulty of combining dynamic contracts and multiple workers in our model. To characterize the optimal dynamic contract for a single worker match, the appropriate state variable for the recursive contracting problem is the value of the contract that the firm promises to its worker. With multiple workers, however, we need to compute the *accumulated value of contracts* that the firm promises to its workers, which depends on the distribution of workers within each firm. We are not aware of an established method to tractably analyze the distribution of accumulated value in a dynamic wage contract setting. Characterizing the optimal dynamic contract in such a setting is beyond the scope of this study.[6]

Our work is also broadly related to the rich and rapidly expanding literature studying the interaction of international trade and labour markets. Early, static models of Davidson et al. (1988) and Hosios (1990) examine the robustness of conventional trade theories in two-sector, small open economy models with search frictions. More recently, Helpman and Itskhoki (2010) demonstrate that labour market flexibility can be a source of comparative advantage and that policy differences across countries can affect the pattern of trade. Using a directed search framework, King and Strähler (2014) similarly characterize how differences in endowments and technologies across countries affect equilibrium unemployment after trade liberalization.

Like many of these studies, we analyze a single industry economy. However, this should not suggest that interindustry reallocation is of less significance. As documented in the structural models of Artuc et al (2010), Coşar (2013) and Dix-Carneiro (2014), trade liberalization can be associated with substantial, long-run, interindustry reallocation. In contrast to these papers, a number

[6]The assumptions of fixed-wage contracts and risk-neutral workers in Kaas and Kircher (2015) and thus Felbermayr et al. (2014) allows these authors to compute an accumulated wage bill, which is a more tractable state variable for the value of the firm.

of reduced-form empirical studies, such as Wacziarg and Wallack (2004) and Goldberg and Pavcnik (2007) find little evidence of interindustry labor reallocation in response to trade liberalization. Though empirical evidence may not be conclusive yet, the single-industry assumption is an important restriction of our work when it comes to evaluating the effects of trade liberalization.

Our paper is naturally related to work which studies output dynamics and entry into export markets. As highlighted in the empirical work of Lopez (2009) and Lileeva and Trefler (2010), growth among new exporters often occurs prior to entry and in response to trade liberalization. Likewise, although we focus on a two country setting, the endogenous export dynamics are consistent with the slow growth of new exporters highlighted by Albornoz et al (2012), Rho and Rodrigue (2016), and Ruhl and Willis (2017).

The next section presents our model that integrates a well-defined product market structure with a search theoretic labour market model. Section 3 describes equilibrium conditions and characterizes the nature of our frictional labour market, dynamic wage contracts, firm dynamics, and endogenous export decisions. Section 4 describes our calibration procedure and documents the calibrated model's quantitative implications in the steady-state. Section 5 concludes.

2 Model Environment

We consider an economy with two identical countries: home and foreign. Time is discrete, continues forever, and is indexed by t. In each country, there is a continuum of firms and a continuum of infinitely lived workers. There are two markets: the product market and labor market. In the product market, a continuum of horizontally differentiated varieties of consumption goods are traded. A worker and a firm meet in a frictional labor market and create a job. All goods are internationally tradable, while labor service is not.

2.1 The Product Market

The real consumption index for the product market takes the constant elasticity of substitution form:

$$Q = \left(\int_{\omega \in \Omega} y(\omega)^{\rho} d\omega \right)^{\frac{1}{\rho}}, \tag{1}$$

where ω indexes varieties and Ω is the set of available varieties. We assume $0 < \rho < 1$ so that these goods are substitutes, and the elasticity of substitution between any two goods is given by $\sigma = 1/(1-\rho) > 0$. The price index corresponding to Q is given by

$$P = \left(\int_{\omega \in \Omega} p(\omega)^{1-\sigma} d\omega \right)^{\frac{1}{1-\sigma}}, \tag{2}$$

where $p(\omega)$ is the price of variety ω. We choose the aggregate good (at home) as the numéraire, and we normalize $P = 1$. Given this market structure, the demand for variety ω is

$$y(\omega) = A^{\sigma} p(\omega)^{-\sigma}, \tag{3}$$

where A is a demand shifter for the home economy. In equilibrium, after normalizing $P = 1$, $A = E^{1-\rho}$ holds where E is the total expenditure on the varieties within the home economy.

In this monopolistically competitive product market each operating firm produces a different variety of product ω. Therefore, given the standard demand function (3), each firm's equilibrium revenue from producing variety ω is

$$r(\omega) = p(\omega)y(\omega) = Ay(\omega)^{\rho}. \tag{4}$$

2.2 The Labour Market

The labor market is modeled in a similar way as in Menzio and Shi (2010) and Tsuyuhara (2016). All workers and firms are *ex-ante* homogenous. Workers are risk-averse and have a periodical utility function of consumption $v(\cdot)$, which is strictly increasing, strictly concave, and twice continuously differentiable. We assume that workers cannot borrow and save, so their consumption equals wage w if employed or unemployment benefit b if unemployed. Employed workers exert effort on the job, and the disutility of effort is given by $c(\cdot)$, which is strictly increasing, strictly convex, and twice continuously differentiable. Each worker maximizes the expected lifetime sum of utilities discounted at rate $\beta \in (0, 1)$.

There is an unbounded number of potential firms entering into the market. Prior to entry, all firms are identical. Each firm has an access to common production technology:

$$y = ze, \tag{5}$$

where z is idiosyncratic productivity and e is effort exerted by the worker employed in the firm. Each firm maximizes the expected sum of profits discounted at the rate β.

Entering firms create a vacancy and post a job offer at a flow cost $k > 0$. A job offer is fully described by the long term wage contract, which specifies wages that the worker receives in each period as a function of tenure on the job, namely the *wage-tenure profile*. The contract also depends on the aggregate state of the economy as described below. We assume the offer is binding and no renegotiation occurs after a firm and a worker agree to create a job.

For a given wage-tenure profile, workers can calculate a discounted lifetime expected utility that the contract would deliver, taking into account the possibility of job destruction and the worker's job-to-job transition. We call it *the promised value*, or simply the *value* of the contract. Workers can calculate the value of the contract not only at the beginning of tenure but also at any point of tenure in terms of the truncated section of the wage profile.

Using this notion of the value of a contract, the labor market is organized in a continuum of submarkets indexed by $x \in X = [\underline{x}, \bar{x}]$, which denotes the value of contract offered in that submarket. That is, submarket x consists of all the firms that promise to deliver value x, irrespective of the shapes of each individual wage profile. Then, following the traditional view of efficiency-wage models, we assume that a worker's effort depends positively on the current value of the contract. That is, we assume an increasing and continuously differentiable function $e : X \to \mathbb{R}_+$ that determines the worker's effort on the job.[7]

[7]Worker effort is usually modeled as a function of wages, instead of the value of a contract, in standard efficiency wage models such as Solow (1979) and Summers (1988). The existence of such a function is shown in Tsuyuhara (2016) with a more explicit incentive structure.

Once a firm incurs the cost of entry and chooses which submarket to enter, it draws its idiosyncratic productivity z from a common distribution $G(z)$. $G(z)$ has positive support over $(0, \infty)$ and has a density $g(z)$. Idiosyncratic productivity is constant throughout the duration of operation. If a firm's realized productivity is too low relative to the value of the contract it promised, the firm can choose to exit immediately before engaging in the matching process.

Both employed and unemployed workers get the opportunity to search for a job with probability λ_e and λ_u, respectively. Observing the available vacancies, searching workers direct themselves to a particular submarket. In submarket x, the ratio between the number of vacancies and the number of searching workers is denoted by $\theta(x) \geq 0$, and we refer to it as *the tightness* of submarket x.

In each submarket, a worker and a firm meet through a frictional matching process, which is summarized by a job finding probability function, $p(\theta)$, and a vacancy filling probability function, $q(\theta)$. We assume that $p : \mathbb{R}_+ \to [0,1]$ is twice continuously differentiable, strictly increasing, and strictly concave, and satisfies $p(0) = 0$ and $p'(0) < \infty$, and that $q : \mathbb{R}_+ \to [0,1]$ is twice continuously differentiable, strictly decreasing, and strictly convex, and satisfies $\theta^{-1}p(\theta) = q(\theta)$ and $q(0) = 1$. In addition, the matching technology is assumed to satisfy the condition that $p(q^{-1}(\cdot))$ is concave.

In each period, an operating firm faces a constant probability $\delta \in (0,1)$ of a bad shock that destroys the job and forces the worker into unemployment. Finally, let $\Phi(x, z)$ be the distribution of workers who are employed at a job that gives the value of contract $X \leq x$ and that have an idiosyncratic productivity $Z \leq z$. Also, let u be the measure of unemployed workers. The state of the labour market is denoted by $\psi_t \equiv (\Phi_t, u_t)$, and the evolution of the state is generically denoted by an operator Ψ so that $\psi_{t+1} = \Psi(\psi_t)$.

2.3 The Worker's Job Search Problem

A worker who gets the opportunity to search chooses which submarket to enter, taking into account the value of the job offer and the probability of actually finding a job in each submarket. The optimal job search decision depends on the reservation value, which is the promised value of the current contract for employed workers, and the value of unemployment for unemployed workers. Consider a searching worker whose reservation value is $\hat{V}$. If he visits submarket x, he finds a vacancy and moves to a new job with probability $p(\theta(x, \psi))$ or fails to find a job and stays with the current job or stays unemployed with probability $1 - p(\theta(x, \psi))$. Therefore, his optimal job search decision maximizes $p(\theta(x, \psi))x + (1 - p(\theta(x, \psi)))\hat{V}$ with respect to x. We denote the maximized net expected value of job search given reservation value $\hat{V}$ by

$$D(\hat{V}, \psi) = \max_{x \in X} p(\theta(x, \psi))(x - \hat{V}). \tag{6}$$

The solution to this maximization problem is denoted by $m(\hat{V}, \psi)$, and we define $\tilde{p}(\hat{V}, \psi) \equiv p(\theta(m(\hat{V}, \psi), \psi))$ as the probability that the worker finds a vacancy in the optimally chosen submarket given the current promised value $\hat{V}$. Given this optimal search decision, the searching worker's gross expected value of search is $\hat{V} + D(\hat{V}, \psi)$, and the worker's expected value for the next period given $\hat{V}$ is $\lambda_k(\hat{V} + D(\hat{V}, \psi)) + (1 - \lambda_k)\hat{V} = \hat{V} + \lambda_k D(\hat{V}, \psi)$ where $k = e, u$.

To define the value of unemployment, let $U(\psi)$ denote the value of unemployment given the state of the labor market ψ. In the current period, unemployed workers receive and consume unemployment benefit b. If an unemployed worker gets the opportunity to search for a job with probability λ_u, he solves the above problem with $U(\hat{\psi})$ as his reservation value, where $\hat{\psi}$ denotes the state of the labor market next period. Then, as described above, the expected value for an unemployed worker entering next period is $U(\hat{\psi})+\lambda_u D(U(\hat{\psi}),\hat{\psi})$. Therefore, if the unemployment benefit is constant over time, the value of unemployment $U(\psi)$ satisfies the following recursive equation:

$$U(\psi) = v(b) + \beta \mathbb{E}_\psi (U(\hat{\psi}) + \lambda_u D(U(\hat{\psi}), \hat{\psi})), \tag{7}$$

where the expectation is taken with respect to the state of the labour market ψ.

2.4 The Firm's Problem

The firm's problem is divided into two parts: 1) its entry and exit decision and 2) the design of the dynamic contract which incorporates its export decision. The solution to the contracting problem gives the value of the firm for a given pair of state variables: the promised value of the contract and firm productivity. For an entering firm, this value function also becomes the value of creating a vacancy in a submarket for a given productivity. Therefore, we describe the firm's problem backwards: we start by describing the firm's export decision, which generates the firm's revenue function. We then describe the dynamic contracting problem, which generates the firm's value function. And finally we explain the entry and exit decision.

2.4.1 Export decision

Let $s \equiv (\psi, \{A_j\}_{d,x})$ denote the aggregate state of the economy, and its evolution is generically denoted by Λ so that $s_{t+1} = \Lambda(s_t)$.[8] Each firm takes s and its evolution as given.

Consider a firm whose current period output is y. Given this amount of output, the firm chooses whether to serve only the domestic market or to engage in exporting as well. In order to export, a firm has to incur a fixed cost $c > 0$ and iceberg-type variable trade costs: $\tau > 1$ units of a variety must be exported for one unit to arrive in the foreign market. If the firm chooses to export, it allocates its output between the domestic and export market (y_d and y_f, respectively) to equate its marginal revenues in the two markets. Following the argument in Helpman et al. (2011), the firm's revenue as a function of its export decision ι and its total production level $(y = y_d + y_f)$ can be expressed as

$$R(\iota, y, s) = A_d[1 + \iota(\Upsilon_x - 1)]^{1-\rho} y^\rho, \tag{8}$$

where ι equals 1 if the firm exports and 0 otherwise. The variable Υ_x captures a firm's "market access," which depends on the demand shifters of both domestic and export markets:

$$\Upsilon_x = 1 + \tau^{-\frac{\rho}{1-\rho}} \left(\frac{A_x}{A_d}\right)^{\frac{1}{1-\rho}} \geq 1. \tag{9}$$

Exporter revenue is decreasing in variable trade costs, τ, and increasing in the ratio of the foreign

[8] By construction, Λ includes Ψ, the evolution of worker distribution.

demand shifter to the domestic demand shifter, $\frac{A_x}{A_d}$.

2.4.2 Dynamic contracts

A job offer specifies the long term wages that the worker receives in each period as a function of tenure on the job. At each point in time, the remaining sequence of wages determines the promised-value of the contract, and it in turn determines the output level, y, through the worker's effort. As described above, because the firm's optimal export decision depends on its output level, its dynamic contract design jointly determines export status as well.

We characterize the optimal dynamic contract recursively. The state of the problem is the current promised value V, the firm's idiosyncratic productivity z, and the aggregate state variable s. Then, the firm's problem is to find the *optimal policy* that determines the current period wage w, the promised utility $\hat{V}(\hat{s})$ that the contract promises to the worker at the beginning of the next period, and its export decision ι, as functions of the state variables. Let $\xi \equiv \{w, \hat{V}(\hat{s}), \iota\}$.

Let $J(V, z, s)$ be the value of the firm for a given pair of promised value V and productivity z in the aggregate state of the economy s. Then, the firm's maximized value function $J(V, z, s)$ satisfies the Bellman equation:

$$J(V, z, s) = \max_{\xi} R(\iota, y(z, V), s) - w - \iota c + \beta \mathbb{E}_{\hat{s}}(1 - \delta(\hat{s}))(1 - \lambda_e \tilde{p}(\hat{V}(\hat{s}), \hat{\psi}))J(\hat{V}(\hat{s}), z, \hat{s}), \quad (10)$$

subject to

$$V = v(w) - c(e(V)) + \beta \mathbb{E}_{\hat{s}}(\delta(\hat{s})U(\hat{s}) + (1 - \delta(\hat{s}))(\hat{V}(\hat{s}) + \lambda_e D(\hat{V}(\hat{s}), \hat{s}))), \quad (11)$$

$$\delta(\hat{s}) = \{\delta \text{ if } J(\hat{V}(\hat{s}), z, \hat{s}) \geq 0, \ \ 1 \text{ otherwise}\}, \text{ and} \quad (12)$$

$$\hat{V}(\hat{s}) \in \{x \in X : J(x, z, \hat{s}) \geq 0\}. \quad (13)$$

It is important to note that the firm's current output and thus revenue is determined by the current state V and the firm cannot change it concurrently. The firm's contract design, however, affects its future output and revenue through the promised value of the contract $\hat{V}$. The value of $\hat{V}$ also affects how likely the worker stays with the firm next period, through $\tilde{p}$, which in turn affects the firm's expected continuation value. If the worker leaves for another job, the job is destroyed and the firm needs to re-enter the market as a new entrant. Therefore, the firm's outside option after separation is zero.[9]

The firm's optimal policy need to be consistent with its promised value V. That is, for a given choice of ξ, the worker evaluates its value according to the right hand side of equation (11). Then, the first constraint requires that the choice of ξ indeed delivers the current promised value V to the worker. The second constraint (12) requires that the job destruction probability, aside from the worker's on-the-job search, needs to be consistent with the exogenous job destruction probability δ and the firm's exit decision, as described in detail below. The last constraint (13) requires that the firm does not offer a self-destructing, empty promise to improve its current payoff.

[9]This is formally derived as an equilibrium condition (12) for the competitive entry of potential firms into the labor market.

2.4.3 Entry and Exit

We now use the firm's optimal value function $J(x, z, s)$ to describe the firm's entry and exit problem. Even though firms do not design a contract that will lead to a negative value of J in a stationary environment, an aggregate state shock in s may cause the realized value of J to be negative so that the firm wishes to exit. To admit this possibility, we allow the firm to exit after an adverse aggregate shock. That is, $\delta(\hat{s}) = 1$ if $J(\hat{V}(\hat{s}), z, \hat{s}) < 0$ in (12).

For an entering firm, $J(x, z, s)$ represents the value when it finds a match with a worker in submarket x and its realized productivity is z. However, because an entering firm draws its productivity only after entering into a submarket, it may choose to exit before engaging in the matching process if its realized productivity is too low relative to the value of contract that the firm wishes to offer. As for an incumbent firm, an entering firm chooses to exit if $J(x, z, s) < 0$. We denote the value of this decision problem by $I(x, z, s)$, and it is defined by

$$I(x, z, s) = \max\{0, J(x, z, s)\}. \tag{14}$$

Then, a firm's expected value from entry into a submarket x conditional on meeting a worker is given by

$$\mathbb{E}_z I(x, z, s) = \int_0^\infty I(x, z, s) g(z) dz, \tag{15}$$

where the expectation is taken with respect to the firm productivity z.

If a firm enters submarket x, a firm can find a match with probability $q(\theta(x, \psi))$. If the expected value of entry $q(\theta(x, \psi))\mathbb{E}_z I(x, z, s)$ is strictly less than the cost of creating vacancy k, no firm enters submarket x. If the product $q(\theta(x, \psi))\mathbb{E}_z I(x, z, s)$ is strictly greater than k, infinitely many firms enter submarket x, which decreases the probability of finding a searching worker in that submarket and thus decreases the expected value. Therefore, in any submarket that is visited by a finite and positive number of workers, the market tightness $\theta(x, \psi)$ is consistent with the firm's optimal job creation strategy if and only if

$$q(\theta(x, \psi))\mathbb{E}_z I(x, z, s) - k \leq 0, \tag{16}$$

and $\theta(x, \psi) \geq 0$, with complementary slackness.

3 Equilibrium

In this economy, because of monopolistic competition and the fixed cost of entry into the frictional labor market, each incumbent firm earns positive *ex-post* profit in equilibrium. We assume that all individuals in the economy hold the same portfolio of shares of firms. That implies that the *ex-post* profits are aggregated across all firms and are redistributed to individuals. Hence, since individuals cannot save, and home and foreign economies are symmetric, balanced trade implies that aggregate revenue must equal aggregate expenditure in equilibrium. Given the distribution of jobs $\Phi(x, z)$, aggregate revenue is computed by $\int R(\iota, y(z, x), s) d\Phi(x, z)$. Then, using the condition $A = E^{1-\rho}$, the demand shifter for the home country is given by

$$A = \left(\int R(\iota, y(z,x), s) d\Phi(x,z) \right)^{1-\rho}. \tag{17}$$

3.1 Definition and solution algorithm

Definition 1. *A recursive equilibrium is a set of functions $\{I^*, J^*, \theta^*, D^*, m^*, U^*, \xi^*\}$ and transition operators Ψ^* and Λ^* for the aggregate variables such that*

1. *the value of job search D^* and the optimal search policy m^* satisfy equation (6),*
2. *the value of unemployment U^* satisfies equation (7),*
3. *the value of incumbent firm I^* satisfies equation (14),*
4. *the value of operating firm J^* and an optimal contract policy ξ^* satisfy equation (10),*
5. *the market tightness θ^* satisfies condition (16), and*
6. *the aggregate value of A is implied by the aggregate of the individual decisions (17).*
7. *Ψ^* and Λ^* are derived from the policy functions ξ^* and m^* and the probability distribution for z.*

In general, search models with aggregate state variables are difficult to compute even at the steady state. In equilibrium, the distribution of workers across different employment values, which is necessary to compute the aggregate variable, is endogenously determined through the individuals' optimal decision making. However, the distribution itself is typically a state variable for the individuals' problem. Since the distribution of workers is an infinite-dimensional object, solving the individuals' optimal decision problem and computing the steady state equilibrium is computationally difficult.

The present model significantly simplifies the process for computing the steady state. In this model, because search is directed, a worker's optimal search depends on the tightness of each submarket but does not depend on the entire distribution of workers. Therefore, a firm's optimal contracting problem also does not depend on the distribution of workers. It depends on the aggregate state variable A only through its effect on the revenue function. Hence, we can compute the labor market equilibrium for each value of the aggregate variable without searching for the steady state distribution of workers as a fixed-point. Instead, we compute the transition operators Ψ and Λ and the resulting stationary distribution as an equilibrium object using the derived optimal policy functions. We then use the computed distribution to update the initial guess of the aggregate variable.

To be precise, we first solve for the labor market equilibrium, namely the firm's optimal contracting problem together with its entry and exit decisions. The labor market and the product market are linked only through the aggregation condition (17), which determines the firm's revenue. Since each firm takes A and τ as given when making its decisions, we treat them parametrically in the first step. If we treat A and τ as parameters, the labor market admits the block recursive structure; that is, equilibrium functions do not (directly) depend on the distribution of workers.[10] After computing the equilibrium functions for a given A (and τ), we can calculate the stationary

[10] As our main objective is quantitative, we do not prove the existence in this paper. See Menzio and Shi (2010) for details.

distribution of workers across employment states as well as the productivity of each job. We use this distribution to compute the updated A from (17). Then, the second step computes the fixed point of this process to find a consistent stationary equilibrium.

3.2 Qualitative properties of the firm's decisions

In this section, we describe key qualitative properties that are useful for describing the model's behavior. Following the above discussion of the solution algorithm, we drop s from the value function of the firm, and focus on $J(x, z)$ that is independent of the aggregate state variables at the steady state. The qualitative properties described below rely on some technical regularity conditions that we assume for illustrative purposes.[11]

Assumption 1.

1. *For all $x \in X$, $J(x, z)$ is increasing in z,*
2. *for all $z \in Z$, $J(x, z)$ is strictly decreasing and concave in x.*

Intuitively, the value function J is an outcome of optimal profit sharing between the firm and the worker. As higher productivity generates larger output for a given level of worker effort, it enables the firm to achieve a higher firm value. Also, at the optimum increasing the value of the contract promised to the worker must lower the value left for the firm.[12]

3.2.1 Zero-profit productivity cutoffs for operation

First, Assumptions 1-2 imply that $\mathbb{E}_z I(x, z)$ is decreasing in x. Therefore, there exists a unique $\tilde{x} \in \mathbb{R}$ such that $\mathbb{E}_z I(x, z) < k$ for any $x > \tilde{x}$. In equilibrium, no firm enters a submarket with $x > \tilde{x}$ from the equilibrium condition (16).

Then, conditional on entering submarket $x < \tilde{x}$, a new entrant needs to decide whether to engage in the hiring process conditional on its productivity draw. It would immediately exit if its value of operation is negative with certain productivity levels. Because $J(x, z)$ is increasing in z, there exists a cutoff productivity for each x, $\underaccent{\tilde}{z}(x)$ such that $J(x, z) < 0$ for any $z < \underaccent{\tilde}{z}(x)$. In each submarket x, if an entering firm draws productivity below $\underaccent{\tilde}{z}(x)$, it immediately exits without hiring a worker. In addition, because $J(x, z)$ is strictly decreasing in x, it is immediate to show that $\underaccent{\tilde}{z}(x)$ is strictly increasing in x. Therefore, only high productivity entrants can maintain a vacancy in a submarket with higher x. The shaded area of Figure 1 is a set of value-productivity combinations that yield a positive value $J(x, z) > 0$, and the firms are operative in this area.

3.2.2 Dynamic contracts

We denote the optimal policy functions associated with J^* by $\xi^* = \{\iota^*(V, z), w^*(V, z), \hat{V}^*(V, z)\}$ as a function of the current state (V, z). The optimal dynamic contract has the following property.

[11] Proving these properties, though possible in principle, goes beyond the purpose of our paper. See Menzio and Shi (2010) for technical details that provide grounds for these assumptions. Our quantitative exercise, described in the next section, always satisfies these assumptions.

[12] When J is not concave, the firm can improve its value by randomizing the contracts as discussed in Hopenhayn and Nicolini (1997), which makes the resulting value function concave. Our numerical solutions always generate globally concave value functions without lotteries. Therefore, for expositional clarity, we avoid using the lottery when expressing the firm's objective function.

Proposition 1. *Under the optimal contract, for any productivity level of the firm, the promised value weakly increases with the tenure of the match, i.e.,* $\hat{V}^*(V,z) \geq V$ *for all* $V \in X$ *and for all* $z \in Z$*. In addition, the optimal policy* $\hat{V}^*(V,z)$ *is increasing in* V*; that is,* $\hat{V}^*(V_1,z) \geq \hat{V}^*(V_2,z)$ *for any* $V_1 \geq V_2$ *and for any* z*.*

The proof is in the appendix. This is a key result of labor search models with dynamic contracting and on-the-job search.[13]

An immediate corollary to this result is that a firm backloads wages as follows.

Proposition 2. *Under the optimal contract, for any productivity level of the firm, the wage weakly increases with the tenure of the match, i.e.,* $w^*(\hat{V}(V,z),z) \geq w^*(V,z)$ *for all* $V \in X$ *and for all* $z \in z$*.*

See Tsuyuhara (2016) for the proof. In each period the firm knows that the worker may have the opportunity to search for a new job. If the worker finds an opportunity to search, the likelihood of losing the worker depends entirely on whether the future value of that same wage contract is greater at the current firm or the new job. As such, firms will optimally dissuade workers from leaving by offering future wage increases. In addition, by promising increasing value and wages, the firm can induce increasing effort by the worker. These two reasons induce the firm to an offer with increasing values.

Proposition 1 also implies an important property for firm dynamics. A worker's effort is induced by the current promised value of contract. Therefore, the firm's choice of promised value implicitly determines its output for the next period. Since the firm optimally chooses a higher promised value next period, its next period output will also be higher.

Proposition 3. *Under the optimal contract, a worker's effort, and thus a firm's output weakly increases with the tenure of the match.*

Hence, the firm optimal contract implicitly determines how its output grows over time conditional on the match continuing.

The primary drivers of firm dynamics in this context are the workers' variable effort and the corresponding dynamic contract. This is a novel mechanism for augmenting the heterogeneous productivity among firms, and it is clearly different from the standard mechanism that is characterized by any *ex-ante* differences in worker skill as in Helpman et al. (2011).

3.2.3 Export decision

Finally, in each period, for a given value-productivity pair (x,z), the firm chooses $\iota^*(x,z) = 1$ if and only if the following condition holds:

$$\begin{aligned}
& R(1, y(z,x), z) - c \geq R(0, y(z,x), z) \\
\Leftrightarrow\ & R(1, y(z,x), z) - R(0, y(z,x), z) \geq c \\
\Leftrightarrow\ & \left[\Upsilon_x^{1-\rho} - 1\right] A_d (ze(x))^\rho \geq c. \qquad (18)
\end{aligned}$$

It is clear that the left hand side monotonically increases in both z and x. Therefore, we can characterize the exporting productivity cutoffs $\underline{z}_{ex}(x)$, such that, for each $x \in X$, firms with productivity

[13] Shi (2009) proves this result for a model with directed on-the-job search. Tsuyuhara (2016) provides the proof for a model with a moral hazard contracting problem.

z greater than $\underline{z}_{ex}(x)$ export. Moreover, it is immediate to show that $\underline{z}_{ex}(x)$ is a decreasing function in x. The shaded area of Figure 2 is a set of value-productivity pairs where the firm optimally chooses to export, conditional on operation.

3.2.4 Firm dynamics

The characterization of the firm's dynamic contracting and productivity cutoffs generates novel implications for firm dynamics in our setting. First, we give an alternative characterization of zero-profit cutoffs; namely, zero-profit *value* cutoffs. For a given z, $J(x, z)$ is strictly decreasing in x. Therefore, there exists $\bar{x}(z)$ such that $J(x, z) \leq 0$ for any $x \geq \bar{x}(z)$. The optimal promised value of the contract increases with the tenure of the match, but it can be shown that the optimal contract in this model does not prescribe a value profile that yields a negative value for the firm.[14] Then, we redefine $\bar{x}(z) = \min\{\{x : J(x, z) = 0\}, \tilde{x}\}$ as the upper bound of the value that a firm promises to deliver for a given productivity level z. A firm with productivity z increases its promised value of the contract up to $\bar{x}(z)$, and once the value reaches it, the firm keeps the constant wage until the match breaks up.

Similar to the zero-profit cutoffs, we can alternatively define the exporting value cutoffs for each level of productivity. For exporting to be profitable, either x and/or z need to be sufficiently large. Even though z is fixed once a firm starts producing, the optimal dynamic contract implies that the firm raises x over time, and the resulting output level increases. Therefore, for each z, there is a threshold $\underline{x}_{ex}(z)$ such that if the contract delivers a value higher than this level, the firm starts exporting. Figure 3 and 4 respectively depict the zero-profit value cutoffs and exporting value cutoffs as a function of z.

Based on these cutoffs and increasing contract values, firms entering the same submarket will exhibit different dynamics depending on the realized productivity level at the point of entry. Figure 5 illustrates some examples. Suppose a firm enters submarket x. If the realized productivity is z_0, the value-productivity pair does not meet the zero-profit cutoff, and it immediately exits. If the realized productivity is z_1, the firm will start producing if it finds a worker. However, z_1 is still too small to make exporting profitable even if it raises the value of contract to the highest possible level ($\bar{x}(z_1) < \underline{x}_{ex}(z_1)$). If the realized productivity is z_2, then the firm serves the domestic market but does not export initially. As its value increases and eventually exceeds $\underline{x}_{ex}(z_2)$, it starts exporting. Finally, if the realized productivity is z_3, the firm exports as well as serves the domestic market from its inception.

4 Quantitative Application: Trade Liberalization

This section conducts a quantitative exercise to illustrate the impact of trade liberalization on wage dispersion in a general equilibrium setting. A rich literature explores the impact of trade on wage differences across skilled and unskilled workers, rather than across homogeneous workers. For example, Helpman et al (2011), Helpman et al (2012), Coşar (2013), and Ritter (2014, 2015), among others, characterize the impact of trade on the difference in wages across workers with differing skills, human capital or occupational expertise. A smaller literature studies the impact of trade liberalization on residual wage dispersion. Felbermayr et al. (2014) studies a framework

[14] There is no present-period gain by committing to a wage profile that yields negative value in the future.

similar to ours; however, in that case there is no role for on-the-job-search. Our work highlights the interplay of firm heterogeneity and on-the-job-search on wage dispersion and firm dynamics. In this sense our work is also closely related to Fajgelbaum (2016) which studies a model of trade with heterogeneous firms allowing for on-the-job-search. However, our paper uses a directed search model to study the impact of trade on wage dispersion, while Fajgelbaum (2016) uses an undirected search model to characterize the impact of trade and on-the-job-search on income growth.

4.1 Model Calibration

For our quantitative exercise, we will treat the global economy as if it is composed of two symmetric countries: the US and the rest of the world (ROW). We restrict attention to the manufacturing sector and continue to assume that workers do not search for work outside of this industry. While this is an admittedly strong assumption it is consistent with the empirical regularity that workers change industries with very low frequency in response to trade liberalization.[15]

We first calibrate our model to match US employment, wage, production and trade data. Given the calibrated model we describe the steady state distribution of wages, employment, production and exporting across *ex-post* heterogeneous firms and workers. We consider the implications of an unexpected trade liberalization, parameterized by a reduction in iceberg transport costs, on wage dispersion and characterize the role of on-the-job-search and its interaction with firm-heterogeneity.

4.1.1 Functional Forms

There are several equations for which we will need to impose functional forms to compute the model's steady-state. The functional forms we choose are collected in Table 1, while our empirical targets and parameter values are documented in Tables 2 and 3, respectively.

We begin by defining the flow utility that a consumer receives, $v(w) - c(e)$. We assume that we can describe the utility a consumer receives from current consumption by a standard, CRRA utility function $v(w) = \frac{w^{1-\eta}}{1-\eta}$, where η is the CRRA coefficient. Since effort is an increasing and continuously differentiable function of the current value of the wage contract, V, we write the effort function as $e(V) = V^{\nu}$ for $\nu > 0$. The effort exerted by the worker is translated into a utility flow through the worker's cost of effort function which is modeled as an increasing convex function $c(e) = \gamma e^2$ where $\gamma > 0$.

[15]Related evidence can be found in Pavcnik et al (2002), Wacziarg and Wallak (2004), Goldberg and Pavcnik (2007), Menezes-Filho and Muendler (2011), Cosar (2013), or Dix-Carneiro (2014). We note, however, that a subset of counterfactual simulation exercises in Artuc et al (2010) and Dix-Carneiro suggest that workers may substantially reallocate across industries in the long-run, under certain conditions. For example, a large degree of cross-industry reallocation is only achieved in the Dix-Carneiro (2014) counterfactual simulation exercise when capital is assumed to also be perfectly mobile across industries.

Table 1: Model Structure: Functional Forms

Description	Functional Form
Flow utility	$v(w) = \frac{w^{1-\eta}}{1-\eta}$
Effort function	$e(V) = V^{\nu}$
Cost of effort	$c(e) = \gamma e^2$
Productivity distribution	$G(z) = 1 - (z/\underline{z})^{\xi}$
Production function	$y = ze$
Matching technology	$p(\theta) = \theta(1 + \theta^{\phi})^{-1/\phi}$

Notes: Table 1 documents the specific functional forms of the quantitative model equations.

Total firm production is determined by firm productivity, the amount effort induced in the current period from the worker's contract, and the shape of the production function. We assume that productivity is drawn from a Pareto distribution with shape parameter ξ, $G(z) = 1 - (z/\underline{z})^{\xi}$, where $\underline{z}$ is the lowest possible productivity draw. The production function is specified, as in the theoretical model, as a multiplicative combination of firm productivity and worker effort, $y = ze$.

Last, we need to specify the matching technology to characterize labor market search. We again chose a standard functional form for the matching technology, $p(\theta) = \theta(1 + \theta^{\phi})^{-1/\phi}$, where θ captures market tightness. Given these functional form choices we next describe the empirical values used to pin down model parameters.

4.1.2 Parameter Calibration

We appeal to two separate data sources to help pin down the 13 parameters needed to compute the model's equilibrium. We first turn to the Consumer Population Survey (CPS) to characterize the degree of residual wage dispersion among US manufacturing workers. Following Lemieux (2006) we retrieve data from the May CPS and compute measures of residual wage dispersion.[16] A key difference for our purposes is that we only study the degree of residual wage dispersion among manufacturing workers. Specifically, we consider a regression of a manufacturing worker's log hourly wage on a host of observable worker characteristics, including age, education, sex and race.[17] Our measures of residual wage dispersion are based on the residuals obtained from these regressions.[18]

Similarly, we also use the CPS to compute manufacturing employment transitions in the same year. We assume the length of a period is a quarter. Following the process described in Shimer (2012), we compute the fraction of manufacturing workers who transition from employment to unemployment, from unemployment to employment and from employment at one job to employment at a new job.

[16]We choose to focus on 2002 since this is the same year for which there is well documented production and export statistics. We also consider a longer time period, 2001-2004, but this had virtually no impact on the target moments. We follow Lemieux (2006) in our manipulation of the data with the exception that we only focus on manufacturing workers. See the Appendix for details.

[17]We have alternatively considered specifications which only use white male workers, but this had little effect on our measures of residual wage dispersion.

[18]Details are provided in the Supplemental Appendix.

To characterize the production side of the economy we appeal to a series of well-established empirical benchmarks from the 2002 Annual Survey of US manufacturers as documented in Bernard et al. (2007) and Bernard, Redding and Schott (2007). In particular, we use moments which characterize the dispersion of manufacturing revenues, the prevalence and intensity of exporting, and empirical wage differences across workers at exporting and non-exporting firms.

Two model parameters can be set to exactly fit their counterpart in the data. First, the iceberg transport cost, τ, is set to 1.12 to match the finding that 14% of total sales originate from exports among US exporters (Bernard et al., 2007). Second, we compute that the quarterly transition rate from employment to unemployment among manufacturing workers is 3% and set exogenous separation rate, δ, to fit this target moment.

Four more parameters not identified by our empirical targets and are fixed by assuming that they take on the same value as those commonly chosen in the literature. In particular, we assume[19] that the CRRA coefficient (η) takes a value of 2, the matching technology parameter (η) is set to 0.5, the elasticity of substitution (σ) across varieties is equal to 3.8, and the quarterly discount factor (β) is fixed to 0.988. Three more parameters are set to 1: λ_u, the probability of searching for employment when unemployed, γ, the cost of effort parameter, and the lowest possible productivity draw, $\underline{z}$.

Table 2: Calibration Targets

Moments	Data Target	OTJS	No OTJS
Mean-Min Wage Ratio	2.60	2.63	1.85
Unemployment to Employment Transition Rate	0.87	0.88	0.97
Employment to Employment Transition Rate	0.08	0.08	0
Unemployment Insurance Replacement Rate	0.40	0.40	0.43
Fraction of Exporting Firms	0.18	0.17	0.19
Exporter Wage Premium	0.06	0.08	0.07
Variance of Manufacturing Revenues	0.60	0.61	0.56

Notes: Table 2 documents the target data moments for model calibration.

The remaining 6 parameters are chosen to match 7 target moments from US manufacturing. The first three moments capture key features of firm-level export behavior. Specifically, we target the fraction of manufacturing firms which export abroad (Bernard et al., 2007), the standard deviation of manufacturing revenues (Bernard, Redding and Schott, 2007), and the export wage premium (Bernard et al., 2007). The export wage premium is defined as the difference paid by exporting and non-exporting firms to similar manufacturing workers. This target moment allows us to capture the difference in wages exporters are willing to pay to induce further output in our model. Likewise, the fraction of exporting firms and the standard deviation of manufacturing revenues are inherently linked to the fixed export cost parameter and the underlying distribution of productivity, respectively.

The final four moments target features of the US labor market for manufacturing workers over

[19]The elasticity of substitution is set to 3.8 as in Bernard, Redding and Schott (2007) and empirically consistent with the estimates in Simonovska and Waugh (2014). The matching technology parameter implies that the elasticity of substitution between vacancies and applicants is $2/3$, and the discount factor is chosen so that the annual interest rate is 5%.

Table 3: Calibrated Results

Parameter	Value OTJS	No OTJS	Parameter	Value OTJS	No OTJS
Entry cost, k	0.001	0.002	Prob. of on-the-job-search, λ_e	0.095	0
Unemployment insurance, b	0.698	0.400	Fixed export cost, f	0.395	0.290
Effort shape parameter, ν	1.320	1.310	Productivity dist. shape parameter, ξ	4.600	2.810

the same period. In particular, we aim to match the fraction of unemployed US manufacturing workers who transition to employment, the fraction of employed workers who transition to employment at a new job, and the degree of wage dispersion among manufacturing workers as measured by the mean-min wage ratio. All three of these values are computed using data taken from the CPS. Last, we also target the average wage replacement rate among US workers as commonly parameterized the labour-search literature (Menzio and Shi, 2011). The observed employment transitions and replacement rate intuitively discipline the degree of on-the-job-search and the firm-level entry cost. Last, the observed mean-min ratio disciplines the relationship between wages, effort, and firm revenues.

The target moments are collected in Table 2, while Table 3 displays the calibrated parameter values. To characterize the importance of on-the-job search (OTJS), we also calibrate the model to a setting where workers cannot search on-the-job for comparison. There are two striking results. First, the entry cost is relatively low, especially with OTJS. In general, this is a result of the relatively high probability of finding new employment among unemployed workers. However, with OTJS firms are less willing to enter the market since they have to pay higher wages to keep workers from leaving the job and, as such, lower entry costs are needed to match the target unemployment-to-employment transition rate. Second, the shape parameter of the productivity distribution is relatively large with OTJS and small without OTJS. The reason for this is that productivity and effort are multiplicative complements in the production function and, as such, both affect the dispersion of revenues. The effort exerted by the worker is, in turn, a function of the dynamic wage contract and inherently influenced by the opportunity to search for new employment on-the-job. For robustness, we also check whether the quantitative model can replicate the observed variance of wages. We find that the observed variance of wages in the model is 0.98, which is very similar to its empirical counterpart of 0.97. In contrast, the model without OTJS only produces a residual wage variance of 0.73.[20]

4.2 Steady State Wage Dispersion

Table 4 presents various statistics describing the degree of wage dispersion in the quantitative model. Across all statistics, the benchmark model with OTJS generates substantially more wage dispersion than the calibrated model without OTJS.

[20] We also compute the Ninety-Ten Ratio, the ratio of the wage of workers in the ninetieth percentile of the wage distribution to those in the tenth percentile of the wage distribution, in the actual data and in that implied by the simulated model. The benchmark model generates a Ninety-Ten ratio (4.6) which is larger than that observed in the data (3.2).

Table 4: Impact of Trade Liberalization on Wage Dispersion

Model	With OTJS	Without OTJS
Wage Variance	0.984	0.738
Mean-Min Ratio	2.630	1.846
Ninety-Ten Ratio	4.640	4.507

Notes: The wage variance is the variance of the wage distribution. The Mean-Min ratio is the ratio of the average wage to the minimum wage. The Ninety-Ten ratio is the ratio of the wage of workers in the ninetieth percentile of the wage distribution to those in the tenth percentile of the wage distribution.

The comparison of these wage dispersion measures across the models reveals the importance of both dynamic wage contracts and OTJS for magnifying the dispersion generated by the underlying productivity distribution. For the model without OTJS to produce the observed wage dispersion, the variance of the underlying productivity distribution needs to be roughly 8 times larger than that with OTJS.[21] Though it produces significantly greater wage dispersion than previous studies, it still cannot match the empirical counterpart in our data. The model with OTJS, however, produces sufficient wage dispersion to match the empirical target and does so with a much less dispersed productivity distribution.

As discussed in Section 3, in the model with OTJS, there are two forces which cause wages to increase over job tenure. First, as in other models with OTJS, firms have an incentive to increase wages to deter workers from leaving the firm through OTJS. Second, and specific to our model, higher future contract values induce greater effort and thus greater output in future periods. These two forces magnify the effect of firm productivity dispersion on wage dispersion, which has not been extensively studied in previous research. In contrast, the model without OTJS has only the second force to induce the increase in wages, and wage variation in that model is significantly smaller. Quantitatively, this impact of having workers move through the wage distribution through job-to job transitions appears particularly important for matching equilibrium wage dispersion.

4.3 Trade Liberalization

Our aim in this section is to characterize the impact of trade liberalization on workers and firms, in general, and wage dispersion, in particular. An important caveat in this experiment is our specification of a single-industry model. Because of on-the-job search, our findings in the following exercise may potentially be affected by whether initially high or low wage workers are disproportionally reallocated across industries after trade liberalization. Although it is beyond the scope of this paper to conjecture how the wage dispersion responds in a multi-industry setting, the following results must be interpreted with this limitation in mind.

In the context of our model, we parameterize the long-run impact of trade liberalization by reducing iceberg trade costs to 1, $\tau \to 1$. Because of the elimination of variable trade costs, lower productivity firms may now grow into export markets, which are less costly to service than before. As the expected profitability of exporting improves, even lower productivity firms offer higher

[21] The variance of the Pareto distribution can be computed as $Var(z) = (\underline{z}/\xi - 1)^2(\xi/(\xi - 2)$ for $\xi > 2$.

wage contracts to increase their output in anticipation of future exporting. This, in turn, drives up current output and wages. Our calibrated example suggests that aggregate revenues and average wages grow by 3% each.

As exporting becomes less costly, high value wage contracts become more profitable and allow less productive firms to survive in high value submarkets. This is illustrated in Panel (a) of Figure 6 where the firm survival threshold rotates downwards in high value submarkets. The reduction in the minimum productivity required to survive in high value submarkets has two important effects. First, for a given productivity level, workers can be promoted into higher wage jobs within the firm than what was previously possible. Second, OTJS allows workers who are searching on-the-job to direct their search to thicker, higher value submarkets relative to what was possible prior to trade liberalization.

Our quantitative exercise indicates that the endogenous firm dynamics and worker mobility not only the shifts the worker distribution to higher wages but also generates larger wage dispersion. Table 5 documents that the mean-min ratio, the total variance of wages, and the ninety-ten ratio all increase by 3-4% after trade liberalization.

Table 5: Impact of Trade Liberalization on Wage Dispersion

Percentage Change After Liberalization		
Mean-Min Ratio	Wage Variance	Ninety-Ten Ratio
3.2%	4.3%	4.0%

Notes: The Mean-Min ratio is the ratio of the average wage to the minimum wage. The wage variance is the variance of the wage distribution. The Ninety-Ten ratio is the ratio of the wage of workers in the ninetieth percentile of the wage distribution to those in the tenth percentile of the wage distribution.

First, the change in wage dynamics leads to greater variation in wages since more firms of different productivities are active in each submarket. This mechanism in general shifts the workers to higher wage jobs. Second, unlike the standard Melitz model, trade liberalization in our model does not eliminate lower productivity-lower wage jobs. These jobs will eventually become more profitable, and knowing the underlying firm dynamics, unemployed workers still search in these submarkets. Therefore, the lowest equilibrium wage in our model does not change before and after trade liberalization. As illustrated in Figure 7, the cumulative wage distribution is generally shifted to the right after trade liberalization even though the minimum wage does not change. These two mechanisms together cause larger wage dispersion after trade liberalization in our model.

As noted above, trade liberalization also has an effect on the export participation decision. As displayed in Panel (b) of Figure 6, the export threshold declines and rotates downwards after liberalization. This has a substantial impact on the number of exporting firms, which increases from 17 to 71%, and the export wage premium, which increases from 8% to 19%, in equilibrium. Nonetheless, export status does not directly increase wages or wage variability, *per se*. Rather, exporting and output growth induced through trade liberalization only affect wages indirectly by allowing firms to survive in previously unprofitable submarkets.

Lastly, total employment is nearly unaffected as the unemployment rate rises mildly from 3.2% to 3.3% after trade liberalization. In our steady state comparison, trade liberalization does not strongly affect worker transition probabilities between unemployment and employment and between jobs.[22] In our model, job separation rate is largely determined by the exogenous destruction rate. In addition, as mentioned above, trade liberalization in our model does not eliminate lower wage jobs. Therefore, unemployed workers' job finding rate remains high and hence the unemployment rate does not change significantly.

5 Conclusion

This paper developed a dynamic, general equilibrium, open economy model with frictional labor markets to study the impact of trade on residual wage dispersion. With dynamic wage contracts and on-the-job search, our model of heterogeneous, single-worker firms generates positive correlation between firm output, exporting status and wages through endogenous worker effort. It also implies that ex-ante homogeneous workers experience widely different labour market histories. Our quantitative model has two key findings. First, firm-heterogeneity and on-the-job-search are strong complements in generating wage dispersion. The model calibrated with on-the-job-search generates 43% more wage dispersion relative to the model without on-the-job-search. Second, trade liberalization *increases* equilibrium wage dispersion by 3-4%. This effect is due to the ability of firms to offer workers wages which were previously unprofitable after trade liberalization.

[22]The job finding rate slightly increased from 88% to 90%, the separation rate is unchanged, and job-to-job transition rate slightly increases from 8.3% to 8.5%.

References

Albornoz, F., H.F. Calvo Pardo, G. Corcos and E. Ornelas (2012) "Sequential Exporting," *Journal of International Economics*, 88(1), 17–31

Amiti, M., and D. Davis (2012) "Trade, Firms and Wages: Theory and Evidence," *Review of Economic Studies*, 79(1), 1–36

Artuç, E., S. Chaudhuri and J. McLaren (2010) "Trade Shocks and Labor Adjustment: A Structural Empirical Approach," *American Economic Review*, 100(3), 1008–45

Bernard, A.B., J. Eaton, J.B. Jensen and S. Kortum (2003) "Plants and Productivity in International Trade," *American Economic Review*, 93(4), 1268–90

Bernard, A.B., S.J. Redding and P.K. Schott (2007) "Comparative Advantage and Heterogeneous Firms," *Review of Economic Studies*, 74(1), 31–66

Bernard, A.B., J.B. Jensen, S.J. Redding and P. K. Schott (2007) "Firms in International Trade," *Journal of Economic Perspectives*, 21(3), 105–30

Burdett, K., and M. Coles (2003) "Equilibrium Wage-Tenure Contracts," *Econometrica*, 71(5), 1377–404

Burdett, K., and M. Coles (2010) "Wage/tenure contracts with heterogeneous firms," *Journal of Economic Theory*, 145(4), 1408–35

Cahuc, P., F. Postel-Vinay and J.-M. Robin (2006) "Wage bargaining with on-the-job search: theory and evidence," *Econometrica*, 74(2), 323–64

Coşar, A.K. (2011) "Human Capital, Technology Adoption and Development," *The B.E. Journal of Macroeconomics: Contributions* 11(1), Article 5.

—— (2013) "Adjusting to Trade Liberalization," mimeo, University of Chicago

Davidson, C., L. Martin and S. Matusz (1988) "The structure of simple general equilibrium models with frictional unemployment," *Journal of Political Economy*, 96(6), 1267–93

Davis, D.R., and J. Harrigan (2011) "Good jobs, bad jobs, and trade liberalization," *Journal of International Economics*, 84(1), 26–36

Dix-Carneiro, R. (2014) "Trade Liberalization and Labor Market Dynamics," *Econometrica*, 82(3), 825–85

Fajgelbaum, P.D. (2016) "Labor Market Frictions, Firm Growth and International Trade," NBER Working Paper 19492

Felbermayr, G., G. Impulliti and J. Prat (2014) "Firm Dynamics and Residual Wage Inequality in Open Economies," mimeo, University of Hohenheim

Goldberg, P., and N. Pavcnik (2007) "Distributional Effects of Globalization in Developing Countries," *Journal of Economic Literature*, 45(1), 39–82

Helpman, E., and O. Itskhoki (2010) "Labor Market Rigidities, Trade and Unemployment," *Review of Economic Studies,* 77(3), 1100–37

Helpman, E., O. Itskhoki and S. Redding (2011) "Inequality and Unemployment in a Global Economy," *Econometrica*, 78 (4), 1239–83

Helpman, E., M. Muendler, O. Itskhoki and S. Redding (2016) "Trade and Inequality: From Theory to Estimation," *Review of Economic Studies*, 84(1), 357–405

Hopenhayn, H.A., and J.P. Nicolini (1997) "Optimal Unemployment Insurance" *Journal of Political Economy*, 105(2), 412–38

Hornstein, A., P. Krusell and G.L. Violante (2011) "Frictional Wage Dispersion in Search Models: A Quantitative Assessment." *American Economic Review*, 101(7), 2873–98

Hosios, A.J. (1990) "Factor Market Search and the Structure of Simple General Equilibrium Models," *Journal of Political Economy*, 98(2), 325–55

Kaas, L., and P. Kircher (2015) "Efficient Firm Dynamics in a Frictional Labor Market," *American Economic Review*, 105(10), 3030–60

King, I. and F. Strähler (2014) "International trade and directed search unemployment in general equilibrium," *Canadian Journal of Economics*, 47(2), 580–604

Lemiuex, T. (2006) "Increasing Residual Wage Inequality: Composition Effects, Noisy Data, or Rising Demand for Skill?" *American Economic Review*, 96(3), 461–98

Lileeva, A., and D. Trefler (2010) "Improved Access to Foreign Markets Raises Plant-Level Productivity ? for Some Plants," *Quarterly Journal of Economics*, 125(3),1051–99

Lopez, R. (2009) "Do Firms Increase Productivity in Order to Become Exporters?" *Oxford Bulletin of Economics and Statistics* 71(5), 621–42

Melitz, M.J. (2003) "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity," *Econometrica*, 71(6), 1695–725

Menzio, G., and S. Shi (2010) "Block Recursive Equilibria for Stochastic Models of Search on the Job," *Journal of Economic Theory*, 145(4), 1453–94

—— (2011) "Efficient Search on the Job and the Business Cycle," *Journal of Political Economy*, 119(3), 468–510

Moen, E.R. (1997) "Competitive Search Equilibrium," *Journal of Political Economy*, 105(2), 385–411

Nagypál, E. (2007) "Labor-Market Fluctuations and On-the-Job Search." Manuscript, Northwestern University

Papp. T.K. (2013) “Frictional wage dispersion with Bertrand competition: An assessment,” *Review of Economic Dynamics*, 16(3), 540–52
Postel-Vinay, F., and J.-M. Robin (2002) “Equilibrium wage dispersion with worker and employer heterogeneity,” *Econometrica*, 70(6), 2295–350
Rho, Y., and J. Rodrigue (2016) “Firm-Level Investment and Export Dynamics,” *International Economic Review*, 57(1), 271–304
Ritter, M. (2014) “Offshoring and Occupational Specificity of Human Capital,” *Review of Economic Dynamics*, 17(4), 780–98
—— (2015) “Trade and Inequality in a Directed Search Model with Firm and Worker Heterogeneity,” *Canadian Journal of Economics*, 48(5), 1902–16
Ruhl, K.J., and J.L. Willis. (2017) “New Exporter Dynamics,” *International Economic Review*, forthcoming
Shi, S. (2009) “Directed Search for Equilibrium Wage-Tenure Contracts,” *Econometrica,* 77(2), 561–84
Shimer, R. (2012) “Reassessing the ins and outs of unemployment,” *Review of Economic Dynamics*, 15(2), 127–48
Simonovska, I., and M. Waugh. (2014) “Trade Models, Trade Elasticities, and the Gains from Trade,” Working Paper, New York University
Solow, R.M. (1979) “Another Possible Source of Wage Stickiness, ” *Journal of Macroeconomics*, 1(1), 79–82
Summers, L.H. (1988) “Relative Wages, Efficiency Wages, and Keynesian Unemployment,” *American Economic Review*, 78(3), 383–88
Topkis, D.M. (1998) *Supermodularity and Complementarity*, Princeton University Press
Tsuyuhara, K. (2016) “Dynamic Contracts with Worker Mobility via Directed On-the-job Search,” *International Economic Review*, 57(4), 1405–24
Wacziarg, R., and J. Wallack. (2004) “Trade Liberalization and Intersectoral Labor Movements,” *Journal of International Economics*, 64(2), 411–39

Appendices

A Proofs

Proof of Proposition 1. We apply the same method as in Tsuyuhara (2016) to show that the value of contract is increasing with tenure by deriving the inverse Euler equation.

We first denote the consistent wage as an implicit function using the first constraint:

$$w(V,\hat{V}) = v^{-1}\big(V + c(e(V)) - \beta(\delta U + (1-\delta)(\hat{V} + \lambda_e D(\hat{V})))\big). \tag{A1}$$

It is clear that the consistent wage function is increasing in V and decreasing in $\hat{V}$. Using this notation, let F denote the objective function of the maximization operator as a function of V and ξ:

$$F(V,\xi) \equiv R(\iota, y(z,V), z) - w(V,\hat{V}) - \iota c + \beta(1-\delta)(1-\lambda_e\tilde{p}(\hat{V}))I(\hat{V},z). \tag{A2}$$

Since concave functions are almost everywhere differentiable, J is almost everywhere differentiable. It implies that the composite function $\hat{p}(\hat{V})$ is almost everywhere differentiable, and its derivative is negative wherever it exists (Menzio and Shi, 2010). Therefore, F is concave and almost everywhere differentiable with respect to V.

Then, the interior solution to the optimal choice of $\hat{V}^*$ satisfies the first order condition:[23]

$$\frac{-\beta(1-\delta)(1-\lambda_e\hat{p}(\hat{V}^*))}{v'(w(V,\hat{V}^*))} + \beta(1-\delta)[(1-\lambda_e\hat{p}(\hat{V}^*))J'(\hat{V}^*,z) - \lambda_e\hat{p}'(\hat{V}^*)J(\hat{V}^*,z)] = 0. \tag{A3}$$

Dividing through by common terms, it simplifies as

$$\frac{1}{v'(w(V,\hat{V}^*))} + J'(\hat{V}^*,z) - \frac{\lambda_e\hat{p}'(\hat{V}^*)J(\hat{V}^*,z)}{1-\lambda_e\hat{p}(\hat{V}^*)} = 0. \tag{A4}$$

By the envelope theorem, we have $J'(V,z) = \frac{\partial R(\cdot)}{\partial V} - \frac{1}{v'(w(V,\hat{V}^*)}(1 + c'(e)e'(V))$, which implies that $\frac{1}{v'(w(V,\hat{V}^*)} = -\frac{1}{\Delta}(J'(V,z) - \frac{\partial R(\cdot)}{\partial V})$, where $\Delta \equiv 1 + c'(e)e'(V)$. Substituting this term into the above condition and rearranging terms yield

$$J'(V,z) - J'(\hat{V}^*,z)\Delta = \frac{\partial R(\cdot)}{\partial V} - \frac{\lambda_e\hat{p}'(\hat{V}^*)J(\hat{V}^*,z)}{1-\lambda_e\hat{p}(\hat{V}^*)}\Delta. \tag{A5}$$

Because $\hat{p}'(\hat{V}^*)$ is negative, the right hand side is positive. Moreover, since $\Delta > 1$, $J'(V,z) - J'(\hat{V}^*,z)\Delta > 0$ implies that $J'(V,z) - J'(\hat{V}^*,z) > 0$. Hence, by the concavity of J, this inequality implies that $\hat{V}^* \geq V$ for all $V \in X$.

For the second statement, by Theorem 2.8.1 in Topkis (1998), if the objective function F is supermodular and has increasing differences in $(V,\hat{V})$ on $X \times X$, then the solution $\hat{V}(V)$ as a

[23] To complete the argument incorporating the possible nondifferentiable points, we apply the theory of nonsmooth analysis to characterize the solution. See Tsuyuhara (2016) for more details.

function of V is increasing.

Lemma 1. *F has increasing differences on $X \times X$.*

Proof: Since F is almost everywhere differentiable function in V, it suffices to show that the derivative $\frac{\partial F}{\partial V}$ is increasing in $\hat{V}$ wherever it exists. In addition, the derivative $\frac{\partial F}{\partial V}$ depends on $\hat{V}$ only through the implicit function $w(V, \hat{V})$. Therefore, F has increasing differences if and only if $\frac{\partial^2 w(V,\hat{V})}{\partial \hat{V} \partial V} \geq 0$.

First, using (A1)

$$\frac{\partial w(V, \hat{V})}{\partial V} = -\frac{1 + c'(e)e'(V)}{v'(w(V, \hat{V}))}, \tag{A6}$$

by the inverse function theorem. Then,

$$\begin{aligned}
\frac{\partial^2 w(V, \hat{V})}{\partial \hat{V} \partial V} &= -\frac{1 + c'(e)e'(V)}{(v'(w(V, \hat{V})))^2} \left\{ -v''(\cdot) \left(\frac{-\beta(1-\delta)(1-\lambda_e \hat{p}(\hat{V}))}{v'(\cdot)} \right) \right\} \\
&= -\frac{1 + c'(e)e'(V)}{(v'(w(V, \hat{V})))^2} \left\{ v''(\cdot) \left(\frac{\beta(1-\delta)(1-\lambda_e \hat{p}(\hat{V}))}{v'(\cdot)} \right) \right\}. \qquad \text{(A7)}
\end{aligned}$$

To compute the derivative in the large parenthesis, we use the result that the differentiability of J implies differentiability of $D(\hat{V})$ and that the derivative of $\hat{V} + \lambda_e D(\hat{V})$ is equal to $1 - \lambda_e \hat{p}(\hat{V})$ (Menzio and Shi, 2010). Since $v'' < 0$ from concavity of V, (A7) implies that $\frac{\partial^2 w(V,\hat{V})}{\partial \hat{V} \partial V} \geq 0$. $\square$

A real-valued function f on $\mathbb{R}^2$ is supermodular on $\mathbb{R}^2$ if and only if f has increasing differences on $\mathbb{R}^2$ (Topkis 1998, Theorem 2.6.1., 2.6.2, and Corollary 2.6.1.). Therefore, the above lemma implies that F is supermodular. Hence, Theorem 2.8.1 in Topkis (1998) implies that the optimal solution $\hat{V}(V)$ as a function of V is increasing. $\square$

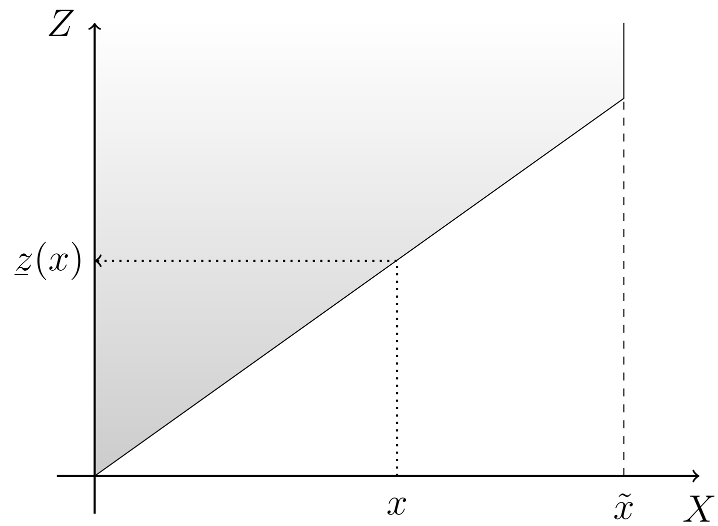


Figure 1: Zero-profit productivity cutoffs.

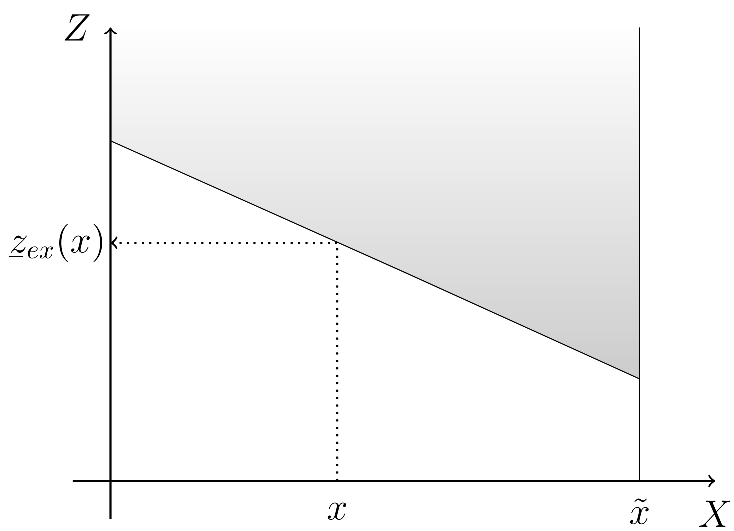


Figure 2: Exporting productivity cutoffs.

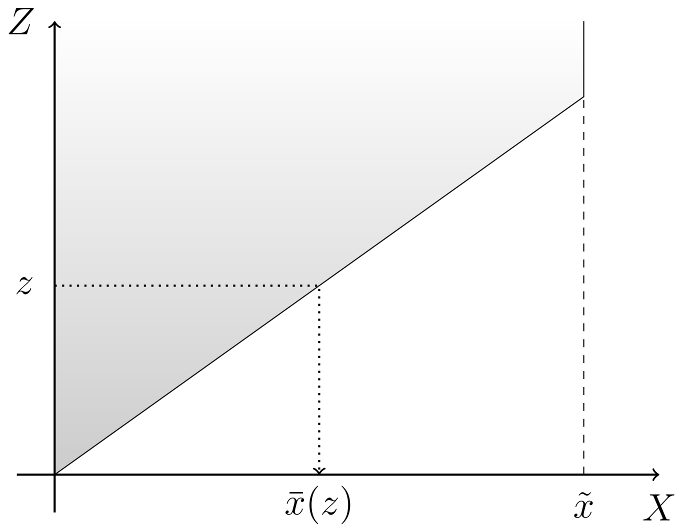


Figure 3: Zero-profit value cutoffs.

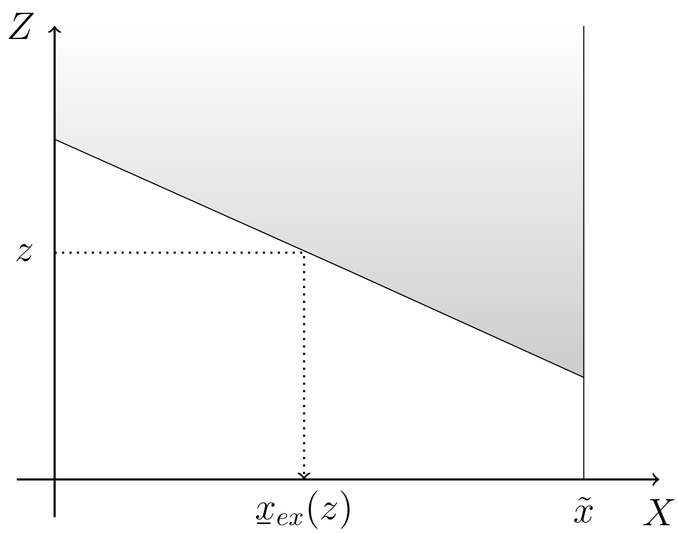


Figure 4: Exporting value cutoffs.

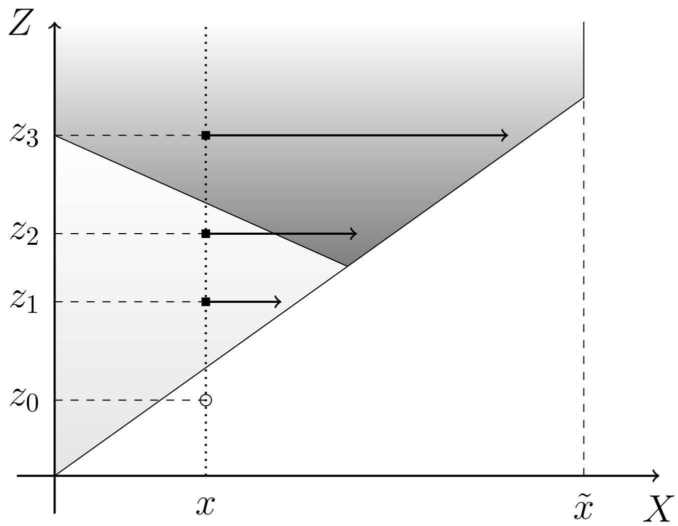
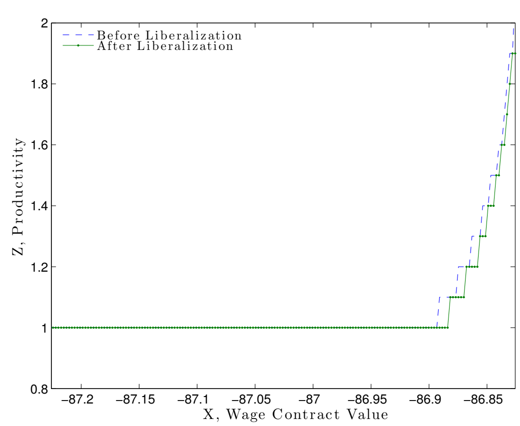
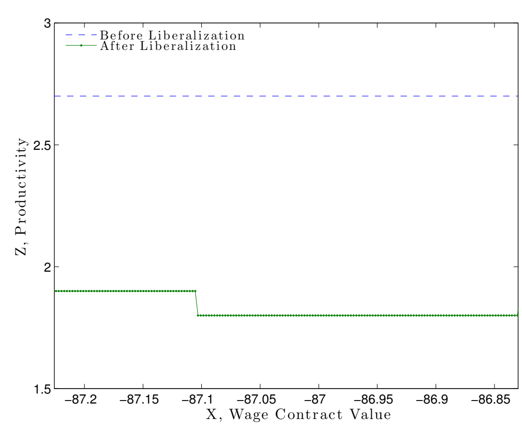


Figure 5: Firm dynamics.

Figure 6: Impact of Trade Liberalizaton on Zero Profit and Export Cutoffs

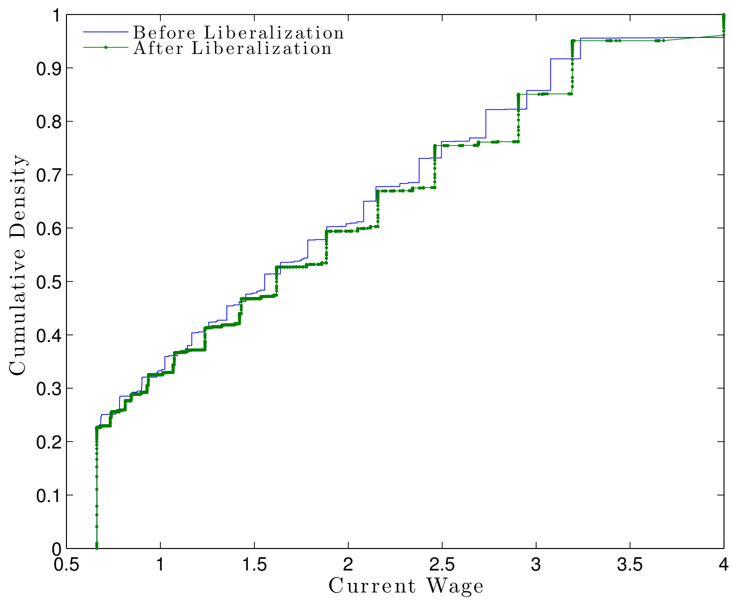


(a) Zero Profit Cutoffs



(b) Export Cutoffs

Figure 7: Trade Liberalization and the Distribution of Wages



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