War model

May 17, 2017

Timeline

1. Today: model of battle; brief history of warfare. For Thursday, review Friedman.
2. Thursday: WWI; Game theory; For Monday, read Ch. in CBB on Strategic Bombing; Akerlof on line.
3. Monday May 22; Game theory cont.; strategic bombing; Dinner at 5:00 PM before "Life of Galileo"
4. Tuesday May 23; Bargaining failure.
5. Wed-Fri Normandy
6. Tuesday: wild card (TBD).
7. Wednesday: exam,

Battle model

- Air war (see CBB)
- Iwo Jima; castle defenses; ; Invasion of UK; Normandy invasion
- Implication: CSF depends on force ratio (no uncertainty)
- When do we expect first-strike advantage?
- Propensity for peace when defense is king

Scenario

- Attacking a castle, an island, or Omaha Beach in Normandy.
- "American casualties, projected with an elaborate formula called Love's Tables, would likely reach 12 percent of the assault force on D-Day, or higher if gas warfare erupted. The 1st Infantry Division, the point of the spear on Omaha Beach, estimated that under "maximum" conditions, casualties would reach 25 percent, of whom almost a third would be killed, captured, or missing. The admiral commanding bombardment forces at Utah Beach told his captains that "we might expect to lose one-third to one-half of our ships." Projected U.S. combat drownings in June, exclusive of paratroopers, had been calculated at a grimly precise 16,726. To track the dead, wounded, and missing, the casualty section under SHAEF's adjutant general would grow to three hundred strong; so complex were the calculations that an early incarnation of the computer, using punch cards, would be put to the task."

Atkinson, Rick (2013-05-14). The Guns at Last Light (pp. 15-16). Henry Holt and Co.. Kindle Edition.

Scenario
Omaha Beach

"... now it was known, and would forever be known, as Omaha. Five miles long, composed of packed sand yielding to shingle sorted in size by a thousand storms, the beach offered but five exits up the hundred-foot escarpment, each following a narrow watercourse to four villages of thick-walled farmhouses a mile or so inland.
... The German defenses were fearsome. Eighty-five machine-gun nests, soon known to GIs as "murder holes," covered Omaha, more than all three British beaches combined. Unlike the obstacles at Utah, many of the 3,700 wood pilings and iron barriers embedded in the tidal flat at Omaha were festooned with mines ... Thirty-five pillboxes and eight massive bunkers ... defended the beach's five exits, while eighteen antitank sites, six Nebelwerfer rocket-launcher pits, and four artillery positions covered the balance of the beach. Guns enfiladed nearly every grain of sand on Omaha, concealed from the sea by concrete and earthen blast shields that aerial photos had failed to find."
Atkinson, Rick The Guns at Last Light, Henry Holt and Co.. Kindle

- Three combat arms
  - Infantry: queen of battle.
    - rifles: semi-automatic in WWII;
    - machine guns: 200-300 rounds per minute in combat.
    - grenades, anti-tank, mortars.
    - Most battle deaths.
    - Companies (200 at full strength); 4 platoons (3 rifle, 1 mortar); 3-4 squads per platoon.
  - Artillery: king of battle; organized in "batteries," shoot shells with max range maybe 15 KM; most kills.
  - Armor: tanks; critical in warfare across open terrain.
- REMF's: as much as 90% of armed forces. Logistics key for sustained operations.

Some military background

A squad of infantry

War (battle) model: A attacks, B defends

- Force attrition through time:

$$
\begin{aligned}
M_A(t+\Delta) - M_A(t) &= -\beta_d M_B(t); \\
M_B(t+\Delta) - M_B(t) &= -\alpha_a M_A(t).
\end{aligned}
$$

- Variables and parameters: M_A, M_B; α_d, β_a; $M_A(0), M_B(0)$
- e.g., $\alpha_a = 0.05, \beta_d = 0.1, M_A(0) = 200, M_B(0) = 100$

A attacks, B defends

time	$M_A(t)$	$-\beta_d M_B(t)(KIA_A)$
0	200	$\overbrace{-.1}^{-\beta_d} \times \overbrace{100}^{M_B(0)} = -10$
1	$\overbrace{200}^{M_A(0)} \overbrace{-10}^{-\beta_d M_B(0)} = \overbrace{190}^{M_A(1)}$	$-.1 \times \overbrace{90}^{M_B(1)} = -9$
2	$\overbrace{190}^{M_A(1)} - \overbrace{9}^{\beta_d M_B(1)} = \overbrace{181}^{M_A(2)}$	$-.1 \times \overbrace{80.5}^{M_B(2)} = -8.05$
3	$\overbrace{181}^{M_A(2)} - \overbrace{8.05}^{\beta_d M_B(2)} = \overbrace{172.95}^{M_A(3)}$	

A attacks, B defends

$$M_B(t) \qquad\qquad -\alpha_a M_A(t)(KIA_B)$$

$$100 \qquad\qquad \overbrace{-.05}^{-\alpha_a} \times \overbrace{200}^{M_A(0)} = -10$$

$$\overbrace{100}^{M_B(0)} - \overbrace{10}^{\alpha_a \times M_A(0)} = \overbrace{90}^{M_B(1)} \qquad -.05 \times \overbrace{190}^{M_A(1)} = -9.5$$

$$\overbrace{90}^{M_B(1)} - \overbrace{9.5}^{\alpha_a \times M_A(1)} = \overbrace{80.5}^{M_B(2)} \qquad -.05 \times \overbrace{181}^{M_A(2)} = -9.05$$

$$\overbrace{80.5}^{M_B(2)} - \overbrace{9.05}^{\alpha_a \times M_A(2)} = \overbrace{71.45}^{M_B(3)}$$

A attacks, B defends

time	M_A	M_B
0	200	100
1	190	90
2	181	80.5
3	172.95	71.45
4	165.805	62.8025
5	159.52475	54.51225
6	154.073525	46.5360125
7	149.4199238	38.83233625
8	145.5366901	31.36134006
9	142.4005561	24.08450556
10	139.9921056	16.96447775
11	138.2956578	9.964872472
12	137.2991705	3.050089583
13	136.9941616	-3.814868944

A attacks, B defends

- Percentage attrition:

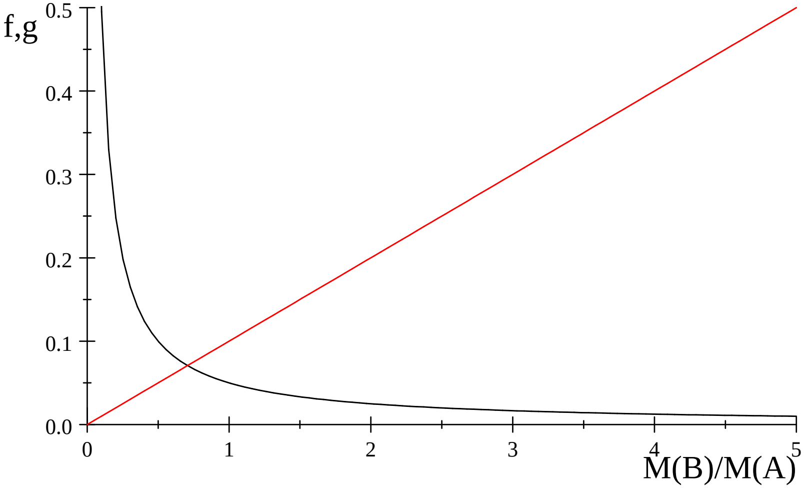
$$\overbrace{\frac{M_A(t+\Delta)-M_A(t)}{M_A(t)}}^{f\left(\frac{M_B}{M_A}\right)} = -\frac{\beta_d M_B(t)}{M_A(t)} = -\beta_d \frac{M_B(t)}{M_A(t)};$$

$$\overbrace{\frac{M_B(t+\Delta)-M_B(t)}{M_B(t)}}^{g\left(\frac{M_B}{M_A}\right)} = -\frac{\alpha_a M_A(t)}{M_B(t)} = -\alpha_a \times \frac{1}{\frac{M_B(t)}{M_A(t)}}.$$

- For A, bigger percentage losses for bigger values of $\frac{M_B}{M_A}$; for B, smaller percentage losses for bigger values of $\frac{M_B}{M_A}$

A attacks, B defends

pictures of the functions



$f\left(\frac{M_B}{M_A}\right); \alpha_a = .1 : rd; \; g\left(\frac{M_B}{M_A}\right); \; \beta_d = .05 : bl$

Three cases: Case 1

$$\begin{aligned} \frac{\Delta M_A}{M_A} &< \frac{\Delta M_B}{M_B}, \; e.g., \; -6\% < -4\%; \\ -\frac{\beta_d M_B(t)}{M_A(t)} &< -\frac{\alpha_a M_A(t)}{M_B(t)}; \\ \frac{M_B(t)}{M_A(t)} &> \sqrt{\frac{\alpha_a}{\beta_d}}. \end{aligned}$$

This means $A's$ percentage losses are greater than $B's$. Who is winning?

A attacks, B defends

Case 2:

$$
\begin{aligned}
\frac{\Delta M_A}{M_A} &> \frac{\Delta M_B}{M_B}, \ e.g., \ -2\% > -4\%; \\
-\frac{\beta_d M_B(t)}{M_A(t)} &> -\frac{\alpha_a M_A(t)}{M_B(t)}; \\
\frac{M_B(t)}{M_A(t)} &< \sqrt{\frac{\alpha_a}{\beta_d}}.
\end{aligned}
$$

This means $A's$ percentage losses are less than $B's$. Who is winning?

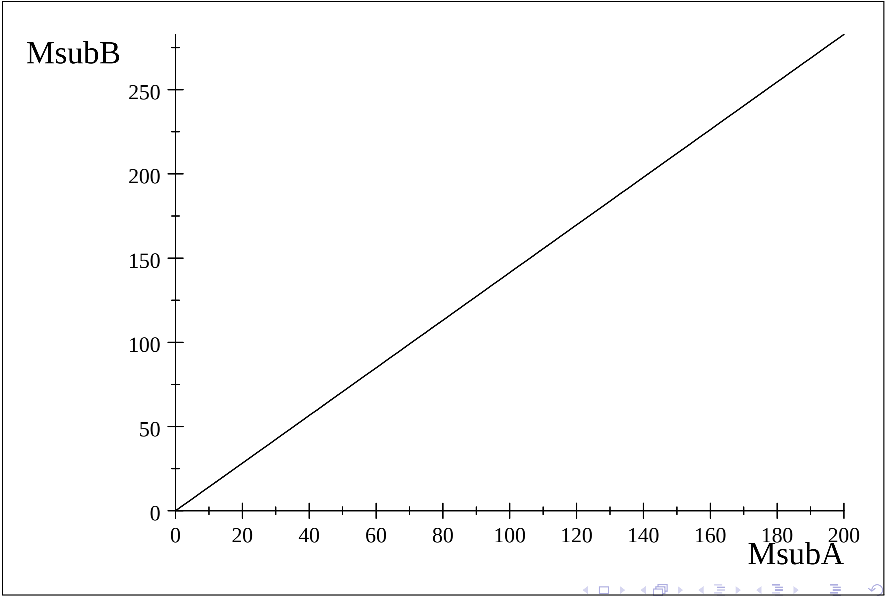
A attacks, B defends: MAD path

- Equal percentage losses

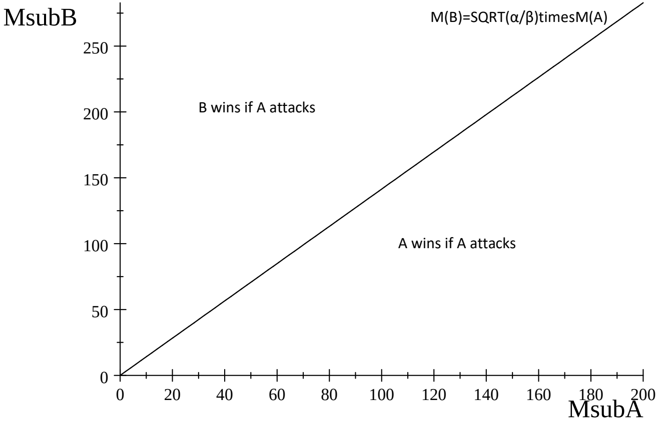
$$
\begin{aligned}
\frac{\beta_d M_B(t)}{M_A(t)} &= \frac{\alpha_a M_A(t)}{M_B(t)}; \\
\alpha_a M_A^2 &= \beta_d M_B^2; \\
\frac{M_A}{M_B} &= \sqrt{\frac{\beta_d}{\alpha_a}}. \\
M_A &= \sqrt{\frac{\beta_d}{\alpha_a}} M_B; \\
M_B &= \sqrt{\frac{\alpha_a}{\beta_d}} M_A.
\end{aligned}
$$

- A dichotomizing line in the $M_B - M_A$ plane: Above the line, A loses, B wins; below, vice versa

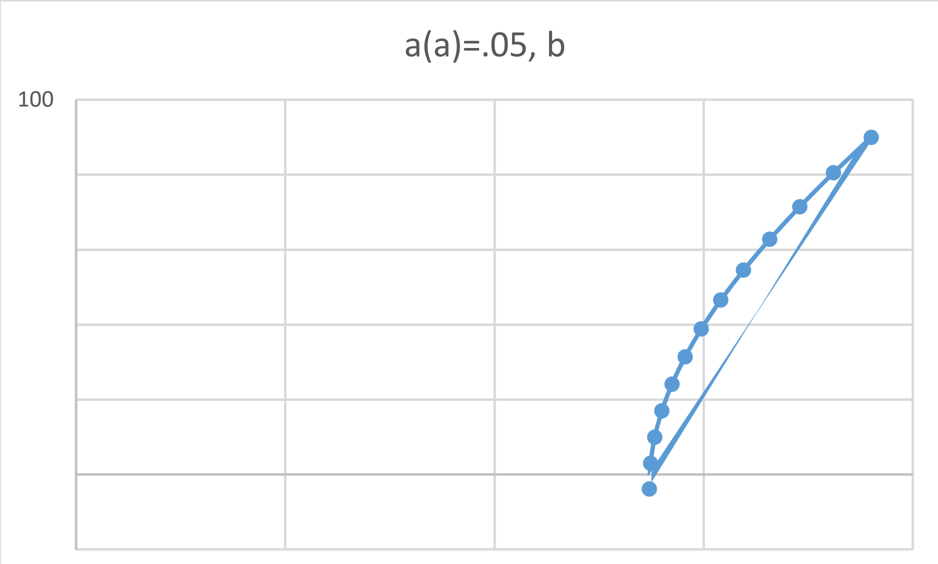
A attacks, B defends



A attacks, B defends



Actual paths



Reverse: A defends B attacks

- Attrition dynamics

$$
\begin{aligned}
M_A(t+\Delta) - M_A(t) &= -\beta_a M_B(t); \\
M_B(t+\Delta) - M_B(t) &= -\alpha_d M_A(t);
\end{aligned}
$$

- In percentage terms:

$$
\begin{aligned}
\frac{M_A(t+\Delta) - M_A(t)}{M_A(t)} &= \frac{-\beta_a M_B(t)}{M_A(t)} \\
\frac{M_B(t+\Delta) - M_B(t)}{M_B(t)} &= \frac{-\alpha_d M_A(t)}{M_B(t)}
\end{aligned}
$$

B attacks, A defends: MAD path

- Equal percentage losses

$$\begin{aligned} \frac{\beta_a M_B(t)}{M_A(t)} &= \frac{\alpha_d M_A(t)}{M_B(t)}; \\ \alpha_d M_A^2 &= \beta_a M_B^2; \\ \frac{M_A}{M_B} &= \sqrt{\frac{\beta_a}{\alpha_d}}. \end{aligned}$$

- A dichotomous line in $M_B - M_A$ plane

Propensity to Peace, War

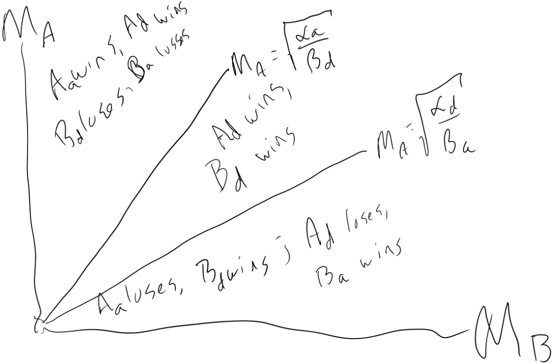
A attacks, B defends: MAD line is

$$M_B = \sqrt{\frac{\alpha_a}{\beta_d}} M_A.$$

For (M_A, M_B) above the line, A wins.
B attacks, A defends: MAD line is

$$M_B = \frac{1}{\sqrt{\frac{\beta_a}{\alpha_d}}} M_A.$$

Propensity to Peace, War



Brief history of warfare from castles to muskets to machine guns

1. Smooth-bore guns, powder horns: 100 yards. Think Waterloo, early US Civil War.
2. Rifling, breech-loading weapons: end of US Civil War; think of siege of Richmond.
3. Add machine guns.
4. Along comes tanks, more effective air weapons.
5. Lesson 1: Generals "fight the last war."
6. Lesson 2: people "see what they want to see."