Class 5

Game theory, SB?

May 28, 2017

Game theory

- Strategy: a complete plan of action (checkers, chess, tic-tac-do)
- Payoffs: each player has a complete numerical scale with which to compare all logically conceivable outcomes of the game, corresponding to each available combination of choices of strategies by all the players. The number associated with each possible outcome will be called that player's payoff for that outcome. Higher payoff numbers attach to outcomes that are better in this player's rating system.
- *At some level*, the players have a common understanding of the rules of the game:
  - the list of players,
  - the strategies available to each player
  - the payoffs of each player for all possible combinations of strategies pursued by all the players,
  - the assumption that each player is a rational maximizer.

Game theory

- Equilibrium: a "persuasive" prediction of the outcome, i.e., choices of strategies by players. "Rest point."
- Dominance (illustrated with PD)
- Nash Equilibrium: a list of strategies, one for each player, such that no player can get a better payoff by switching to some other strategy that is available to her while all the other players adhere to the strategies specified for them in the list. (Dixit, Avinash K.; Skeath, Susan; Reiley, David H.. Games of Strategy (Fourth Edition) (Page 95). W. W. Norton & Company. Kindle Edition.
- Nash concept has some subtle issues: in simultaneous move games like we consider, how does each player think about what the other player thinks?
- One idea: thinking through the others' thinking. You put yourself in the position of other players and think what they are thinking, which of course includes their putting themselves in your position and thinking what you are thinking. The logic seems circular. But best seen via example.

Game theory: PD and ID, Generals' nightmare

- Friedman's scenario: Two soldiers: if both brave, probabilities of living (.8, .8) ; if R brave and C shirks, R's prob. goes down (heroically provides cover) while C's goes up (escapes under the covering fire of R); if both shirk, (.6, .6): with neither providing covering fire, both of their probabilities of living fall.

Ray/Charley	B(rave)	S(hirk)
B(rave)	(.8, .8)	(.6, .9)
S(hirk)	(.9, .6)	(.7, .7)

- Easy to motivate a prediction: dominance. R thinks to self: whatever C does, my best choice is Shirk. C says to self, no matter what I do, R's choice is shirk; he will only play shirk, so my best choice to him shirking is myself shirking.

Game theory: another scenario (StagHare)

- If both brave, probabilities of living $(.8, .8)$; if R brave and C shirks, R's prob. goes down *a lot* (heroically provides cover) while C's goes down a little (escapes under the covering fire of R):$(.1, .7)$; if both shirk, $(.6, .6)$: with neither providing covering fire, both of their probabilities of living fall.

R/C	B	S
B	$(\underline{.8}, \underline{.8})$	$(1, .7)$
S	$(.7, .1)$	$(\underline{.6}, \underline{.6})$

- Two pure Nash equilibria, (B, B) and (S, S). In the absence of pre-play communication, each one has something going for it; (B, B)

  is *Pareto-dominant*, but (S, S) is much "safer."

Game theory SH

Indeed, since the players cannot communicate, Ray may well be uncertain that Charley will play B; she might therefore wish to play S, which assures her .6, whereas with B she may get .1. Moreover, if she takes into account that C may reason in the same way, she is all the more likely to play S; this makes it still more likely that C, too, will play S, and so on. We do not, however, assert that reasonable players must play S; only that they may do so, that S is not unreasonable or foolish. And for the time being, we assert this only when there is no pre-play communication.

Game theory SH

Permit pre-play communication. On the face of it, it seems that the players can then "agree" to play (B, B); though the agreement is not enforceable, it removes each player's doubt about the other one playing S. But does it indeed remove this doubt?

R/C	B	S
B	$(\underline{.8}, \underline{.8})$	$(1, .7)$
S	$(.7, .1)$	$(.\underline{6}, \underline{.6})$

Game theory SH

Suppose that Ray is a careful, prudent person, and in the absence of an agreement, would play S. Suppose now that the players agree on (B, B), and each retires to his "corner" in order actually to make a choice. R is about to choose B, when she says to herself: "Wait; I have a few minutes; let me think this over. Suppose that C doesn't trust me, and so will play S in spite of our agreement. Then he would still want me to play B, because that way he will get .7 rather than .6. And of course, also if he does play B, it is better for him that I play B. Thus he wants me to play B no matter what. So he wants the agreement to play (B, B) in any case; it doesn't bind him, and might increase the chances of my playing B. That doesn't imply that he will necessarily play S, but he may; since he wants the agreement no matter what he plays, the agreement conveys no information about his play. In fact, he may well have signed it without giving any thought as to how actually to play. Since he can reason in the same way about me, neither one of us gets any information from the agreement; it is as if there were no agreement. So R will choose now what she would have chosen without an agreement, namely S."

The game of Figure 1 is sometimes called the "stag hunt".5 Two men agree to hunt a stag. To succeed, they must go along separate paths, giving the task their un.divided attention. On the way, each has the opportunity to abandon the stag hunt and hunt rabbits instead. If he does so the number of rabbits he bags increases if the other continues to hunt the stag. Both would prefer it if both hunted the stag, since it is more valuable than a bag of rabbits. But each fears that each mistrusts the other, that the mistrust breeds more mistrust, and so on.
In the international relations literature, the game has been called the "security dilemma" (Jervis, 1978). Two countries between which there is tension are each considering the development of a new, expensive weapons system. Each is best off if neither has the system, but would be at a serious disadvantage if only the other had it. Can either side afford not to develop the system?

Game theory: How to get the Generals what they want

Assurance games: relative numbers the important thing

- Medals, training, etc.: payoffs not just prob. of living

R/C	B	S
B	(.95, .95)	(.6, .9)
S	(.9, .6)	(.7, .7)

- Is there a dominant strategy? If C is Brave, R should be brave. But if C Shirks, I should shirk.
- Two *Nash equilibria*!
- Solution to Friedman's problem: how to get coordination on (.95, .95) .

Lagniappe: required part over!

2017: what follows is NOT required or tested upon.

Some games in normal form

Agrression game

Scenario: Hitler is "Ray;" His most-prefrerred outcome is if he aggresses and Chamberlain ("Charley") plays strategy CESR aka "Cheese-Eating Surrender Monkey" aka Being French. He gets payoff of 3. If he refrains, his payoff is the status quo, 2. His worst outcome is if he plays "Aggress" and Chamberlain plays "Bow Up," in which he gets 1. Chamberlain has bad choices if Hitler agresses: he gets worse than the status quo no matter what he does. The payoff matrix is:

	CESR	Bow Up
Aggress	$(3, 1.5)$	$(1, 1)$
Refrain	$(2, 2)$	$(2, 2)$

What's not captured in payoff matrix: sequential nature.

By our usual mehtod of looking for a NE, we get two: (Aggress, CESR) and (Refrain, Bow Up)

	CESR	Bow Up
Aggress	$(\underline{3}, \underline{1.5})$	$(1, 1)$
Refrain	$(2, \underline{2})$	$(\underline{2}, \underline{2})$

- Iterated dominance and backward induction: which strategy should be thrown out?
  - If Hitler plays Aggress, he could end up with less than if he played Refrain, and if he played Refrain, he could end up with less than if he played Aggress: no dominated strategy.
  - If Chamberlain played CESR, he gets either 1.5 or 2; if he plays Bow Up, he gets 1 or 2. He has no reason to play Bow Up.

	CESR	Bow Up
Aggress	$(3, 1.5)$	$(1, 1)$
Refrain	$(2, 2)$	$(2, 2)$

- Key assumption: Hitler knows this about the payoff matrix and knows that Chamberlain is rational. Thus he can deduce that "Bow Up" will never be played, and thus knows that his best play is Aggress.
- What have most people taken away from this episode? Appeasement always bad (VN, ISIS,Ukr.)

In the movie "Dr. Strangelove," American Air Force General Jack Ripper, believing the Russkies have used water flouridation to sap American's "precious bodily fluids," wants the U.S. to hit the USSR with an all-out first-strike nuclear attack. He has successfully tricked the small nuclear-armed bomber group under his command to believe the US has been attacked and has sent them on a bombing mission to Russia. Only he knows the secret code to recall them. Ripper plays a game with the U.S. president to get him to launch the all-out first-strike attack that insures victory-clearly at a high cost, but in Ripper's mind worth it-over the Russkies.

While his bomber group is still short of crossing Soviet airspace, he makes sure the U.S. President knows what he has done. He explains his reasoning: Once his bombers cross Soviet airspace, the Soviets will launch a retaliatory attack. This will really be a "first strike," as Ripper's bombers are few in number, and their effect on the USSR will be small. Thus, if the U.S. President does not immediately launch a first strike, the U.S. will be hit with an ensuing Russkie first strike and lose the ensuing nuclear war. The president's only rational choice is to now launch a U.S. first strike.

Some normal form games

Dr. Strangelove

- Does this payoff matrix capture the scenario?

	First Strike Now	Lose Nuclear War
Not Recall	$(3, 1.5)$	$(1, 1)$
Recall	$(2, 2)$	$(2, 2)$

- This says Ripper values status quo at 2. (Not Recall, First Strike Now) is preferred by Ripper to status quo. Pres. values status quo at 2, but values (NR, FSN) less than status quo but greater than Lose Nuclear War.

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- Is Ripper's thinking sound? No matter what Ripper does, "Lose Nuclear War" is a dominated strategy. See the movie to see what happened.