Lectures 5-6

Collective action and game theory

May 21, 2018

Miscellany and review

- Review of some ideas (fear as lack of commitment)
  - "Because of this distrust amongst men, the most reasonable way for any man to make himself safe is to strike first, that is, by force or cunning subdue other men - as many of them as he can, until he sees no other power great enough to endanger him. This is no more than what he needs for his own survival, and is generally allowed." (Hobbes, *Leviathan*)
  - "Effectively there is nothing stopping someone from grabbing resources except fear of retaliation. Hobbes goes on to suggest that reasonable people can come to realize the inherent difficulties with anarchy and cede their rights to a Leviathan in order to live in peace. However, such social contracts do not generally appear in the international arena, and hence for an agreement to endure it has to be balanced in such a way as to be self-enforcing." (Jackson-Morelli)
  - "I could just go and rob someone here in London-they don't have guns." (Shelby Moats, 6–foot–nine–inch basketball player.)
- Truth as war casualty: "gas explosions."

Review: strategic bombing, moral hazard

- "I can't imagine a more demoralizing act than to turn an entire city to ash with just one bomb. After the war was over, many of the scientists working on the Manhattan project became pacifists and sought that nuclear weapons may never be used again. It's not hard to imagine why."
- "Cost sharing (via copays, deductibles, etc.) is a technique aimed at deterring beneficiaries of health insurance from consuming excessive healthcare because they are insured.

Miscellany review

- My goals for this class:
  - To enhance your ability to think critically, i.e., to use cohesive and logical reasoning patterns that lead to careful and deliberate decisions of whether to accept, reject, or suspend judgment about the issues in question.
  - Expand horizons–take advantage of being abroad!
- In comparison to earlier versions:
  - **less equations**, more of an emphasis on seeing whether and how economic concepts can organize thought about conflict.
  - an emphasis on complexity and contradiction.
    - killing prisoners, bombing civilians, holocaust: how can we understand why people have done what they did; what might you have done?
    - Allied invasion of North Africa: first real combat was against Vichy, i.e., French, troops!
    - Cognitive biases: availability and affirmation.

More principles: review

Moral hazard

But moral hazard exists in that the men ordering battle are not the men to die. Those who give orders must be subject to a set of incentives (the possibility of being relieved from duty, reassigned, court-martialed, etc.) that induces them to deploy resources under their command to best effect. The fighting men depend on it with their lives. Moral hazard is an aspect of information asymmetry: only the officer knows whether his men really need to be sent into this or that particular battle. ... what are a commander's real intentions when he gives orders? What benevolent or malevolent purposes are hidden beneath the veneer of his uniform ...? To overcome this incentive alignment problem, hierarchies must provide for oversight and recourse. These may include appeal to higher authorities up the rank, but more effective is the simple requirement that commanding officers fight with their men. If the officer is to face death, he will think twice about being heedless; if he is truly mad, a mutiny may well be sanctioned upon inquiry. In World War II, rear-area officers were prone to go along on bombing missions. In addition, ineffective commanders were

Miscellany

- Measuring success: an exam question:

"... they generally take it for granted that the playing pieces will go where they are moved. In real battles they frequently do not. The economic problem is why they do not and what can be done about it." Explain, in a way that could be understood by someone who has not had this course, e.g., your parents, why Friedman saw this as the central problem to economists who want to understand conflict, and discuss the various approaches we have seen that attempt to "solve" this basic problem. Use all appropriate rhetorical devices you deem necessary.

- Stirrups (a random remembrance of an earlier point).

Getting people to fight

Getting soldiers to fight

Notes on "Economics of War" by D. Friedman
What is thesis?

End of 2nd para:.
"The economic problem is why they do not and what can be done about it."

Basic issue he raises: the logic of collective action: an *externality* that cannot be internalized; micromotives and macrobehavior.
Game theory is one tool.

Friedman again

In part a PA problem: Generals vs grunts "The soldier, however, is likely to rank his own survival a good deal higher and the general's survival a good deal lower in importance than the general does. One consequence of that disagreement is that the general may rationally tell the soldier to do something and the soldier may rationally not do it. Neither is necessarily making a mistake; each may be correctly perceiving how to achieve his ends."(also Generals vs Head of State as in the desire for glory for a general at expense of overall victory); also patrols (note monitoring issue that arises as well: tickets from police? Body counts?)
"But rationality is an assumption about individuals, not about groups. Each individual, in my simple example of the economics of war, is making the correct decision about how he should act in order to keep himself alive. It so happens that the correct decision for me (running away) decreases the chance of being killed for me but increases it for everyone else on my side, and similarly for everyone else's correct decision; individually, each of us is better off (given what everyone else is doing) than if he stood and fought, but we are all worse off than we would be if each of us had failed

Friedman again

Other examples? Schelling rubbernecking; who plays what position on a team (Hunter Bledsoe)

Key elements: misaligned incentives. But note he assumes soldiers all want to live, which IS aligned with standing and fighting and not running, a la traffic problem. So somewhat more subtle (barge-pullers paying the whip?)

Solution? punishment (why would not soldiers want this, a la barge example?); changing the culture (heroism); aligning incentives through committment (burning bridges).

Supporting evidence of basic proposition (individual motives/rationality might not be aligned with agent's or with collective goals).

1. Shooting your weapon: smaller units mean more likely to have good reward/cost ratio; BAR; Note: less need for punishment? Is this consistent with Army studies of why soldiers fight?

Game theory

- Strategy: a complete plan of action (checkers, chess, tic-tac-do)
- Payoffs: each player has a complete numerical scale with which to compare all logically conceivable outcomes of the game, corresponding to each available combination of choices of strategies by all the players. The number associated with each possible outcome will be called that player's payoff for that outcome. Higher payoff numbers attach to outcomes that are better in this player's rating system.
- *At some level*, the players have a common understanding of the rules of the game:
    - the list of players,
    - the strategies available to each player
    - the payoffs of each player for all possible combinations of strategies pursued by all the players,
    - the assumption that each player is a rational maximizer.

Game theory

- Equilibrium: a "persuasive" prediction of the outcome, i.e., choices of strategies by players. "Rest point."
- Dominance (illustrated with PD)
- Nash Equilibrium: a list of strategies, one for each player, such that no player can get a better payoff by switching to some other strategy that is available to her while all the other players adhere to the strategies specified for them in the list. (Dixit, Avinash K.; Skeath, Susan; Reiley, David H.. Games of Strategy (Fourth Edition) (Page 95). W. W. Norton & Company. Kindle Edition.
- Nash concept has some subtle issues: in simultaneous move games like we consider, how does each player think about what the other player thinks?
- One idea: thinking through the others' thinking. You put yourself in the position of other players and think what they are thinking, which of course includes their putting themselves in your position and thinking what you are thinking. The logic seems circular. But best seen via example.

PD and Iterated Dominance, Generals' nightmare

- Friedman's scenario: Two soldiers: if both brave, probabilities of living $(.8, .8)$; if R brave and C shirks, R's prob. goes down (heroically provides cover) while C's goes up (escapes under the covering fire of R); if both shirk, $(.6, .6)$: with neither providing covering fire, both of their probabilities of living fall.

Ray/Charley	B(rave)	S(hirk)
B(rave)	$(.8, .8)$	$(.6, \underline{.9})$
S(hirk)	$(\underline{.9}, .6)$	$(\underline{.7}, \underline{.7})$

- Easy to motivate a prediction: dominance. R thinks to self: whatever C does, my best choice is Shirk. C says to self, no matter what I do, R's choice is shirk; he will only play shirk, so my best choice to him shirking is myself shirking.

Game theory: another scenario (StagHare)

- If both brave, probabilities of living $(.8, .8)$; if R brave and C shirks, R's prob. goes down *a lot* (heroically provides cover) while C's goes down a little (escapes under the covering fire of R):$(.1, .7)$; if both shirk, $(.6, .6)$: with neither providing covering fire, both of their probabilities of living fall.

R/C	B	S
B	$(\underline{.8}, \underline{.8})$	$(1, .7)$
S	$(.7, .1)$	$(\underline{.6}, \underline{.6})$

- Two pure Nash equilibria, (B, B) and (S, S). In the absence of pre-play communication, each one has something going for it; (B, B)

  is *Pareto-dominant*, but (S, S) is much "safer."

Game theory SH

Indeed, since the players cannot communicate, Ray may well be uncertain that Charley will play B; she might therefore wish to play S, which assures her .6, whereas with B she may get .1. Moreover, if she takes into account that C may reason in the same way, she is all the more likely to play S; this makes it still more likely that C, too, will play S, and so on. We do not, however, assert that reasonable players must play S; only that they may do so, that S is not unreasonable or foolish. And for the time being, we assert this only when there is no pre-play communication.

Game theory SH

Permit pre-play communication. On the face of it, it seems that the players can then "agree" to play (B, B); though the agreement is not enforceable, it removes each player's doubt about the other one playing S. But does it indeed remove this doubt?

R/C	B	S
B	$(\underline{.8}, \underline{.8})$	$(1, .7)$
S	$(.7, .1)$	$(\underline{.6}, \underline{.6})$

Game theory S(tag)H(are)

Suppose that Ray is a careful, prudent person, and in the absence of an agreement, would play S. Suppose now that the players agree on (B, B), and each retires to his "corner" in order actually to make a choice. R is about to choose B, when she says to herself: "Wait; I have a few minutes; let me think this over. Suppose that C doesn't trust me, and so will play S in spite of our agreement. Then he would still want me to play B, because that way he will get .7 rather than .6. And of course, also if he does play B, it is better for him that I play B. Thus he wants me to play B no matter what. So he wants the agreement to play (B, B) in any case; it doesn't bind him, and might increase the chances of my playing B. That doesn't imply that he will necessarily play S, but he may; since he wants the agreement no matter what he plays, the agreement conveys no information about his play. In fact, he may well have signed it without giving any thought as to how actually to play. Since he can reason in the same way about me, neither one of us gets any information from the agreement; it is as if there were no agreement. So R will choose now what she would have chosen without an agreement, namely S."

Game theory SH

The game of Figure 1 is sometimes called the "stag hunt".5 Two men agree to hunt a stag. To succeed, they must go along separate paths, giving the task their un.divided attention. On the way, each has the opportunity to abandon the stag hunt and hunt rabbits instead. If he does so the number of rabbits he bags increases if the other continues to hunt the stag. Both would prefer it if both hunted the stag, since it is more valuable than a bag of rabbits. But each fears that each mistrusts the other, that the mistrust breeds more mistrust, and so on.
In the international relations literature, the game has been called the "security dilemma" (Jervis, 1978). Two countries between which there is tension are each considering the development of a new, expensive weapons system. Each is best off if neither has the system, but would be at a serious disadvantage if only the other had it. Can either side afford not to develop the system?

Game theory: How to get the Generals what they want

Assurance games: relative numbers the important thing

- Medals, training, etc.: payoffs not just prob. of living

R/C	B	S
B	(<u>.95</u>, <u>.95</u>)	(.6, .9)
S	(.9, .6)	(<u>.7</u>, <u>.7</u>)

- Is there a dominant strategy? If C is Brave, R should be brave. But if C Shirks, I should shirk.
- Two *Nash equilibria*!
- Solution to Friedman's problem: how to get coordination on (<u>.95</u>, <u>.95</u>) .

Motivating soldiers: Akerlof

- "...It is relatively cheap to impart identity to soldiers and officers, since many self-select into the armed services and thus are open to its methods, and it is very costly to quit (for example, Lipsky reports that West Point is the only economically viable college education for many cadets). In the military, it is hard to observe effort, especially when it is most crucial—in battle. In addition, military personnel are especially susceptible to indoctrination because of their isolation. Hence, the model would predict that the military would rely more on identity than on monetary compensation, and this prediction is consistent with described differ-ences between military and civilian organizations, which we discuss further below"
- Draftees

How do they do it?

- "cognitive dissonance theory suggests why people who are led to choose an unpleasant experience will change their image of themselves: they need a consistent explanation why they made such a choice. Such changes in self-perception can be used to manipulate identities, as in fraternity initiations that induce loyalty to the fraternity. A large number of other perceptual biases can be used to alter subjects' self-perceptions."
- "*The American Soldier*, a study of combat soldiers in World War II, finds soldiers' major incentive to fight came from adherence to the ideal fostered in the combat unit of being "a man." It meant showing "courage, endurance and toughness, ..., avoidance of display of weakness in general, reticence about emotional or idealistic matters, and sexual competency"

But remember:

1. NYT, Tuesday Dec 1: "To curb Repeat Hospital Stays, Pay Doctors," by Sandeep Jauhar, M.D.

   "You have to motivate doctors to do the right thing. You can appeal to professionalism or altruism, to doing well for patients or serving a greater purpose, but nothing influences behavior like money..."

Article is about the cost of not discharging medicare patients in a timely manner. The conflict comes from doctors getting paid if the patient is in the hospital but not if they are discharged and not there, while the hospital is under pressure to discharge quickly becuase they get paid a lump sum based on the patient's diagnosis.

Scenario: Hitler is "Ray;" His most-prefrerred outcome is if he aggresses and Chamberlain ("Charley") plays strategy CESR aka "Cheese-Eating Surrender Monkey" aka Being French. He gets payoff of 3. If he refrains, his payoff is the status quo, 2. His worst outcome is if he plays "Aggress" and Chamberlain plays "Bow Up," in which he gets 1. Chamberlain has bad choices if Hitler agresses: he gets worse than the status quo no matter what he does. The payoff matrix is:

	CESR	Bow Up
Aggress	$(3, 1.5)$	$(1, 1)$
Refrain	$(2, 2)$	$(2, 2)$

What's not captured in payoff matrix: sequential nature.

Some games in normal form

Aggression game

By our usual mehtod of looking for a NE, we get two: (Aggress, CESR) and (Refrain, Bow Up)

	CESR	Bow Up
Aggress	$(\underline{3}, \underline{1.5})$	$(1, 1)$
Refrain	$(2, \underline{2})$	$(\underline{2}, \underline{2})$

- Iterated dominance and backward induction: which strategy should be thrown out?
  - If Hitler plays Aggress, he could end up with less than if he played Refrain, and if he played Refrain, he could end up with less than if he played Aggress: no dominated strategy.
  - If Chamberlain played CESR, he gets either 1.5 or 2; if he plays Bow Up, he gets 1 or 2. He has no reason to play Bow Up.

	CESR	Bow Up
Aggress	$(3, 1.5)$	$(1, 1)$
Refrain	$(2, 2)$	$(2, 2)$

- Key assumption: Hitler knows this about the payoff matrix and knows that Chamberlain is rational. Thus he can deduce that "Bow Up" will never be played, and thus knows that his best play is Aggress.
- What have most people taken away from this episode? Appeasement always bad (VN, ISIS,Ukr.)

In the movie "Dr. Strangelove," American Air Force General Jack Ripper, believing the Russkies have used water flouridation to sap American's "precious bodily fluids," wants the U.S. to hit the USSR with an all-out first-strike nuclear attack. He has successfully tricked the small nuclear-armed bomber group under his command to believe the US has been attacked and has sent them on a bombing mission to Russia. Only he knows the secret code to recall them. Ripper plays a game with the U.S. president to get him to launch the all-out first-strike attack that insures victory-clearly at a high cost, but in Ripper's mind worth it-over the Russkies.

While his bomber group is still short of crossing Soviet airspace, he makes sure the U.S. President knows what he has done. He explains his reasoning: Once his bombers cross Soviet airspace, the Soviets will launch a retaliatory attack. This will really be a "first strike," as Ripper's bombers are few in number, and their effect on the USSR will be small. Thus, if the U.S. President does not immediately launch a first strike, the U.S. will be hit with an ensuing Russkie first strike and lose the ensuing nuclear war. The president's only rational choice is to now launch a U.S. first strike.

- Does this payoff matrix capture the scenario?

	First Strike Now	Lose Nuclear War
Not Recall	$(3, 1.5)$	$(1, 1)$
Recall	$(2, 2)$	$(2, 2)$

- This says Ripper values status quo at 2. (Not Recall, First Strike Now) is preferred by Ripper to status quo. Pres. values status quo at 2, but values (NR, FSN) less than status quo but greater than Lose Nuclear War.

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- Is Ripper's thinking sound? No matter what Ripper does, "Lose Nuclear War" is a dominated strategy. See the movie to see what happened.

Suppose a group of 25 people must all arrive at a suggested time of 13 : 00 before the group can depart. Each individual can choose an effort level s_i that gets them close to the 13 : 00 suggested departure time from the set of effort levels $\{1, 2, 3, 4, 5, 6, 7\}$ where effort increases with the number. The benefit (payoff) each individual gets from leaving on time at 13 : 00 is a "weakest link" function: The payoff depends on *lowest* effort level expended by a member of the group (because the group cannot leave until the last person gets there)

$$B_i = 50 + 20 \times \min \{s_1, s_2, ..., s_{25}\}.$$

E.G., if the lowest effort level is 2, the payoff for each individual is 90. If the lowest level is 7, the payoff is 190.
The cost to each individual is proportional to the effort level, and is given as

$$C_i = 10s_i.$$

That is, if someone chooses $s_i = 5$, their cost is 50.

- Nash Equilibrium: Each person takes the strategy of every other person as given, and tries to do best for herself.
- Because benefits depend on the minimum effort of the group, but costs are proportional to effort, all people will pick the same effort level, call it $s\prime$.
- Person 1 chooses s_1 to maximize her own Benefits minus Costs, assuming $s_2 = s_3 = s_4 = ... = s_{25} = s\prime$:

$$\max_{s_1} NB_1 = \{50 + 20 \times \min(s_1, s\prime\} - 10 \times s_1;$$

- suppose she chooses $s_1 = s\prime$:

$$NB\left(s_1 = s\prime\right) = 50 + 20s\prime - 10s\prime = 50 + 10s\prime$$

- Suppose she chooses $s_1 > s\prime$:

$$NB_1\left(s_1 > s\prime\right) = 50 + 20s\prime - 10s_1.$$

This cannot be a maximum, because she could choose $s\prime < s_1$ and save on costs without sacrificing benefits:

$$NB(s_1 = s\prime) - NB_1\left(s_1 > s\prime\right) = -10s\prime + 10s_1 = 10(s_1 - s\prime) > 0$$

- Suppose she chooses $s_1 < s\prime$:

$$\begin{aligned} NB_1\left(s_1 < s\prime\right) &= 50 + 20s_1 - 10s_1; \\ NB_{1,s\prime} - NB_{1,s_1} &= 20(s\prime - s_1) > 0. \end{aligned}$$

- Upshot: *any* common level of security is a NE.
- Like other coordination games: NB's are different for different equilibria:
  -
  $$NB_i = 50 + 10s\prime$$
  - This is increasing in effort level: best is maximum effort!
- Experimental evidence; race to the bottom!
- Other applications: Fight or Flee, NDP's. airlines, early versus late planting.

- Fish population dynamics:
  - At time t, e.g., 1990, we start period with x_t units of fish;
  - fisherpeople catch $c_{i,t}$ units of fish
  - Fish population at beginning of next period is

$$
\begin{aligned}
x_{t+1} &= (x_t - c_{1t} - c_{2t})^{\alpha},\ 0 < \alpha < 1;\ e.g.,\ \alpha = .5; \\
x_{t+1} &= (x_t - c_{1t} - c_{2t})^{.5}
\end{aligned}
$$

- Fish-catching: each angler chooses an effort level that leads to a proportion of the fish stock that is caught

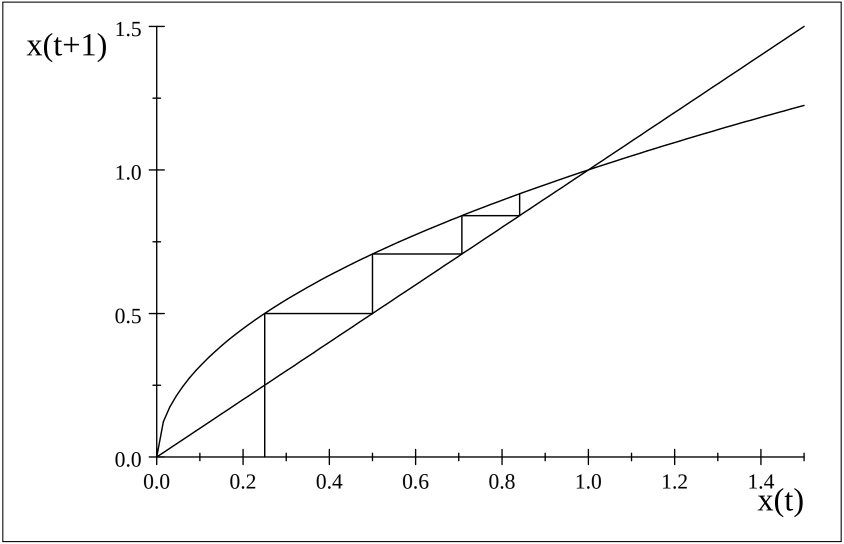
$$c_{i,t} = \beta_i x_t$$

- Fish dynamics

$$x_{t+1} = \{x_t (1 - \beta_1 - \beta_2)\}^{.5}$$

Common pool games

Great Fish Wars



- Steady-state stock of fish:

$$x = (1 - \beta_1 - \beta_2)^{\frac{\alpha}{1-\alpha}}$$

- $\alpha = .5:$

$$x = (1 - \beta_1 - \beta_2)$$

- Objective: taking β_2 as given, find your best response in terms of β_1 to maximize your catch, subject to the steady-state fish constraint:

$$\max_{\beta_1} y \equiv \beta_1 x = \beta_1 (1 - \beta_1 - \beta_2)$$

- Solution and reaction curves:

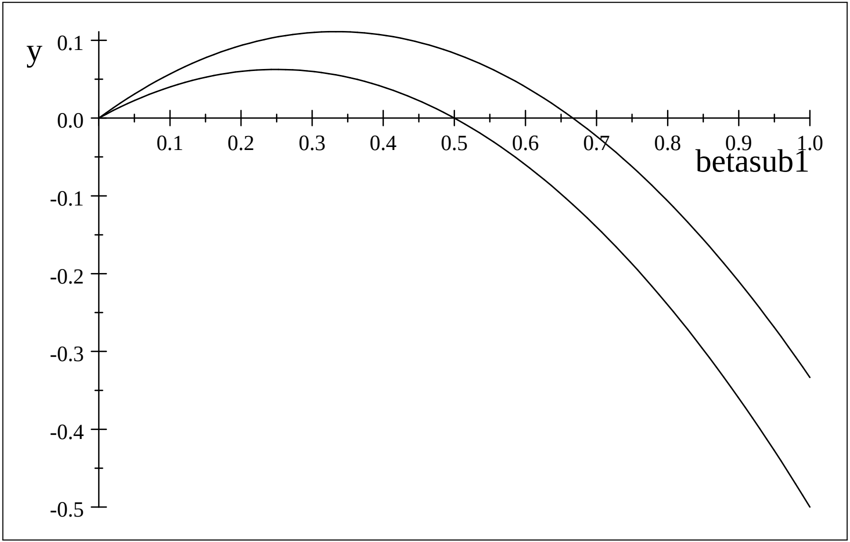
$$\beta_1 = \frac{1 - \beta_2}{2};\ \beta_2 = \frac{1 - \beta_1}{2}$$

- Nash equilibrium:

$$\beta_1 = \beta_2 = \frac{1}{3}; \beta_1 + \beta_2 = \frac{2}{3}.$$

Common pool games

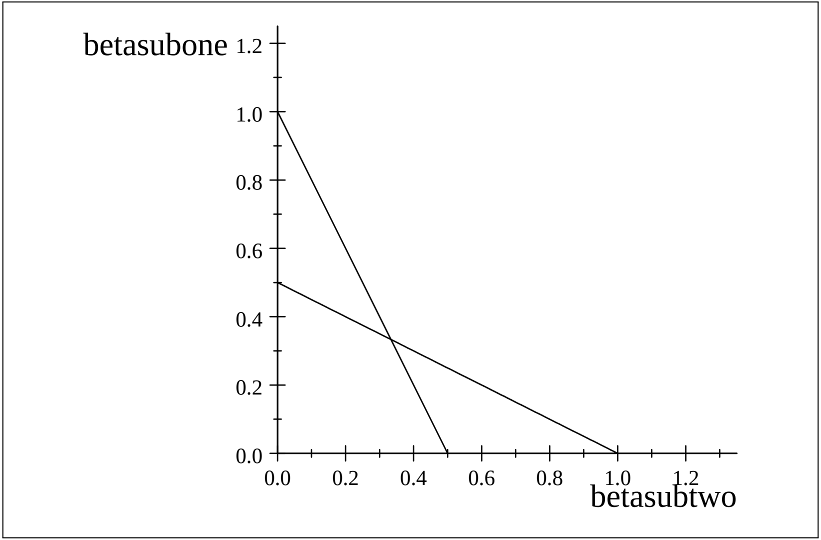
Great Fish Wars: individual maximization



$\beta_2 = .5, .333...$

Common pool games

Great Fish Wars: reaction curves



- Joint maximization

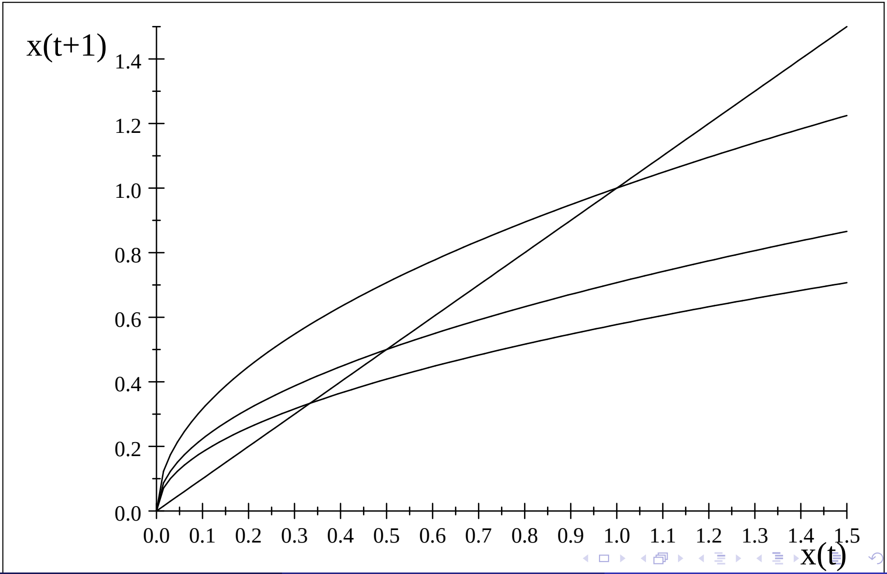
$$\max(\beta_1 + \beta_2)(1 - (\beta_1 + \beta_2))$$

- Solution:

$$\begin{aligned} \beta_1 &= \beta_2 = \frac{1}{4} \\ \beta_1 + \beta_2 &= \frac{1}{2} \end{aligned}$$

Common pool games

Great Fish Wars: depiction



- Now consider a two-period version in prep of bargaining failure, deepen BI understanding:
  - At T, each angler gets a fraction, e.g., one-half, of the stock of fish, x_T.
  - Fish population at T:

$$x_T = (x_{T-1} - c_{1,T-1} - c_{2,T-1})^{\alpha}$$

  - Preferences:

$$\begin{aligned} U_i &= \ln(c_{i,T-1}) + \rho \ln(c_{i,T}) \\ &= \ln(c_{i,T-1}) + \rho \ln \left(\left(\frac{1}{2}\right) \overbrace{((x_{T-1} - c_{1,T-1} - c_{2,T-1})^{\alpha})}^{x_T} \right) \\ &= \ln c_{i,T-1} + \rho \ln \frac{1}{2} + \rho\alpha \ln(x_{T-1} - c_{1,T-1} - c_{2,T-1}). \end{aligned}$$

- Maximization (for angler 1taking $c_{2,T-1}$ given):

$$\frac{1}{c_{1,T-1}} - \rho\alpha\left(\frac{1}{x_{T-1} - c_{1,T-1} - c_{2,T-1}}\right) = 0$$

- RC:

$$c_{1,T-1} = \frac{1}{\rho\alpha + 1}\left(x_{T-1} - c_{2,T-1}\right)$$

- Solution:

$$c_{1,T-1} = c_{2,T-1} = \overbrace{\beta_1}^{\frac{1}{\rho\alpha+2}} x_{T-1}$$

- Let's name these two-period solutions as $c^*_{i,T-1}$:

$$c^*_{i,T-1} = \beta_i x_{T-1}$$

- Goal: maximize utility

$$
\begin{aligned}
U_i &= \ln c_{i,T-2} + \rho \ln c^*_{i,T-1}(x_{T-1}) \\
&+\rho^2 \ln \left(\left(\frac{1}{2}\right) \overbrace{\left(\left(x_{T-1} - c^*_{1,T-1}(x_{T-1}) - c^*_{2,T-1}(x_{T-1})\right)^{\alpha}\right)}^{x_T(x_{T-1})} \right)
\end{aligned}
$$

- We emphasize that the best choices of each angler at $T-1$ are *functions* of the *state* at $T-1$, namely x_{T-1}.
- Now,

$$
x_{T-1} = \left(x_{T-2} - c_{1,T-2} - c_{2,T-2}\right)^{\alpha}.
$$

- So, each angler's problem at time $T-2$ is a choice of fish-catching, i.e., $c_{i,T-2}$, as a function of the other angler's catch and the stock of fish available at $T-2$:

$$
c_{1,T-2} = f_{1,T-2}(x_{T-2}, c_{2,T-2})
$$

- Three-pd solution:

$$c^*_{i,T-2} = \beta_{i,T-2} x_{T-2}.$$

- Work backward for four-period problem, five-period problem, and so forth, take limit at $T \to \infty$:

$$\begin{aligned} c^*_{i,t} &= \beta_i x_t; \\ \beta_i &= \frac{\rho\alpha(1-\rho\alpha)}{1-(1-\rho\alpha)^2}. \end{aligned}$$

This motivates our initial attack on this problem: choosing a β instead of a choice of c.

	C	F
BKBQ	$(0, \underline{0})$	$(\underline{1}, -1)$
BKFQ	$(\underline{\frac{1}{2}}, -\frac{1}{2})$	$(0, \underline{0})$

- No Nash Eq. in pure strategies.
- Mixed strategies:
  - Ray: play "Bluff," i.e., BKBQ, with probability p, play "truth-telling," i.e., BKFQ, with prob. $1 - p$.
  - Charley: play Call with probability q and Fold with probability $1 - q$
- A Nash Eq: a pair of numbers (p, q) such that if Ray plays p Charley has no reason to change his play from q, and vice-versa.
- How to find? Compute EV_C, EV_R, Figure out best responses interms of p, q

Payoff matrix with general probability π of drawing a King:

$$
\begin{array}{cccc}
 & & q & 1-q \\
 & & \text{C} & \text{F} \\
p & \text{BKBQ} & (0,\underline{0})/(4\pi-2,2-4\pi) & (\underline{1},-1)/(1,-1) \\
1-p & \text{BKFQ} & (\underline{\tfrac{1}{2}},-\tfrac{1}{2})/(3\pi-1,1-3\pi) & (0,\underline{0})/(2\pi-1,1-2\pi)
\end{array}
$$

Mixed-strategy equilibria
SDP

	C	F
BKBQ	$(0, \underline{0})$	$(\underline{1}, -1)$
BKFQ	$(\underline{\frac{1}{2}}, -\frac{1}{2})$	$(0, \underline{0})$

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- Mixed strategies:
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Mixed-strategy equilibria
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