Capitalization, Decentralization, and Intergenerational Spillovers in a Tiebout Economy with a Durable Public Good

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September 2013

Abstract: We consider an overlapping generations model with a Durable Local Public Good (DLPG). We establish a Tiebout Theorem (equilibrium exists and is first best) as well as a Second Welfare Theorem in this dynamic DLPG economy. We also establish conditions under which local provision of durable public goods, as Tiebout suggested, results in the full internalization of the intergenerational spillovers that durability entails. We show that housing price bubbles or busts can result in a violation of these conditions. However, if housing prices fall within a "normal range", they give agents correct incentives for optimal investments in DLPG. In contrast, when durable public good are provided by the national government, internalization does not take place and under-provision of public goods will result. This sets up an institutional trade-off between national and local provision of public goods that balances the relative strength of intergenerational and interjurisdictional spillovers. (JEL Classification: H4, D9, H0, D7)

Keywords: Durable Local Public Goods, Capitalization and Intergenerational Spillover Effects, Dynamic Tiebout Equilibrium, Welfare Analysis.

Acknowledgment: We would like to thank Marcus Berliant, Hideo Konishi, Tom Nechyba, Antonio Rangel, Robert Rosenthal, Steve Tadelis, an anonymous referee, an associate editor and an editor, as well as participants of the Public Economic Theory Meetings, the Midwest Economic Theory Meetings, and the National Tax Conferences for useful discussions. Needless to say, the usual disclaimer applies.

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Co-author Notes

This revision focuses on the following most crucial changes:

- infinite horizon OLG: done in section 8
- presence of another store of value: add a remark in section 8
- maintaining nonnegativity constraints: more formal proof in section 6
- understanding conditions under which welfare theorems fail: added to the end of section 6
  - First Welfare Theorem: SJ, extreme prices
  - Second Welfare Theorem: high levels of optimal steady state DLPG
- heterogeneous agents: more formal proof in section 9.

Some minor ones incorporated are:

- Comment 1 on richer structure: We are abstracting from heterogeneous landscape and agglomerative externalities. Adding such would increase the complexity significantly. See a sentence added to the first paragraph of section 2.
- Comment 6 on the old voting as well: When the young born in t vote on how much g_t to add, this g_t adds to G_{t+1}, which is to be enjoyed by the young born in $t+1$ and is totally irrelevant to the old in t. Thus, in the interest of the current paper, we are excluding the old from voting. A footnote is added to section 3 under stage 2.
- Comment 7 on social planner's objective function: We adopt the Benthamite welfare function without considering inequality aversion because it is linked well to Pareto optimality as elaborated in Negishi (1960). See section 3.
- Specify choice variables in (3): done.
- Correct some typos, including the utility functions and the definitions of FM and PE (ρ replaced by β): done.

The remaining crucial issue to be addressed is:

- Several comments related SJ by editors and referee:
  - back to primitives
  - economic justification
  - whether replacable by government policy
  - other refinements without unique implementation
  - note: need to say why a continnum of locations would not work

1 Introduction

Many goods provided by various levels of government, such as police and health services, food assistance, and redistributional transfers, are intended for immediate consumption. Others, such as highways, public buildings, ports, parks and research and development are forms of public capital intended to last for many years. Providing these Durable Public Goods (DPGs) at efficient levels is difficult not only for the ordinary "free riding" reasons common to all public goods, but also because they produce intergenerational spillovers. Even if all existing agents truthfully revealed their willingness to pay and made their appropriate Lindahl contributions, they still would have no incentive to take into account the benefits that future generations receive from the stock of DPGs they leave behind. Clearly, unless this intergenerational spillover is somehow internalized, DPGs will be systematically under-provided.

Tiebout (1956) observed that many public goods such as police services, fire protection, and primary education, are provided by cities and counties rather than the federal government. He suggested that this created a kind of market in which different localities would offer different bundles of taxes and local public goods. Agents would then "vote with their feet", and by choosing their favorite jurisdiction, reveal their willingness to pay for public good and thereby solving the free riding problem. His focus (and the focus of most of the subsequent literature), however, was on static coalition formation, optimal sorting of agents by taste, and overcoming free-riding though tax/public good bundles offered by competing jurisdictions. See Conley and Wooders (1998, 2001) for an extensive discussion of this literature.

Of course, many of the public goods provided by cites and counties, such as city streets, storm drains, and school buildings, are durable as well. The focus of this paper is to ask whether Tiebout's argument can be extended to show that interjurisdictional competition also prevents free riding (and the consequent under-provision of public goods) between different generations.

To address this, we consider a simple model with multiple jurisdictions and an overlapping generations demographic structure. To emphasize the role of intergenerational spillovers, we suppose that every jurisdiction is identical and every agent has the same taste for public goods. Agents go through their life-cycle by buying a house in a particular jurisdiction when young and then selling it when old. The generation that lives in a jurisdiction in any given period enjoys the services of the Durable Local Public Good (DLPG) stock they inherit from the previous generation and then, in turn, chooses how much to add to the stock to be inherited by the next generation.

The main contribution of this paper is to establish reasonable conditions under which the value of any DLPG left at the end of a period is fully "capitalized" into the value of houses. Such capitalization causes the present generation to internalize the intergenerational spillover and therefore to invest optimally. This result does not depend on the presence of an "outside offer" such as developable land at the jurisdictions' periphery to pin down land prices.

In contrast, we show that capitalization does not take place when decision making is centralized. This is because when DPG is provided at the national level, all the jurisdictions are identical and the price of housing is not responsive to public investment. In this sense, decentralization is both necessary and sufficient to assure first best outcomes in the presence of intergenerational spillovers. At a formal level, this paper provides a Tiebout theorem (equilibrium exists and is first best) as well as a Second Welfare Theorem for a DLPG economy.

Our analysis also has interesting implications for the impact of housing price booms and busts. While competition between jurisdictions pins down the relative price between different locations, it says nothing about what the absolute prices must be. Thus, we show that both booms and busts in housing prices can take place without affecting the level, and in particular, the efficient provision of DLPG in jurisdictions. The effect of such booms and busts is simply to redistribute wealth across generations. However, if the absolute price level of housing gets either too high or too low (compared to income), it can become impossible to support the efficient DLPG levels through prices and the First Welfare Theorem fails. Although we argue that prices seen in the real world are not likely to leave the range that supports efficient outcomes, it is interesting that both relative and absolute prices play a role in achieving socially optimal policies.

2 The Model

Consider a simple finite-horizon, overlapping generations (OLG) economy with one private consumption good, c, and one DLPG, G. The DLPG is provided by a set of local jurisdictions indexed $j \in \{1, \ldots, J\} \equiv \mathcal{J}$. Each jurisdiction contain L plots of indivisible land.[1] We are abstracting from heterogeneous landscape and agglomerative externalities because adding such would increase analytic complexity significantly.

Time is indexed by $t \in \{1, \ldots, T\} \equiv \mathcal{T}$. In each "ordinary" period, $t \in \{2, \ldots, T-1\} \equiv \mathcal{T}^O$, a generation of 2-period lived young agents, indexed by $i \in \{1, \ldots, I\} \equiv \mathcal{I}$, where $I = J \times L$, and endowed with ω units of private good, is born, and a similar set of two-period lived old agents sell their land and die. On the other hand, in period $t = 1$, there exists a set of old agents with "time" index 0 (referred to as the "initial old") endowed with land only, and in period $t = T$, a final cohort of young agents (referred to as the "terminal young") are born endowed with ω units of private good but who live only for one period and then die.

The benefits of adopting this finite-horizon OLG framework are three fold. First, it avoids the typical "transfer from infinity" problem often seen in infinite horizon models (cf. Shell 1971), thus permitting us to use quasi-linear utility functions which allow for great analytic tractability in characterizing the willingness to pay for the public good (cf. Bergstrom and Cornes 1983). Second, it ensures the validity of using the conventional definition of Pareto optimality without requiring modifications such as forward-looking Pareto optimality.[2] Finally, it allows us to compare our results to those obtained in dynamics models of DLPG by Wildasin and Wilson (1996) and Sprunger and Wilson (1998) on an equal footing.

There is no storage technology for the private good and so the total social endowment in any given period must be divided between investment in DLPG and private good consumption for the young and old agents currently alive.

All agents except the initial old and the terminal young are identical and receive utility from consuming both private good and DLPG in the first period of their lives, but from private good alone obtained from selling their land in the second period. Thus, all agents

[1] This approach to indivisible land follows Fujita (1985), Dunz (1985) and Nechyba (1996). See also McCallum (1983), Wang (1987), and Glomm (1992) for introducing land into OLG models.

[2] We shall return to both the transfer from infinity and forward looking Pareto optimality issues in Section 8. We will also discuss the difficulty associated with infinite horizon OLG structure and the generality of some of our key results.

born between periods 1 and $T-1$ have the following quasi-linear utility functions:

$$U(c_{t,t}, c_{t,t+1}, G_t) = c_{t,t} + \beta c_{t,t+1} + V(G_t) \quad \text{for} \quad t = 1, ..., T-1.$$

where V is a strictly increasing and strictly concave C^2 function, $\beta \in (0,1)$ is the exogenous discount factor, and $c_{t,t'}$, is the level of private good consumption for an agent born in period t but consumed in period $t' = t, t+1$. Agents of the initial old and terminal young cohorts have utility functions that account for the timing of their consumption:

$$\begin{aligned} U(c_{0,1}) &= \beta c_{0,1} \\ U(c_{T,T}, G_T) &= c_{T,T} + V(G_T). \end{aligned}$$

We denote additions (or subtractions, in some cases) to the DLPG stock by g and assume one unit of private good produces one unit of DLPG. We assume that DLPG requires one period to build and depreciates over time at rate $\delta \in (0,1)$. This implies that DLPG evolves according the following rule:

$$G_t = (1-\delta)(G_{t-1} + g_{t-1}) \text{ for } t = 2, \ldots, T,$$

with an exogenous initial level assumed to be identical across jurisdictions of $G_1 \geq 0$.

Young agents must buy a plot of land from old agents at prices p_t^j and thereafter enjoy the services of the DLPG level that is currently in place. They decide how much private good to consume and how much to add to the existing stock of the DLPG, which in turn, will be enjoyed by the next generation. In the next period, the now old agents sell their land to the newly born young agents, consume the proceeds, and leave the economy. Old agents do not consume DLPG.

A *feasible allocation* consists of $(\mathbf{c}, \mathbf{g}, \mathbf{G})$ where

$$\begin{aligned} \mathbf{c} &= (c_{0,1}^1, \ldots, c_{0,1}^I, \ldots, c_{t,t+1}^i, \ldots, c_{T-1,T}^1, \ldots, c_{T-1,T}^I, c_{1,1}^1, \ldots, c_{1,1}^I, \ldots, c_{t,t}^i, \ldots, c_{T,T}^1, \ldots, c_{T,T}^I) \\ \mathbf{g} &= (g_1^1, \ldots, g_1^J, \ldots, g_t^j, \ldots, g_T^J, \ldots, g_T^J) \\ \mathbf{G} &= (G_1^1, \ldots, G_1^J, \ldots, G_t^j, \ldots, G_T^J, \ldots, G_T^J), \end{aligned}$$

such that:

$$I\omega = \sum_{i \in \mathcal{I}} c_{t-1,t}^i + \sum_{i \in \mathcal{I}} c_{t,t}^i + \sum_{j \in \mathcal{J}} g_t^j \text{ for } t \in \mathcal{T} \tag{1}$$

$$G_t^j = (1-\delta)(G_{t-1}^j + g_{t-1}^j) \text{ for } t = 2, \ldots, T \text{ and } G_1^j = G_1, \text{ for } j \in \mathcal{J}. \tag{2}$$

We will also include nonnegativity constraints on DLPG investment and private good consumption in some of the analysis below:

$$\begin{aligned} g_t^j &\geq 0 \ \text{ for } t \in \mathcal{T} \text{ and } j \in \mathcal{J} \\ c_{t,t}^i &\geq 0 \text{ and } c_{t-1,t}^i \geq 0 \ \text{ for } t \in \mathcal{T} \text{ and } i \in \mathcal{I}. \end{aligned}$$

The reader may notice that we omit to the describe how agents are allocated to jurisdictions in the definition of feasibility above. Since all agents are identical and there are exactly as many residential locations as agents each period, it will not make any difference. On the planner's side, any mapping of agents to locations gives the same welfare level. On the market side, all agents face the same prices and have the same initial allocations. Thus, equal treatment must prevail. We therefore omit this notational detail in the interest of simplicity.

To summarize, each period t evolves as follows:

Stage 1: Young agents are born with a private good endowment of ω, choose a jurisdiction, j in which to live, purchase a parcel of land from old agents, and enjoy the services of the current DLPG stock G_t^j that they inherit from the previous generation. Simultaneously, old agents sell their land at equilibrium prices, consume the proceeds as $c_{t-1,t}$ and leave the economy. Agents who are old in period 1 simply sell their land and consume the proceeds.

Stage 2: Young agents in each jurisdiction participate in a majority vote over how much to add to (and in some cases, subtract from) the DLPG stock for the next generation.[3] We denote this investment by g_t^j and assume the cost is equally shared over all the agents in the jurisdiction. Note that the investment of the young generation does not effect the level of DLPG that they, themselves, enjoy, but only the levels that are inherited by the young in the next period.[4]

Stage 3: Young agents consume $c_{t,t} = \omega - p_t^j - \frac{g_t^j}{L}$, where p_t^j is the price of a plot of land in jurisdiction j in period t. This is the amount of private good that remains from their endowment after buying land and paying for their share of the new investment in the DLPG stock for future generations.

[3]When the young born in t vote on how much g_t to add, this g_t adds to G_{t+1}, which is to be enjoyed by the young born in $t+1$ and is totally irrelevant to the old in t. Thus, in the interest of the current paper, we are excluding the old from voting.

[4]Stage 2 is equivalent to a notion defined later at a more formal level we call "political equilibrium".

Stage 4: The current DLPG stock depreciates at a rate δ and generation $t+1$ inherits $(1-\delta)(G_t + g_t)$.

3 The Planner's Problem

We take the planner's objective to be maximizing the sum of the discounted utilities of all agents over all periods. We adopt this Benthamite welfare function without considering inequality aversion because it is linked well to Pareto optimality as elaborated in Negishi (1960). Given the concavity of V and the symmetry of agents and jurisdictions, this is equivalent to maximizing the sum of utilities of a representative agent from each period at an equal treatment allocation. Thus, the planner solves the following optimization problem:

$$\max_{c_{0,1},c_{1,1},\ldots,c_{T-1,T},c_{T,T},g_1,\ldots,g_T,G_2,\ldots,G_T} W \equiv \sum_{t=0}^{T} \beta^{t-1} U_t \tag{3}$$

subject to

$$\begin{aligned} \omega &= c_{t-1,t} + c_{t,t} + \frac{g_t}{L} \text{ for } t \in \mathcal{T} \\ G_t &= (1-\delta)(G_{t-1} + g_{t-1}) \text{ for } t = 2, \ldots, T \\ g_t &\geq 0 \text{ for } t \in \mathcal{T} \\ L\omega - g_t &\geq 0 \text{ for } t \in \mathcal{T} \end{aligned}$$

where

$$\begin{aligned} U_0 &= \beta c_{0,1} \\ U_t &= c_{t,t} + \beta c_{t,t+1} + V(G_t) \text{ for } t = 1, \ldots, T-1 \\ U_T &= c_{T,T} + V(G_T). \end{aligned}$$

The nonnegativity constraints given in (3) imply that DLPG investment is irreversible and that agents must consume nonnegative levels of private good. The problem becomes much cleaner if we relax these constraints or, more precisely, assume that *the nonnegativity constraints are removed*. It will be useful to do so at certain points below to compare the results of the finite time case to the those of the infinite horizon case and to show the role that these constraints play in both the optimal and market outcomes in the economy.

To solve (3), we can utilize the quasi-linear property of utility functions to set up the following Lagrangian:

$$\begin{aligned}\max_{g_1,\ldots,g_T,G_2,\ldots,G_T} W &= \sum_{t=1}^{T}\beta^{t-1}\left(w-\frac{g_t}{L}+V(G_t)\right) \\ &+\sum_{t=1}^{T}\beta^{t-1}\lambda_t\left((1-\delta)\left(G_{t-1}+g_{t-1}\right)-G_t\right) \\ &+\sum_{t=1}^{T}\beta^{t-1}\theta_t g_t+\sum_{t=1}^{T}\beta^{t-1}\phi_t(L\omega-g_t).\end{aligned} \tag{4}$$

where λ_t, θ_t and ϕ_t are the respective Lagrangian multipliers associated with (2) and the two sets of nonnegativity constraints.

Denote the solution to the planner's problem as g_t^* for $t = 1, \ldots, T-1$ and G_t^* for $t = 2, \ldots, T$. We then solve (3) using (4) to derive the socially optimal steady-state level of DLPG, G_{ss}, and the socially optimal steady-state value of DLPG investment, g_{ss}.

Lemma 1. *The socially optimal steady-state level of DLPG is determined by:*

$$V'(G_{ss}) = \frac{1}{\beta(1-\delta)L} - \frac{1}{L} \tag{5}$$

and the socially optimal steady-state value of DLPG investment g_{ss} by:

$$g_{ss} = \frac{\delta G_{ss}}{1-\delta}. \tag{6}$$

Proof. All proofs are relegated to the Appendix. ■

We are now able to provide a full characterization of the solution to the planner's problem.

Theorem 1. *Assume $g_{ss} < L\omega$, and $G_{ss} > G_1$. Then the socially optimal levels of DLPG relate to the socially optimal steady-state level in the following manner:*

$$\beta\left(V'(G_{ss}) - V'(G_t)\right) = \frac{\theta_{t-1}}{1-\delta} - \beta\theta_t - \frac{\phi_{t-1}}{1-\delta} + \beta\phi_t \text{ for } t = 1, \ldots, T-1. \tag{7}$$

Moreover, the solution to the planner's problem is the following:

(i) *$g_t^* = L\omega$ from $t = 1$ to some t' (note that t' may equal 1 or $T-1$)*

(ii) *$L\omega > g_{t'+1}^* \geq 0$*

(iii) *$g_t^* = g_{ss}$ for period $t'+2$ to some period $t'' \geq t'$ or $g_{t'+2}^* = 0$*

(iv) $g_t^* = 0$ *for period* $t'' + 1$ *to* T.

What Theorem 1 says in essence is that the socially optimal plan is to start by investing the entire endowment of private good until the steady-state level of DLPG is reached, maintain this level by investing g_{ss} for the next interval of periods, but at some point in time, stop investing entirely and let the DLPGs depreciate until the final period.[5]

As a thought experiment, suppose that we removed the nonnegativity constraints. This means that agents are assumed to be able to consume negative levels of private good (as is typical in economies with quasi-linear utility functions such as ours). It also implies that investments in DLPG are reversible, that is, g_t^j is allowed to be less than zero. Given the timing of consumption, if a generation decides to be net-negative investors in DLPGs it does not affect the level of DLPG that they themselves enjoy. Increments and decrements to the stock take place after the services of the existing stock have been experienced and so only affect future generations. We will require that $g_t^j \geq -G_t^j$ so that the DLPG does not itself become negative (for which an economic interpretation would be difficult to imagine). It will turn out that we will find an interior optimum, and so we will not need to impose this as an additional Kuhn-Tucker condition. The planner's solution can now be characterized as follows:

Theorem 2. *Suppose that the nonnegativity constraints on DLPG investments and private good consumption are removed. Then the solution to the planner's problem becomes:* $g_1^* = \frac{G_{ss}-(1-\delta)G_1}{1-\delta}$, $g_t^* = g_{ss} = \frac{\delta G_{ss}}{1-\delta}$ *and* $G_t^* = G_{ss}$ *for* $t \in \mathcal{T}^O$, *and* $g_T^* = -G_T^* = -G_{ss}$.

Theorem 2 says that if the non-negativity constraints are removed, the planner invests enough in period 1 to get to the optimal steady state DLPG level immediately in period 2, then over all of the following ordinary periods $t \in \mathcal{T}^O$, invests enough to keep the DLPG stock at the optimal steady state, and then in the final period T cannibalizes the DLPG stock by choosing the optimal largest feasible negative investment level $g_T^* = -G_{ss}$. We will show

[5]Note that depending on the economic parameters and the size of T, the first or the second interval could be degenerate. However, if T is large enough and $g_{ss} < L\omega$ (which ensures the steady state is feasible without violating the non-negativity constraints), then both will be non-degenerate. Also note that, provided that T is large enough to make the second (steady state) interval non-degenerate, the number of periods of decumulation (that is, the length of the interval from $t'' + 1$ to T) is independent of T. Finally, note that there may or may not be one transitional period between maximal investment and the steady state interval when investment is something positive, but less than the endowment.

below that this solution to our finite horizon OLG model mimics the infinite horizon steady-state equilibrium in all the ordinary periods.

4 Dynamic Tiebout Equilibrium

In our economy, agents choose where to live and then vote over how much to add the current stock of DLPG. The price system will affect both of these and so it is worth spending some time discussing it. At a formal level, the price system specifies the cost of housing for each period and for every possible level of DLPG in each jurisdiction. We denote prices in each period as follows:

$$p_t(G_t) = (p_t^1(G_t), \ldots, p_t^J(G_t))$$

where $G_t = (G_t^1, \ldots, G_t^J)$, and a price system by:

$$\mathbf{p} = (p_1(G_1), \ldots, p_T(G_t)).$$

Note that an agent of generation t must consider prices for both periods he is alive. First, he must compare both the level of inherited DLPG and the cost of buying a house under period t prices across jurisdictions to make an optimal location choice. Second, he must consider the impact on period $t+1$ prices (when he will try to sell his house) when choosing a level of public investment, g_t^j, to add to the current DLPG stock. In other words, an agent must anticipate the effect of public investment, *both in his own community and in others*, on his property values. We will see below that without further constraints, commonly held beliefs among agents about the relationship of public good levels to property values can generate a wide variety of equilibria. One of the contributions of this paper will be to show that a simple economically motivated refinement gets rid of all socially suboptimal outcomes.

The reader may object that we are constraining the price of housing to depend only upon the current state of DLPG by specifying this form. This excludes the possibility that prices might depend on the history of DLPG levels or anticipations of future levels. We have two defenses. First, from an economic standpoint, it really should not matter how the current state evolved. Agents should be indifferent between jurisdictions if they have the exact same DLPG levels. Future levels are determined by future generations, so the current generation has neither certain knowledge of nor any degree of control over what they might be (although they may speculate that unborn agents will choose the optimal path). Second, we will demonstrate in the next section that given "free mobility", defined below, at least relative prices between jurisdictions must depend only on the current state of DLPG. Absolute prices, however, are not pinned down even with this restriction. While this has some interesting implications, it also means that allowing prices to depend on either past or future states does not change the set of equilibrium allocations that prices will support. We therefore choose the more intuitive form for the price functions.

Now we turn to defining our equilibrium concept. A feasible allocation $(\mathbf{c}, \mathbf{g}, \mathbf{G})$ and a price system $\mathbf{p}$ constitute a *Dynamic Tiebout Equilibrium* (DTE) if the following two conditions are met:

> **Free Mobility** (FM): Given the prices and politics, the allocation assigns every agent i who is born in any given period $t \in \mathcal{T}$, to his favorite jurisdiction. Formally, for $t = 1, \ldots, T-1$, for all $i \in \mathcal{I}$ where agent i chooses to live in jurisdiction j, it holds for all $\bar{j} \in \mathcal{J}$ that:
>
> $$V(G_t^j)+\omega-p_t^j(G_t)-\frac{1}{L}g_t^j+\beta p_{t+1}^j(G_{t+1}) \geq V(G_t^{\bar{j}})+\omega-p_t^{\bar{j}}(G_t)-\frac{1}{L}g_t^{\bar{j}}+\beta p_{t+1}^{\bar{j}}(G_{t+1}).$$
>
> and for T and all $i \in \mathcal{I}$ where agent i chooses to live in jurisdiction j, it holds for all $\bar{j} \in \mathcal{J}$ that:
>
> $$V(G_T^j) + \omega - p_T^j(G_T) - \frac{1}{L}g_T^j \geq V(G_T^{\bar{j}}) + \omega - p_T^{\bar{j}}(G_t) - \frac{1}{L}g_T^{\bar{j}}.$$
>
> **Political Equilibrium** (PE): *Given the mapping of agents to jurisdictions and the price of housing, g_t^j arises as a political equilibrium. Formally, we require for $t = 1, \ldots, T-1$, for all $j \in \mathcal{J}$ and all $\bar{g}$,*
>
> $$\beta p_{t+1}^j(G_{t+1}) - \frac{g_t^j}{L} \geq \beta p_{t+1}^j(\delta(\bar{g} + G_t^j), G_{t+1}^{-j}) - \frac{\bar{g}}{L},$$
>
> *where $G_{t+1}^{-j} \equiv (G_{t+1}^1, \ldots, G_{t+1}^{j-1}, G_{t+1}^{j+1}, \ldots, G_{t+1}^J)$, and for T and all $j \in \mathcal{J}$, g_T^j is the lowest number that is feasible (either 0 or $-G_T$).*

The FM assumption requires that the housing market clears each period. Given that all agents are identical, this is equivalent to stating that the inequalities given in FM are in fact, equalities. The PE assumption requires that g_t^j is chosen such that the increment to utility from selling land in period $t+1$ less the decrement to utility from paying for additional DLPG in period t, is maximized. Since T is the terminal period, minimal investment is trivially optimal for period T. In a strict sense, PE is not needed as a separate condition since this is exactly what agents are required to do at stage 2 of each period. We think it is is useful to define these political actions formally in order to emphasize the role they play in establishing equilibrium. Note that since we only treat the case of identical agents, majority rule, the Condorcet winner, and unanimity are all equivalent to allowing a representative voter in each jurisdiction choose the level of public goods investment. We discuss generalizations below.

Unfortunately, FM and PE are not sufficient to guarantee that all DTE are Pareto optimal. The next Lemma shows that in general, there will exist price systems that support many nonoptimal equilibria.

Lemma 2. *Consider any arbitrarily chosen steady-state level of DLPG, $\bar{G}$. Suppose that the nonnegativity constraints on DLPG investments and private good consumption are removed. Then there exists a price system* $\mathbf{p}$ *such that for all* $j \in \mathcal{J}$

$$\bar{g}_t^j = \begin{cases} \frac{\bar{G}-(1-\delta)G_1}{1-\delta} & t = 1 \\ \frac{\delta\bar{G}}{1-\delta} & t \in \mathcal{T}^O \\ -G_T^j = -\bar{G} & t = T \end{cases}$$

and $\mathbf{p}$ *supports this plan and satisfies FM and PE.*

In effect, any commonly held beliefs about the effect of public investment on land prices that respect the differences in the attractiveness between jurisdictions are a self-fulfilling prophecy. This multiplicity is similar to the phenomena of sunspot equilibria in macroeconomics (see, for example, Cass and Shell 1983). Sunspots can arise in dynamic models with multiple stages if there are multiple equilibria in the spot markets. In this case, the equilibrium behavior on earlier stages might depend on the selection of the continuation equilibrium. In our case, the problem is that while equilibria exist, almost all of them are inefficient. For example, if all agents alive at some time t believe that putting a subway in every city and town, no matter how small, will result in the cost of subway construction plus $\$1,000,000$ being added to the price of every house in the country, then all locations will choose to build the subway. In period $t+1$, the housing markets will clear. Thus, the price system induces agents in period t to over-invest in public good. Of course, other equilibrium price systems exist that cause period t agents to under-invest public goods, or to consume them efficiently. The point is that there is no reason to expect that the market should induce optimal investment decisions in general.

A closer look at these sunspots, however, shows that they depend on price expectations that may not be very plausible. Sunspots arise only if the choice of DLPG that a jurisdiction makes affects the price in every other jurisdiction. In our example above, it only takes the failure of one small town to build a subway to cause every house in the country to lose $\$1,000,000$ in "extra" value. This seems highly unlikely when the number of jurisdictions is large and each jurisdiction is a tiny part of the economy. It turns out that making the small refinement on the formation of price expectations that this observation suggests is enough to eliminate all of the implausible and inefficient equilibria. Formally, the assumption we make is the following:

> **Small Jurisdictions** (SJ): *Suppose for any $t \in \mathcal{T}$, G_t and $\bar{G}_t$ differ only in the amount of DLPG in single jurisdiction $j \in \mathcal{J}$. Then there exists a jurisdiction $\bar{j} \neq j$ such that $p_t^{\bar{j}}(G_t) = p_t^{\bar{j}}(\bar{G}_t)$.*

This is a fairly weak assumption. All it says is that if any single jurisdiction changes its DLPG level, there is at least one other jurisdiction in which land prices are unaffected. For example, this implies that if San Diego builds a new airport, the price of housing in Boston should not change, or at any rate, there should be at least one city somewhere in the world that is not affected.

This refinement dramatically reduces the set of allocations that can be supported as DTE. Under SJ we will also be able to prove First and Second Welfare Theorems. We begin by characterizing of the set of DTE under SJ.

Theorem 3. *Let $(\mathbf{c}, \mathbf{g}, \mathbf{G})$ and $\mathbf{p}$ be a DTE for an economy satisfying SJ. Suppose that the nonnegativity constraints on DLPG investments and private good consumption are removed. Then for all $j \in \mathcal{J}$, $g_1^j = \frac{G_{ss}-(1-\delta)G_1}{1-\delta}$, $g_t^j = g_{ss} = \frac{\delta G_{ss}}{1-\delta}$, $G_t^j = G_{ss}$ for $t \in \mathcal{T}^O$, and $g_T^j = -G_T^j = -G_{ss}$.*

In proving Theorem 3, we show that PE and FM imply the following key necessary condition for equilibrium prices for all $t \in \mathcal{T}$, and all $j, \bar{j} \in \mathcal{J}$, which we refer to as the *relative price condition*:

$$p_t^j(G_t) - p_t^{\bar{j}}(G_t) = \left(V(G_t^j) - V(G_t^{\bar{j}})\right) + \frac{1}{L}\left(G_t^j - G_t^{\bar{j}}\right). \tag{8}$$

This shows that even if we included the entire history of DLPG levels in each period and every jurisdiction as arguments in the price function, the only thing that could have an effect on the differences in price between jurisdictions in any period t is the current state of DLPG. Thus, the relative price of jurisdictions in a given period depends only on the current state and is pinned by PE and FM.

5 Welfare Theorems without Nonnegativity Constraints

In this section, we show welfare theorems for DTE. Our first step is to show that the set of planner's solutions is identical to the set of Pareto optimal allocations.

Lemma 3. *Assume $G_{ss} \geq G_1$ and suppose that the nonnegativity constraints on DLPG investments and private good consumption are removed. Then a feasible allocation, $(\mathbf{c}, \mathbf{g}, \mathbf{G})$, is Pareto efficient if and only if it is also a solution to the planner's problem.*

A First Welfare Theorem follows almost immediately.

Theorem 4. (First Welfare Theorem) *Suppose that the nonnegativity constraints on DLPG investments and private good consumption are removed. Then if $(\mathbf{c}, \mathbf{g}, \mathbf{G})$ and $\mathbf{p}$ are a DTE for an economy satisfying SJ, $(\mathbf{c}, \mathbf{g}, \mathbf{G})$ must also be Pareto optimal.*

A Second Welfare Theorem also holds. In fact, it is possible to implement any equal treatment Pareto optimal allocation solely through the price system without redistributing endowments at all. By equal treatment we mean that agents in a given period get identical levels of private good, though this level may differ across periods. Formally,

> **Equal Treatment in Private Goods** (ET): *A feasible allocation $(\mathbf{c}, \mathbf{g}, \mathbf{G})$ satisfies ET if for all $t \in \mathcal{T}$, and all $i, \bar{\imath} \in \mathcal{I}$, $c^i_{t-1,t} = c^{\bar{\imath}}_{t-1,t} \equiv c_{t-1,t}$ and $c^i_{t,t} = c^{\bar{\imath}}_{t,t} \equiv c_{t,t}$.*

Theorem 5. (Second Welfare Theorem) *Suppose that the nonnegativity constraints on DLPG investments and private good consumption are removed and that a feasible allocation $(\mathbf{c}, \mathbf{g}, \mathbf{G})$ is Pareto optimal and satisfies ET. Then there exists a price system $\mathbf{p}$ such that $(\mathbf{c}, \mathbf{g}, \mathbf{G})$ and $\mathbf{p}$ are a DTE.*

The prices that support these equilibria take the form:

$$p^j_t(G_t) = V(G^j_t) + \frac{G^j_t}{L} + K_t = V(G^j_t) + \frac{G^j_t}{L} + c_{t-1,t} - V(G_{ss}) - \frac{G_{ss}}{L} = c_{t-1,t}$$

for some $K_t \geq 0$. This implies that as long as $(\mathbf{c}, \mathbf{g}, \mathbf{G})$ satisfies the nonnegativity constraints, the supporting prices are nonnegative. If we are willing to relax this and allow negative prices, we would be imposing an economic assumption that says that old agents cannot walk away from housing with negative value and would therefore be forced to pay young agents take housing off their hands. In this case, however, we could support any feasible allocation $(\mathbf{c}, \mathbf{g}, \mathbf{G})$, equal treatment or not, with prices and a redistribution of initial allocations. To do so, we would need to add a set of individualized transfer constants K_t^i to each agent's endowments where

$$K_t^i = c_{t-1,t}^i - V(G_{ss}) - \frac{G_{ss}}{L},$$

and

$$p_t^j(G_t) = V(G_t^j) + \frac{G_t}{L}. \tag{9}$$

The same logic applied to prove Theorem 5 applies here. Making these transfers does not change the relative price equation and so agents would choose the Pareto optimal steady-state levels of DLPGs in each period. It is easy to verify that these individualized transfers both leave agents with the consumption levels specified in $\mathbf{c}$ and are feasible. The problem is that for some private good allocations, $\mathbf{c}$, the transfers might be so large that agents end up with negative endowments.

Therefore, if we are willing to allow endowments and prices to be negative, then all Pareto optimal allocations can be supported as DTE for some reallocation of endowments. If we require prices and endowments to be nonnegative, then the set of allocations that can be supported as DTE for some reallocation of endowments is larger than the set of equal treatment Pareto optimal allocations, but smaller then the entire set of Pareto optimal allocations.

The Second Welfare Theorem is also a constructive proof that equilibrium exists. Thus, the two Welfare Theorems together imply that DTE exists and is first best. This means that Tiebout's basic insight that if agents vote with their feet to choose tax/public good combinations, then the outcome will be first best carries over to overlapping generations economies with a DLPG (as least under the conditions above). Thus, we have a Dynamic Tiebout Theorem.

6 Optimality and Decentralization with Nonnegativity Constraints

Recall that Theorem 1 shows that an optimizing planner builds to G_{ss} as fast as he can by investing the entire endowment of private good until the steady-state level of DLPG is reached. He maintains this level by investing g_{ss} for the next interval of periods, but at some point in time, stops investing entirely and let the DLPG depreciate until the final period. In contrast, if we drop the nonnegativity constraints, the planner invests enough private good to build to G_{ss} in period 2. He maintains this level by investing g_{ss} the all the way until period T, and then has the agents born in period T consume all the remaining DLPGs through disinvestment. Thus, the only differences are in the build up and build down periods at the start and finish of time.

In this section, we will maintain the nonnegativity constraints and the requirement that all prices be nonnegative in order to explore the implications for decentralizing the planner's solution. The focus is on establishing a "normal range" of housing prices within which agents will have correct incentives for optimal investments in DLPG and on establishing price support for a socially optimal steady state DLPG when its level is not too high.

Theorem 6. *Suppose that the nonnegativity constraints on DLPG investments and private good consumption are maintained. Then we have the following:*

(i) *If* $(\mathbf{c}, \mathbf{g}, \mathbf{G})$ *and* $\mathbf{p}$ *constitute an interior DTE for an economy satisfying SJ,* $(\mathbf{c}, \mathbf{g}, \mathbf{G})$ *must also be Pareto optimal.*

(ii) *Very high or very low absolute price levels for housing in a DTE can result in suboptimal investment in DLPG, even at a steady state.*

(iii) *If the socially optimal steady state DLPG level is not too high, it can be support as a DTE.*

Proof of Theorem 6 **(to be moved to the Appendix later)**

Part (i) is trivial. This is because of the fact that if the investments in DLPG are strictly positive and the prices are such that the young still consume a strictly positive level of private good in the first period of their lives, then the proof of Theorem 4 would go through and hence a DTE is Pareto optimal.

To prove Parts (ii) and Part (iii), define γ such that $V'(\frac{1-\delta}{\delta}\gamma L\omega) = \frac{1}{\beta(1-\delta)L} - \frac{1}{L}$. It is sufficient to restrict our attention to the case where $\gamma \in (0, 1)$ so that $g_{ss} = \gamma L\omega$ is well defined. Suppose this steady state level of DLPG investment is socially optimal and reached in period t. Then, $c_{t,t} = (1-\gamma)\,\omega - p_t^j$ and hence, in order for $c_{t,t} \geq 0$, it must be that $p_t^j \leq (1-\gamma)\,\omega$. Moreover, with quasi-linear utility, the marginal rate of substitution between $c_{t,t}$ and $c_{t,t+1}$ is β. Thus, to induce the incentive to invest at the amount $g_{ss} = \gamma L\omega$, it must be that $p_{t+1}^j = c_{t,t+1} \geq \gamma\beta\omega$. Otherwise, an agent would be better off investing $g_t^j = 0$ and accepting a price $p_{t+1}^j = 0$ for their land in period $t+1$. Thus, for prices higher than

$(1-\gamma)\,\omega$ or lower than $\gamma\beta\omega$, the optimal steady state level of DLPG cannot be supported, which verifies Part (ii).

To proof Part (iii), notice that when γ is not too large, it is find supporting prices in the range $(\gamma\beta\omega, (1-\gamma)\,\omega)$. ■

Intuitively, absolute prices can force all agents to invest less when young, and consume more in old age than is socially optimal. If capital markets were perfect, it is possible that agents could borrow from the future to set things right. However, to the extent they are imperfect, we should expect to see lower levels of public investment the higher land prices become. Thus, it is not only relative prices that must be set correctly to induce optimal behavior, but also absolute prices. A strong housing market might very well starve the public sector for funds. On the contrary, low absolute prices make it impossible to give young agents a sufficient incentive to invest optimally. In our strict model, this causes relatively high private good consumption when young at the expense of consumption when they are old. If capital markets were perfect, young agents could save optimally, but this would not solve the DLPG investment problem. Saving by the young will not change absolute prices and thereby give them correct incentives to invest. All the agents who would benefit from investments in the next period are yet to be born, and so even in the presence of perfect capital markets, there is nothing these unborn agents can do to incentivize an increase in investment before they exist.

Remark. To further illustrate the required range of prices, consider the following examples.

1. (Very high absolute price levels for housing can result in suboptimal investment in DLPG) To see this, suppose that $g_{ss} = \frac{1}{3}L\omega$. Also suppose that the price of housing in each jurisdiction for each period is $p_t^j(G_{ss}) = \frac{3}{4}\omega$, and for other values of investment, satisfies the relative price condition. The most that agents born in period t would be able to individually invest in DLPG without violating the nonnegativity constraint is $\frac{1}{4}\omega$. Of course, since the relative price equation rewards investing as long as it generates more benefits than costs, agents would therefore invest until they hit the nonnegativity constraint (and so $g_t = \frac{1}{4}L\omega$). It follows that the higher the absolute prices level, the lower the level of DLPG that can be supported as a steady state.

2. (Very low absolute price levels for housing can also result in suboptimal investment in DLPG) To see this, suppose that $g_{ss} = \frac{2}{5}L\omega$ and the price of housing in jurisdiction j for each period t is $p_t^j(g_{ss}) = \frac{1}{5}\omega$ and satisfied the relative price condition otherwise. Prices are bounded below by zero, and so we have a situation very much like the one

described in Observation 1. The most that agents born in period t would be willing to invest in DLPGs is $\frac{\beta}{5}\omega$. Rather than investing more than this, they would be better off investing zero, consuming all the potential investment today, and accepting a zero price for land in the future.

We thus have a kind of Goldilocks situation. Prices can be too high or too low. If they are just right, then they may be able to support the optimal steady state. In places with high prices like New York City and San Francisco, we might expect to see young people being "house-poor" and voting against a large public sector because it would take a significant portion of the small disposable income that remains after housing costs. In places with low prices like Detroit or Rochester, young people might have relatively high disposal income, but find that investing in a city with low and falling property values not to be worthwhile.

We suspect that generally, that the "normal price" range for housing falls within this Goldilocks zone. In most places, property taxes are on the scale of .5% to 5% of property values. This means that we are a long way from hitting a nonnegativity constraint on the low price side. If we follow the rule of thumb that one can afford a house costing about four times gross income, then even at a 5% local property tax rate, DLPG investments are about 20% of income, well below the 50% sustainability cutoff. Within this Goldilocks range, absolute prices can shift up or down in ways that are either anticipated or unanticipated by current and future generations without reducing the efficiency of the current generation's DLPG investment choice. The only effect is that wealth is transferred between generations.

Before concluding the section, we shall provide some qualification on price supportable bound for a socially optimal steady state DLPG.

Remark. It is *never* possible to support an investment level higher than $g = \frac{1}{2}L\omega$ with prices in any jurisdiction for two consecutive periods. To see this, suppose that the price system induced agents in period t in jurisdiction j to invest $g_t^j = \frac{1}{2}L\omega$. Then price in period $t+1$ for jurisdiction j must be at least $\frac{1}{2}\beta\omega$ since this exactly compensates agents born in period t for their investment. If the price was lower than this, these agents would be better off investing nothing and accepting a price of zero for their land in period $t+1$. Thus $p_{t+1}^j \geq \frac{1}{2}\beta\omega$. But then agents born in period $t+1$ who live in jurisdiction j have less than half their endowment left over once they buy their land. As a result, they simply do not have enough private good left over to invest $\frac{1}{2}\omega$ without violating the nonnegativity constraint. An immediate implication is that, in general, we should not expect to be able to decentralize the planner's solution with prices if we do not allow them to be negative. At best, DLPG should accumulate at something less than half the rate that would be socially optimal in a free market equilibrium. This also means that the free market could never support a steady-state DLPG level such that $g_{ss} > \frac{1}{2}L\omega$.

In conclusion, we have shown that if we add the non-negativity constraints, a DTE, if not interior, may not always be Pareto optimal even at a steady state. We can support any socially optimal steady state DLPG level as a DTE if the steady state level of DLPG is not too high, requiring something less than half the endowment to be invested each period depending on the discount rate. As a result of the possibility of underinvestment in the DLPG, the build up to the steady state will take longer in a DTE than a social optimum.

7 Centralization versus Decentralization

In this section, we compare the performance of centralized and decentralized institutions in the presence of intergenerational spillovers. We also discuss how allowing agents to have heterogeneous preferences might affect the results. Previous studies of decentralization have emphasized the role of differences in the taste for public goods.[6] In these papers, decentralization is valuable because it allows agents to sort into jurisdictions populated by agents with similar tastes. Here we provide a new case for decentralization that is based *solely* on the capitalization effect.

The model of centralization we use is a straightforward variation of the decentralized one outlined in previous sections. The only difference is that the level of DLPG is chosen in a national election and is *identical* across jurisdictions.[7] Let G_t denote the common level of DLPG in each jurisdiction. Note that agents have identical tastes and so the level of DLPG that agents would like to consume is the same as in the decentralized case and is still unanimously agreed upon. If there is any difference in the outcome of the vote, it is because centralization has distorted the capitalization effect through the price system.

Since the DLPG levels are the same in each jurisdiction (and thus, per capital investment

[6] See Section 9 for a more complete discussion of this literature.

[7] Note that the results in this section would also hold if we had agents in many jurisdictions voting collectively for the national level of a pure public good like defense or research and development. To make direct comparison to the previous sections clear, however, we set this up as a kind of national vote over grants in aid to local governments to build a common specified level of DLPG such as city streets or school buildings in each.

is also the same) it is immediate that a price system $\mathbf{p}$ satisfies FM if and only if for all $t \in \mathcal{T}$, any $j, \bar{j} \in \mathcal{J}$, and any $G_t \in \Re^1_+$,

$$p^j_t(G_t, \ldots, G_t) = p^{\bar{j}}_t(G_t, \ldots, G_t).$$

Thus, FM has no bite since we can never have price or DLPG level differences between jurisdictions within a given period. The SJ assumption has no bite either for the same reason. There is no possibility of agents in a single jurisdiction contemplating the effect on local land prices of increasing or decreasing DLPG provision within their own city alone. SJ no longer makes any sense under centralized government.

This implies that arbitrary sunspots can arise, and anything can be an DTE under centralization.

Theorem 7. *Suppose that the nonnegativity constraints on DLPG investments and private good consumption are removed but that agents vote in a central election over a common level investment for all jurisdictions each period. Consider an arbitrary path of DLPG levels for each period:* $(\bar{G}_2, \ldots, \bar{G}_T) \in \Re^{T-1}_+$ *(not necessarily a steady state). Then there exists a price system* $\mathbf{p}$ *that satisfies PE, FM which supports this path.*

Notice that sunspots can arise if local land prices depend on the national level of DLPG. But why should this be so? Agents have a taste for DLPG, but their taste is not based on how the DLPG interacts with land. With decentralization, agents bid up the price of jurisdictions with higher levels of DLPG because they want *access* to this DLPG. With centralization, access is not tied to land because the DLPG is provided at the national level. Thus, the only economic force behind these sunspots are self-fulfilling beliefs. Since the plots of land are identical in every jurisdiction *and* DLPG levels are also identical by assumption, it might make sense to remove the dependence of land prices on centrally provided DLPG. Formally,

> **No Sunspots** (NS): *For all* $t \in \mathcal{T}$, *all* $j \in \mathcal{J}$, *and all* $G_t \in \Re^1_+$, *prices take the form:* $p^j_t(G_t, \ldots, G_t) = K_t$.

The next Theorem shows that although the no-sunspot refinement gets rid of the problem of multiple equilibria, the one that remains is inefficient.

Theorem 8. *Suppose that the nonnegativity constraints on DLPG investments and private good consumption are removed but that agents vote in a central election over common level investment for all jurisdictions each period. Then any price system* $\mathbf{p}$ *that satisfies PE, FM and NS results in zero provision of DLPG in each period.*

8 On the Infinite Horizon Case

We have elaborated on the value of using finite horizon OLG framework for establishing welfare theorems and characterizing the equilibrium and the optimal allocation of DLPG investments over time. In this section, we examine the robustness of our results and the extension of our benchmark economy to an infinite horizon OLG setting.

Specifically, we maintain the assumption that agents live two periods, but note that the "terminal young" no longer exist since time is now unbounded. Thus, the utility functions are now:

$$U(c_{0,1}) = \beta c_{0,1}; \quad U(c_{t,t}, c_{t,t+1}, G_t) = c_{t,t} + \beta c_{t,t+1} + V(G_t) \quad \text{for} \quad t \geq 1.$$

We begin by reexamining the planner's problem, where the social welfare function is: $W \equiv \sum_{t=0}^{\infty} \beta^{t-1} U_t$. Since resources are not growing and V is strictly concave, there must exist $\bar{U} < \infty$ such that $U_t \leq \bar{U}$ $\forall t$. Given $\beta \in (0,1)$, W is thereby bounded. Under quasi-linear preferences, one may again write the planner's problem parallel to (4) with T replaced by ∞. Bearing this in mind, one may then work through the proof to re-establish Lemma 1. In particular, the socially optimal steady-state level of DLPG G_{ss} is still determined by (5) and the socially optimal steady-state value of DLPG investment g_{ss} by (6). Next, the proof of Theorem 1 can also go through, except there is no terminal period. This implies that while part (i) and part (ii) of Theorem 1 continue to hold true, part (iii) and part (iv) should be modified as:

Theorem $1'$. *Assume $g_{ss} < L\omega$, and $G_{ss} > G_1$. Then the socially optimal levels of DLPG relate to the socially optimal steady-state level in the following manner:*

$$\beta \left(V'(G_{ss}) - V'(G_t)\right) = \frac{\theta_{t-1}}{1-\delta} - \beta\theta_t - \frac{\phi_{t-1}}{1-\delta} + \beta\phi_t \text{ for } t \geq 1.$$

Moreover, the solution to the planner's problem is the following:

(i) $g_t^* = L\omega$ *from* $t = 1$ *to some* $t' \geq 1$

(ii) $L\omega > g_{t'+1}^* \geq 0$

(iii) $g_t^* = g_{ss}$ *for period* $t' + 2$ *and onward.*

Similarly, Theorem 2 should be modified as:

Theorem 2$'$. *Suppose that the nonnegativity constraints on DLPG investments and private good consumption are removed. Then the solution to the planner's problem becomes:* $g_1^* = \frac{G_{ss}-(1-\delta)G_1}{1-\delta}$, $g_t^* = g_{ss} = \frac{\delta G_{ss}}{1-\delta}$ *and* $G_t^* = G_{ss}$ *for* $t \geq 2$.

Additionally, Lemma 3 also go through without any problem. That is, Pareto optimality and social optimality are equivalent when the nonnegativity constraints are removed.

Recall that we adopt the Benthamite welfare function because it is linked well to Pareto optimality. However, this argument is no longer valid as a result of the "transfer from infinity" problem when there is no final period. Specifically, with linear utility in the private good, a small arbitrary transfer from the old to the young is always Pareto improving. To fix this, we first cut off this transfer channel via the private good by assuming that all agents consume the private good only when they are old. That is, the utility functions are modified to be:

$$U(c_{0,1}) = \beta c_{0,1}; \quad U(c^j_{t,t+1}, G^j_t) = \beta c^j_{t,t+1} + V\left(G^j_t\right) \quad \text{for} \quad t \geq 1.$$

Of course, such a transfer may also be via the trade-off between the private good and the DLPG. To prevent this from happening, we further impose a Gaussian Curvature condition (GC) on the utility function following Balasko and Shell (1980). Basically, this curvature condition requires sufficiently strong diminishing marginal rates of substitution ($MRS = \frac{V'(G^j_t)}{\beta}$), which ensures that for some distant future generations (large t), such a transfer leads to an inferior allocation and hence a Pareto suboptimal outcome. Despite the above modification, quasi-linear preferences still enable us to write the planner's problem parallel to (4) with T replaced by ∞. As a consequence, Lemmas 1 and 3 as well as Theorems $1'$ and $2'$ all remain valid.

Next, we turn to reexamining the dynamic Tiebout equilibrium. By adopting the assumption that all agents consume the private good only when they are old, the decentralized optimization problem shall be modified as well. To be more specific, note that agents are endowed with ω units of private good only when young but consume it only when old. This implies a "forced saving" via the investment in the DLPG. It is not the case with agents consuming the private good in both periods and with the private good consumption when young and when old being perfect substitutes. Thus, to be consistent with the original economy, we allow for perfect intertemporal borrowing and lending so that agents can fully optimize between the private good and the DLPG. Without loss of generality, let such borrowing/lending be through a risk-free bond b_t at a market interest r_{t+1} over the periods from t to $t+1$. So the budget constraints become:

$$\begin{aligned} b_t^j + p_t^j &= \omega - \frac{g_t^j}{L} \\ c_{t,t+1}^j &= (1 + r_{t+1}) b_t^j + p_{t+1}^j, \end{aligned}$$

and the lifetime budget constraint is given by:

$$p_t^j + \frac{c_{t,t+1}^j}{1 + r_{t+1}} = y_t^j = \omega - \frac{g_t^j}{L} + \frac{p_{t+1}^j}{1 + r_{t+1}}. \tag{10}$$

By substituting out $c_{t,t+1}^j$ and the DLPG evolution condition, the decentralized optimization problem facing an agent with an inherited G_t^j can then be written as:

$$\max_{g_t^j} V\left(G_t^j\right) + \beta\left(1 + r_{t+1}\right)\left[\omega - \frac{g_t^j}{L} + \frac{p_{t+1}^j((1-\delta)G_t^j + g_t^j)}{1 + r_{t+1}} - p_t^j\left(G_t^j\right)\right] \tag{11}$$

Of course, in equilibrium, FM and PE (taking T to ∞) must hold. Since all net borrowing/lending across all agents in all jurisdictions must sum to zero in equilibrium, the resource constraint is simply:

$$I\omega = \sum_{i \in \mathcal{I}} c_{t-1,t}^i + \sum_{j \in \mathcal{J}} g_t^j \qquad \text{for} \qquad t \geq 1. \tag{12}$$

It is not surprising that the proof of Lemma 2 and Theorems 3-5 no longer works because backward induction cannot be used without a finite terminal date T. Nonetheless, we are able to establish decentralized price support for centralized optimal allocation under ET. We will show that this supporting price is consistent with the DTE price derived under the finite horizon setting.

Theorem 9. *There exists a price system* $\mathbf{p}$ *that supports the solution to the planner's problem satisfying ET. The supporting prices take the form:*

$$p_t^j(G_t) = V(G_t^j) + \frac{G_t^j}{L} + K_t \tag{13}$$

for some $K_t \geq 0$ *for all* $j \in \mathcal{J}$. *When private good consumption is interior, the supporting prices are positive.*

Proof of Theorem 9 **(to be moved to the Appendix later)**

Problem (11) yields the following First Order Condition: for $g_t^j \in (0, L\omega)$,

$$\frac{dp_t^j(G_t^j)}{dG_t^j} = \frac{1 + r_{t+1}}{(1-\delta)L}.$$

To establish price support for the solution to the social planner's problem, first take $r_{t+1} = \frac{1}{\beta} - 1$, i.e., the market rate of interest is set as the subjective rate of time preference. Thus, the condition above becomes:

$$\frac{dp_t^j(G_t^j)}{dG_t^j} = \frac{1}{\beta(1-\delta)L}. \tag{14}$$

To verify the price support (13), straightforward differentiation leads to:

$$\frac{dp_t^j(G_t^j)}{dG_t^j} = V^{'}(G_t^j) + \frac{1}{L}$$

By Lemma 1 and Theorem 1′, the solution to the social planner's problem satisfies (5) for period $t' + 2$ and onward. So the above equation reduces to (14), which is consistent with the First Order Condition of the decentralized problem (11). Since (5) holds for all jurisdictions, the level of $G_t^j = G_{ss}$ and hence $g_t^j = g_{ss}$ must be identical for all $j \in \mathcal{J}$. Also by ET, $c_{t-1,t}^i = c_{t-1,t}$ for all $i \in \mathcal{I}$. Thus, the supporting prices must have identical K_t for all $j \in \mathcal{J}$. One may then go backward from period $t' + 1$ to period 1 as in the finite horizon case, which completes the proof. ■

Thus, while the stronger Welfare Theorems results (specifically, First Welfare Theorem) in the finite horizon setting cannot be reproduced here, the main properties regarding optimal allocation and supporting prices remain valid even under the infinite horizon setup. Also, since K_t is the same across all jurisdictions, the relative price condition (8) continues to hold true. Accordingly, the difference in the supporting price of any two jurisdictions in a given period depends only on the current state of DLPG levels in these jurisdictions.

It is noted that to restore the version of First Welfare Theorem in the finite horizon model is far from obvious even after removing the transfer from infinity problem. On the one hand, one may need to adopt the concept of forward-looking Pareto optimum, defined as a feasible allocation that cannot be dominated by any other feasible allocation for agents born after some time $t \geq \bar{t}$, where $\bar{t}$ is finite (cf. Wang 1987, 1993, and papers cited therein). This may help by focusing on the optimality of decentralized allocation and pricing closer to the steady state. On the other hand, one may need to use a much stronger refinement than SJ, which may not be desirable to impose. Despite these difficulties, the source of suboptimality of a DTE is unchanged: in the absence of the transfer from infinity problem, the inefficiency is due to the phenomena of sunspot equilibria with which the decentralized market need not induce optimal investments in the DLPG.

Remark. It is interesting to note that to be consistent with the original economy, we have introduced perfect intertemporal borrowing and lending so that agents can fully optimize between the private good and the DLPG. In so doing, we have incorporated another form of durable good, a risk-free bond, into the infinite horizon model. Nonetheless, the theorem above indicates that our findings concerning optimal allocation and supporting prices are all unchanged. Thus, while the durability of the LPG is required, the assumption that the DLPG is the sole store of value in the benchmark model is not essential for the properties established.

9 On Heterogeneous Agents

One may inquire what happens if agents have heterogeneous tastes for the DLPG. With heterogeneous agents, the model would become much more complicated. For example, taste heterogeneity can lead to very different equilibrium outcomes from our homogeneous agent benchmark.

The case we build in this paper for decentralization is based purely on the capitalization effect. In this section, we shall focus on examining whether this insight would remain with heterogeneous agents. To explore this, we consider the following economy, where agents have a complete order of their degree of preference for DLPGs:

$$\begin{aligned} U^i(c_{0,1}) &= \beta c_{0,1} \\ U^i(c_{t,t}, c_{t,t+1}, G_t^j) &= c_{t,t} + \beta c_{t,t+1} + \rho(i) V(G_t^j) \text{ for } \quad t = 1, ..., T-1 \\ U^i(c_{T,T}, G_T) &= c_{T,T} + \rho(i) V(G_T). \end{aligned}$$

where $\rho(i) > \rho(\bar{\imath})$ if $i > \bar{\imath}$, $i, \bar{\imath} \in \mathcal{I}$ and i is called to have higher taste for DLPG than $\bar{\imath}$.

An immediate task is to modify the concept of DTE. Since the presence of taste heterogeneity generates disagreement about the optimal level of DLPG, the political equilibrium now becomes nontrivial. We shall assume that the investment in DLPG is determined by the median voter. Define $\mathcal{I}^j \equiv \{(j-1)L+1, \ldots, jL\}$. A DTE is called *completely stratified* if agent $i \in \mathcal{I}^j$ resides in j with G^j increasing in j for all $j \in \mathcal{J}$. We begin by showing the following.

Theorem 10. *Suppose that the nonnegativity constraints on DLPG investments and private good consumption are removed. Then a DTE with DLPG investments determined by median voters is completely stratified.*

Proof of Theorem 10 **(to be moved to the Appendix later)**

Suppose not. Consider any other distribution of agents across jurisdictions. Since by complete preference ordering no two median voters are identical. Regardless of who the median voter in each jurisdiction happens to be, for any two jurisdictions $j, \bar{j} \in \mathcal{J}$, $j \neq \bar{j}$, we must have $G^j \neq G^{\bar{j}}$. This means it is possible to order the jurisdictions from lowest to highest provision of DLPG. Suppose that agents were not stratified by taste in a DTE. Then there must be at least one pair of median voters $(i, \bar{\imath})$ with $i \in \mathcal{I}^j$ and $\bar{\imath} \in \mathcal{I}^{\bar{j}}$ such that $\rho(i) > \rho(\bar{\imath})$ and $G^j < G^{\bar{j}}$. But this implies that the higher taste voter i would be willing to pay more than the lower taste voter $\bar{\imath}$ to own the land in jurisdiction $\bar{j}$ with a higher provision of DLPG. By FM, i would have chosen to reside in $\bar{j}$ regardless of prices, which contradicts to the concept of DTE. ■

Thus, in a DTE, agents are completely stratified, sorting into jurisdictions based on their tastes for DLPG in a stratified way with the agents who like DLPG the least taste (those in $\mathcal{I}^1$) together in jurisdiction 1, agents with the next lowest taste (those in $\mathcal{I}^2$) together in jurisdiction 2, and so on. Notably, the nonnegativity constraints bind, there may exist other types of equilibria and complete stratification need not be the unique equilibrium outcome.

The above theorem helps establish the following First Welfare Theorem.

Theorem 11. *Suppose that the nonnegativity constraints on DLPG investments and private good consumption are removed. Then if a feasible allocation and a price system are a DTE with DLPG investments determined by median voters, this feasible allocation must also be Pareto optimal.*

Proof of Theorem 11 **(to be moved to the Appendix later)**

By Theorem 10, a DTE features complete stratification. Thus, all agents are sorted based on their tastes and Pareto improvements cannot be achieved by relocating agents between different jurisdictions. Thus, to complete the proof, it remains to rule out possible Pareto improvements by reallocation between agents in the same jurisdiction. This is trivial because agents in the same juridiction face the same level of DLPG provision and value the private good identically, so such Pareto dominant reallocation cannot be feasible. ■

Thus, even with heterogeneous agents, decentralized provision of DLPG still results in the full internalization of the intergenerational spillovers, leading to Pareto efficient equilibrium outcomes.

Remark. How would one compare the median-voter DLPG investments with a planner solution with planner in each jurisdiction maximizing per capita utility, $W^j \equiv \sum_{t=0}^{T} \beta^{t-1} \left(\frac{1}{L} \sum_{i \in \mathcal{I}^j} U^i\right)$? This is equivalent to comparing the decisions between the median and the mean voters, which need not be the same. Yet, with a large number of jurisdictions J and a relatively flat gradient of $\rho(i)$, each jurisdiction would cover a small segment of tastes with agents approximately homogeneous. In the limit, the DTE and the planner investments in DLPG would be approximately the same.

10 Literature Review

This paper attempts to tie together the literature on intergenerational goods and Tiebout economies. In general, intergenerational goods have been treated as private goods that are voluntary transferred either forward or backward across generations in the context of a dynamic and unified (that is single jurisdiction) economy. The central question of this literature is if present generations are selfish, why should they make such transfers? See, among others, Kotlikoff, Persson, Svensson (1988), Rangel (2003, 2005), Boldrin and Montes (2005), and Hatfield (2008). There are policy implications for a wide range of issues including social security, education spending, global warming, and research and development, to name only a few.

It may be possible to induce self-interested agents to provide backward intergenerational goods (BIGs) through a game in which all generations play the trigger strategy that they will transfer goods to the currently old only if the currently old made similar transfers when they were young. As long as these intergenerational transfers grow at least as fast as the interest rate, the agents are best served by not defecting from this strategy. See Rangel (2005) for details. Forward intergenerational goods (FIGs) can also be sustained in equilibrium, but only if they are linked to the provision of BIGs. The problem is that both optimal and nonoptimal levels of BIGs and FIGs can be supported in these games. Thus, although institutions exist that can incentivize selfish generations to make transfers under centralization, they do not assure optimal outcomes.

Turning now to Tiebout (1956), recall that his main argument was that interjurisdictional competition should lead to efficient provision of public goods. In the early literature especially, the focus was on static coalition formation in economies without land. See, for example, Buchanan (1965), Pauly (1970), McGuire (1974), Berglas (1976), Wooders (1978), and Bewley (1981), or Conley and Wooders (1998) for a survey. Other work has added either divisible land (Rose-Ackerman 1979 and Epple, Filimon and Romer 1984, for example) or indivisible land (Dunz 1985 and Nechyba 1996, for example) to the model. See Konishi (1996) for an excellent discussion of this literature and additional results.

An element which is common to the literature mentioned above is that the models they treat are static. The conclusion in general is that under some conditions, interjurisdictional competition is sufficient to cause agents to internalize spillovers between agents within the same jurisdiction concerning contributions to local public good provision. In contrast, our focus is on dynamic economies and concerns whether or not interjurisdictional competition working through the mechanism of capitalization is sufficient to cause intergenerational spillovers to be internalized.

From an empirical standpoint, there is a great deal of evidence that capitalization of some type is an important phenomenon. Such studies go back at least as far as the famous paper of Oates (1969) who confirms that both property taxes and spending get capitalized into property values. See Nguyen-Hoang and Yinger (2011) for a recent survey. The correct econometric treatment of this question is quite subtle, however. For example, it is unclear if spending is strongly correlated to public goods quality (especially school quality), and thus, it is not immediate what exactly should be capitalized. See Hanushek (1986), Hayes and Taylor (1998) and Black (1999). How to test for capitalization in a steady state is especially troublesome and we will not attempt to deal with this issue here. We refer the reader to Epple, Zelenitz and Visscher (1978) Yinger (1982, 1995) Brueckner (1982), and Starrett (1997) for enlightening discussions on this topic.

In addition to the papers above, there is a small theoretical literature of capitalization in a static economy. Notable contributions include Wildasin (1979), Stiglitz (1983), Brueckner and Wingler (1984) de Bartolome (1990) and more recently Wildasin and Wilson (1998). For the most part, this work considers the optimality of equilibrium local public good and tax levels, and the response of property values in specific economic contexts. For example, Brueckner and Wingler are concerned about public goods as intermediate inputs, de Bartolome is interested in how peer groups affect the value of school districts, Wildasin looks at how capitalization affects the possibility of risk pooling. It is not immediate how these results might be extended to dynamic economies in which public goods are durable.

Unfortunately, the theoretical literature on dynamic Tiebout economies with DLPG is similarly small. The earliest paper of which we are aware is Kotlikoff and Rosenthal (1993) who consider a two period model with two jurisdictions and discover that one should not expect competition to generate efficient provision of such goods.[8] Schultz and Sjöström,

[8] It is also worth calling the reader's attention to the literature of dynamic Tiebout models without DLPG or capitalization. See especially Kotlikoff and Raffelhueschen (1991), Glomm and Lagunoff (1999), Benabou (1996), and Brueckner (1997), Hatfield (2007), and more recently Chen, Peng and Wang (2009) and Epple,

(2001) also treat a two period, two jurisdiction model with public debt and free mobility. They find that the equilibrium is generally inefficient, but their model does not allow either DLPG or debt to be capitalized into land prices (also see Schultz and Sjöström, 2004). Other papers consider these questions in the context of an overlapping generations model. For example, Wildasin and Wilson (1996) consider such an economy with imperfectly mobile agents but with local public goods that are nondurable. They discover that the capitalization mechanism may not induce efficient provision of local public goods. Similarly, Sprunger and Wilson (1998) consider how the desire of governments to exploit imperfectly mobile households may be expressed when public goods choices are made a period before the goods are consumed. These goods fully depreciate the period they are produced, however, so may have more of a flavor of a standard intergenerational good than of a DLPG.

11 Conclusions

We have constructed an overlapping generations model with a durable local public good and established a Tiebout Theorem and an equal-treatment Second Welfare Theorem. Without the nonnegative constraints on private good consumption and DLPG investments, we have shown that a dynamic Tiebout equilibrium is Pareto optimal under the Small Jurisdiction assumption (which requires that if a single jurisdiction changes its DLPG level, there must be at least one other jurisdiction somewhere in the economy whose land prices are unaffected). If, in addition, we allow endowments and prices to be negative, then all Pareto optimal allocations can be supported as DTE for some reallocation of endowments (that is, a general Second Welfare Theorem holds).

Our main conclusion is that capitalization is indeed an effective mechanism to cause agents to internalize intergenerational spillovers. The effectiveness of this mechanism is, however, limited by the degree to which there are more general spillovers across jurisdictions. The establishment of a Tiebout Theorem for a simple economy with DLPG is largely in contrast to the existing Tiebout literature, which either shows that equilibria exist, or that equilibria are efficient, but typically not both (see Conley and Konishi 1999 for further discussion). Our finding is important because current studies of DLPG generally include economic distortions in various forms (e.g., uncertainties, incomplete information, and market power). Unless we have a baseline case of a competitive economy for which a First Welfare Theorem applies, it

Romano and Sieg (2012). Also see Conley and Rangel (2001) for a simple two period approach.

is hard to know if the inefficiencies in these models come from the distortions in question, or are simply a result of the underlying economic structure.

If one takes the view, perhaps because of real world frictions, that jurisdictions of fixed size and indivisible land are a reasonable approximation to reality, this paper shows that there is an essential trade off between intergenerational spillovers which can be internalized by competing jurisdictions through capitalization, and interjurisdictional spillovers which may be internalized when agents vote centrally over public goods levels. This suggests the following policies for optimal public good provision:

	Durability	
Rivalry	nondurable	durable
local	by jurisdictions	by jurisdictions
pure	by central government	cannot be provided optimally

(i) Nondurable local public goods should be provided by jurisdictions. Examples include police and fire protection, local services and fireworks displays. This is because of heterogeneous tastes only.

(ii) Durable local public goods should be provided by jurisdictions. Examples include city streets and local infrastructure. This is because of heterogeneous tastes and intergenerational spillovers.

(iii) Nondurable purely public goods should be provided nationally. This also includes private goods and public services with widespread externalities. Examples include medical care, poverty relief, and research relating to immediate problems like what this year's flu shot should contain. This is because of interjurisdictional and interpersonal spillovers. Of course, efficiency requires that some sort of mechanism be used to figure out the right levels of public goods and to set the correct tax rates.

(iv) Durable purely public goods cannot be provided optimally at any level. Examples include defense, environmental protection, abatement of global warming and most types of pure research. This is because of the conflict between internalizing intergenerational and interjurisdictional spillovers. It is interesting to note that questions of how to deal with goods of this type seem to be at the center of many of the most politically contentious issues today. It may be that there is a kind of continuing crisis surrounding these goods because of the failure of any institution to provide them efficiently.

Finally, we show that moderate property value booms and busts, whether anticipated or not, do not affect the result that the value of the existing DLPG stock will be capitalized into local housing prices. This in turn means that agents continue to have the correct incentives to internalize the intergenerational spillovers that are produced by investing in DLPG. However, if these booms or busts raise prices too high or depress them too low in absolute terms relative to income, then this result breaks down. Thus, both relative and absolute prices play a role in generating efficient market outcomes when local public goods are durable.

Appendix

In this appendix, we present all the detailed mathematical proofs of Lemmas and Theorems. A significant portion of the Appendix is not intended for publication.

Lemma 1. *The socially optimal steady-state level of DLPG is determined by:*

$$V'(G_{ss}) = \frac{1}{\beta(1-\delta)L} - \frac{1}{L} \tag{15}$$

and the socially optimal steady-state value of DLPG investment g_{ss} by:

$$g_{ss} = \frac{\delta G_{ss}}{1-\delta} \tag{16}$$

Proof of Lemma 1

Problem (4) gives the following First Order Conditions:

$$\begin{aligned}
\frac{\partial W^*}{\partial g_t} &= 0 = -\frac{1}{L} + \theta_t - \phi_t + \delta\lambda_t \quad \text{for } t = 1, \ldots, T & (17)\\
\frac{\partial W^*}{\partial G_t} &= 0 = -\lambda_{t-1} + \beta\delta\lambda_t + \beta V'(G_t) \quad \text{for } t = 2, \ldots, T-1 & (18)\\
\frac{\partial W^*}{\partial G_T} &= 0 = -\lambda_{T-1} + \beta V'(G_T) & (19)\\
\frac{\partial W^*}{\partial \lambda_t} &= (1-\delta)(G_{t-1} + g_{t-1}) - G_t = 0 \text{ for } t = 2, \ldots, T
\end{aligned}$$

and Kuhn-Tucker Conditions associated with the nonnegativity constraints on g and $L\omega - g$:

$$\begin{aligned}
\theta_t g_t &= 0 \quad \text{for } t \in \mathcal{T}\\
\theta_t &\geq 0 \quad \text{for } t = 1, \ldots, T-1\\
\phi_t(L\omega - g_t) &= 0 \quad \text{for } t = 1, \ldots, T-1\\
\phi_t &\geq 0 \quad \text{for } t = 1, \ldots, T-1.
\end{aligned}$$

Rearranging (17), (18) and (19), respectively, we get

$$\begin{aligned}
\lambda_t &= \frac{1}{(1-\delta)}\left(\frac{1}{L} - \theta_t + \phi_t\right) \quad \text{for } t = t \in \mathcal{T} & (20)\\
\lambda_{t-1} &= \beta\delta\lambda_t + \beta V'(G_t) \quad \text{for } t \in \mathcal{T}^O & (21)\\
\lambda_{T-1} &= \beta V'(G_T). & (22)
\end{aligned}$$

Using this, we can characterize the stationary state of the planner's problem. We define an (interior) optimal stationary state the level of DLPG, G_{ss}, that solves the first-order conditions of the planner's problem when $\lambda_{t-1} = \lambda_t$ and $\phi_{t-1} = \phi_t = \theta_{t-1} = \theta_t = 0$ for $t \in \mathcal{T}$. Substituting this into equation (20) gives us:

$$\frac{\partial W^*}{\partial g_t} = 0 \Rightarrow \lambda_t = \frac{1}{(1-\delta)\,L}.$$

Since $\lambda_{t-1} = \lambda_t = \frac{1}{(1-\delta)L}$, we can put this into equation (21) to get:

$$\overbrace{\frac{1}{(1-\delta)\,L}}^{\lambda_{t-1}} - \overbrace{\frac{\beta}{L}}^{\beta\delta\lambda_t} = \beta V'(G_t).$$

which yields (15).

Finally, if we have G_{ss} DLPG at the end of a period, $(1-\delta)G_{ss}$ survives into the next period. Thus, to maintain the steady state, $\frac{G_{ss}-(1-\delta)G_{ss}}{1-\delta G_{ss}}$. It immediately follows that:

$$g_{ss} = \frac{\delta G_{ss}}{1-\delta}.$$

■

Theorem 1. *Assume $g_{ss} < L\omega$, and $G_{ss} > G_1$. Then the socially optimal levels of DLPG relate to the socially optimal steady-state level in the following manner:*

$$\beta\left(V'(G_{ss}) - V'(G_t)\right) = \frac{\theta_{t-1}}{1-\delta} - \beta\theta_t - \frac{\phi_{t-1}}{1-\delta} + \beta\phi_t \text{ for } t = 1,\ldots,T-1. \tag{23}$$

Moreover, the solution to the planner's problem is the following:

(i) *$g_t^* = L\omega$ from $t = 1$ to some t' (note that t may equal 1, or $T-1$)*

(ii) *$L\omega > g_{t'+1}^* \geq 0$*

(iii) *$g_t^* = g_{ss}$ for period $t'+2$ to some period $t'' \geq t'$ or $g_{t'+2}^* = 0$*

(iv) *$g_t^* = 0$ for period $t''+1$ to T.*

Proof of Theorem 1

The Kuhn-Tucker conditions immediately imply for all $t = 1,\ldots,T$ that one of the two following things is true:

$$\phi_t \geq 0 \text{ and } \theta_t = 0 \quad \text{if } g_t^* > 0$$

or

$$\phi_t = 0 \text{ and } \theta_t \geq 0 \quad \text{if } g_t^* < L\omega$$

We will use this fact in the proof below.

Inserting (20) into (21) for λ_{t-1} and λ_t gives the following:

$$\frac{\frac{1}{L}-\theta_{t-1}+\phi_{t-1}}{1-\delta}=\beta\delta\left[\frac{\frac{1}{L}-\theta_t+\phi_t}{1-\delta}\right]+\beta V'(G_t) \text{ for } t=1,\ldots,T-1.$$

Rearranging and using equation (15) gives:

$$\overbrace{\frac{1}{(1-\delta)\,L}-\frac{\beta}{L}}^{\beta V'(G_{ss})}=\frac{\theta_{t-1}}{1-\delta}-\beta\theta_t-\frac{\phi_{t-1}}{1-\delta}+\beta\phi_t+\beta V'(G_t) \text{ for } t=1,\ldots,T-1.$$

which we can rewrite to obtain the key equation (23) in the Theorem.

Using this, we show a series of simple claims:

Claim (a): For all $t=1,\ldots,T-1$, if $g^*_{t-1}<L\omega$ and $g^*_t=L\omega$, then $G_t\geq G_{ss}$. Suppose for some $t\in\{1,\ldots,T-1\}$, $g^*_{t-1}<L\omega$ and $g^*_t=L\omega$. Then, $\phi_t\geq 0$, and $\theta_t=0$ and $\phi_{t-1}=0$, and $\theta_{t-1}\geq 0$. From equation (23):

$$\beta\left(V'(G_{ss})-V'(G_t)\right)=\frac{\theta_{t-1}}{1-\delta}+\beta\phi_t\geq 0$$

which in turn implies

$$G_t\geq G_{ss}.$$

Claim (b): For all $t=1,\ldots,T-1$, if $g^*_{t-1}=L\omega$ and $g^*_t<L\omega$, then $G_t\leq G_{ss}$. Suppose for some $t\in\{1,\ldots,T-1\}$, $g^*_{t-1}=L\omega$ and $g^*_t<L\omega$. Then, $\phi_t=0$, and $\theta_t\geq 0$ and $\phi_{t-1}\geq 0$, and $\theta_t=0$. From equation (23):

$$\beta\left(V'(G_{ss})-V'(G_t)\right)=-\beta\theta_t-\frac{\phi_{t-1}}{1-\delta}\leq 0$$

which in turn implies

$$G_t\leq G_{ss}.$$

Claim (c): For all $t=1,\ldots,T-1$, if $g^*_{t-1}=0$ and $g^*_t>0$, then $G_t\geq G_{ss}$. Suppose for some $t\in\{1,\ldots,T-1\}$, $g^*_{t-1}=0$ and $g^*_t>0$,. Then, $\phi_t\geq 0$, and $\theta_t=0$ and $\phi_{t-1}=0$, and $\theta_{t-1}\geq 0$. From equation (23):

$$\beta\left(V'(G_{ss})-V'(G_t)\right)=\frac{\theta_{t-1}}{1-\delta}+\beta\phi_t\geq 0$$

which in turn implies

$$G_t\geq G_{ss}.$$

Claim (d): For all $t = 1, \ldots, T-1$, if $g^*_{t-1} > 0$ and $g^*_t = 0$, then $G_t \leq G_{ss}$. Suppose for some $t \in \{1, \ldots, T-1\}$, $g^*_{t-1} > 0$ and $g^*_t = 0$. Then, $\phi_t = 0$, and $\theta_t \geq 0$ and $\phi_{t-1} \geq 0$, and $\theta_{t-1} = 0$. From equation (23):

$$\beta \left(V'(G_{ss}) - V'(G_t)\right) = -\beta\theta_t - \frac{\phi_{t-1}}{1-\delta} \leq 0$$

which in turn implies

$$G_t \leq G_{ss}.$$

Claim (e): If $g^*_{T-1} > 0$ then $G_T < G_{ss}$. From the first order conditions, we know:

$$\begin{aligned} \lambda_{T-1} &= \beta V'(G_T); \\ \lambda_{T-1} &= \frac{1}{(1-\delta)L} - \frac{\theta_{T-1}}{1-\delta} + \frac{\phi_{T-1}}{1-\delta} \\ \Rightarrow \overbrace{\beta V'(G_T)}^{\lambda_{T-1}} &= \frac{1}{(1-\delta)L} - \frac{\theta_{T-1}}{1-\delta} + \frac{\phi_{T-1}}{1-\delta}. \end{aligned}$$

Suppose first that $g^*_{T-1} = L\omega$. Then $\theta_{T-1} = 0$, and so

$$\frac{1}{(1-\delta)L} + \frac{\phi_{T-1}}{1-\delta} = \beta V'(G_T)$$

Remember,

$$\frac{1}{(1-\delta)L} - \frac{\beta}{L} = \beta V'(G_{ss}).$$

So,

$$V'(G_{ss}) < V'(G_T)$$

which implies

$$G_T < G_{ss}.$$

Suppose now that $L\omega > g^*_{T-1} > 0$. Then $\phi_{T-1} = \theta_{T-1} = 0$, and so

$$\frac{1}{(1-\delta)L} = \beta V'(G_T).$$

But

$$\frac{1}{(1-\delta)L} - \frac{\beta}{L} = \beta V'(G_{ss})$$

so again

$$G_T < G_{ss}.$$

Using these claims, we can derive the following implications:

(I-1) For all $t \in \mathcal{T}^O$, if $g_t^* = L\omega$ then $g_{t-1}^* = L\omega$. Suppose for some $t \in \mathcal{T}^O$, $g_{t-1}^* < L\omega$, but $g_t^* = L\omega$. Then by (a) $G_t \geq G_{ss}$. But since $g_t^* = L\omega > g_{ss}$ we are adding more than the steady-state level of investment and so it must be that $G_{t+1} > G_{ss}$ Suppose $t_1 \in \mathcal{T}^O$ and $g_{t+1}^* < L\omega$. Then by (b), $G_{t+1} \leq G_{ss}$, a contradiction. Thus, $g_{t+2}^* = L\omega$. By the same argument, for all $t' \in \mathcal{T}^O$, $g_{t'}^* = L\omega > g_{ss}$ and $G_t > G_t$. In particular, $G_T > G_{ss}$. However, by (e), $G_T < G_{ss}$, a contradiction. It follows that if $g_t^* = L\omega$ then $g_{t-1}^* = L\omega$.

(I-2) For all $t \in \mathcal{T}^O$, if $g_{t-1}^* = 0$ then $g_t^* = 0$. Suppose for some $t \in \mathcal{T}^O$, $g_{t-1}^* = 0$, but $g_t^* > 0$. Then by (c) $G_t \geq G_{ss}$. Consider period $t-2 > 1$. Suppose that $g_{t-2}^* > 0$, then from (d) $G_{t-1} \leq G_{ss}$. This is impossible since nothing was added to the public good stock in period $t-1$, and yet $G_{t-1} \leq G_{ss} \leq G_t$. It follows that $g_{t-2}^* = 0$. Now consider period $t-3 > 1$. Suppose that $g_{t-3}^* > 0$, then by (d) $G_{t-2} \leq G_{ss}$. This is similarly impossible since nothing was added to the public good stock in period $t-2$ or $t-1$, and yet $G_{t-2} \leq G_{ss} \leq G_t$. It follows that $g_{t-3}^* = 0$. By the same argument, for all $t' = 1, \ldots t-1$, $g_{t'-1}^* = 0$ and $G_{t'} \geq G_{ss}$. In particular, $G_1 \geq G_{ss}$. This contradicts the assumption that $G_{ss} > G_1$. It follows that if $g_{t-1}^* = 0$ then $g_t^* = 0$.

(I-3) For all $t \in \mathcal{T}^O$, if $L\omega > g_{t-1}^* > 0$ and $L\omega > g_t^* > 0$, then either $G_t = G_{ss}$ *and* $g_t^* = g_{ss}$ or $g_t^* = 0$. Suppose for some $t \in \mathcal{T}^o$, $L\omega > g_{t-1}^* > 0$ and $L\omega > g_t^* > 0$, Then we are at an interior optimum, and $\phi_t = \theta_t = \phi_{t-1} = 0$, and $\theta_t = 0$. From equation (15)

$$\beta\left(V'(G_{ss}) - V'(G_t)\right) = 0$$

which in turn implies

$$G_t = G_{ss}.$$

Suppose $g_{t+1}^* > 0$. Since since $g_t^* > 0$, by the same argument $G_{t+1} = G_{ss}$, which is only possible if $g_t^* = g_{ss}$. it is immediate that for all $k \geq 1$ if $t+k < T$ and $g_{t+k}^* > 0$, then $G_{t+k+1} = G_{ss}$, and $g_{t+k}^* = g_{ss}$. Suppose for some $k \geq 1$, $g_{t+1}^* = 0$. This is possible and by implication I-2, investment would stay at zero until T. Thus, from some (possibly degenerate) interval from t' to t'', $G_{t+1} = G_{ss}$, and $g_t^* = g_s$. In addition, $g_t^* = 0$ for periods $t''+1$ to T.

It is clear that (I-1) directly implies part (i) of the Theorem and (I-2) directly implies part (iv) of the Theorem. To see the remainder, consider period t' as mentioned in the statement of the Theorem. Note that $g_{t'+1}^* < L\omega$ or else we would still be in case (i). Suppose $g_{t'+1}^* = 0$. Then $t' = t''$ and case (ii) is satisfied by assumption, case (iv) obtains in the next period, and case (iii) is vacuous. Finally suppose $L\omega > g_{t'+1}^* > 0$. Then case (ii) is satisfied by assumption and (I-3) implies that the optimal investment level stays at g_{ss} unless and until it drops to zero at some period t'' and stays rest of the future, that is, part (iii) of the Theorem obtains. ■

Theorem 2. *Suppose that the nonnegativity constraints on DLPG investments and private good consumption are removed. Then the solution to the planner's problem becomes:* $g_1^* = \frac{G_{ss}-(1-\delta)G_1}{1-\delta}$, $g_t^* = g_{ss} = \frac{\delta G_{ss}}{1-\delta}$ *and* $G_t^* = G_{ss}$ *for* $t \in \mathcal{T}^O$, *and* $g_T^* = -G_T^* = -G_{ss}$.

Proof of Theorem 2

Relaxing the nonnegativity constraints allows agents to make negative investments bounded only by the current level of DLPG. In effect, this allows agents the option of consuming the existing stock. Given the timing of periods, agents first enjoy the services of DLPGs and only afterwards decide how much to add or subtract from the current DLPG stock. This immediately implies that any DLPG remaining at time T should be consumed by setting $g_t^* = -G_t$. It also allows us to state the planner's problem in a very simple way. Imagine for a moment that agents in each period consume all the current stock of DLPG, but afterwards invest enough private good to get to the planner's chosen level of DLPG for the next period. Then we can directly incorporate the capital evolution constraint into the problem as follows:

$$\max_{G_2,\ldots,G_T} W = \sum_{t=1}^{T} \beta^{t-1}\left(\omega + \frac{G_t}{L} - \frac{G_{t+1}}{(1-\delta)\,L} + V(G_t)\right)$$

This gives the following First Order Conditions:

$$\frac{\partial W^*}{\partial G_t} = 0 = \frac{1}{L} - \beta\frac{1}{(1-\delta)\,L} + V'(G_t) \text{ for } t = 2,\ldots,T$$

or

$$\frac{1}{(1-\delta)\,L} - \frac{\beta}{L} = \beta V'(G_t) \text{ for } t = 2,\ldots,T$$

Since we know that G_{ss} is the solution to this equation, we conclude that the planner jumps to the steady state by investing whatever is necessary in period 1. Thus, $g_1^j = \frac{G_{ss}-(1-\delta)G_1}{1-\delta}$. The planner then maintains this until period T and so: $g_t^* = g_{ss} = \frac{\delta G_{ss}}{1-\delta}$ and $G_t^* = G_{ss}$ for $t \in \mathcal{T}^O$. Finally, in the last period, the planner allows the current stock to be completely consumed and so: $g_T^* = -G_{ss}$

■

Lemma 2. *Consider any arbitrarily chosen steady-state level of DLPG,* $\bar{G}$. *Suppose that the nonnegativity constraints on DLPG investments and private good consumption are removed. Then there exists a price system* **p** *such that for all* $j \in \mathcal{J}$

$$\bar{g}_t^j = \begin{cases} \frac{\bar{G}-(1-\delta)G_1}{1-\delta} & t = 1 \\ \frac{\delta\bar{G}}{1-\delta} & t \in \mathcal{T}^O \\ -G_T^j = -\bar{G} & t = T. \end{cases}$$

and **p** *supports this plan and satisfies FM and PE.*

Proof of Lemma 2

Define prices as follows:

$$p_t^j(G_t) = \begin{cases} V(G_t^j) + \frac{1}{L}G_t^j + K & \text{if } G_t^{\bar{j}} = \bar{G} \text{ for all } \bar{j} \in \mathcal{J} \\ V(G_t^j) + \frac{1}{L}G_t^j & \text{otherwise} \end{cases}$$

where K is a large constant. Consider period T. First suppose that all agents born from period 1 to $T-1$ have followed the plan. This implies that $G_T^j = \bar{G}$ for all $j \in \mathcal{J}$. Working backwards, Suppose that all period T agents have chosen a jurisdiction. It is immediate that at stage 2 of the period, these agents determine that it is optimal to set $\bar{g}_T^j = -G_T^j = -\bar{G}$ which is also what is required by PE.

Since

$$p_T^j(\bar{G}) = V(\bar{G}) + \frac{1}{L}\bar{G} + \bar{U},$$

the price of land in each jurisdiction is equal across all jurisdictions Thus, for all $j, \bar{j} \in \mathcal{J}$, the net utility is equal:

$$V(\bar{G}) + \omega - p_T^j(\bar{G}) + \frac{1}{L}\bar{G} = V(\bar{G}) + \omega - p_T^{\bar{j}}(\bar{G}_T) + \frac{1}{L}\bar{G}.$$

It follows that agents are equally well off regardless of where they decide to buy a house, and so FC is satisfied in period T.

Consider any period $t \in \mathcal{T}^O$. Suppose that all jurisdictions followed the plan from period 1 to $t-1$. Again, working backwards, suppose that all period t agents have chosen a jurisdiction. If any jurisdiction j decides to deviate from the plan, the price of land for period $t+1$ for both jurisdiction j and all other jurisdictions drops by K. If K is chosen to be large enough, this loss in period $t+1$ consumption is more than enough to offset any potential utility gain from choosing any other investment level. Thus, investing according to plan gives higher utility than any other choice and so the investment decision in period t satisfies the PE requirement.

This implies for all $j, \bar{j} \in \mathcal{J}$, $G_t^j = G_{t+1}^j = G_t^{\bar{j}} = G_{t+1}^{\bar{j}} = \bar{G}$ and therefore $p_t^j(\bar{G}) = p_{t+1}^j(\bar{G}) = p_t^{\bar{j}}(\bar{G}) = p_{t+1}^{\bar{j}}(\bar{G})$. Thus, the utility received by an agent born in period t is equal to

$$V(\bar{G}) + \omega - p_t^j(\bar{G}) - \frac{1}{L}\bar{g} + \beta p_{t+1}(\bar{G})$$

regardless of where he chooses to live and so FM is satisfied.

Since the same argument holds for period 1, we conclude that for all $j \in \mathcal{J}$, $g_1^j = \frac{G_{ss}-(1-\delta)G_1}{1-\delta}$, and $g_t = g_{ss} = \frac{\delta G_{ss}}{1-\delta}$ for $t \in \mathcal{T}^O$, since these are the only investment levels that support the specified DLPG plan. Therefore, $\mathbf{p}$ supports this plan and satisfies FM and PE, which proves the Theorem. ■

Theorem 3. *Let* $(\mathbf{c}, \mathbf{g}, \mathbf{G})$ *and* $\mathbf{p}$ *be a DTE for an economy satisfying SJ. Suppose that the nonnegativity constraints on DLPG investments and private good consumption are removed.*

Then for all $j \in \mathcal{J}$, $g_1^j = \frac{G_{ss}-(1-\delta)G_1}{1-\delta}$, $g_t^j = g_{ss} = \frac{\delta G_{ss}}{1-\delta}$ *and* $G_t^j = G_{ss}$ *for* $t \in \mathcal{T}^O$, *and* $g_T^j = -G_T^j = -G_{ss}$.

Proof of Theorem 3

We start in period T. Since T is the last period, $g_T^j = -G_T^j$, $j \in \mathcal{J}$, by PE. By FM, for all $j, \bar{j} \in \mathcal{J}$

$$\omega - p_T^j(G_T) + V(G_T^j) + \frac{G_T^j}{L} = \omega - p_T^{\bar{j}}(G_T) + V(G_T^{\bar{j}}) + \frac{G_T^{\bar{j}}}{L}$$

which implies the following about the equilibrium prices system $\mathbf{p}$:

$$p_T^j(G_T) = p_T^{\bar{j}}(G_T) + V(G_T^j) - V(G_T^{\bar{j}}) + \frac{G_T^j}{L} - \frac{G_T^{\bar{j}}}{L}$$

This is the relative-price equation that will constrain agents born in period $T-1$

Now consider the problem for agents born in period $T-1$. Working backwards, suppose that all agents have chosen a jurisdiction. Consider any particular jurisdiction j and consider what level of DLPG the agents in j would optimally choose to pass on to the next generation T. The implicit maximization problem is the following:

$$\max_{G_T^j} \beta p_T^j(G_T) - \frac{1}{(1-\delta)L}(G_T^j - G_{T-1}^j)$$

Substituting the relative price equation gives:

$$\max_{G_T^j} \beta \left(p_T^{\bar{j}}(G_T) + V(G_T^j) - V(G_T^{\bar{j}})\right) + \left(\frac{G_T^j}{L} - \frac{G_T^{\bar{j}}}{L}\right) - \frac{1}{(1-\delta)L}(G_T^j - G_{T-1}^j)$$

Taking the derivative with respect to G_T^j gives:

$$\beta\frac{dp_T^{\bar{j}}(G_T)}{dG_T^j} + \beta\frac{dV(G_T^j)}{dG_T^j} - \beta\frac{dV(G_T^{\bar{j}})}{dG_T^j} + \frac{\beta}{L} - \frac{1}{(1-\delta)L} = 0$$

The key observation is that $\frac{dp_T^{\bar{j}}(G_T)}{dG_T^j} = 0$ by the SJ. Since $\frac{dV(G_T^{\bar{j}})}{G_T^j} = 0$ by construction, the First Order Condition becomes:

$$\beta V'(G_T^j) = \frac{1}{(1-\delta)L} - \frac{\beta}{L}$$

Thus, by PE, $G_T^j = G_{ss}$ for all $j \in \mathcal{J}$. On the other hand, FM requires:

$$\omega - p_{T-1}^j(G_{T-1}) + V(G_{T-1}^j) + \frac{G_{T-1}^j}{L} - \frac{G_T^j}{(1-\delta)L} = \omega - p_{T-1}^{\bar{j}}(G_{T-1}) + V(G_{T-1}^{\bar{j}}) + \frac{G_{T-1}^{\bar{j}}}{L} - \frac{G_T^{\bar{j}}}{(1-\delta)L}.$$

Noting that we have shown that whatever generation $T-2$ leaves to generation $T-1$, generation $T-1$ will adjust investment such that $G_T^j = G_T^{\bar{j}} = G_{ss}$, we can solve this to get:

$$p_{T-1}^j(G_{T-1}) = p_{T-1}^{\bar{j}}(G_{T-1}) + V(G_{T-1}^j) - V(G_{T-1}^{\bar{j}}) + \frac{G_{T-1}^j}{L} - \frac{G_{T-1}^{\bar{j}}}{L}$$

which is the relative price equation for generation $T-2$ and is identical in form to the relative pricing equation for generation $T-1$. By the same argument we made above and applying the SJ, we conclude:

$$\beta V'(G_{T-1}^j) = \frac{1}{(1-\delta)\,L} - \frac{\beta}{L}$$

and so $G_{T-1}^j = G_{ss}$ for all $j \in \mathcal{J}$.

Now suppose for any $t = 1, \ldots, T-1$, and all $j \in \mathcal{J}$, $G_{t+1}^j = G_{ss}$. Then again,

$$p_t^j(G_t) = p_t^{\bar{j}}(G_t) - V(G_t^j) - V(G_t^{\bar{j}}) + \frac{G_t^j}{L} - \frac{G_t^{\bar{j}}}{L}$$

and so by SJ,

$$\beta V'(G_t^j) = \frac{1}{(1-\delta)\,L} - \frac{\beta}{L}.$$

Thus, by backwards induction, for all $t \in \mathcal{T}^O$, and all $j \in \mathcal{J}$, $G_t^j = G_{ss}$.

Finally, the only levels of investment that support this DLPG plan are for all $j \in \mathcal{J}$, $g_1^j = \frac{G_{ss}-(1-\delta)G_1}{1-\delta}$, and $g_t = g_{ss} = \frac{\delta G_{ss}}{1-\delta}$ for $t \in \mathcal{T}^O$, which proves the Theorem. ■

Lemma 3. *Assume $G_{ss} \geq G_1$ and suppose that the nonnegativity constraints on DLPG investments and private good consumption are removed. Then a feasible allocation, $(\mathbf{c}, \mathbf{g}, \mathbf{G})$, is Pareto efficient if and only if it is also a solution to the planner's problem.*

Proof of Lemma 3

From Lemma 2, we know that if an allocation solves the planner's problem, then $G_t^j = G_{ss}$ for $t = 2, \ldots, T$, and $g_T^j = -G_{ss}$ for all $j \in \mathcal{J}$. Suppose that there existed a feasible plan $(\hat{\mathbf{c}}, \hat{\mathbf{g}}, \hat{\mathbf{G}})$ that Pareto dominated $(\mathbf{c}, \mathbf{g}, \mathbf{G})$. It is immediate that such a Pareto dominant allocation could not be found by altering $\mathbf{c}$ to some other $\hat{\mathbf{c}}$ alone. Utility is linear in private good for all agents, so if any agent gets more, another must necessarily gets less. Thus, the new allocation could not be Pareto dominant. It follows that if $(\hat{\mathbf{c}}, \hat{\mathbf{g}}, \hat{\mathbf{G}})$ Pareto dominates $(\mathbf{c}, \mathbf{g}, \mathbf{G})$, it must be that for at least one jurisdiction j for at least one period $t \in \{2, \ldots, T\}$, $\hat{G}_t^j$ is different from G_{ss}.

Note that if residents of any jurisdiction j at some time t invest an extra unit of consumption good in DLPG in period $t-1$, but also receive an extra $1/\beta$ units of consumption good in period t, their net utility would be unchanged. Similarly, they would be just as well off if they invested one less unit of consumption good in period $t-1$ and were given $1/\beta$ fewer units of consumption good in period t

Keeping this in mind, what is the best that the planner can do for the generation born in period t living in jurisdiction j while leaving all other generations and jurisdictions exactly as well off? The planner must solve the following equation for g^{Δ}:

$$\max V(\hat{G}_t^j + \delta g^{\Delta}) + \frac{\delta \hat{g}^{\Delta}}{L} - \frac{\hat{g}^{\Delta}}{\beta L}.$$

In words, the planner chooses an optimal increment or decrement to period $t-1$ investment in jurisdiction j, transfers $\frac{g^{\Delta}}{\beta L}$ to each agents in generation $t-1$ from generation t in period t, but has generation t agents invest the same net amount g_t^j by either topping up or consuming the extra DLPG ($\frac{\delta g^{\Delta}}{L}$) in order to leave generation $t+1$ in jurisdiction j exactly as well off as under $(\mathbf{c}, \mathbf{g}, \mathbf{G})$. The First Order Condition is the following:

$$\delta V_t' = \frac{1}{\beta L} - \frac{\delta}{L}$$

which gives

$$\beta V_t' = \frac{1}{(1-\delta) L} - \frac{\beta}{L},$$

the same equation that defines G_{ss}. Altering $(\hat{\mathbf{c}}, \hat{\mathbf{g}}, \hat{\mathbf{G}})$ such that $\hat{G}_t^j = G_{ss}$ and the transfers just described take place is therefore a Pareto improvement. Moreover, making the same alteration in DLPG along with compensating transfers for every period and jurisdiction for which the DLPG level in $(\hat{\mathbf{c}}, \hat{\mathbf{g}}, \hat{\mathbf{G}})$ is not G_{SS} is also a Pareto improvement. Denote the feasible allocation resulting from all of these alterations in $(\hat{\mathbf{c}}, \hat{\mathbf{g}}, \hat{\mathbf{G}})$ as $(\tilde{\mathbf{c}}, \tilde{\mathbf{g}}, \tilde{\mathbf{G}})$. Then $(\tilde{\mathbf{c}}, \tilde{\mathbf{g}}, \tilde{\mathbf{G}})$ is a feasible allocation that Pareto dominates $(\hat{\mathbf{c}}, \hat{\mathbf{g}}, \hat{\mathbf{G}})$ which in turn Pareto dominates $(\mathbf{c}, \mathbf{g}, \mathbf{G})$ by hypothesis. This implies that that $(\hat{\mathbf{c}}, \hat{\mathbf{g}}, \hat{\mathbf{G}})$ Pareto dominates $(\mathbf{c}, \mathbf{g}, \mathbf{G})$. But $\tilde{G}_t^j = G_{ss}$ for $t = 2, \ldots, T$ and all $j \in \mathcal{J}$, and so $(\tilde{\mathbf{c}}, \tilde{\mathbf{g}}, \tilde{\mathbf{G}})$ and $(\mathbf{c}, \mathbf{g}, \mathbf{G})$ only differ the private good allocations $\tilde{\mathbf{c}}$ and $\mathbf{c}$, contradicting the argument made above.

We can also conclude that if an allocation is Pareto efficient, then $G_t^j = G_{ss}$ for $t = 2, \ldots, T$, and $g_T^j = -G_{ss}$ for all $j \in \mathcal{J}$. Otherwise we could do the same exercise of alternating the DLPG level to G_{ss} along with compensating transfers to find a Pareto dominant allocation. Note that if we start from any feasible allocation and make any set of feasible transfers of private good over agents alive within a given period (that is, any private good consumption levels that satisfy $\sum_i c_{t-1,t}^i + \sum_i c_{t,t}^i = I\omega - Jg_{ss}$ for $t \in \mathcal{T}$), the resulting allocations are Pareto unranked since utility is quasilinear). Therefore, any allocation, $(\mathbf{c}, \mathbf{g}, \mathbf{G})$, such that for all $j \in \mathcal{J}$, for $t = 1$, $g_1^j = \frac{G_{ss} - (1-\delta) G_1}{1-\delta}$, all $t = 2, \ldots, T-1$, $g_t^j = g_{ss}$, and $G_t^j = G_{ss}$ and for $t = T$, $g_T^j = -G_{ss}$ and $G_T^j = G_{ss}$, is Pareto optimal regardless of $\mathbf{c}$.

Turning to the planner's problem, we see immediately that any division between old and young agents in a given period of what private good remains after optimal investments are made leaves the value of the social welfare function unaffected. Therefore, any allocation, $(\mathbf{c}, \mathbf{g}, \mathbf{G})$, such that for all $j \in \mathcal{J}$, for $t = 1$, $g_1^j = \frac{G_{ss} - (1-\delta) G_1}{1-\delta}$, all $t = 2, \ldots, T-1$, $g_t^j = g_{ss}$, and $G_t^j = G_{ss}$ and for $t = T$, $g_T^j = -G_{ss}$ and $G_T^j = G_{ss}$, is a solution to the social planner's problem regardless of $\mathbf{c}$.

We conclude that the set of Pareto efficient allocations and the set of solutions to the social planner's problem are identical. ■

Theorem 4. (First Welfare Theorem) *Suppose that the nonnegativity constraints on DLPG investments and private good consumption are removed. Then if* $(\mathbf{c}, \mathbf{g}, \mathbf{G})$ *and* $\mathbf{p}$ *are a DTE for an economy satisfying SJ,* $(\mathbf{c}, \mathbf{g}, \mathbf{G})$ *must also be Pareto optimal.*

Proof of Theorem 4

By Theorem 3, if $(\mathbf{c}, \mathbf{g}, \mathbf{G})$ and $\mathbf{p}$ are a DTE under these conditions, then $G_t^j = G_{ss}$ for $t = 2, \ldots, T$ and $j \in \mathcal{J}$. Then by Lemma 3, $(\mathbf{c}, \mathbf{g}, \mathbf{G})$ is Pareto optimal. ■

Theorem 5. (Second Welfare Theorem) *Suppose that the nonnegativity constraints on DLPG investments and private good consumption are removed and that a feasible allocation* $(\mathbf{c}, \mathbf{g}, \mathbf{G})$ *is Pareto optimal and satisfies ET. Then there exists a price system* $\mathbf{p}$ *such that* $(\mathbf{c}, \mathbf{g}, \mathbf{G})$ *and* $\mathbf{p}$ *are a DTE.*

Proof of Theorem 5

By Lemma 3, if a feasible allocation $(\mathbf{c}, \mathbf{g}, \mathbf{G})$ is Pareto optimal $G_t^j = G_{ss}$ for $t = 2, \ldots, T$ and $j \in \mathcal{J}$. Then for all $t \in \mathcal{T}$, $j \in \mathcal{J}$, define the price system as follows

$$p_t^j(G_t) = V(G_t^j) + \frac{G_t^j}{L} + K_t$$

where K_t is a (positive or negative) constant defined below. We start with period T. Since T the is the last period, it is optimal for agents in every jurisdiction to choose $g_T^j = -G_T^j$. By construction of the price system, for all $j, \bar{j} \in \mathcal{J}$

$$p_T^j(G_T) = p_T^{\bar{j}}(G_T) + V(G_T^j) - V(G_T^{\bar{j}}) + \frac{G_T^j}{L} - \frac{G_T^{\bar{j}}}{L}$$

Now consider the problem for agents born in period $T-1$. Working backwards, suppose that all agents have chosen a jurisdiction. Consider any particular jurisdiction j and consider what level of DLPG the agents in j would optimally choose to pass on to the next generation T. The implicit maximization problem is the following:

$$\max_{G_T^j} \beta p_T^j(G_T) - \frac{1}{(1-\delta)L}(G_T^j - G_{T-1}^j)$$

Substituting the relative price equation gives:

$$\max_{G_T^j} \beta \left(p_T^{\bar{j}}(G_T) + V(G_T^j) - V(G_T^{\bar{j}}) \right) + \left(\frac{G_T^j}{L} - \frac{G_T^{\bar{j}}}{L} \right) - \frac{1}{(1-\delta)L}(G_T^j - G_{T-1}^j)$$

Now take the derivative with respect to G_T^j,

$$\beta \frac{dp_T^{\bar{j}}(G_T)}{dG_T^j} + \beta \frac{dV(G_T^j)}{dG_T^j} - \beta \frac{dV(G_T^{\bar{j}})}{G_T^j} + \frac{\beta}{L} - \frac{1}{(1-\delta)\, L} = 0$$

This time, $\frac{dp_T^{\bar{j}}(G_T)}{dG_T^j} = 0$ by construction rather than the SJ. Also by construction, $\frac{dV(G_T^{\bar{j}})}{G_T^j} = 0$ and so the First Order Condition becomes

$$\beta V'(G_T^j) = \frac{1}{(1-\delta)\, L} - \frac{\beta}{L}.$$

Thus, agents chose $G_T^j = G_{ss}$ for all $j \in \mathcal{J}$ under the price system defined above. This implies

$$p_{T-1}^j(G_{T-1}) = p_{T-1}^{\bar{j}}(G_{T-1}) - V(G_{T-1}^j) - V(G_{T-1}^{\bar{j}}) + \frac{G_{T-1}^j}{L} - \frac{G_{T-1}^{\bar{j}}}{L}$$

By the same argument we made above we conclude:

$$\beta V'(G_{T-1}^j) = \frac{1}{(1-\delta)\, L} - \frac{\beta}{L},$$

and so $G_{T-1}^j = G_{T-1}^{\bar{j}} = G_{ss}$ for $j \in \mathcal{J}$.

Now suppose for any $t \in \{1, \ldots, T-1\}$, and all $j \in \mathcal{J}$, $G_{t+1}^j = G_{ss}$. Then again,

$$p_t^j(G_t) = p_t^{\bar{j}}(G_t) - V(G_t^j) - V(G_t^{\bar{j}}) + \frac{G_t^j}{L} - \frac{G_t^{\bar{j}}}{L}$$

and so,

$$\beta V'(G_t^j) = \frac{1}{(1-\delta)\, L} - \frac{\beta}{L}.$$

Thus, by backwards induction, for all $t = 2, \ldots, T$, and all $j \in \mathcal{J}$, $G_t^j = G_{ss}$. Note that this result is independent of K_t.

Finally, define the constant in each period as follows:

$$K_t = c_{t-1,t} - V(G_{ss}) - \frac{G_{ss}}{L}$$

Observe that without the constant added to prices, private good consumption levels would have been $\bar{c}_{t-1,t}^i = V(G_{ss}) + \frac{G_{ss}}{L}$ for all $t = 1, \ldots, T$, and all $i \in \mathcal{I}$. Therefore, if an old agent i gets an extra K_t when he sells his land, his consumption becomes $c_{t-1,t}$ the specified equal treatment level. Since by hypothesis, $(\mathbf{c}, \mathbf{g}, \mathbf{G})$ is feasible, it must be that $c_{t,t} = \omega - \frac{g_{ss}}{L} - c_{t-1,t}$, and so adding the constant to prices also results in young agents getting the specified equal treatment consumption level in each period. We conclude that

$$p_t^j(G_t) = V(G_t^j) + \frac{G_t^j}{L} + K_t$$

for all $t \in \mathcal{T}$, $j \in \mathcal{J}$ supports $(\mathbf{c}, \mathbf{g}, \mathbf{G})$ as a DTE. ■

Theorem 7. *Suppose that the nonnegativity constraints on DLPG investments and private good consumption are removed but that agents vote in a central election over a common level investment for all jurisdictions each period. Consider an arbitrary path of DLPG levels for each period: $(\bar{G}_2, \ldots, \bar{G}_T) \in \Re_+^{T-1}$ (not necessarily a steady state). Then there exists a price system $\mathbf{p}$ that satisfies PE, FM which supports this path.*

Proof of Theorem 6

Define prices as follows:

$$p_t^j(G_t, \ldots, G_t) = \begin{cases} \bar{K} & \text{if } G_t = \bar{G}_t \\ 0 & \text{otherwise} \end{cases}$$

where $\bar{K}$ is a large constant. Consider any period $t \in \mathcal{T}^o$. Working backwards, suppose that all period t agents have chosen a jurisdiction. If the young agents alive in period t vote in favor of an investment level that results in DLPG level next period of $G_{t+1} = \bar{G}_t$, they can sell their land for $\bar{K}$. If they vote for anything else, they get can get some finite increment to the utility by choosing G_{t+1} optimally while accepting a price of zero in the next period for their land. Clearly, it is possible to choose $\bar{K}$ to be large enough to exceed any potential gain from this strategy. Therefore $G_{t+1} = \bar{G}_{t+1}$ is a PE under these prices. In addition, since prices and DLPG levels are the same in every jurisdiction in period t, all jurisdictions are equally attractive and so these prices clear the housing market and therefore satisfy FM. ■

Theorem 8. *Suppose that the nonnegativity constraints on DLPG investments and private good consumption are removed but that agents vote in a central election over common level investment for all jurisdictions each period. Then any price system $\mathbf{p}$ that satisfies PE, FM and NS results in zero provision of DLPG in each period.*

Proof of Theorem 7

Given the assumption of the NS, the political decision faced by period t voters becomes the following:

$$\max_{g_t} V(\delta G_t) - \frac{g_t}{L} - K_t + \beta K_{t+1}$$

This gives the following first order condition:

$$-\frac{1}{L} = 0.$$

In words, investing in DLPGs is a pure gift to the next generation and so it is optimal to invest nothing. ■

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