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Demand, supply, the PPF, and the signaling power of prices

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Demand and supply

Let's look at two-person world: **A**lex and **H**arry. Why two? To keep things initially manageable and to understand the mechanism that generates inverse supply functions. With only two, we know that bargaining problems exist (failure, indeterminacy which implies resources expended on non-directly-productive activities; see the earlier example in which there were **multiple** voluntary exchanges that benefited both parties). But let's suspend our concerns on this, and think of Alex and Harry as believing they interact in a *thick* market with lots of other buyers and sellers. That is, we assume Alex and Harry believe themselves to be *price takers*, i.e., they take prices as *exogenous* to themselves.

We illustrate our little model with actual numbers for values of variables. These numbers are illustrative: they are chosen for ease of exposition. They are "good" numbers, though, in that they do not violate any of the theoretical ideas or empirical facts that we think we know apply to actual economies. Some of the numbers may seem "odd" in that they are not whole numbers or simple fractions–the kinds of numbers you are used to from everyday life. These somewhat different numbers– 0.1826, for example–are used only because they make some of the manipulations of the model easy (for me as I construct this example).

Alex and Harry each have only a fixed amount of time in a day, e.g., 8 hours, available for work (or a week, or some other unit of time; we'll stick with "day' to be concrete). They each can allocate this fixed amount of time to either producing V or producing C. Their rates of production for C and V are, respectively: Alex produces one (1) unit of C per unit of time (in this example, a day) spent in C production, and 0.1826 units of V per unit of time spent in V production, while Harry produces produces one (1) unit of C per unit of time (in this example, a day) spent in C production, and one (1) unit of V per unit of time spent in V production. Another way of putting this is:

$$\begin{aligned} C_A^S &= (1) \times L_C^A; \\ V_A^S &= 0.1826 \times L_V^A, \end{aligned}$$

$$C_H^S = (1) \times L_C^H;$$
$$V_H^S = (1) \times L_V^H.$$

where L_C^A (L_C^H) denotes the amount of time ("**L**abor time") devoted by Alex (Harry) to C production, and L_A^V (L_V^H) denotes the amount of labor time devoted by Alex (Harry) to V production. The total amount of work time available to both Alex and Harry (separately) is exogenously given as one unit, e.g., one day, or one week. We represent this symbolically as

$$L_A^V + L_A^C = 1;$$
$$L_H^V + L_H^C = 1.$$

We subscript C and V by "A" or "H" to indicate this is output by Alex or Harry, respectively, and we superscript them by "S" to indicate this is the amount "supplied" or produced by Alex (Harry); this distinguishes this variable from amounts bought and consumed by Alex (Harry).

Alex's opportunity cost of producing one unit of C is 0.1826 units of V. Alternatively, Alex's opportunity cost of producing one more unit of V is $\frac{1}{0.1826}$ units of C ($\frac{1}{0.1826} = 5.4765$). If we denote Alex's opportunity cost of producing one more unit of C as OC_{VC}^A, we would write the first sentence of this paragraph in symbolic form as

$$OC_{VC}^A = 0.1826 \textit{ units of } V/\textit{Unit of } C.$$

If Alex spends all his available work-time making C, he produces one unit; if he spends all his available time producing V, he produces 0.1826 units. If he spends half of his time at each task, he produces one–half unit of C and 0.0913 units of V. If he spends $\frac{1}{4}$ of his time producing C and $\frac{3}{4}$ of his time producing V, he produces $\frac{1}{4}$ unit of C and 0.04565 units of V. Assuming he can divide his time as finely as he might like between the two tasks, the possible pairs of output he can produce in a day are described by the function (we refer to this as Alex's *individual* Production Possibilities Frontier, aka PPF):

$$V_A^S = 0.1826 - .1826 C_A^S.$$

Harry produces one (1) unit of C per unit of time spent in C production, and (1) unit of V per unit of time spent in V production. That is, if Harry spends all his available work-time making V, he produces one unit; if he spends all his available time producing C, he produces one unit. Alternatively, we can say:

Harry's opportunity cost of producing one more unit of C is one (1) unit of V, and vice-versa. Writing this sentence in symbolic form, we have

$$OC_{VC}^H = 1 \textit{ unit of } V/\textit{unit of } C$$

By the same logic we used with Andy, we can describe the possible pairs of output Harry can produce in a day by the function:

$$V_H^S = 1 - C_H^S.$$

If these two could work fractions of time on each task, their potential pairs of output would lie along the two *individual* PPF's depicted in Figure 1:

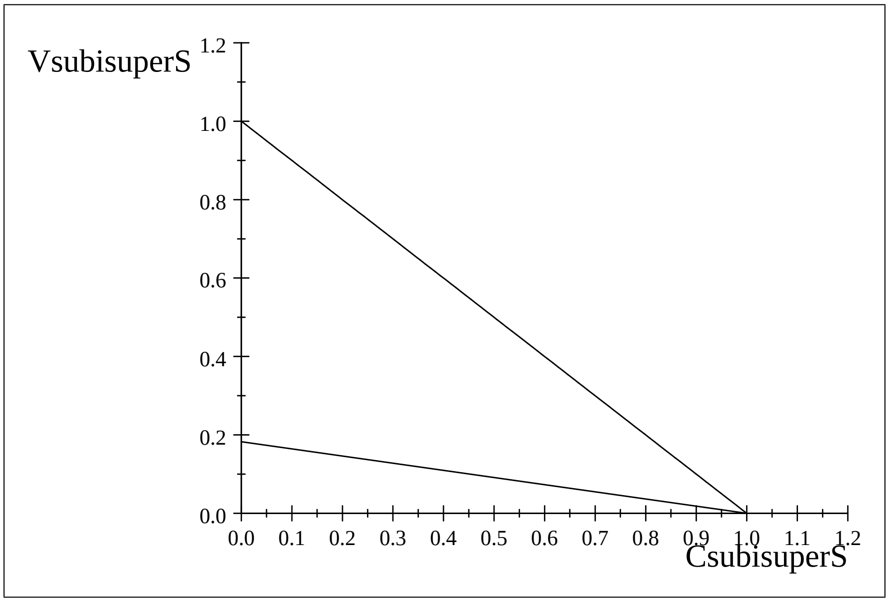

Figure 1

To motivate what follows, first suppose Alex and Harry cannot trade with each other. Alex (likewise for Harry), can, by making choices about how much of his time to allocate to each production activity, have at the end of the day any ordered pair (C_i, V_i) from along his/her PPF (reproduced here for convenience).

$$\begin{aligned} V_A &= 0.1826 - .1826C_A \\ V_H &= 1 - C_H. \end{aligned}$$

If each person is an island, how much of each product does each individual produce? Until now, we have been fairly silent about preferences, and we will continue to be so (you will learn more in Intermediate Micro). But for illustrative purposes, suppose in this *autarkic* state Alex likes to consume the feasible ordered pair $(\frac{1}{2}, 0.0913)$ and Harry likes to consume $(\frac{1}{2}, \frac{1}{2})$. These points are displayed in Figure 2.

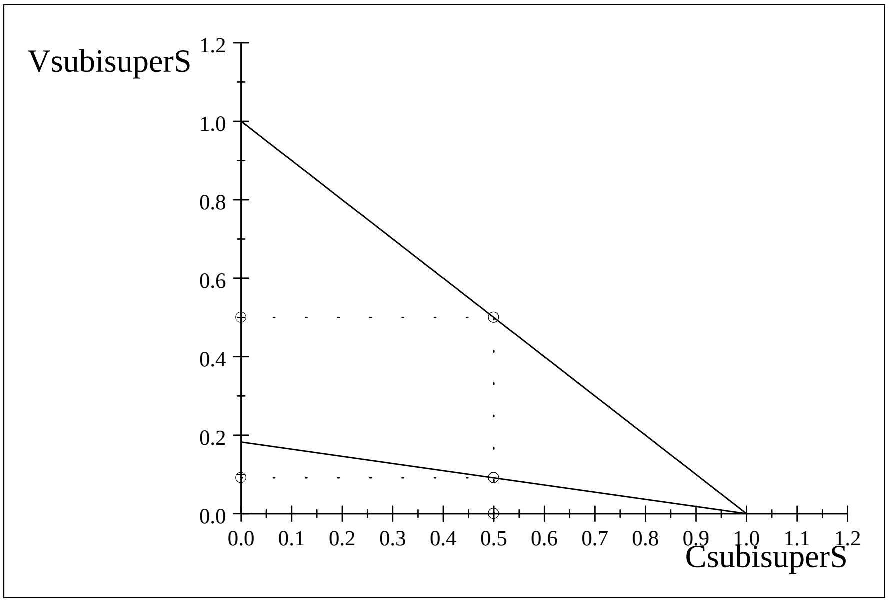

Figure 2

$\frac{1}{2\times0.1826} = 2.738\,2$

Suppose now someone comes to them and says,

> Y'all each own whatever you produce, but can exchange with each other at rate $\frac{1}{2}$ unit of V for 1 unit of C, i.e., at the "market price" $\frac{P_C}{P_V} = \frac{1}{2}$.

Alex says in self-conversation: "If I produce one unit of C, i.e., if I allocate all my time to C production, I can exchange half of it with Harry for $\frac{1}{4}$ unit of V. This gives me the ordered pair $(\overbrace{\frac{1}{2}}^{C_A}, \overbrace{\frac{1}{4}}^{V_A})$ which has the same amount of C and more of V than I had in autarky. I am better off specializing in C and exchanging C for V at the relative price $\frac{P_C}{P_V} = \frac{1}{2}$ (Alex knows that $\frac{1}{4} > 0.0913$) than if I produced both of them myself. I would be better off–more is better." In Figure 3, the red line displays Alex's budget constraint when he specializes in C and can trade at $\frac{P_C}{P_V} = \frac{1}{2}$. Alex's new possible consumption point $(\overbrace{\frac{1}{2}}^{C_A}, \overbrace{\frac{1}{4}}^{V_A})$ is also displayed.

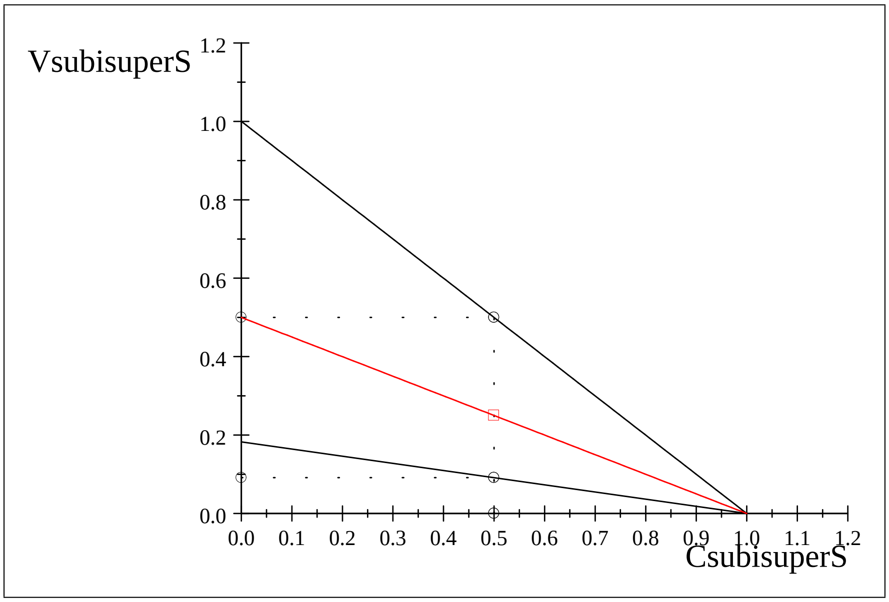

Figure 3

Harry, on the other hand, says in self-conversation: " If I produce one unit of V, i.e., if I allocate all my time to V production, I can exchange $\frac{1}{4}$ of it with Alex for $\frac{1}{2}$ unit of C, leaving me to consume the ordered pair $(\frac{1}{2}, \frac{3}{4})$, which is the same amount of C that I had in autarky but more V $(\frac{3}{4} > \frac{1}{2})$. I would be better off–more is better!" Harry's budget constraint with her specialization in V and with $\frac{P_C}{P_V} = \frac{1}{2}$, and with her consumption point $(\frac{1}{2}, \frac{3}{5})$ is displayed in Figure 4.

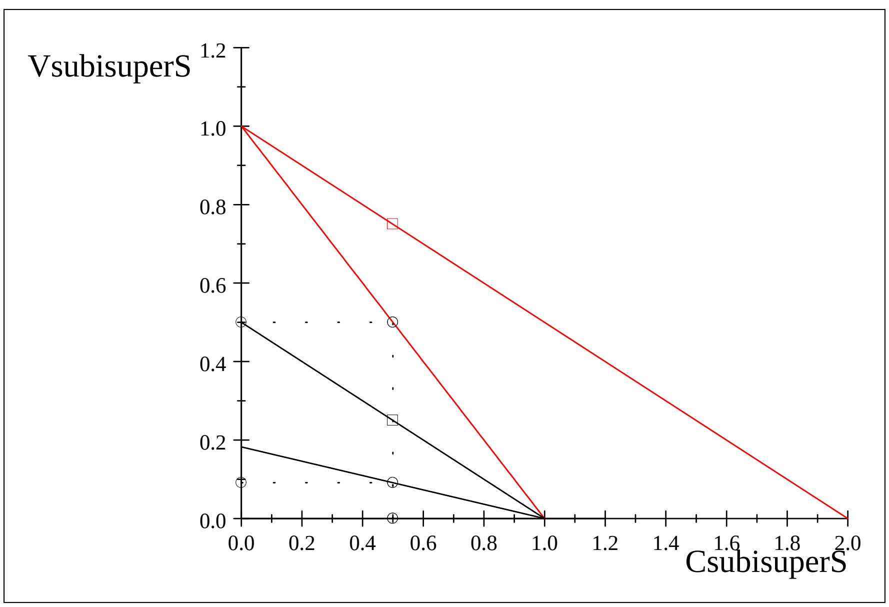

Figure 4

1 Constructing the inverse supply function

Now consider other **relative price values**, that is, other rates at which one can exchange V for C. As long as this rate of exchange is greater than .1826 but less than 1, both Alex and Harry will want to specialize in production (allocating all of their time to producing one good): Alex specializing in producing C (producing one unit) while Harry specializes in producing V (producing one unit). If this relative price is less than .1826, *both* people specialize in fish production (Alex producing 0.1826 fish and Harry producing one fish, for a total of 1.1826 fish); if the price is greater than one, both specialize in coconut gathering (producing one coconut each). If the relative price is 0.1826, Harry specializes in fish, and Alex is indifferent between producing any amount of coconuts between zero and one. If the relative price is one (1), Alex specializes in coconuts, producing one (1), and Harry is indifferent between producing any amount of coconuts between zero and one (1). We can summarize this as an *inverse supply function*, which answers the question: For any permissable value of $\frac{P_c}{P_V}$, what quantity of coconuts will be produced?

1. If $\frac{P_C}{P_V} < 0.1826,\ C = 0$
2. If $\frac{P_C}{P_V} 0.1826,\ C \in (0,1)$
3. If $0.1826 < \frac{P_C}{P_V} < 1, C = 1$
4. If $\frac{P_C}{P_V} = 1,\ C \in (1,2)$
5. If $\frac{P_C}{P_V} > 1,\ C = 2$

We can depict all this in the graph of the *inverse supply function*, depicted in Figure 5.

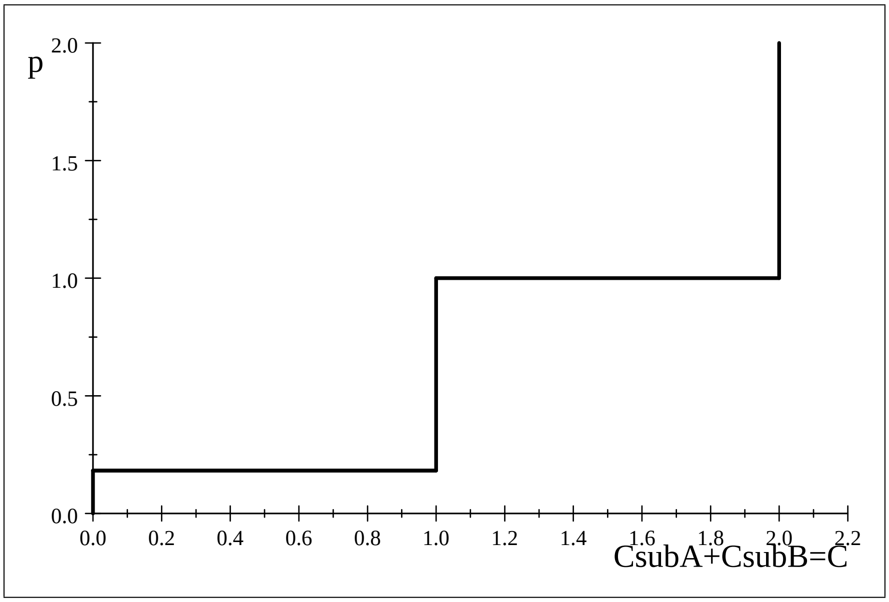

Figure 5

2 Demand and market equilibrium

We don't do much about preferences in this class; a full-blown model, based on axioms generated by introspection and observation, will come your way if you take intermediate micro. But we can appeal to the law of demand, and assume a downward-sloping inverse demand curve. We superimpose one such curve on the inverse supply curve diagram in Figure 6.

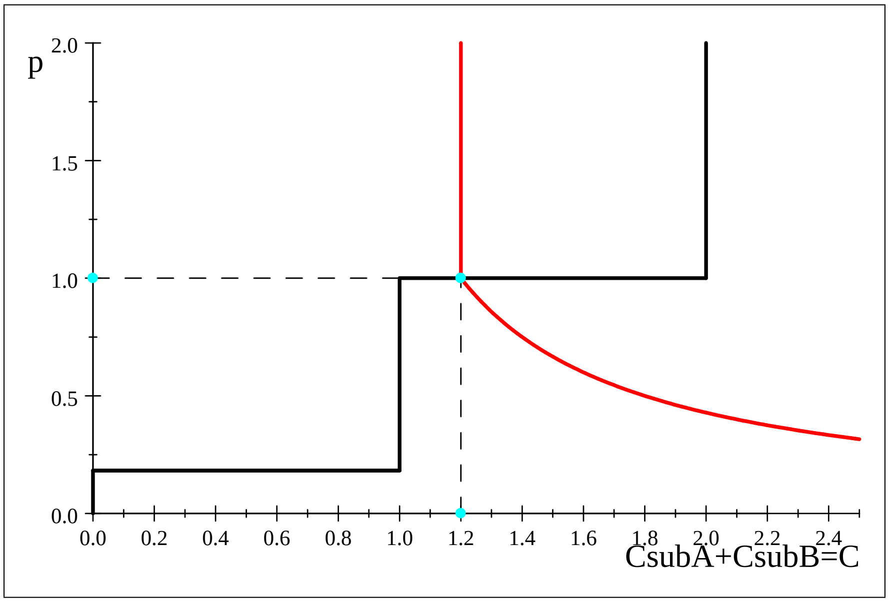

Figure 6

Equilibrium is a pair (C, p) that satisfies (simultaneously) both demand and supply. In this case, it is $(\overbrace{1.2}^{C}, \overbrace{1}^{p})$.

2.1 Optional: a little about demand

Are you a real nerd, and want to know how we get this demand curve? We cannot do the entire derivation here, but the key new idea is that preferences are represented by the utility function $U_i = (C_i)^{.6}(V_i)^{1-.6}$. This implies each individual has a family of indifference curves, i.e., a family of non-intersecting downward-sloping curves in the $C - V = C - V$ plane, in which each individual curve describes all those pairs of coconuts and fish which the individual is equally happy consuming. They look like this (Figure 7):

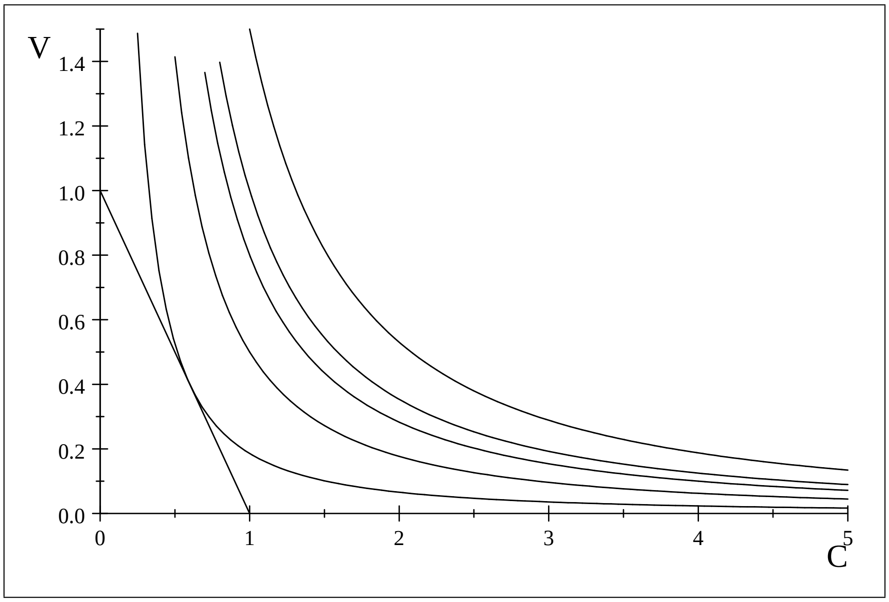

Figure 7

We have superimposed these indifference curves on Harry's PPF. Where his PPF "just touches" an indifference curve (at the point $(.6, .4)$ in this picture) is the one point on his PPF that gives him the most satisfaction.

Intrigued? Take intermediate micro!

3 Economy PPF:

Think of yourself as a planner: How would you maximize the amount of V you could have this two-person economy produce, given resources and technology, for any given amount of C?

The thought experiment: Think at the margin; think in terms of opportunity cost: First, have Alex and Harry devote all their time to V production, yielding two units of V and no units of C. Then think about producing, say, $\frac{1}{10}$ unit of C. What is the most V our economy can produce given it produces $\frac{1}{10}$ unit of C? Let us reproduce here the information on each individual's opportunity cost of producing cloth:

$$\begin{aligned} OC_{VC}^{A} &= 0.1826 \textit{ units of } V/\textit{Unit of } C; \\ OC_{VC}^{H} &= 1 \textit{ unit of } V/\textit{Unit of } C \end{aligned}$$

Alex's opportunity cost of producing one unit of cloth is $\frac{1}{0.1826} = 0.1826$ units of V. Hence, Alex's opportunity cost of producing $\frac{1}{10}$ unit of C is $(0.1826)\left(\frac{1}{10}\right) = .01826$ units of V. For Harry, though, the opportunity cost of producing 1 unit of C is one (1) unit of V. Hence, the opportunity cost of having Harry produce $\frac{1}{10}$ unit of C is $\left(\frac{1}{10}\right)(1) = .1$ unit of V.

If the planner had Alex produce the $\frac{1}{10}$ unit of C, our economy would give up .01826 units of V. But if the planner had Harry produce the $\frac{1}{10}$ unit of C, our economy would give up 0.1 units of V. Clearly, a planner should keep Harry producing V while using Alex to produce the $\frac{1}{10}$ unit of C. And he should continue to use Alex to produce more and more C until Alex has no more time left to produce anything. At that point, our planner would start having Harry use his time to produce more C (and less V) until our economy is producing all C and no V.

The economy PPF would be composed of the two black line segments in Figure 8.

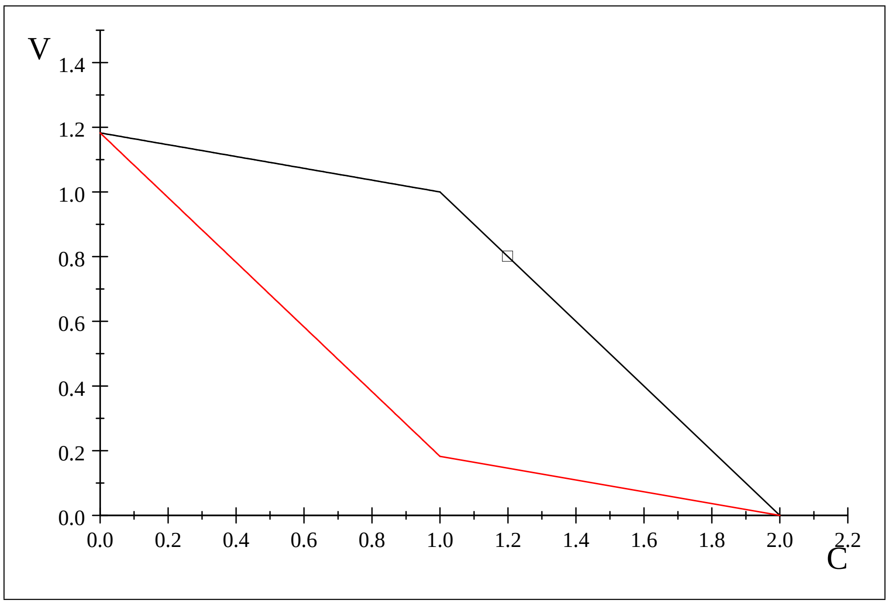

Figure 8

4 Price as guide

Suppose you, as a central planner, want to make sure your economy produces the maximal amount of fish for any stipulated amount of coconuts. That is, you want to make sure your economy is somewhere on the PPF. Perhaps you want to have your economy produce 1.2 coconuts, and the maximal amount of fish associated with that amount of coconuts. To do so, you would need to find out who has the lower opportunity cost of producing coconuts. For two people, this might not be a difficult task. But if we expand our imaginations and think of a world with thousands of individuals (or even 8), this becomes a mighty task. The two red lines in Figure 8 would be all those pairs of coconuts and fish produced if we put the wrong individual producing the wrong stuff.

With a market, though, there is no need to study the relative opportunity costs of different individuals: the relative price, $\frac{P_C}{P_V}$, sorts that out. Pursuing their own self-interest, they respond to the price signal and specialize so as to insure that the economy's point of production is on the PPF, not inside of it. The point of production generated by the intersection of the inverse demand curve we drew in Figure 6 with the inverese supply curve generates 1.2 coconuts and .8 fish: with $\frac{P_C}{P_V} = 1$, Alex knows to specialize in coconuts, and Harry produces .8 coconuts and .8 fish.

Note also that the price that equates demand to supply–one unit of fish for one coconut–is also the opportunity cost this economy faces if it wants to have one more coconut produced. This is a general feature of "perfect" competitive markets, e.g., no monopolies, no externalities (like traffic congestion), no neccesity for public goods.

5 Eight People

We now know how to generate on inverse supply function. Let's expand our toy model to eight people. Each has one day (say, eight hours) to spend in working on either coconut collecting or fishing. In Table 1 we show their outputs/day of the two goods if they specialize, their respective opportunity costs of producing an extra coconut, and their individual PPF's.

	C_i/day	V_i/day	$OC_i(V_i/C_i)$	$PPF's$
Alex	1	0.182 6	0.182 6	$V_A = 0.1826 - 0.1826C_A$
Bobby	1	0.196 3	0.196 3	$V_B = 0.1963 - 0.1963C_B$
Charley	1	0.213 4	0.213 4	$V_C = 0.2134 - 0.2134C_C$
Danny	1	0.236 1	0.236 1	...
Evelyn	1	0.267 9	0.267 9	...
Fran	1	0.317 9	0.317 9	...
Georgie	1	0.4142	0.4142	...
Harry	1	1	1	$V_H = 1 - C_H$

As $\frac{P_C}{P_V}$ rises above 0.1826,but is below 0.1963, Alex specializes in coconuts but everyone else specializes in fish. As $\frac{P_C}{P_V}$ rises above 0.1963 but is below 0.2143, Alex and Bobby specialize in coconuts and the rest in fish. Continuing this thought experiment (contemplating higher and higer prices, and how this induces more people to specialize in coconuts), we can trace out the inverse supply function dispalyed in Figure 9. Superimposed upon it is an inverse demand function. The equilibrium price is $\frac{P_C}{P_V} = 0.4142$.

We can plot the economy-wide PPF from these numbers as well. This is depicted in Figure 10. Note how it now looks more like a "smooth" concave function like Figure 2.1 (p. 31) in the textbook. Note also that the slope of the PPF–measured as the vertical distance between two adjacent points divivded by the horizontal distance (which is one coconut)– is the opportunity cost to this economy of producing one more coconut. Finally, note that the equilibrium price–the amount of fish that exchange for one coconut–is in fact the slope of the PPF at the point $C = 7$, i.e., the point where the equilibrium quantity of coconuts is being produced.

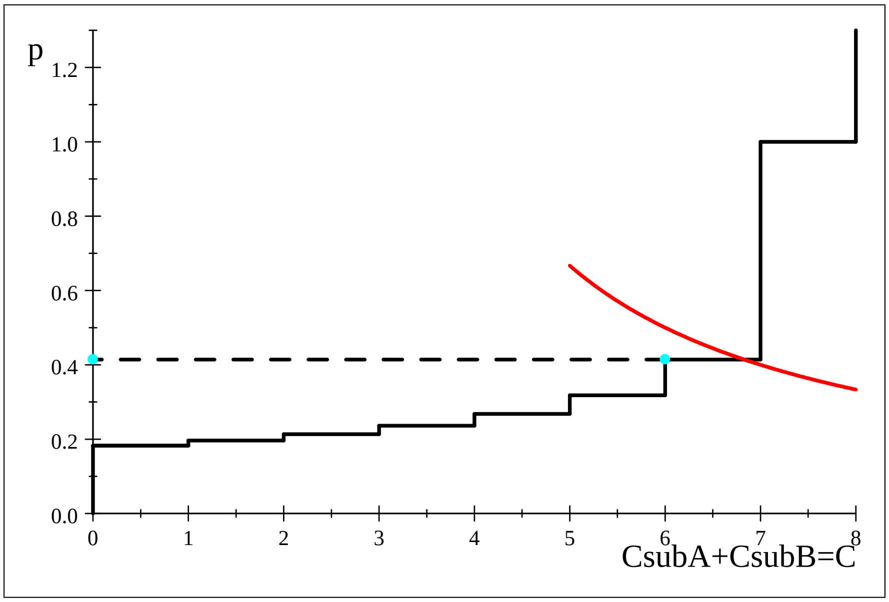

Figure 9

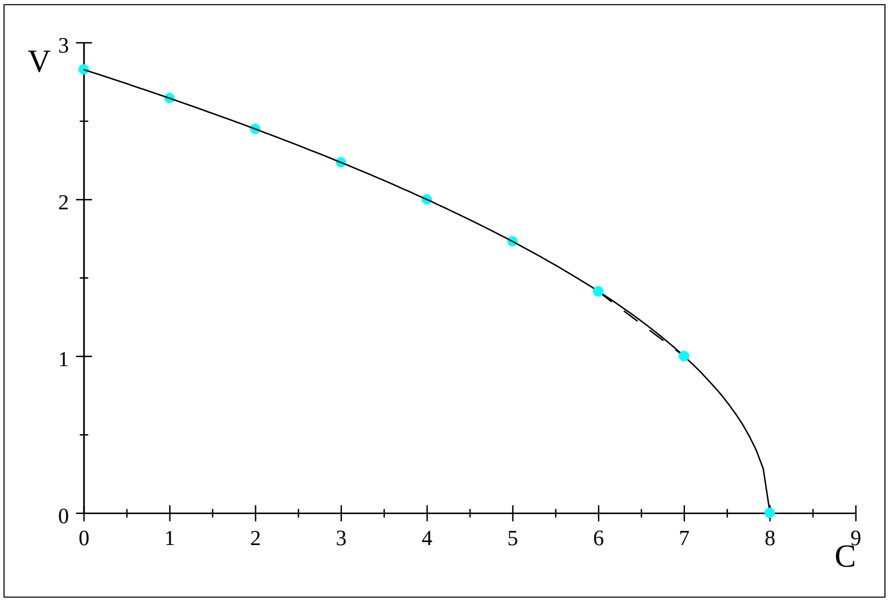

Figure 10

6 Shifting the supply function

Suppose everybody gets better at fishing:

	C_i/day	V_i/day	$OC_i(V_i/C_i)$	$PPF's$
Alex	1	0.2	0.2	$V_A = 0.2 - 0.2\,C_A$
Bobby	1	0.3	0. 3	$V_B = 0.3 - 0.3C_B$
Charley	1	0.4	0. 4	$V_C = 0.4 - 0.4C_C$
Danny	1	0.5	0.5	...
Evelyn	1	0.6	0.6	...
Fran	1	0.7	0.7	...
Georgie	1	0.8	0.8	...
Harry	1	1.1	1.1	$V_H = 1.1 - 1.1C_H$

This shifts the inverse supply function "up" or "to the left."

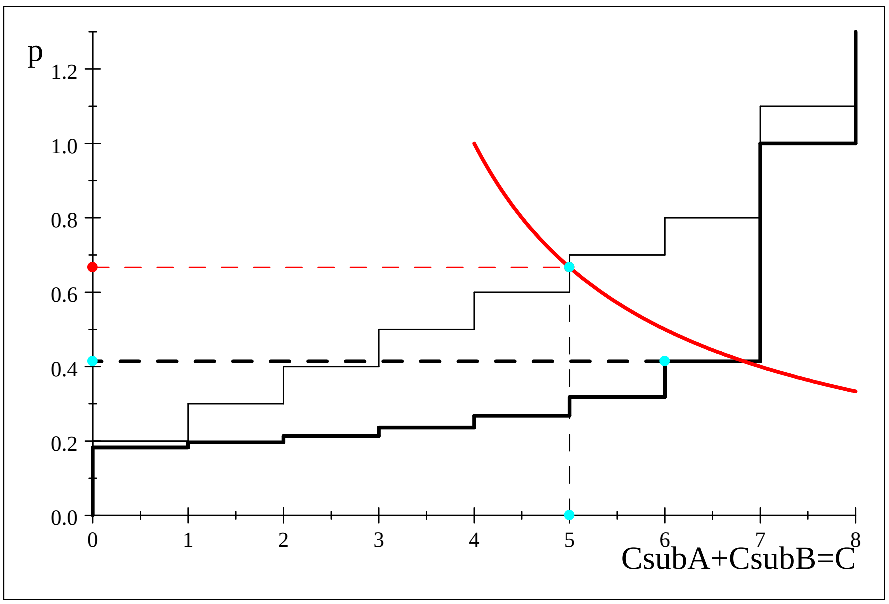

Figure 11

New equilibrium is $(5, \frac{2}{3})$.

7 Moving to the $C - P_C$ plane

Up until now, our inverese supply function has been designed to answere the question: Ceterus paribus, for any feasible value of $\frac{P_C}{P_V}$, i.e., the market-determined amount of fish that exchange for a unit of coconuts, what is the quantity of coconuts that are supplied? Note: we assume Alex and Harry take $\frac{P_C}{P_V}$ as given, or equivalently, as exogenous. Also note, the ceterus paribus assumption means we are assuming that the other exogenous components of the model are unchanging: technology doesn't change, the amount of resources available (the amount of time available to work) doesn't change.

What would our inverse supply function look like if we followed the practice in our book and used our inverse supply function to ask instead: Ceterus paribus, for any feasible value of P_C, i.e., the market-determined **currency price** of coconuts, what is the quantity of coconuts that are supplied? Now, though, the ceterus paribus assumption includes the assumption that P_V, the currency price of fish, is held constant.

We continue to assume individuals take all prices as given. But now we separate out the two currency prices. That is, Alex and Harry take both P_C and P_V as given. To fix ideas, suppose that P_V is fixed one: $P_V = 1$. This implies that $\frac{P_C}{P_V} = P_C$. Hence, our inverse supply function sits in the exact same position in the $P_C - C$ plane as it did in the $\frac{P_C}{P_V}$ plane!

Now, what happens if P_V changes? Again to fix ideas, suppose it doubles, to $P_V = 2$. What happens to our inverse supply function?

Remember, Alex's decision to either produce any amount of coconuts between zero (0) and one (1), or to only specialize in coconut production, depends only on whether $\frac{P_C}{P_V}$ is equal to or greater than her opportunity cost of producing one more unit of coconuts. If, as in our most recent example, Alex only produces coconuts if $\frac{P_C}{P_V} \geq .2$, then with $P_V = 2$, she requires $P_C \geq .4$.

Likewise for everyone else: because the decision to produce any coconuts depends only on whether the **relative** price of coconuts is greater than or equal to the individual's opportunity cost, the associated threshold level of P_C has doubled.

So, our inverse supply function in the $C - P_C$ plane "shifts up" or, equivalently, "shifts back", as illustrated in Figure 12.

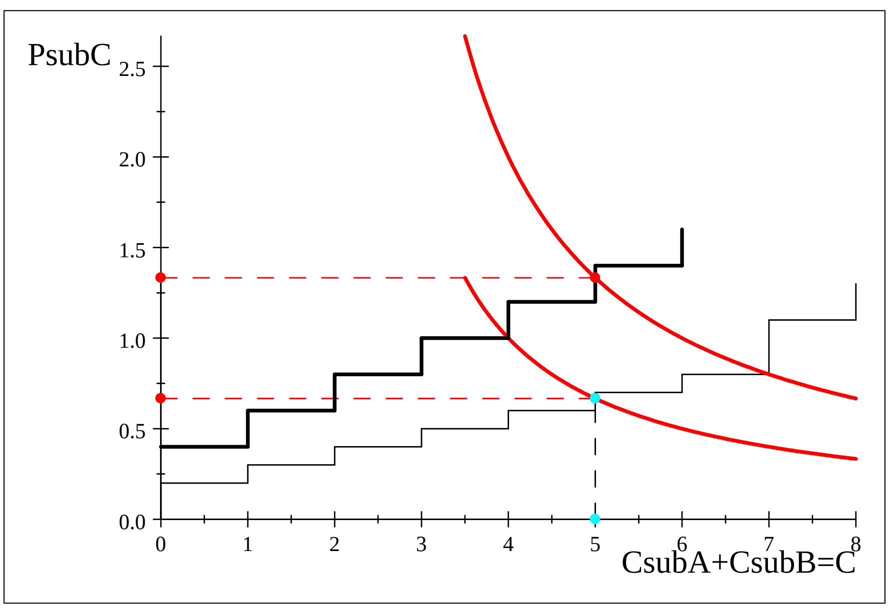

Figure 12

How about demand? The same logic applies: the quantity of coconuts demanded depended on $\frac{P_C}{P_V}$. If P_V doubled from one (1) to two (2), the price P_C

associated with a particular quantity of coconuts demanded will have doubled. The inverese demand function will have "shifted up" or equivalently "shifted out" by an amount that will make demand equal supply at the same quantity of coconuts and an equilibrium price P_C that ahs also doubled. This is illustrtated as well in Figure 12.