The Specific and Mobile Factors Model

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1 Learning Objectives

1. Understand how economists model production.
2. Understand the concept of comparative advantage.
3. Understand how trade affects the distribution of income among the specific factors of production and the mobile factor of production.
4.

2 Introduction

Because Great Britain is an island, its people and governments have long been concerned with whether the country could feed itself if a hostile power were to impose effective sanctions such as a blockade that would limit food imports. In the period surrounding the Napoleonic wars, this concern led to passage of the so-called Corn Laws. The Corn Laws imposed a tariff on imports of grain, and were designed to increase production of grain within the island. A tariff raises the domestic price of a good and thus increases its domestic production.[1]

No doubt the Corn Laws did lead to more grain production than would otherwise have occurred. They also led to a shift in the distribution of income. In particular, landowners prospered relative to manufacturers. This redistribution of income in turn became part of a political struggle to eliminate the Corn Laws in the post-Napoleonic era.[2]

This episode highlights the effects of trade and trade policy on the distribution of income to various factors of production in an economy in which

[1] In fact, the story of Britain's Corn Laws is much more nuanced and fascinating than this short description might suggest. The end in 1815 of the Napoleonic Wars between France and Great Britain coincided with a steep increase in tariffs on grains. This policy was motivated largely by a desire on England's part for food self-sufficiency, a desire born of wartime shortages. See Mancur Olson's interesting account of this time in his *The Economics of the Wartime Shortage* (1963, Duke University Press, Durham, NC)

[2] One might wonder why the tariffs weren't immediately removed once the reason for such tariffs, namely fear of interference by France, had disappeared. As we will learn in a later chapter on the politics of trade, this difficulty in changing a policy back after the precipitating event is over that caused the policy in the first place is both common and foreseeable.

different goods are produced using varying amounts of these factors. Such issues command headlines to this day. Recall, for example, the issue of granting Permanent Normal Trade Relations status to China referred to in chapter one. Granting such status led to increased imports of textiles from China, decreased production of textiles in the U.S., and a reduction in the incomes of those people whose livelihoods are tied to the specific factors used to produce textiles. To understand such phenomena, we obviously need a model richer than that of the endowment economy, a model in which production occurs.

Such a model not only allows us to understand issues of income distribution but also gives us insight into one of the oldest ideas about why nations trade and why they benefit from such trade. This idea is the concept of comparative advantage. It was developed by the English economist David Ricardo (1772-1823) in his classic 1817 book, *The Principles of Political Economy and Taxation*. Ricardo developed this concept in part as a tool to be deployed in the fight to repeal the Corn Laws. As we will see, the concept is much more general than Ricardo's initial examples might suggest, and connects with one of the few truly key ideas in economic thought, the concept of opportunity cost.

To elucidate these ideas, we develop a model here in which, in contrast to the endowment economy model, production occurs. Like the endowment economy model, it assumes a market economy characterized by perfect competition, and it is a general equilibrium model. The key difference, of course, is the introduction of production, which entails a new, additional sub-model. This sub-model is linked and combined with a sub-model of consumer behavior to create the complete general-equilibrium model for an autarkic economy. The exogenous components of this complete model are: consumer preferences, just as with the endowment economy model; production technologies, which we can think of as blueprints or technical specifications that tell us the various ways that labor and capital can be used to produce outputs; and resources, which are the total amounts of labor and capital available to the economy. The endogenous variables in this model are the relative prices that equate demand and supply for commodities, the equilibrium quantities of commodities produced and consumed, the prices of factors of production that equate demand and supply in factor markets, the allocation of factors among different productive activities, and the incomes of various factors of production.

As with the endowment economy chapter, our strategy here will be to build two autarkic economy models and then to analyze the effects on the various endogenous variables of a change from autarky to free trade. Our goals will be to see what can be said about the features of each individual economy that determine the pattern of trade between the two and to see how trade leads to gains from trade (in the same sense in which we used this phrase in the chapter on the endowment economy).

To start, we introduce the way in which economists model production in a market economy. First we discuss the idea of a firm and how we model a firm's decision-making processes. We then introduce the concept of a production function, and show how a firm's profit-maximizing decisions lead to demands for labor that are contingent on the relative price of the final outputs. This allows us to show how labor market equilibrium determines a *general equilibrium* supply function and a distribution of income contingent on the relative price of output. We then construct our submodel of demand. In contrast to the endowment economy model, income for individuals in this sub-model is derived from and depends upon their role in the production process. Equilibrium in the market for commodities then determines the relative price of commodities. Knowing the value of the relative price as a function of only exogenous factors, the values of all other endogenous variables can then be determined.

This is a complicated model. Based on exogenous specifications of preferences, resources, and technologies, we determine the equilibrium values of all quantities produced and the relative price at which they exchange, the equilibrium values of incomes that accrue to the distinct factors of production, and the equilibrium values of quantities consumed by each individual. This sets the stage for an analysis of the concept of comparative advantage and an analysis of gains from trade.

Because by its very nature this is a complicated model, we, as usual, keep things as simple as possible by limiting the dimensionality of our model. Hence, we assume that in each of only two countries only two goods are produced - wine, symbolized by V when produced in the home country and V^* when produced in the foreign country, and cloth, symbolized by C when produced in the home country and C^* when produced in the foreign country. We use wine and cloth as our examples instead of coffee and tea as was used in the endowment economy model as a way of remembering that production adds new phenomena to the issues of international trade. It is also a way to honor the original contribution of Ricardo, whose example of comparative advantage used wine and cloth as the products produced in England and Portugal. Production of each good requires only two factors of production: labor, a perfectly homogeneous factor that is *mobile* between sectors within a country but not across national borders; and capital, a factor that is *specific* to each industry in each country. That is, wine, for example, is produced in the home country with labor that is at work in the home-country wine industry, the quantity of which is symbolized by L_V, and with a fixed and *specific* factor, "wine production capital", the quantity of which is denoted by $\overline{K}_{V.}$

3 Modeling production in a market economy: general equilibrium supply functions

3.1 Firms and firm behavior

In contrast to the endowment economy, the goods that people consume in a production economy are somehow created from more primordial material. Most of us have personal experience with this process and understand that "inputs" are used in a process known as production to produce an output that people consume. For example, your local garage uses the labor of mechanics, various types of machines, tools, and buildings, and the labor of other workers such as bookkeepers, to produce a final output, "car repairs and maintenance." Even for such a relatively small enterprise, the decisions made by the garage owner are myriad and complex: How many mechanics to hire, how many bays to operate, what range of services and repairs to offer, and so on. Furthermore, other decisions were made that led to the existence of the garage in the first place: Why have a garage with many employees rather than have every mechanic operating his or her own shop; why have a garage and not a fast-food emporium or a carpet-cleaning service? How do economists abstract from all this complexity and model the production process?

The first building block of this model of production is a firm. Again, most of us have experience with this concept. For economists, the key questions about firms are: why do they exist, what are their motivations, and what decisions do they make and how do they make them?

3.1.1 Why firms exist

In a commercial economy - an economy in which economic interactions are mostly impersonal transactions -the question arises of why people are organized into firms. Why don't people (or small groups of related people such as households) simply produce and sell items for the marketplace? The basic reason they don't, but rather become members of a firm, is that many of the interactions among people that are necessary for production to take place are not well-suited to market exchanges. An enormous literature explores and elaborates on this idea. For our purpose of providing the skeletal framework necessary to understand and appreciate the economist's approach to international trade issues, a simple analogy provides the basic insight.

First note that some things are not produced by what we think of as firms. For example, many people have their grass cut by one person who acts as an independent contractor. Jobs like this are easily handled by one or a few people because they simply don't call for much coordination. The lone entrepreneur can purchase all the inputs for the production process–gasoline, lawn mowers, brooms, etc., in the marketplace, and make all decisions by him or her self.

In contrast, think about production of something like an automobile. While cars can be produced by one or a few individuals (some cars are in fact "handmade" in small shops), mass-produced cars require a lot of coordination and trust between many people. Running a production line involves having a large number of people at work at the same time and place. Furthermore, even for low-skill assembly-line jobs, a person can't be hired off the street on a day-today basis to do these jobs: most of these jobs require workers to have about three weeks of practice before they are competent. In such a situation, workers and firms must have a level of mutual trust that the employment relationship, once started, will not be arbitrarily and quixotically severed.

For production like this to take place, people must act as if they are members of a team. A team requires members to trust one another, to "do their part" without constant negotiation and renegotiation, and to accept instructions from a "coach" who places the interests of the team as a whole above the interests of any one member of the team.

Consider a sports analogy that may help convey this concept. A basketball team, for example, never develops proper "chemistry" if the players don't stay together as a team for a long enough time. The players need to learn to work together, and this takes time. A good team can't be had by simply hiring new people every day.

There is another aspect of a basketball team that is analogous to a business firm. On a basketball team there may be one player who is better in an absolute sense than any of his teammates regardless of position. That is, he or she is the best center, the best forward, the best guard, and so forth. Furthermore, this person's own individual self-interest might be best served by playing the position at which he or she is more accomplished - say point guard, for example. This player may prefer this for a variety of reasons: point guards might get paid more around the league than shooting guards, or the player just likes doing what he or she does best. Because this individual is a member of a team, though, the coach may assign this player a different role: one that maximizes the *team's* chances of winning. For example, the team's second-best point guard might be better than the team's second-best power forward, and the team's success is better served by having the star play power forward.

All of this discussion is designed primarily to motivate the idea that it makes sense to think of production as being organized by an entity we call a "firm" that purchases inputs and transforms them into output. Such an entity can be thought of as having a goal and/or motivation independent of the goals and motivations of individuals who work for the firm.

3.1.2 Goals of a firm

What are the goals of a firm? Economists assume that firms attempt to maximize profits. The justification for this starkly simply assumption is that it has proved to be a most useful simplification in that it has led to a multitude of verified predictions about firm behavior. Profits are simply total revenues from sale of output minus total costs incurred from purchasing inputs such as labor and raw materials.

3.2 How firms maximize profits: technology and the profit-maximizing rule

3.2.1 Production technology: outputs and inputs

Consider E&J Vintners, a (fictitious) firm we assume was started and is run by two Italian brothers (Ernest and Julio, hereafter known as E and J). We assume E&J's less-expensive products, if they really existed, would undoubtedly be well-known to college students everywhere. This firm produces a variety of wines, e.g., high-priced varietals and low-priced blends, using a variety of inputs: low-skilled workers, complex machinery that crushes and strains grapes, vineyards on which the grapes are grown, skilled labor in the form of winemakers, and a host of other things. We abstract from this large array of outputs and inputs and assume that a uniform product, i.e., one type of wine, is produced using just two inputs: capital (which we denote by K_V, the K a mnemonic for Karl Marx's *Das Kapital*, and the V subscript to identify this capital as being used in wine-"vino"- production) and labor (which we denote by L_V). Labor is assumed to consist of workers who all have identical skills, skills that are not specific to the wine industry. That is, these workers all have the same *general skills*, e.g., literacy, that allow them to work in virtually any industry. Capital, on the other hand, is tailor-made for the wine production process, and can't be used in any other industry. We call such an input a *specific* factor of production.

3.2.2 Production functions

E and J are assumed to have access to a best-practice technology for producing wine from their inputs. That is, they are assumed to know how to organize their production processes so as to get the maximum possible amount of wine per unit of time from any amount of capital and labor. We can think of this technology as being a set of engineering plans and an associated table which lists the maximal output of wine associated with every permissible pair of values of labor and capital. Each element of this table can be thought of as an ordered triplet, where the first number is the wine output per unit of time, the second number is the amount of labor used during the time interval, and the third element is the amount of capital used during the time interval. The set of all these ordered triplets is a three-variable function. Such a function is denoted as a *production function*.

As we have emphasized before, tables are cumbersome ways to represent functions. More insightful for our purposes is to assume that the variables in a function, both the arguments of a function and the variable that is the function of the arguments, can take values from the non-negative segment of the real line. Again, we should think of these continuous variables as approximations of discrete variables. By abstracting from the discreteness of most economic variables, we allow ourselves to use the tools of analytic geometry and to avoid the cumbersome requirements of keeping track of a large but finite number of units.

Hence, we represent our production function in mathematical notation as

$$V = f(L_V, K_V), L_V \geq 0; K_V \geq 0. \tag{6.1}$$

where L_V and K_V can take on values represented by any non-negative real number. This is a three-variable function, and the ordered triplets that are its members can be depicted as points in a three-dimensional diagram. There are a few key properties that we assume any production function must have. To understand these properties, we first illustrate them with a parametric example.

Production function properties illustrated with the Cobb-Douglas function The example we choose is the Cobb-Douglas production function.

This parametric example has the following specific functional form:

$$V = A_V \left(K_V\right)^{1-\theta_V} \left(L_V\right)^{\theta_V}, 0 < \theta_V < 1,\ A_V > 0. \tag{6.2}$$

In this example, parameters - numbers given to us exogenously - are A_V and θ_V. For expository simplicity, let us set $A_V = 1$ and $\theta_V = \frac{1}{2}$. The key properties we want to illustrate hold for any values of $A_V > 0$ and $0 < \theta_V < 1$, but actual numbers may be helpful as an aid to understanding. With these particular parameter values, the wine production function is now:

$$V = \left(K_V\right)^{\frac{1}{2}} \left(L_V\right)^{\frac{1}{2}} \tag{6.3}$$

The three-dimensional picture of this function is displayed in 1.

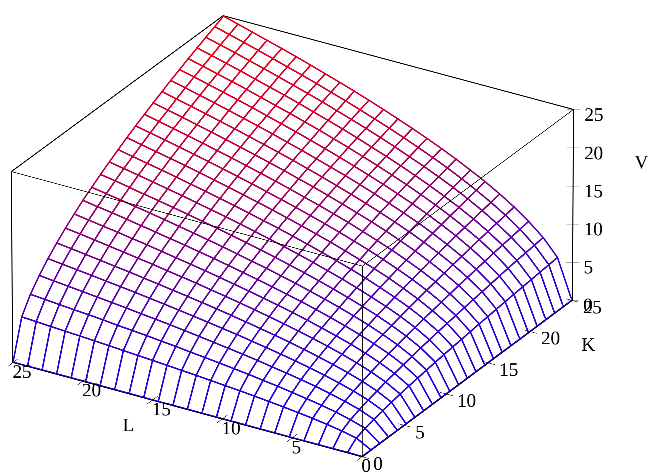


Figure 1: Cobb-Douglas production function

Notice that more inputs are associated with more output. That is, if we compare any two ordered pairs of inputs $((L_V)_1, (K_V)_1)$ and $((L_V)_2, (K_V)_2)$ such that both $(L_V)_2 > (L_V)_1$ and $(K_V)_2 > (K_V)_1$, then $V((L_V)_2, (K_V)_2)$, the output associated with $((L_V)_2, (K_V)_2)$, is greater than $V((L_V)_1, (K_V)_1)$, the output associated with $((L_V)_1, (K_V)_1)$. This is a general feature of any production function.

A second general feature illustrated by this example is the property of *constant returns to scale*. If both inputs are increased in the same proportion, then output increases in the same proportion. For example, if we have an initial condition in which capital and labor are sixteen (16) and twenty-five (25) units, respectively, then output is twenty (20) units of wine per unit of time:

$$V = (\sqrt{16})(\sqrt{25}) = 4 \times 5 = 20.$$

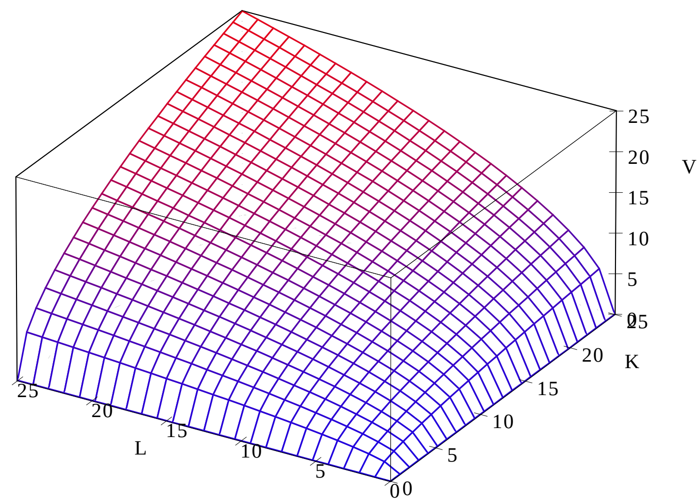


Figure 1: Cobb-Douglas Production Function

Now imagine that both factors increase by a factor of four (4), so that there are now sixty-four (64) units of labor and one hundred (100) units of capital. Output has also increased by a factor of four:

$$V = (\sqrt{64})(\sqrt{100}) = 8 \times 10 = 80.$$

One reason economists believe production functions should exhibit constant returns to scale, at least over the long run and for most enterprises, is because we can always envision replicating the exact conditions of production. That is, if we observe a production facility with so many units capital and so many units of labor and so many units of every other type of input used in the production process, and if we observe the associated level of output, we can always imagine creating an exact replica of that production facility. This means that we would have doubled both the levels of all inputs and the level of output.

Production functions with one variable input Also notice that if we hold constant one of the inputs, say K_V, and just vary the level of the other input, then output is higher at higher levels of the variable input. For example, set $K_V = 100$, so that

$$V = 10(L_V)^{\frac{1}{2}}. \tag{6.4}$$

This function is depicted as the black line in Figure 2. The red line depicts the relation between L_V and V when $K_V = 144$.

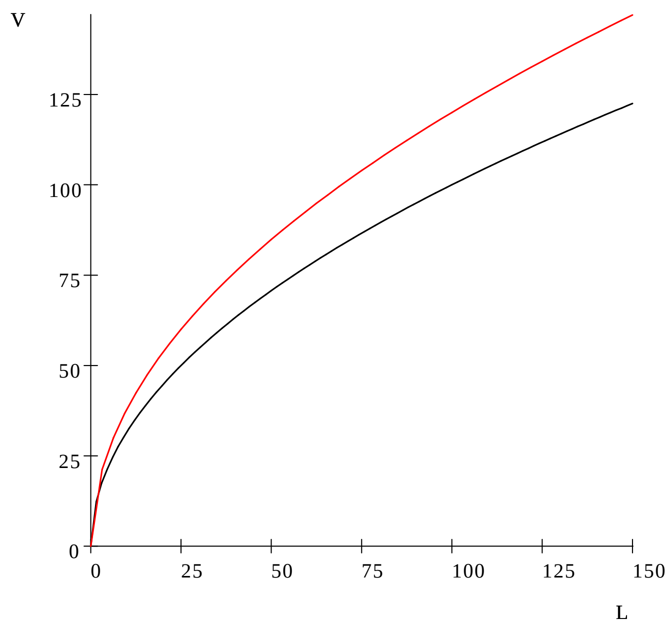


Figure 2: $V(L_V;\overline{K}_V)$

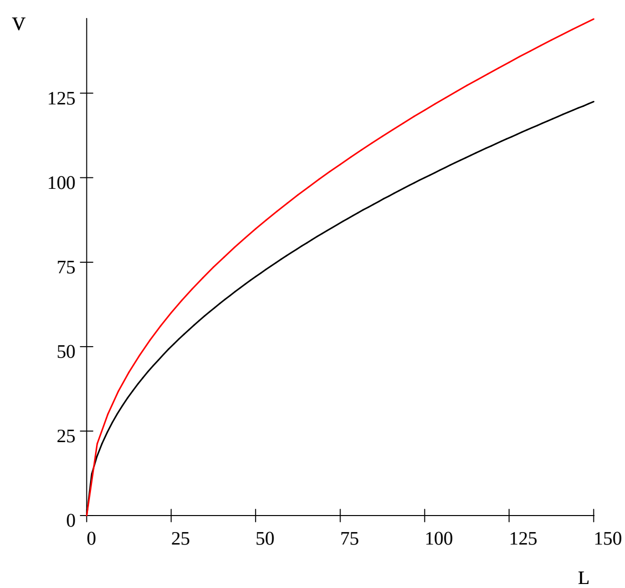


Figure 2.a: $V = 10(L_V)^{\frac{1}{2}}; V = 12(L_V)^{.5}$

Notice that for every value of L_V, output of wine is greater for the production function with the greater amount of capital. This illustrates in another way the general feature of production functions mentioned earlier: more of both inputs means more output.

We can also envision the relationship between the production function written as a function of both capital and labor and the associated production function of one variable input that is formed by holding constant the other input by looking at a cut-away picture of the two-input production function. That is, if we imagine slicing off the production function at some fixed value of the capital stock, the "edge" of this cut-away function depicts the values of output for that fixed value of capital and the variable values of labor. This is depicted in Figure 3.

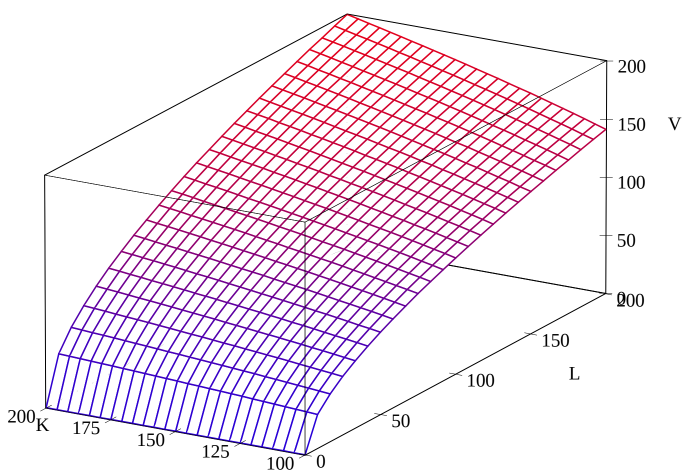


Figure 3: Cut-away Cobb-Douglas production function: $\overline{K}_V = 16$

3.2.3 The marginal product of labor

Each of these one-variable-input production functions also illustrates a general feature of such production functions, a feature often called the law of diminishing marginal productivity. The basic idea is that with one factor, such as capital, fixed in quantity, then additions of the variable input increases output, but these increases per unit of increase of labor are smaller the greater the level of labor at work in the productive process. It is as if with a fixed size capital stock, higher levels of labor suffer “congestion effects” in comparison with lower levels: they have to share the same amount of floor space, or the same number of tools, with more people. This leads to a reduction in *average* output per worker, even though total output is still higher. This means that the increment in output associated with an additional worker is smaller the larger the existing labor force.

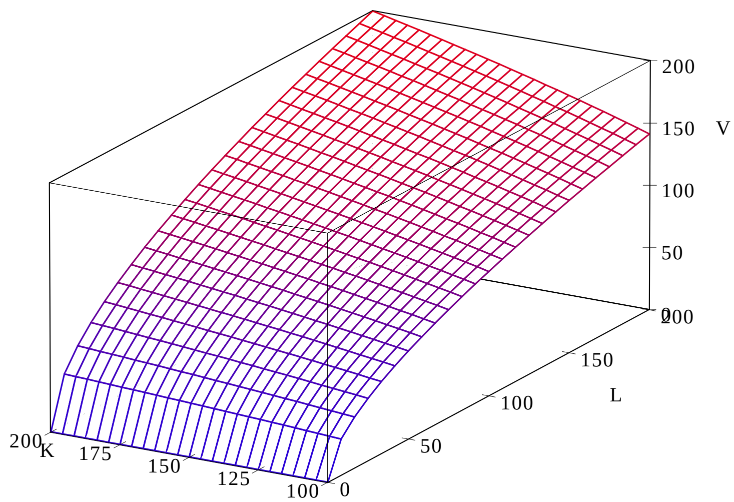


Figure 3: CD cutaway

This feature is exhibited in the graphs of the above production functions by the behavior of the slopes of the functions. Remember that the slope of a curve at a particular point is defined as the slope of the straight line tangent to the curve at that point. For both of the above examples of one-variable-input production functions, note that the slope of each function gets "flatter" as the amount of L_V gets larger.

We denote the change in output associated with "as small as possible" a change in labor as the *marginal product of labor*. Abstracting from indivisibilities, it is equal to the slope of the production function. The production function $V = 10(L_V)^{\frac{1}{2}}$ and slopes of this production function at $L_V = 36$ and at $L_V = 100$, as depicted by the slopes of tangent lines at those points, are displayed in Figure 4. This relationship between changes in output per unit change in input at different levels of input applies at every pair of points along the production function, not just at the two illustrated: the change is smaller the larger is the value of L_V.

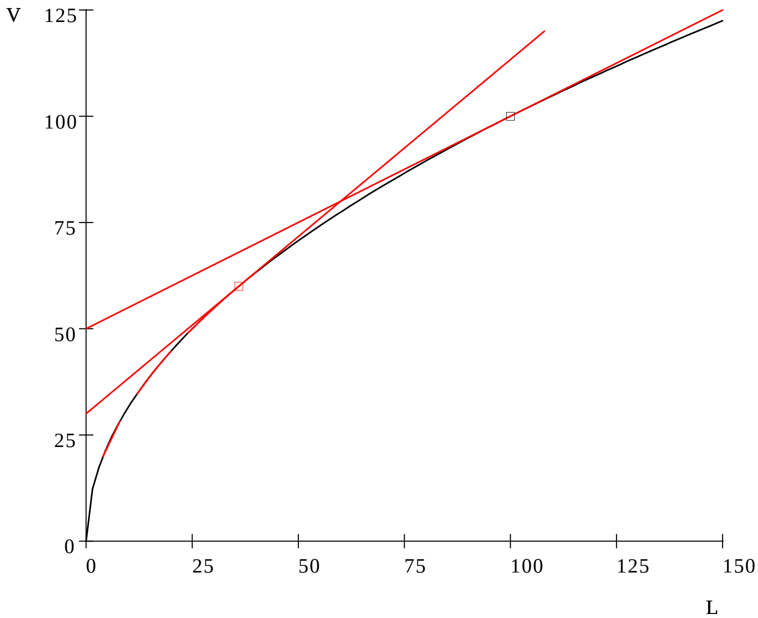


Figure 4: MP as slope of prod. fnctn.

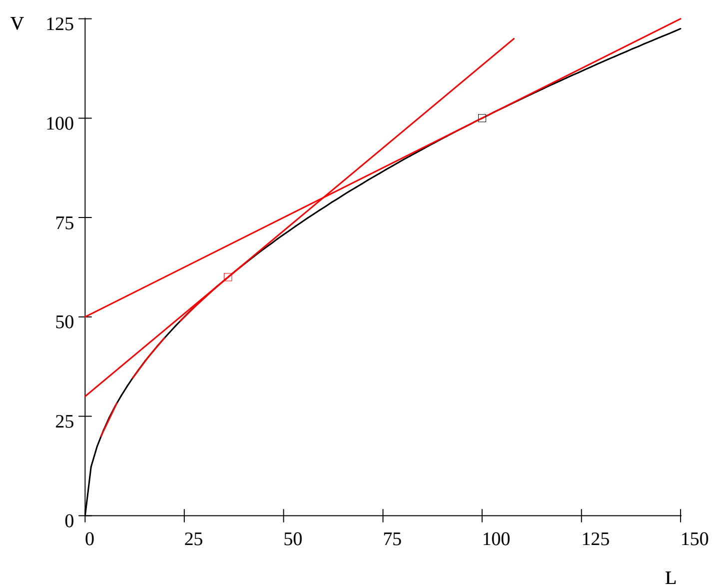


Figure 4: $V = 10(L_V)^{\frac{1}{2}}; \frac{dV}{dL_V} = \frac{1}{2}(L_V)^{-\frac{1}{2}}$

Note that this illustration implies that the marginal product of labor is a **function** of L_V. Hence, for any production function, we could plot the values of the marginal product of labor as a function of the amount of labor being used in the production process. For the two production functions used above, the value of these slopes as a function of L_V are depicted in Figure 6.4, with the black curve depicting the slope of the function $V = 10(L_V)^{\frac{1}{2}}$ and the red line depicting the slope of the function $V = 12(L_V)^{\frac{1}{2}}$. These slope functions are known as *marginal product of labor* functions, and are denoted as $MPL_V(L_V)$.

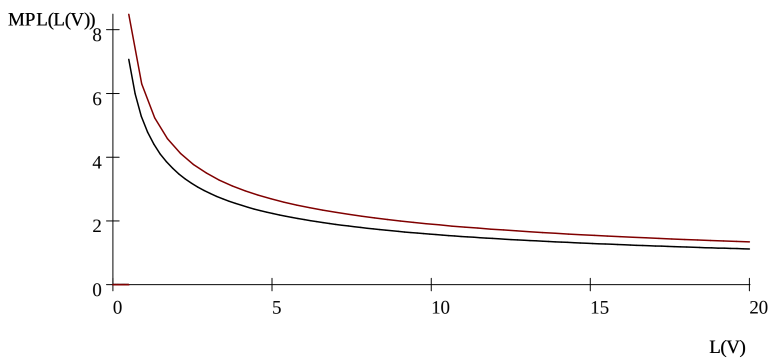


Figure 5: Marginal product functions

Notice that the marginal product of labor in the wine sector is higher at every value of L_V for the function associated with the higher value of the other input, K_V. This is also a general feature of any constant returns to scale production function: an increase in the amount of one factor increases the marginal product of the other factor.

Over relatively long periods of time, say intervals of five to ten years, Ernest and Julio can adjust the size of their capital stock, i.e., they can build more fermentation vats, put more land to use in growing grapes, or purchase more of anything of the sort. These adjustments take so long in part because of the nature of building such things. Over much shorter periods of time, though, even on a week-to-week basis, they can adjust the size of their labor force. For now, we will focus on time periods long enough that E and J can adjust their labor force but cannot adjust their capital stock. Given this *decision horizon* for E and J, we call their capital a fixed factor of production and their labor a variable factor of production. We signify that capital is fixed by putting an overbar over the symbol for capital. Thus the wine production function is represented symbolically as

$$V = f(L_V, \overline{K}_V). \tag{6.5}$$

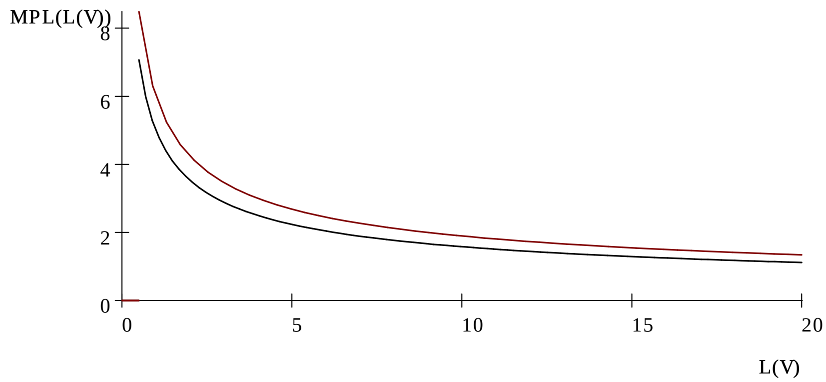


Figure 5: MPL functions

3.2.4 The profit-maximizing rule

The competitive assumption What variables do we assume to be exogenous to the firm, and what variables do we assume are under a firm's control? The key assumption that defines a perfectly competitive firm is the assumption that such a firm takes as exogenous both the price at which it sells its output, P_V in the case of wine, and the prices it pays for inputs. The motivation for such an assumption is that a perfectly competitive market is one in which there are many independent participants, and it is reasonable to assume they don't take into account the effects their choices might have on prices, which are determined by the choices of all of the many participants.

That being said, we will assume throughout this analysis that our firms are such price-takers in both output and input markets. But for expositional ease, we will also assume that there is only one firm in each sector. That is, we assume that E&J vintners is the only firm in the wine sector, and we will assume there is only one firm in the cloth sector. This assumption is obviously hard to motivate in the usual fashion of noting the large number of agents participating in the market, because we assume there is only one! But this assumption of just one firm per sector is innocuous for our purposes: we could carry out the analysis with many firms and get the same results as we do with an assumption of just one firm. The benefit of assuming just one firm accrues from not having to clutter up the analysis with notation that keeps track of individual firm variables.

What are the variables whose values can be chosen by E and J? Because of our assumption of a sufficiently short decision horizon, only one variable is under E and J's control: L_V. Their job, then, can be characterized as choosing an amount of labor per unit of time so as to maximize their profits.

The definition(s) of profits Let us begin by defining profits per unit of time as measured in currency units. We are thinking of profits as what is left over from revenues for the owner of the specific factor after variable costs have been paid. That is, we think of E and J as owning the capital stock used in the production of wine, $\overline{K}_V$, and paying wages to the workers they hire. The definition of their profits, measured in units of currency and denoted by the uppercase Greek letter "pi" (Π), is thus

$$\Pi_V = P_V V(L_V) - wL_V$$

where P_V is the currency price of wine and w is the wage rate. We write the quantity of wine, V, with L_V in parenthesis directly after it so as to emphasize that the quantity of wine produced depends on the amount of labor used in the production process.

We can also measure profits in "real" units, that is, in units of either wine or cloth. We do this by dividing all currency prices by the currency price of the good in whose units we want to measure things. Remember, a currency price of, say, labor, has units of currency per unit of labor. It tells us how many currency units exchange for one unit of labor in the marketplace. If we want to measure how many units of wine that a currency wage can purchase, we can divide the currency wage by the currency price of wine.

This distinction between things measured in units of goods and things measured in units of a currency is frequently confusing to non-economists. A very concrete example may help. Imagine a wage rate of $10/hour. Notice that a currency price-in this case a dollar price-is expressed in terms of how many dollars it takes to purchase a unit of some good or service. Now, what people are really interested in-as noted before-is not how many dollars they get for selling something or how many dollars they spend to purchase something, but how many units of some good or service they can *g*et with those dollars. We call the purchasing power of a certain amount of dollars in terms of some good the *real* value of those dollars in terms of that good. To continue with our example, if the wage rate was $10/hour, and the price of wine was $2/bottle, the real wage measured in units of wine would be five (5) bottles of wine/hour. That is, an hour's worth of work earns five (5) bottles of wine, or, alternatively, five (5) bottles of wine are needed to purchase one unit of work.

Notice how the wine price of labor and the dollar price of labor have analogous dimensions: the dollar price is expressed as how many dollars, i.e., how many units of the currency, it takes to purchase a unit of labor, and the wine price is expressed as how many "wines," i.e., how many units of wine, it takes to purchase a unit of labor. Because a real price expresses how many units of a real commodity or service exchanges for another real commodity or service, real prices are referred to as relative prices.

Symbolically, we depict the transformation of a currency price such as dollars/hour into a real or relative price by dividing the currency price of the good in question by the currency price of the good we are using as the measuring stick. Hence, the real wage measured in units of wine would be $\frac{w}{P_V}$. The real wage measured in units of cloth would be $\frac{w}{P_C}$ The relative price of cloth in terms of wine, then, would be denoted as $\frac{P_C}{P_V}$.

As usual, our interest is not in the value of things measured in currency units, but rather the value measured in real goods and services. Hence, we will want to measure profits in units of either wine or cloth. Here, we will use wine as our measuring stick. Either good would do; the only thing to remember is that for comparison purposes, it is important to measure everything in the same units.

With all this said, we denote real wine-sector profits measured in units of wine by π_V, where π is the lower case Greek letter "pi." The definition of real wine-sector profits measured in units of wine is thus

$$\pi_V = V(L_V) - \frac{w}{P_V} L_V.$$

This just says that revenue measured in units of wine is the actual amount of wine produced, and costs measured in units of wine are the real wage measured in units of wine times the amount of labor hired.

The optimal choice of variable input: the demand for labor For exogenously given (to E and J) P_V and w, E and J are assumed to choose the one variable under their direct control, L_V, so as to maximize profits. Note that by choosing L_V, they implicitly choose the amount of wine they produce, $V(L_V)$. To understand what the profit-maximizing choice of L_V must be, we can think of E and J as carrying out the following thought experiment. They start this experiment by contemplating hiring one unit of labor. A "unit" of labor is interpreted as the "smallest possible increment" that can be purchased. With this contemplated choice of input, they compute their revenue, $P_V \times V(1)$, where $V(1)$ denotes output produced with one unit of labor, and their costs, $w \times 1$. If revenues exceed costs with this one unit of input, they would certainly conclude that they should hire at least this one worker. If revenues were less than costs, though, they would not hire this first worker. Note that $V(1) = MPL_V(1)$: The output produced from the first unit of labor is also the marginal product of labor when the variable input changes from zero to one.

Now, if revenue exceeded cost for hiring one worker, E and J's thought experiment continues with E and J's contemplation of the hiring of an additional, i.e., second, unit of labor. They calculate the extra, or *marginal* addition to their revenues from this hire. This extra revenue is just the marginal product associated with two units of labor in the production process, i.e., $MPL_V(2)$, multiplied times the price received per unit of wine, P_V. For example, if

$MPL_V(2) = 2$ units of wine, and $P_V = \$6$ per unit of wine, then the additional revenue that accrues from hiring two units of labor instead of one would be 2 units of wine, $MPL_V(2)$, times \$6 per unit of wine, P_V. This of course equals \$12. The extra cost of hiring this second unit of labor is just the wage rate, w. If the extra revenue that accrues from hiring two units instead of one is greater than the extra, or marginal, cost of hiring two units of labor instead of one, then they would want to hire at least these two units of labor.

We can imagine this sequential decision-making process continuing as long as the marginal revenue from hiring an additional unit of labor, namely the price of output times the marginal product associated with the amount of labor being used in the production process, exceeds the cost, which is the wage rate. Eventually, because the marginal product diminishes with increases in the level of input usage, the marginal revenue of hiring one more unit of labor will just equal the marginal cost.[3] We would express this profit-maximizing rule for E and J as

$$P_V \times MPL_V(L_V) = w. \tag{6.6}$$

This is implicitly a demand-for-labor function for E and J: for exogenously stipulated values of P_V and w, equation (6.6) tells us the profit-maximizing choice of L_V. We could also rearrange this profit-maximizing rule by dividing both sides by P_V, thus emphasizing that the profit-maximizing rule calls for hiring workers up to the point at which the physical marginal product equals the **real** wage measured in units of wine:

$$MPL_V(L_V) = \frac{w}{P_V} \tag{6.7}$$

We could thus write the demand for labor function as

$$L_V^d = H(\frac{w}{P_V}). \tag{6.8}$$

We add the superscript "d" on L_V to indicate that it is the choice variable of E and J, and that their optimal choice depends upon the values of the exogenous-to-E and J variables w and P_V. This function $H(\frac{w}{P_V})$ is downward-sloping: a lower real wage must be matched by a lower marginal product, which occurs with a larger labor force at work in the production process.

[3]In the presence of individibilities, equality might not be reached. The rule in such situations would be to stop hiring when an additional unit would add more cost than revenue.

Example

An example might clarify this concept of a demand function for labor. If, for example, the production function were given as $V = 10(L_V)^{\frac{1}{2}}$, then the marginal product function would be $MPL_V(L_V) = 5(L_V)^{-\left(\frac{1}{2}\right)}$. The *value of the marginal product* function would just be

$$P_V \times MPL_V(L_V) = P_V \times 5(L_V)^{-\left(\frac{1}{2}\right)}$$

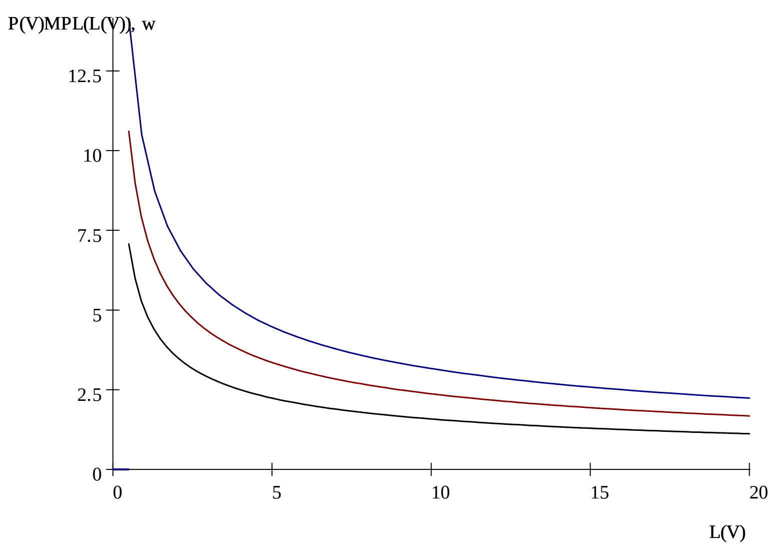
The profit-maximizing rule requires that E and J hire a workforce of size such that

$$P_V \times 5(L_V)^{-\left(\frac{1}{2}\right)} = w.$$

We can solve this equation explicitly for L_V as a function of the exogenous variables w and P_V by dividing both sides of this equation by $P_V \times 5$ and then raising both sides to the power (-2) :

$$L_V^d = \left(\frac{1}{5}\right)^{-2} \left(\frac{w}{P_V}\right)^{-2}$$

First let us depict this relationship as a graph with L_V^d on one axis and w on another by holding constant the value of P_V. As usual in economics, though, we graph inverse demand functions, so we put w on the vertical axis and L_V^d on the horizontal axis. In Figure 6, three members of this function are displayed. The uppermost curve is drawn for $P_V = 2$; the middle curve is drawn for $P_V = 1.5$; the lowest curve is drawn for $P_V = 1$. What any of these curves tell us is that, for the given value P_V, the optimal choice of L_V can be read off of the horizontal axis for any given value of w.



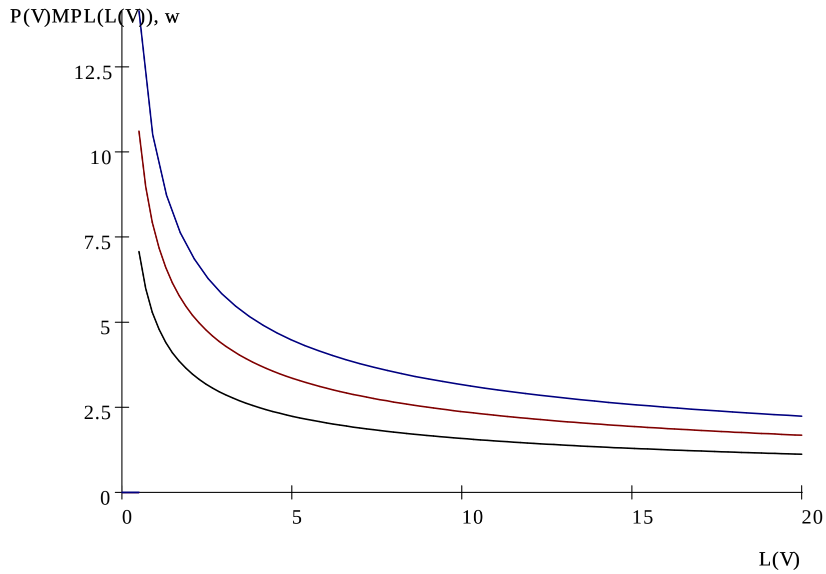


Figure 6: Value of MP functions

Figure 6: Value of the marginal product function

Note that we can also depict this information in "real" terms. That is, we can depict the quantity of labor demanded as a function of the real wage measured in units of wine. In inverse form, we want to graph the relation with the real wage on the vertical axis and the quantity of labor demanded on the horizontal axis. In this case, as highlighted by equation (6.7), the inverse demand curve is just the physical marginal product function. The function is depicted in Figure 7.

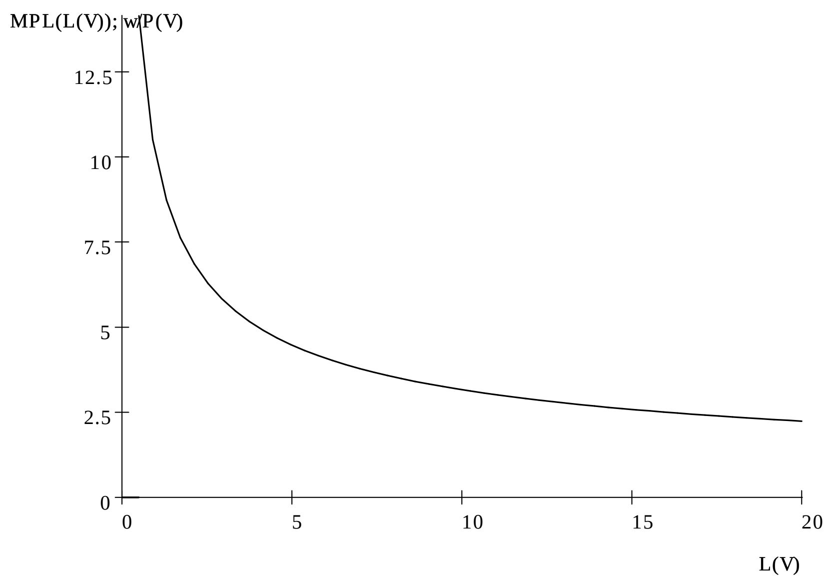


Figure 7: Wine-sector demand for labor function

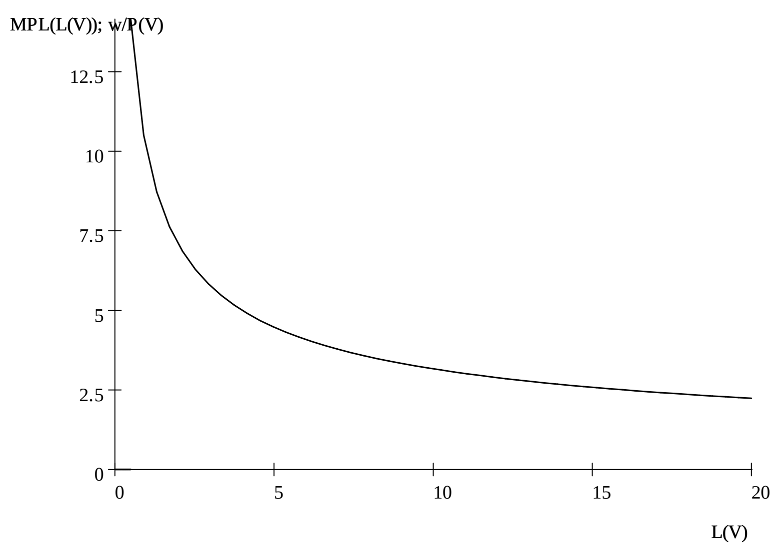


Figure 7: Demand for labor in the wine sector as a function of the real wage

We are now ready to introduce a second sector into this economy and derive a general-equilibrium supply function for the goods produced by each sector.

3.3 Introducing cloth production

Consider LW clothing, a fictitious firm we assume was started and owned by someone much like Les Wexner, the real-life founder of such clothing stores as The Limited and Victoria's Secret. For the same reasons we appealed to in our development of the wine sector, we make the simplifying assumption that cloth is produced by this single firm under conditions of perfect competition. Technology is described by a production function with the same qualitative properties as the one used by E and J:

$$C = g(L_C, \overline{K}_C) \tag{6.8}$$

where C denotes the output of cloth per unit of time, L_C denotes the amount of labor used in the production of cloth, and $\overline{K}_C$ is the fixed, specific factor of production used in cloth production. The variable factor about which LW makes decisions is thus L_C.

LW's profits per unit of time in nominal terms are just revenues minus costs:

$$\pi_C = P_C C(L_C) - wL_C.$$

LW's profit-maximizing rule is:

$$P_C \times MPL_C(L_C) = w. \tag{6.9}$$

That is, LW hires a workforce of a size such that the value of the marginal product equals the wage rate. Let us now express the value of the marginal product and the wage in units of wine, i.e., in real terms, by dividing both sides of the profit-maximizing rule by the nominal price of wine:

$$\frac{P_C}{P_V} \times MPL_C(L_C) = \frac{w}{P_V}. \tag{6.10}$$

This relationship implicitly defines a demand for labor function for labor by LW. This function tells us that LW's demand for labor depends on both the real wage measured in units of wine and the relative price of cloth. In particular, for any given real wage, the demand for labor by LW increases as $\frac{P_C}{P_V}$ increases. We express this symbolically as:

$$L_C^d = h(\frac{P_C}{P_V}, \frac{w}{P_V}). \tag{6.11}$$

Example

Again, a numerical example may help here. Imagine the cloth production function is given as follows:

$$C = 10\,(L_C)^{\frac{1}{2}}\,.$$

The associated marginal productivity function is $MPL_C(L_C) = 5(L_C)^{-(\frac{1}{2})}$. The value of the marginal product function measured in units of wine is thus

$$\left(\frac{P_C}{P_V}\right) \times 5(L_C)^{-(\frac{1}{2})}.$$

The inverse demand for labor function for the cloth sector is thus:

$$\left(\frac{P_C}{P_V}\right) \times 5(L_C)^{-(\frac{1}{2})} = \frac{w}{P_V}$$

This can be thought of as a family of demand curves, each member of the family corresponding to a different value of $\frac{P_C}{P_V}$. For notational simplicity, we will denote the relative price of cloth as p. The curve associated with a higher relative price p is farther out from the vertical axis. In Figure 8, the highest demand function is drawn for $p = 2$, the middle one is drawn for $p = 1.5$, and the lowest is drawn for $p = 1$

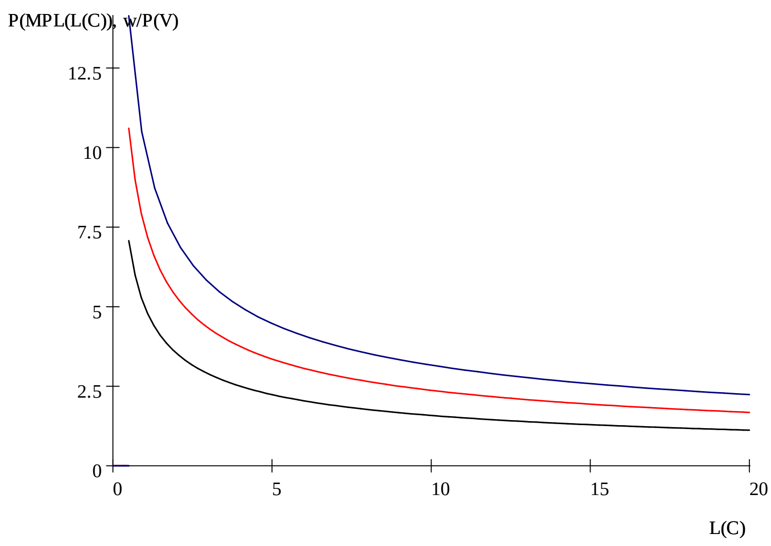


Figure 8: Inverse cloth-sector labor demands: $p = 1; 1.5; 2$.

Having derived demand functions for labor for both the wine sector and the cloth sector, we are now ready to derive the aggregate or economy-wide demand curve for labor. We can then equate demand to supply to find the equilibrium

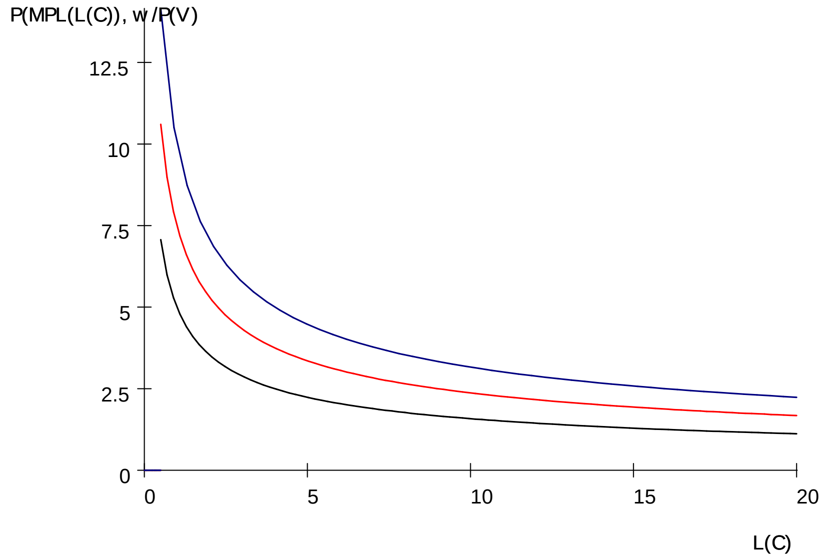


Figure 8: Inverse cloth-sector labor demand functions

real wage and the equilibrium allocation of labor across the two sectors as a function of the relative price of cloth, p. Knowing how much labor is used in the production of each good for any given relative price, we can then substitute this back into the production functions of each sector determine output as a function of relative price.

3.4 Labor-market equilibrium

3.4.1 The aggregate labor-demand function

To derive an aggregate demand function for labor, we must add up the individual demand functions from each sector. That is, we must add up the two functions given in equations (6.8) and (6.11):

$$L^d\left(\frac{w}{P_V}, p\right) = \overbrace{H\left(\frac{w}{P_V}\right)}^{L_V^d} + \overbrace{h\left(\frac{w}{P_V}, p\right)}^{L_C^d} \tag{6.12}$$

The key feature to note here is that the aggregate demand for labor depends on the both real wage (measured in units of wine) and the relative price of cloth, p.

3.4.2 The supply of labor

In reality,the supply of labor is a complex function of many variables: demographic characteristics of the population, wage rates, individual circumstances, and so on. We abstract from all this and simply assume that the supply of labor is fixed and exogenous at some level $\overline{L}$.

3.4.3 Equating demand to supply

Equilibrium in the labor market is a "rest point," that is, an ordered pair $(\widehat{\frac{w}{P_V}}, \widehat{L}^d)$ such that, for any given value of p and $\overline{L}$ and given values of any other parameters, e.g., the parameters that describe the production function, no worker has an incentive to try and move from one sector to another and no firm has an incentive to try and hire or fire any workers. Such an ordered pair is depicted by the intersection of the inverse aggregate demand for labor function and the inverse labor supply function.

Diagrammatically, we can depict the inverse aggregate demand function for labor by "adding up horizontally" the inverse demand functions for each sector. This, along with the (vertical) inverse labor supply function, depicted as a vertical black line, is depicted in Figure 9. The wine sector's inverse demand function for labor is depicted by the single downward-sloping black curve. Two representative members of the cloth sector's inverse labor demand function family are depicted by the downward-sloping red lines of increasing thickness, with the thicker line representing the cloth-sector inverse labor demand function associated with the higher relative prices of cloth. The aggregate inverse labor-demand functions are depicted by the blue lines. The thinner blue line depicts the aggregate inverse labor-demand function associated with the lower relative price of cloth, and the thicker line depicts the aggregate inverse labor-demand function associated with the higher relative price of cloth. Notice that the wine sector inverse labor-demand function does not shift as p changes.

Labor market equilibrium is depicted by the boxed point where demand equals supply. For the case with a higher value of p, the intersection occurs at a higher value of $\frac{w}{P_V}$. From this depiction of equilibrium, we can also see the employment levels in each sector. Once we have depicted the equilibrium wage, we can simply "read off" the quantities of labor demanded by each sector from their individual demand curves. These quantities are depicted by the red and black boxed points.

Also depicted in the Figure is an equilibrium for a lower value of p. In this case, the relevant inverse aggregate demand curve is the thinner blue line. The equilibrium real wage, $\frac{w}{P_V}$, associated with this lower value of p is lower relative to the case of a higher value of p, and employment in the wine sector is higher. This implies, as depicted, that employment in the cloth sector has gone down. Heuristically, it makes sense that a lower value of p, the relative price of cloth, should lead to less employment in the cloth industry. Because we have assumed a fixed supply of labor, this means that employment in the wine sector must have increased.

Again, a numerical example may help fix these ideas. In Figure 9, the inverse demand function for labor in the wine sector is specified as $\frac{w}{P_V} = \frac{2}{\sqrt{L_V^d}}$, the inverse demand function for cloth is specified as $\frac{w}{P_V} = p\frac{3}{\sqrt{L_C^d}}$. The lower value of p is specified as $p = 1$, and the higher value of p is specified as $p = 1.5$. First consider the case when $p = 1.5$. For this value of p, the inverse demand function for labor in the cloth sector is depicted by the thicker red line, and the inverse aggregate labor demand function is depicted by the thicker blue line. When $\frac{w}{P_V} = 2$, for example, $L_V^d = 1$, and $L_C^d = 5.0625$. Aggregate labor demand, L^d, thus equals 6.0625 when $p = 1.5$. Conveniently, we have specified $\overline{L} = 6.0625$, so these values are also equilibrium values.

Now consider the case when $p = 1$. For this value of p, the inverse demand function for labor in the cloth sector is depicted by the thinner red line, and the inverse aggregate labor demand function is depicted by the thinner blue line. The figure depicts an equilibrium real wage (measured in units of wine) that is approximately equal to 1.46 units of wine per unit of work, an equilibrium quantity of labor employed in the wine sector sector approximately equal to 1.865 units of labor, and as equilibrium quantity of labor employed in the cloth sector approximately equal to 4.197 units of labor.

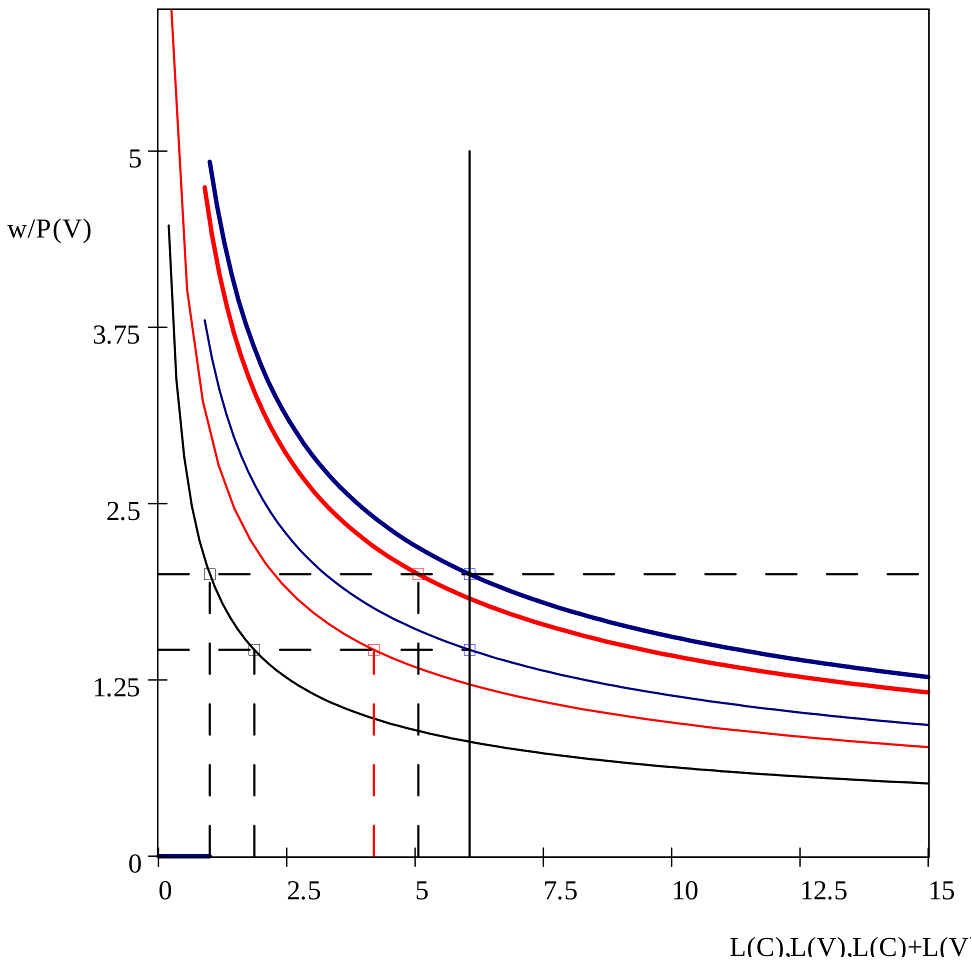


Figure 9 :Labor mkt equil.:$\overline{L} = 6.0625; p = 1, 1.5$

We can also depict labor market equilibrium in another diagram that depicts more clearly the relationship between the relative price of cloth, p, and the allocation of labor across the two sectors. Such a diagram has a horizontal length equal to $\overline{L}$, and measures increasing values of L_V from the left-hand endpoint of this horizontal line segment. Increasing values of L_C, though, are measured going right to left from the right-hand endpoint of the horizontal line segment. Any point along this segment thus divides it into two segments, the segment extending from the left endpoint to the interior point measuring an amount of L_V and the segment extending from the interior point to the right endpoint measuring an amount L_C, such that the amounts L_V and L_C sum to $\overline{L}$. The left-hand vertical axis measures the real wage measured in units of wine and the marginal product of labor in the wine sector, while the right-hand vertical axis measures the real wage measured in units of wine and the value of the marginal product of labor in the cloth sector (measured in units

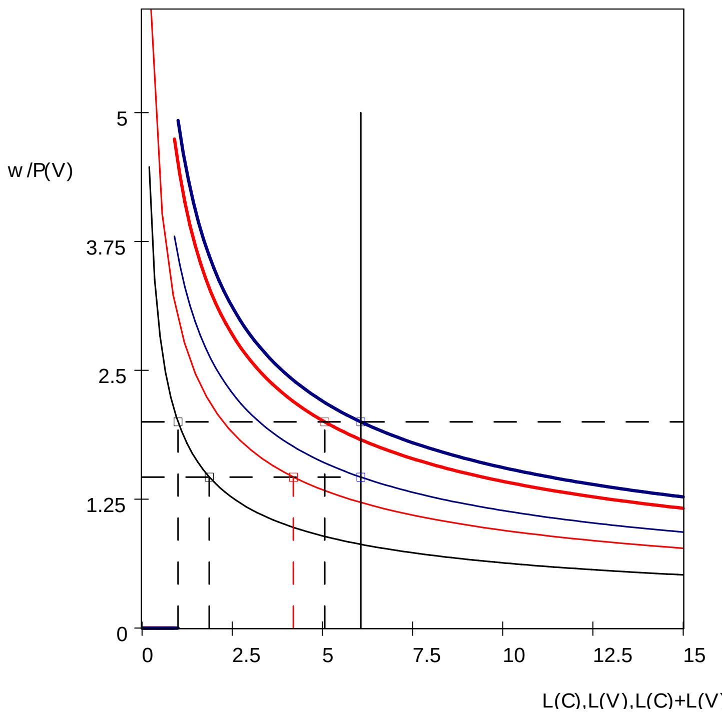
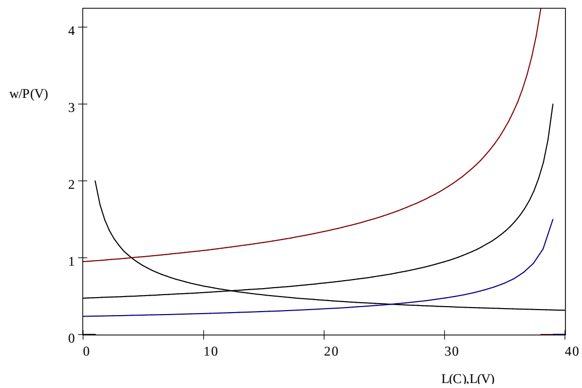


Figure 9: Labor market equilibrium

of wine). It is as if we rotated the diagram of the inverse cloth-sector labor demand function one-hundred eighty degrees out from the plane, so that its vertex was on the right. Such a diagram is pictured in Figure 10 below, with three (3) labor-sector inverse demand functions, each associated with a different value of p, and one wine-sector inverse labor demand function. The intersection of a wine-sector and a cloth-sector inverse labor demand function thus depicts labor-market equilibrium: a triplet of numbers, $\widehat{\frac{w}{P_V}}(p)$, $\widehat{L}_V(p)$, and $\widehat{L}_C(p)$. The "hat" over each variable indicates it is an equilibrium variable for this sub-model of the labor market, while the "p" in parentheses next to each variable reminds us that these equilibrium values are calculated for a particular value of p, i.e., are a function of p.



Labor mkt equil.:$\overline{L} = 40; p = .5, 1, 2$

3.5 Miller Time! The general equilibrium supply function for cloth

The hard work is done (this is what is meant by "Miller Time:" in old television commercials for Miller Beer, workers were portrayed as stopping off at a bar on their way home after a hard day's work and having a Miller brand beer, and this time was called "Miller Time"). We can now derive what we will call the general equilibrium supply function for cloth. We add the modifier "general equilibrium" to distinguish this from what is traditionally described as the supply function: a schedule that tells us the quantity supplied for any given price of output and price of inputs. A general equilibrium supply function "solves out" the input price (in this case the price of labor) as a function of the relative price of output. Thus, a general equilibrium supply function tells us the quantity per unit of time of a good that is produced for any exogenously given price. Because we so seldom use "regular" supply functions in this treatment of international economics, we will usually refer to general equilibrium supply curves as just "supply curves."

From our analysis of labor market equilibrium, we know that at relatively higher values of p, (the relative price of cloth), we have higher levels of equi-

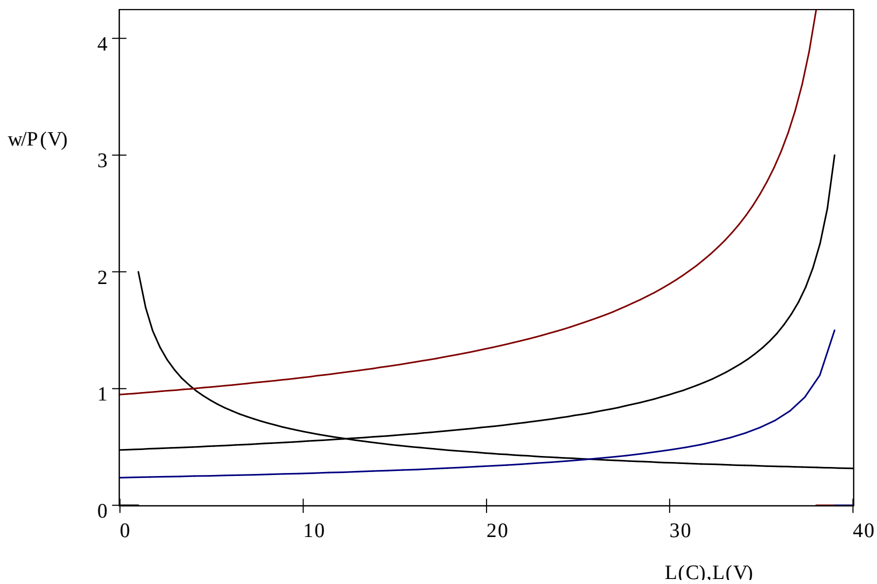


Figure 10: Labor market equilibrium

librium employment in the cloth sector. Hence, because higher inputs create higher outputs, we must have higher levels of output of cloth associated with higher values of p. Symbolically, we express this as follows:

$$C^S = F(p), F\prime > 0.$$

Such a relationship, in inverse form, is depicted in Figure 11.

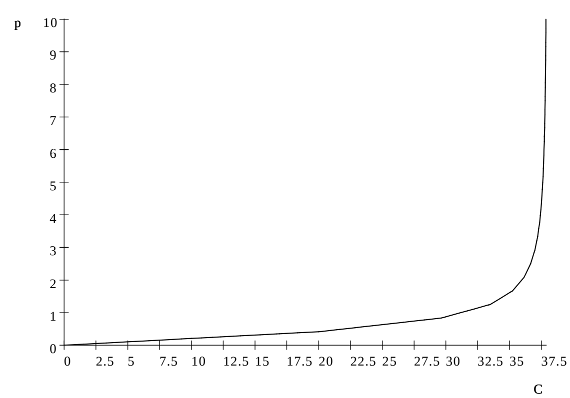


Figure 11: $C^S(p)$

This completes a sub-model of the supply side of this general equilibrium economy. This is, in Alfred Marshall's terms, "one blade of the scissors" needed

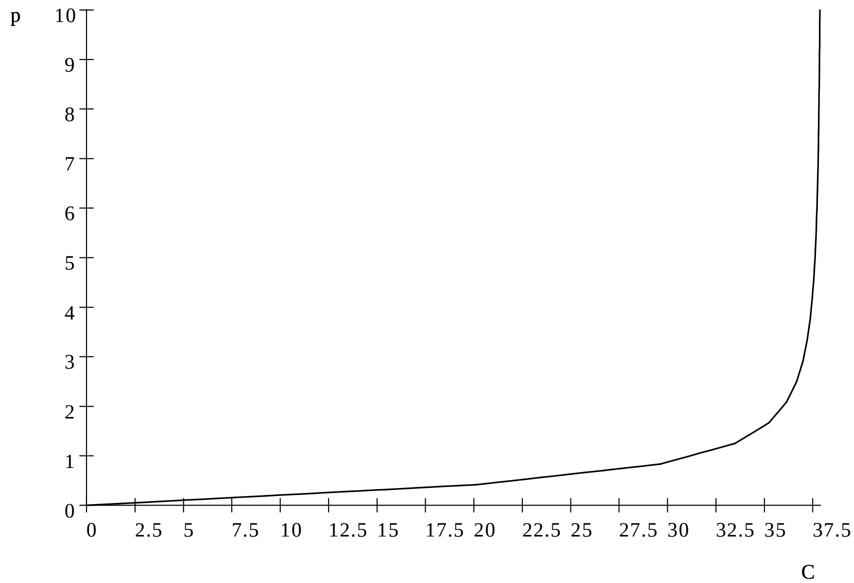


Figure 11: Cloth supply

to analyze full equilibrium in a model. The other blade is the demand function. In contrast to the analysis of demand in the endowment economy, though, in a model with production real incomes of various members of the economy are not independent of the production decisions. Nonetheless, most of the hard work of deriving demand functions is done in the endowment economy model.

We might note that the above analysis also provides a relationship between relative prices and the production of wine. Just as a higher relative price of cloth generates a larger allocation of the labor supply to the cloth sector and consequently a higher quantity of cloth supplied, it also must generate a smaller allocation of the labor supply to wine production. Hence, a higher relative price of cloth is associated with a smaller quantity of wine supplied. The intuition is much as with cloth: a higher relative price of cloth is by definition a lower relative price of wine, and we would expect a lower relative price of wine to be associated with a smaller quantity supplied.

Just as in our analysis of the endowment economy, we have no need to analyze demand and supply in the market for wine as well as in the market for cloth because of Walras' Law: in a two-good economy, if one market is in equilibrium, then so must be the other market.

What shifts the supply function? What shifts the supply curve in the (C, p) plane? Anything that *exogenously* shifts out the marginal product of labor in the wine production function implies an equilibrium allocation of labor with more workers in the wine sector at any given p. This would shift "back" or "up" the inverse supply function of cloth: at any given p, less cloth would be produced. An increase in capital in the wine sector would be one thing that could shift out the $MPL_V(L_V)$ function, as would some types of technical change.

By the same token, anything that exogenously shifts out the $MPL_C(L_C)$ function would lead to an allocation of labor with more workers in the cloth sector. This, again, would include such things as an increase in the capital in the cloth sector and some types of technical change.

4 Demand and the distribution of income

In contrast to the sub-model of demand in the endowment economy in which income the amount of goods received as income for each individual was exogenous, the goods received as income by individuals in a market economy with production is *endogenous*. To develop a sub-model of demand in a production economy, we must first determine the distribution of the goods produced as payments to factors of production as a function of the relative price of cloth, p.

In our market economy with production of two goods, the income generated from the sale of output is distributed to three distinct groups: labor, the owners of the factor specific to the production of wine, and the owners of the factor specific to the production of cloth. It may help to imagine that the output produced in each sector is physically distributed to the factors of production for each sector. That is, in the cloth sector workers are paid their marginal physical product *in cloth*, and what's left over from the production process after the wage bill is paid is the income, in cloth, that accrues to the owner of the specific factor used in cloth production (LW in our terminology). Likewise, in the wine sector, workers are paid their physical marginal product *in cloth*, and whatever is left over from the production process after paying the wage bill accrues to E&J. In labor market equilibrium, the **value** of the real wage paid to workers must be the same, whether measured in units of cloth or wine, but we can think of the workers as getting paid in terms of the commodity produced in the sector in which they work.

The most important feature of the market-determined distribution of income among these three groups is its dependency on relative prices.

4.1 Labor income (the real wage)

First let us take up the dependency of labor income on the relative price of cloth. From our analysis of labor market equilibrium, we know that the real wage *measured in units of wine* is an increasing function of the relative price of cloth. That is, if we contemplate an increase in the relative price of cloth, we know this would shift out the value of the marginal product function for the cloth sector, which in turn implies a higher real wage measured in units of wine, i.e., $\frac{w}{L_V}$. But what happens to the real wage measured in units of cloth?

First note that the real wage measured in units of cloth would be symbolized as $\frac{w}{P_C}$. This measures how many units of cloth exchange for one unit of labor. We can also express this, or "convert" the real wage measured in units of wine into the real wage measured in units of cloth, by dividing the real wage measured in units of wine by the relative price of cloth:

$$\frac{w}{P_C} = \frac{w}{P_V} \times \frac{1}{\frac{P_C}{P_V}}$$

Now, our analysis of equilibrium in the labor market showed that a higher value of p is associated with a higher equilibrium value of $\frac{w}{P_V}$. Put another way, a lower value of $\frac{1}{p}$ is associated with a higher value of $\frac{w}{P_V}$. Hence, whether the value of the product of $\frac{w}{P_V}$ and $\frac{1}{p}$ increases or decreases when p increases is not obvious: $\frac{1}{p}$ gets smaller, but $\frac{w}{P_V}$ gets larger.

We can show, though, that the result is unambiguous: an increase in p leads to a *decrease* in the real wage measured in units of cloth. Remember that the profit-maximizing condition requires Les Wexner to hire a labor force of such a size that the value of the marginal product equals the real wage. We are free to measure the value of the marginal product and the real wage in either units of wine or units of cloth. Earlier, we measured these entities in terms of wine. Now let us measure in units of cloth. To do this, we divide the currency price of labor and the currency value of the marginal product by the currency price of cloth and equate the real wage measured in units of cloth to the value of the marginal product measured in units of cloth:

$$\begin{aligned} \frac{w}{P_C} &= \frac{P_C}{P_C} \times MPL_C(L_C) \\ &= MPL_C(L_C). \end{aligned}$$

This expression emphasizes that the equilibrium real wage measured in units of cloth must equal the physical marginal product of labor in the cloth sector.

Now, from our preceding analysis we know that an increase in p increases employment in the cloth sector. Hence, the marginal product of labor in the cloth sector associated with this higher level of employment will be smaller because of the law of diminishing returns. That is, we have moved down the cloth-sector marginal product of labor curve.

So, an increase in p leads to an increase in the real wage measured in units of wine but leads to a decrease in the real wage measured in units of cloth. We want to analyze what this implies for a worker's optimal choices of wine and cloth and also what it implies about a worker's well-being, i.e., is he or she better off or worse off after an increase in p?

To analyze these questions, consider any individual worker's budget constraint:

$$V_l + pC_l = \frac{w}{P_V}$$

where we have subscripted V and C to indicate this is the l^{th} worker's (l a mnemonic for "laborer") choices of wine and cloth. Writing this budget constraint in slope-intercept form, we have

$$V_l = \widehat{\frac{w}{P_V}}(p) - pC_l$$

where we have put p in parentheses after the real income measured in units of wine to emphasize the functional dependence of $\frac{w}{P_V}$ on p and we have put a "hat" over the real wage to indicate this is the equilibrium value from our submodel of the labor market. Now, an increase in p, we deduced, increases $\frac{w}{P_V}$, and so moves the vertical intercept of the budget constraint up. But the increase in p makes the slope of the budget constraint steeper. Furthermore, we can deduce that the horizontal intercept of this budget constraint gets smaller, i.e., moves closer to the origin. We know this because the horizontal axis intercept is determined by setting $V_l = 0$ in the budget constraint and solving for C_l. Remembering that $p = \frac{P_C}{P_V}$, we see that the intercept is just $\frac{w}{P_C}$, the real wage measured in units of cloth. We showed that this goes down as p goes up. The graph of a worker's budget constraint, then, rotates in a clockwise direction in the wine-cloth plane, with the intercept on the vertical axis-the wine axis-increasing, and the intercept on the horizontal axis-the cloth axis-decreasing. This is depicted in Figure 12, with the budget constraint associated with the higher value of p drawn as a red line and the budget constraint associated with the lower value of p drawn as a black line.

$y = 4 - x$

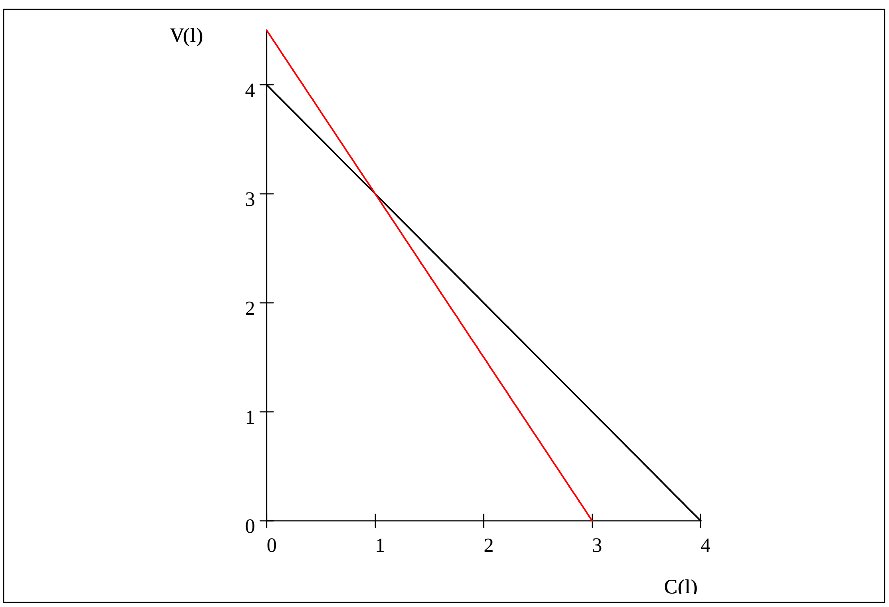


Figure 12: worker's budget constraint

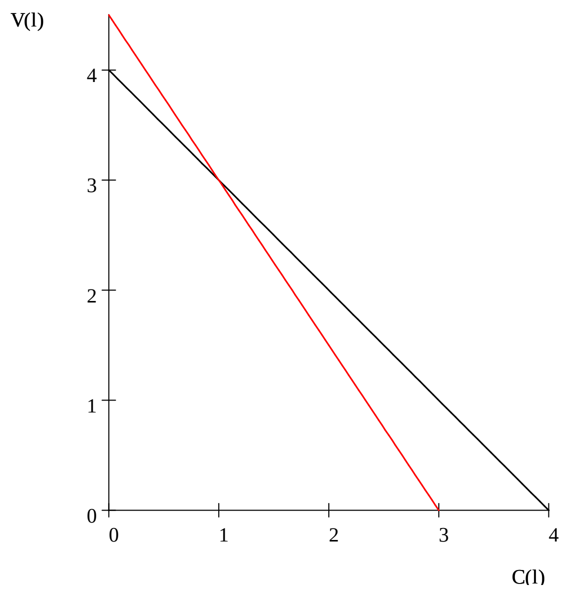


Figure 12: Worker's budget constraint

4.2 A worker's optimal choices

What does this change imply about the worker's optimal choice of wine and cloth? The only thing we can say without more information on the exact shape of his indifference curves is that the optimal pair chosen will change. But in one way this is enough for us to know, because it implies a functional relationship between the quantity of cloth per unit of time chosen by each worker and the relative price of cloth. That is, it describes a demand function for each worker.

But consideration of two different specification of preferences for a worker might help understanding by providing a more concrete depiction of these possibilities. First consider a worker with preferences represented by the indifference curves superimposed on the two (2) possible budget constraints in the preceding figure. The black budget constraint represents a lower value of p and the red budget constraint represents a higher value of p. For workers with preferences represented by the indifference curves in this figure (call these workers "clothes-lovers" for reasons of comparison with the following example), an increase in p makes them worse off and leads to a reduction in the quantity of clothing purchased.

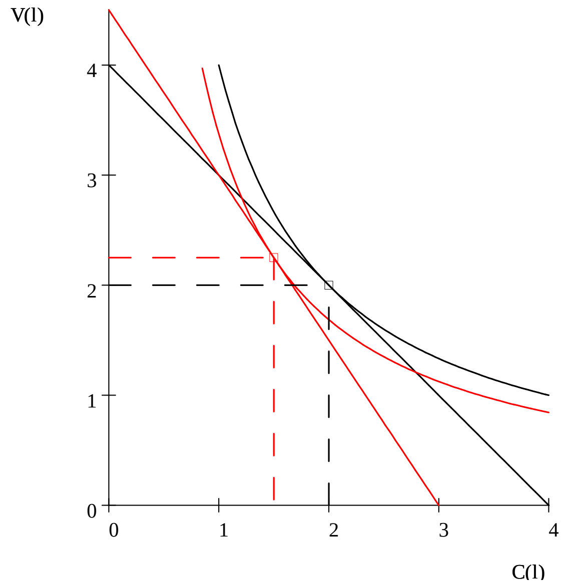


Figure 13: Cloth-lover's bud. constraint and opt. choice

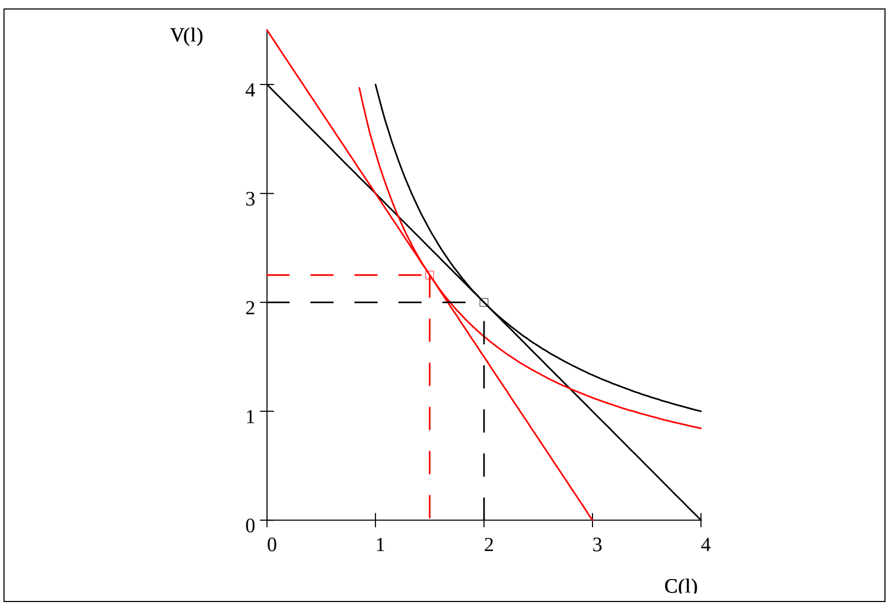


Figure 13: clothes-lover's budget constraints and optimal choices

Now consider a different group of worker's whose preferences lead them to be characterized as "wine-lovers" vis a vis the "clothes lovers." Indifference curves of a representative of this group are superimposed on the two budget

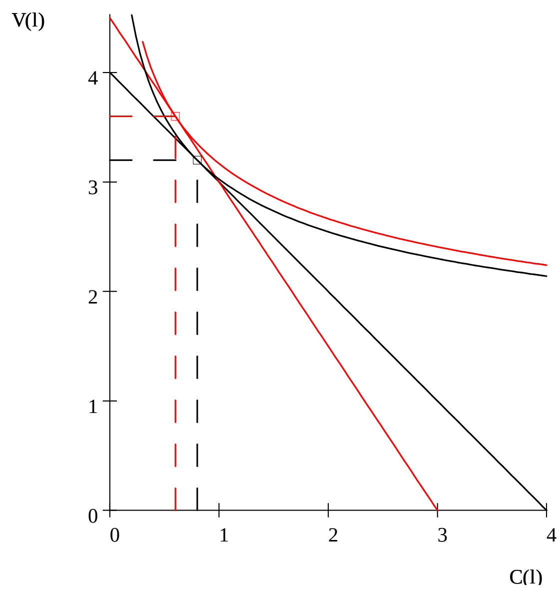


Figure 14: Wine-lover's bud. constraint and opt. choice

constraints associated with two different values of p in Figure 14. For these individuals, an increase in p made them better off.

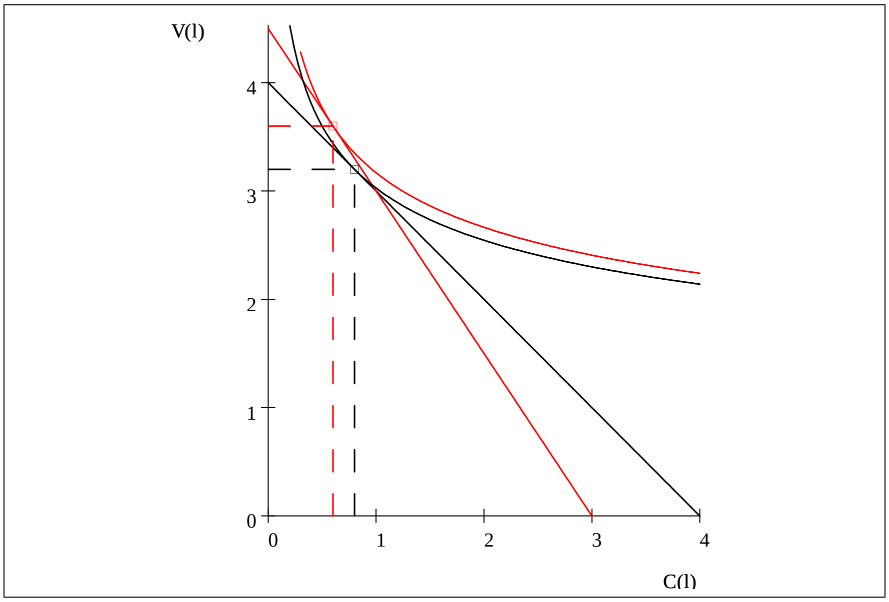


Figure 14: Wine-lover's budget constraint and optimal choices.

4.3 Income of owners of specific factors

4.3.1 The income of E&J

Consider first the real income measured in units of wine of E&J, owners of the factor specific to the production of wine. This is the profits measured in wine that E&J receive after selling their production and paying their workers. Thus, for any choice of labor L_V, this is output as determined by the production function $V = f(L_V, \overline{K}_V)$ minus the wage bill, $\frac{w}{P_V} \times L_V$.

Our interest is in how this income is affected by changes in p. From our sub-model of the labor market, we know that an increase in p from some initial value increases cloth-sector employment and thus decreases wine-sector employment by the same amount. How does this affect E&J's profits?

First note that E&J are producing less wine than at the lower relative price of cloth: lower wine-sector employment implies, via the production function relationship, lower output. Thus E&J's total revenue measured in units of wine decreases in response to a higher relative price of cloth.

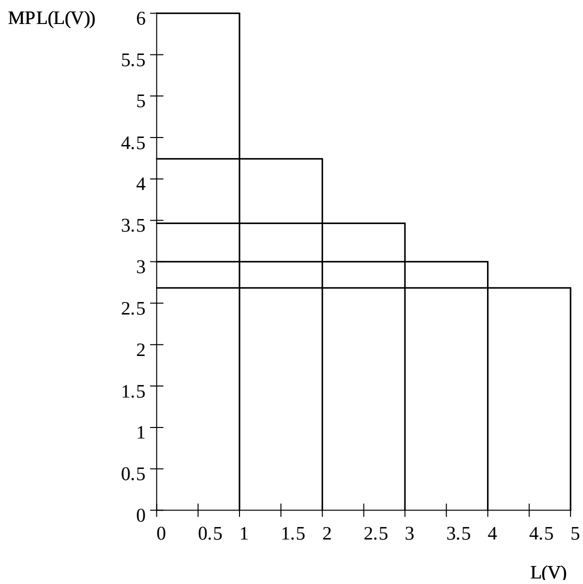
But total revenue is only part of the profit equation. We also must know what happens to total costs, or, in equivalent terminology (with only labor as the variable factor), the **wage bill**. The lower wine-sector employment is accompanied by a higher real wage measured in units of wine: there is a movement along the $MPL_V(L_V)$ curve from a higher value of L_V to a lower value. Thus, the effect on the **wage bill**, which is the product of L_V and $\frac{w}{P_V}$, is ambiguous.

To understand the net effect of an increase in p on E&J's profits, then, we need some additional analytic apparatus. In particular, we need to know the relationship between the marginal product of labor function and total output. The key relationship between these entities that we will demonstrate is that the total output produced by a given amount of labor can be depicted geometrically as the area under the marginal product of labor function.

Imagine constructing a marginal product of labor function from a production function by adding sequentially to the production process small increments, i.e., one of the "smallest possible" units, of labor, aka "a worker," starting with one unit. The first unit creates total output equal to the marginal product of one worker. The geographical depiction of this would be a rectangular strip with a base one unit of labor wide and a height equal to the marginal product of the first worker.

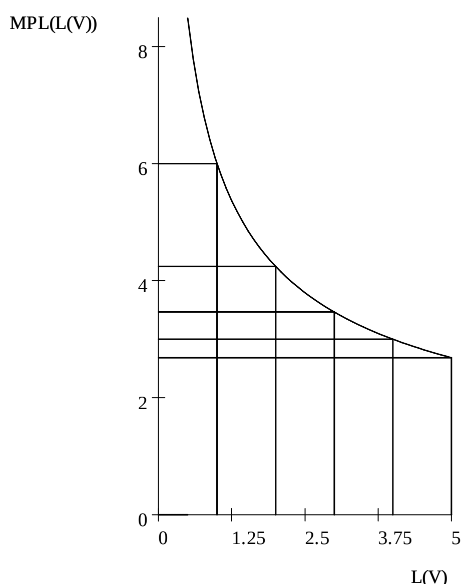
Now imagine adding a second unit of labor to the production process. This second worker would add to total production the marginal product of this second worker. The geometric representation of this addition would be a rectangular strip one unit wide and with height equal to the marginal product of the second worker. Adding this second strip to the one generated by the first worker, we would have a geometric representation of total output produced by two workers.

We could obviously continue this thought experiment for more and more increments of labor. The addition of the rectangular strips that represent the extra output, i.e., marginal product, associated with additional workers, is depicted in the figure below. The total output produced by all these workers is represented geometrically by the combined area of all the strips.



15: Total product from marginal products: I

As usual, we find it easier to describe our models as smooth or continuous functions, which allows us to depict our models with smooth curves. To this end, imagine smoothly connecting the above rectangles with a smooth curve, as depicted below:



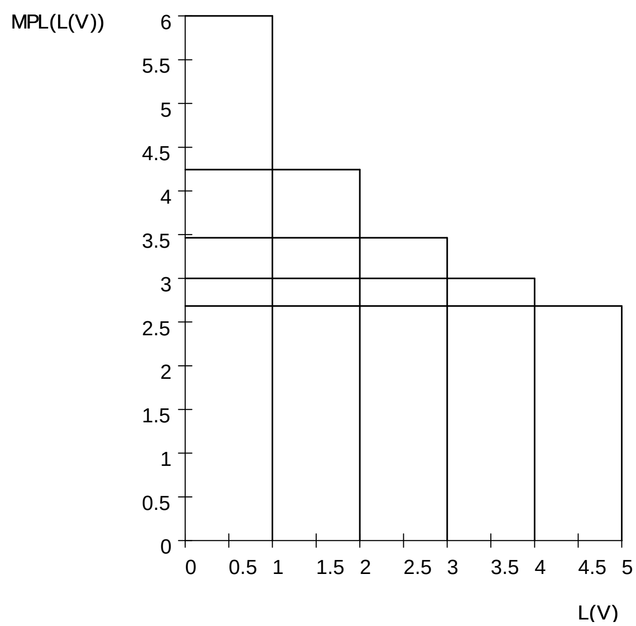


Figure 15: Total output from marginal product

16: Total product from marginal products

We can think of this smooth curve as representing a smooth marginal product of labor function. The area under this curve over the interval $(0, 5)$ is thus approximately the sum of the areas of the individual rectangles, each of which is one unit wide.

Now let's go back to the problem of determining how E&J's income varies with changes in p, the relative price of cloth. From our labor market equilibrium diagram we know that for any particular value of p there is an amount of labor $L_V(p)$ working in the wine sector, and that each worker receives a wage equal to the marginal product of labor associated with that particular level of L_V, namely $MPL_V(L_V(p))$. We put p in parentheses after L_V to emphasize that this value of L_V is a *function* of p, and we put $L_V(p)$ in parentheses after MPL_V to emphasize that this is a function of the amount of labor at work in the wine sector.

We can depict E&J's wage bill - $L_V \times w_V$ - as a rectangle with height equal to the real wage measured in units of wine and width equal to L_V, and then place this rectangle under the $MPL_V(L_V)$ curve. This is displayed in Figure 17, with the vertically striped area depicting the wage bill. For example, if $L_V = 4$, and the associated $MPL_V = 3$, then the wage bill equals twelve (12) units of wine. That is, each of the four (4) workers gets paid a real wage measured in units of wine equal to the marginal product of labor, in this case three (3) units

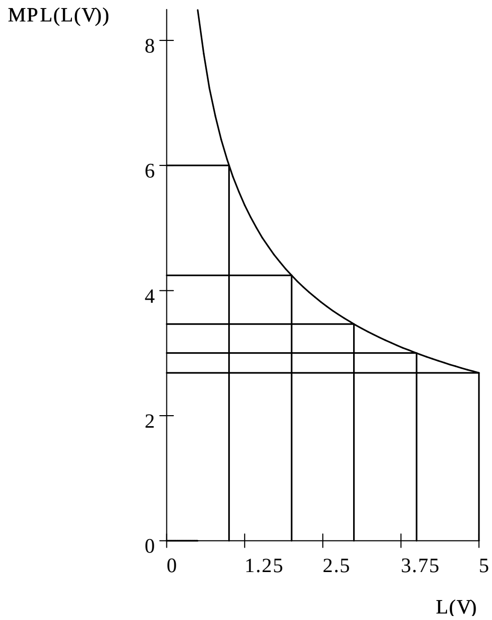
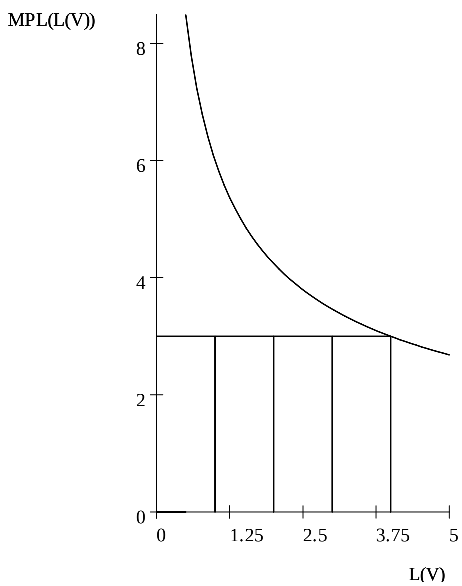


Figure 16: Total output from MP: smooth case

of wine per unit of work. Hence, the wage bill is $3 \times 4 = 12$.



17: The wage bill

Now note that total production of wine is the area under the marginal prod-

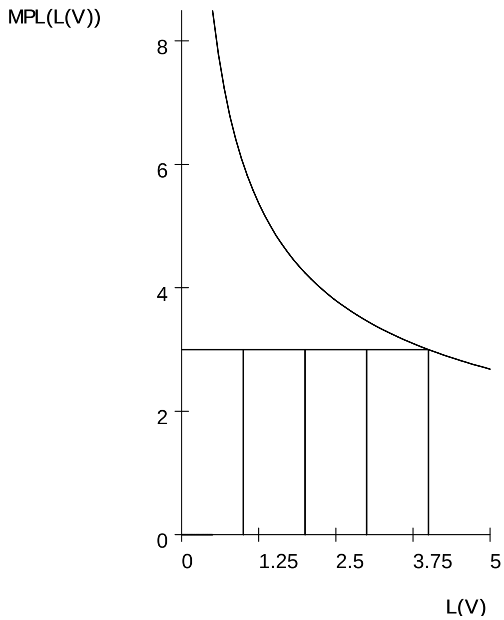
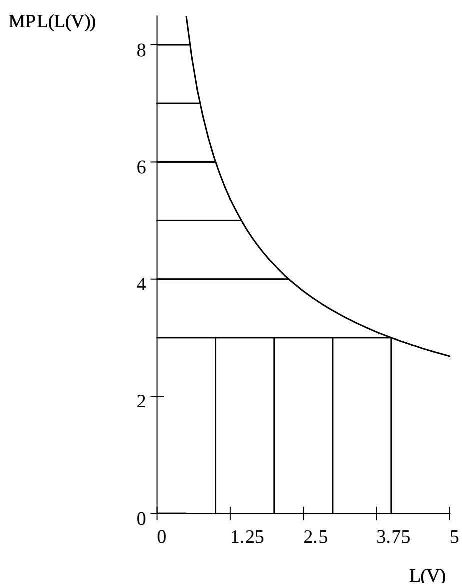


Figure 17: The wage bill

uct curve. After subtracting the wage bill representation form this area, we have E&J's income left. This is depicted as the vertical striped area in the Figure 18.



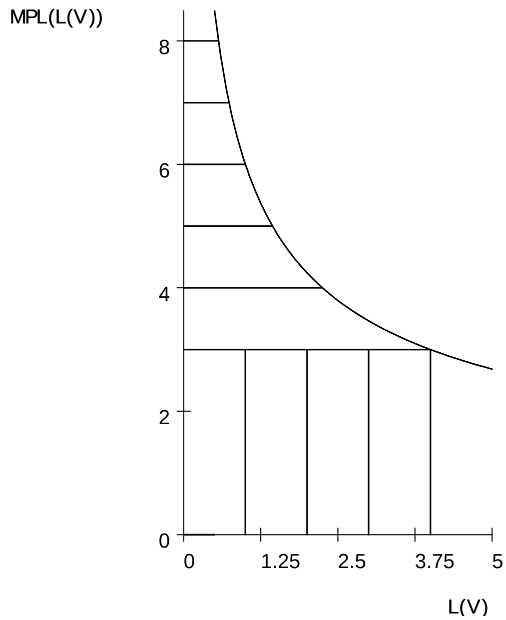


Figure 18: Wage bill and spec. factor income

18: The wage bill and specific factor income

Now consider what happens when p increases. From the labor-market equilibrium diagram, we know that an increase in p increases w_V, the equilibrium real wage measured in units of wine, and decreases the amount of labor at work in the wine sector. Such a change is depicted in Figure 20, with the new wage bill area and new depiction of E&J's income displayed in red, and with the cross-hatched area depicting E&J's loss in income measured in wine.

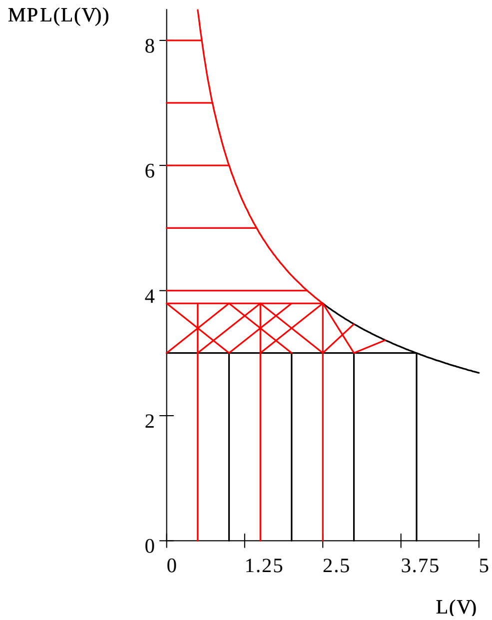
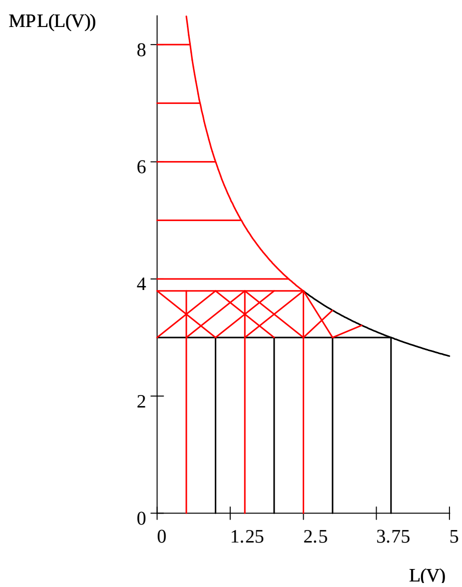


Figure 19: Effects of higher p



20: Effects of a higher value of p

What does E&J's budget constraint (depicted in the cloth-wine plane) look like now compared to what it looked like with the original lower value of p? The vertical intercept of the budget constraint is their real wage measured in wine, and it has decreased. The slope of the budget constraint is $-p$, and has thus gotten steeper: each unit of wine exchanges for fewer units of cloth. Hence, E&J's new budget constraint is everywhere below what it was at the lower value of p. This is depicted in the Figure 21, with the new budget constraint in red.

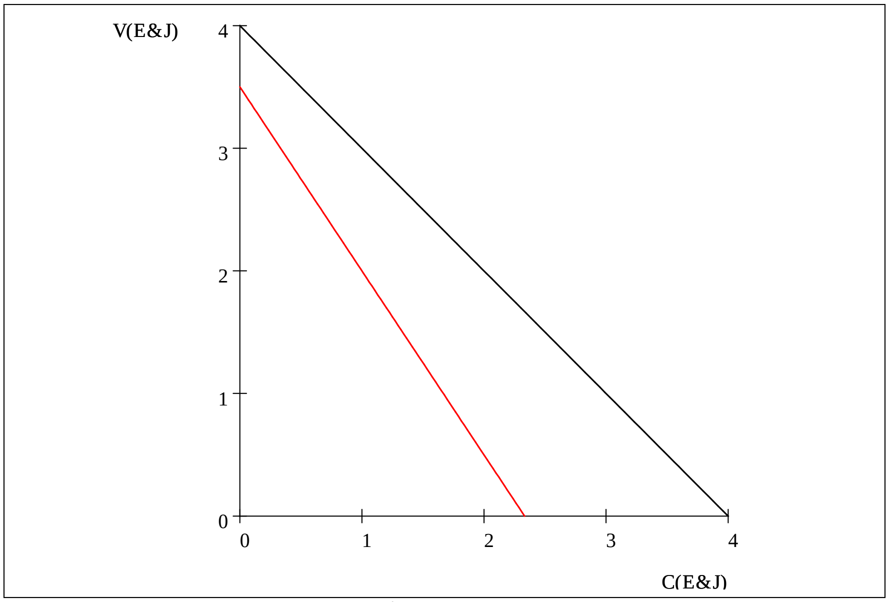


Figure 21: E&J's budget constraints

E&J are unambiguously worse off with the higher value of the relative price of cloth. This example represents a general feature of S and M models: the owner of a factor specific to production of a good whose relative price has fallen is unambiguously worse off.

4.3.2 E&J's optimal choice

What happens to E&J's most-preferred choice of cloth an wine as p changes? Without further information about preferences we can't in general say anything. What is likely, though, is a reduction in the quantity demanded of cloth as p increases. This is because the increase in p has not just the the usual *substitution effect* but also reduces E&J's income. This probable situation is depicted in the Figure 22, with the red curves representing the case of a higher p and the black curves representing the case of a lower p.

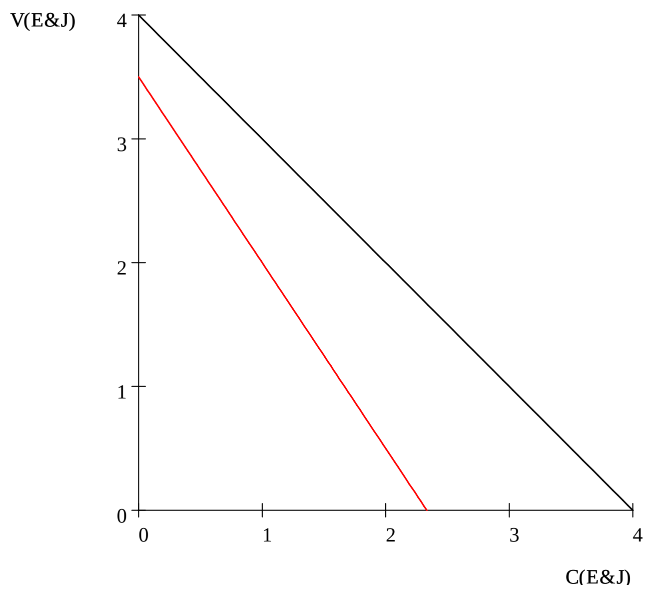


Figure 20: E&J's budget constraints

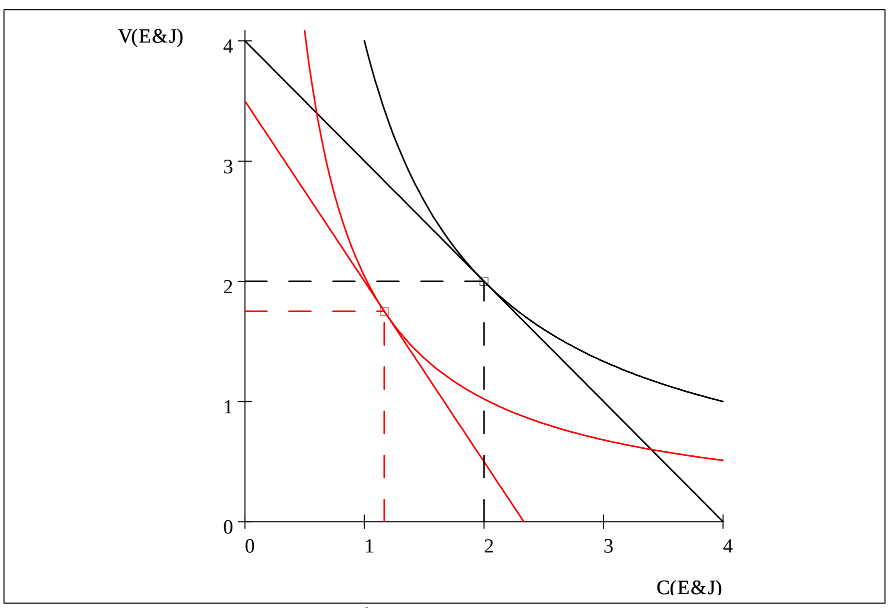


Figure 22 : E&J's budget optimal choices

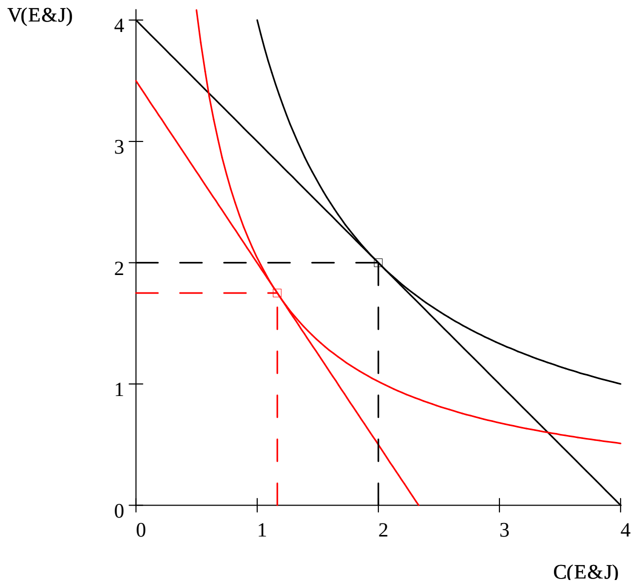


Figure 21: E&J's optimal choices

4.3.3 LW's income

Now consider what happens to LW, owner of the factor specific to the production of cloth, when p increases. First remember, as noted earlier, that an increase in p increases employment in the cloth sector and thus decreases $MPL_C(L_C)$. We can think of workers in the cloth sector as getting paid in units of cloth, so that they get paid the marginal product associated with the level of employment in the cloth sector. As noted earlier, this means a higher value of p leads to a lower real wage measured in units of cloth. We can depict the wage bill in the cloth sector, measured in units of cloth, again by a rectangle with height equal to the marginal product of labor at that level of employment and width equal to L_C. The area under the marginal product function minus the wage bill thus represents LW's income measured in units of cloth.

If p increases, then, and L_C increases as LW hires more workers, the effects on LW's real income **measured in units of cloth** have effects in the reverse direction of the effects of an increase in p on E and J's real income **measured in units of wine**. This is depicted in the Figure 23, in which the cross-hatched area represents the **increase** in LW's real income measured in units of cloth, and the red lines represent the new situation, i.e., the situation following the increase in p.

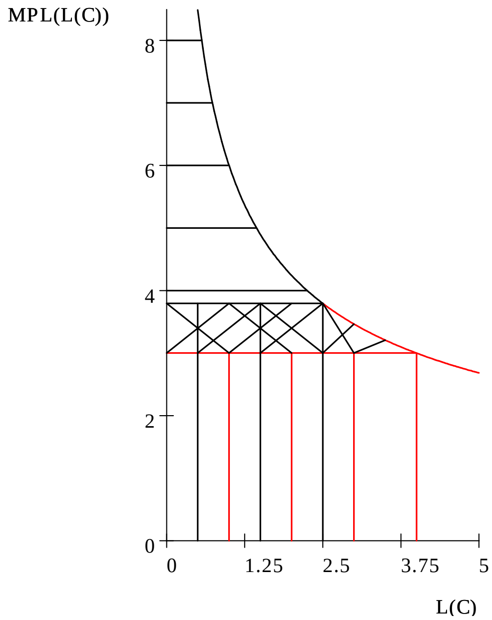
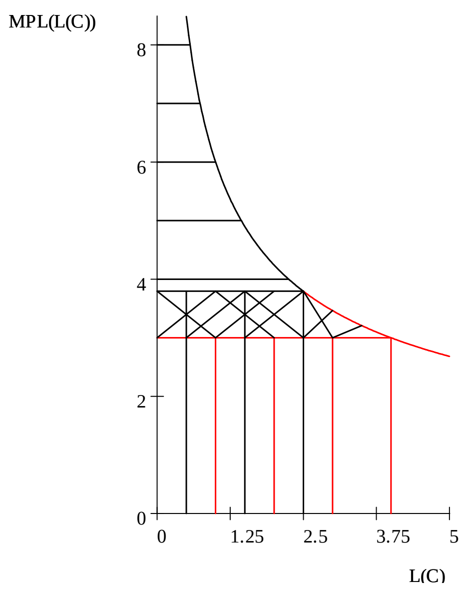


Figure 22: Effect of higher p on $R(E\&J)$



23: Effects of a higher value of p on $R(C)_{LW}$

What about the effect of an increase in p on LW's real income measured in units of wine? Because the hypothesized change is an increase in the relative price of cloth, this means a unit of cloth now exchanges for more units of wine (remember, the relative price of cloth is units of wine/unit of cloth). LW has more cloth, and this cloth is worth more wine, so his real income measured in units of wine has also increased.

We can depict this as a shift in LW's budget constraint. The **horizontal** intercept of his budget constraint measures his real income measured in units of cloth. This has increased. And, because p has increased, the budget constraint has a steeper slope. Hence, the vertical intercept, which measures LW's real income measured in units of wine, must also have increased. This shift is depicted in the Figure 24, in which the black line reflects the budget constraint for the lower value of p and the red line reflects the budget constraint for the higher value of p.

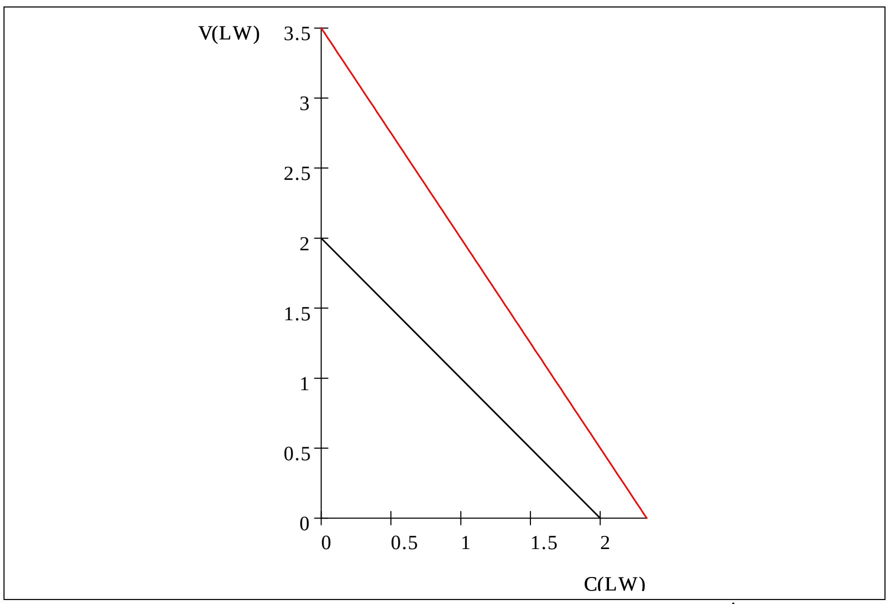


Figure 24: LW's budget constraints for different $p's$

The conclusion we draw, then, is that LW is unambiguously better off with a higher relative price of cloth, irrespective of his tastes. This, again, is a general feature of the specific and mobile factors model: the owner of factors specific to production of a product for which the relative price has increased is unambiguously better off.

4.3.4 LW's optimal choices

Again, little can be said about the most general possible case (that is, the case in which the only restriction on preferences are that they satisfy the four axioms of choice). Nonetheless, there will be a relationship between p and the choice of a most-preferred bundle, as depicted in the Figure 25, with, again, the red

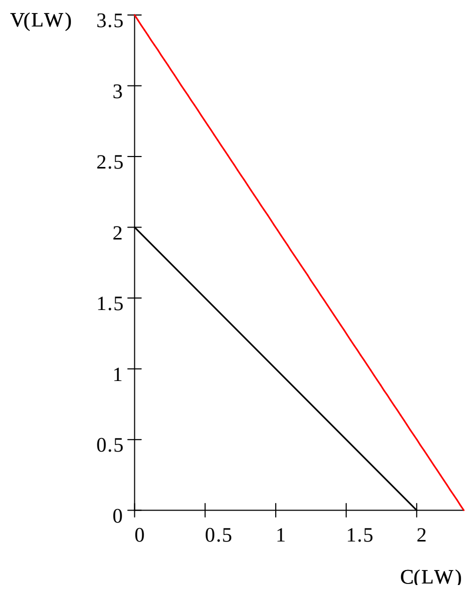


Figure 23: LW's budget constraint for different $p's$

curves representing the case of a higher p and the black curves representing the case of a lower p.

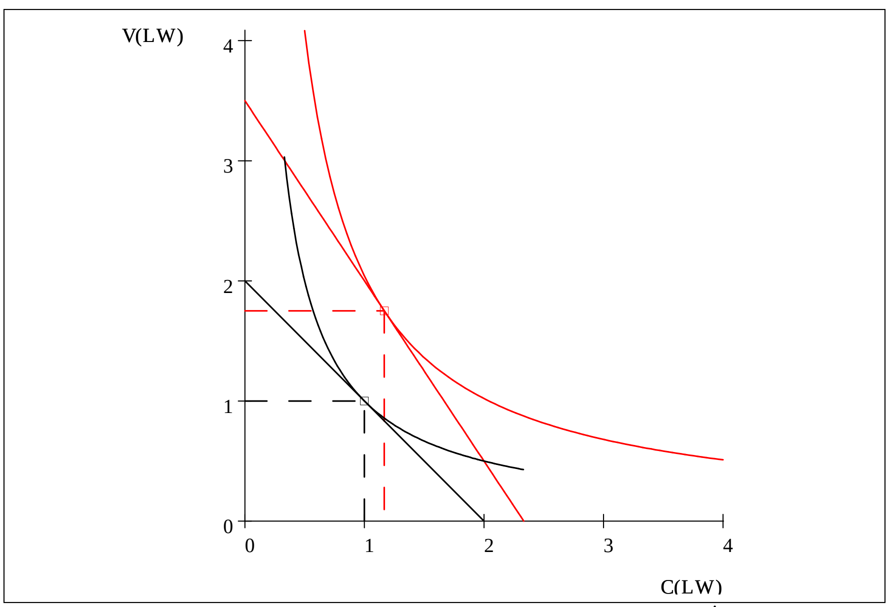


Figure 25: LW's optimal choices for different $p's$

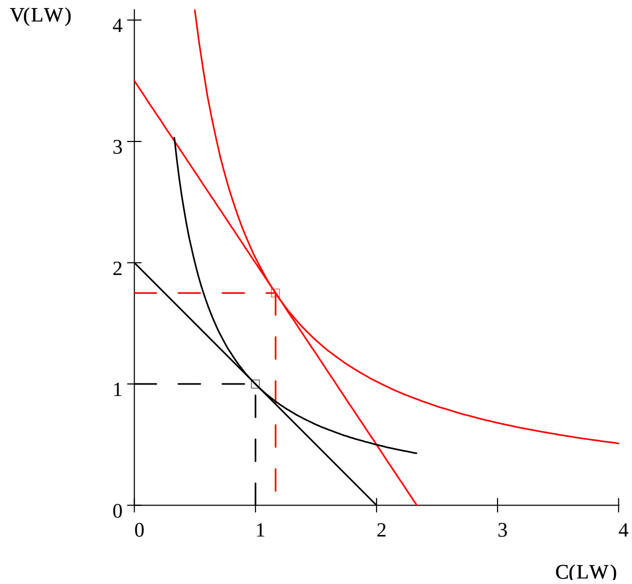


Figure 24: LW's optimal choices

Note that for preferences as depicted in this picture the increase in p led LW to *increase* cloth consumption: the effect on income swamped the substitution effect.

4.4 Market demand

The market demand curve is just the sum of the individual demand curves and is symbolically expressed as

$$C^d(p) = \sum_{l=1}^{L} C_l^d(p) + C_{E\&J}^d(p) + C_{LW}^d(p).$$

This provides the other blade of the scissors, and allows us to solve for the autarkic equilibrium relative price of cloth by equating demand to supply. An inverse demand curve example is depicted below:

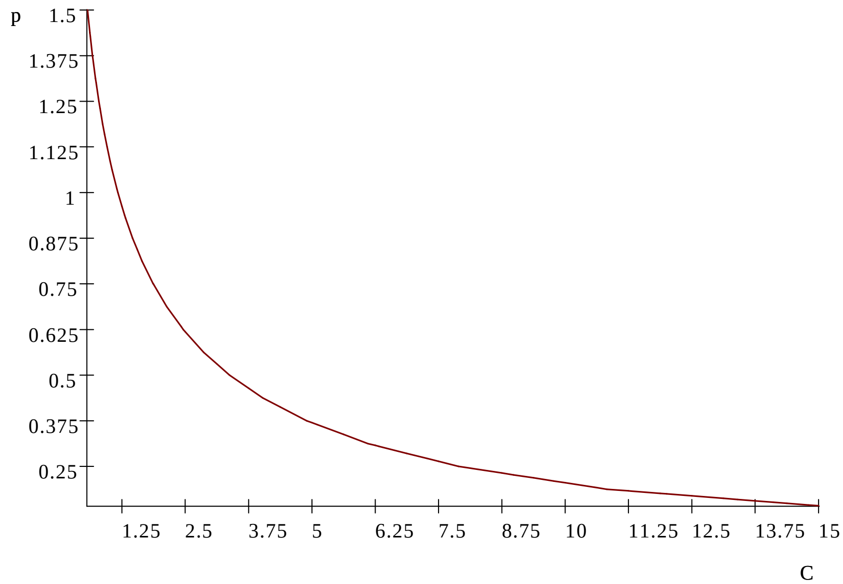
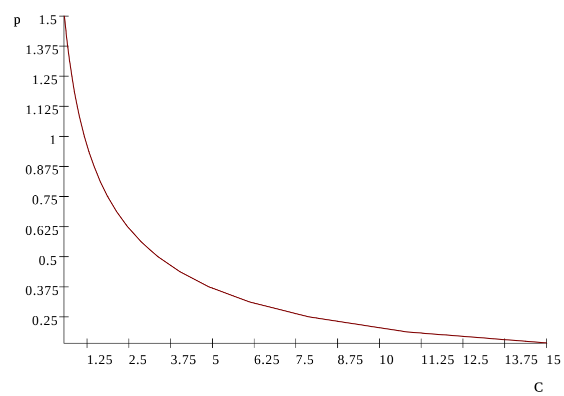


Figure 25: $p(C^d)$



26: $p(C^d)$

As should be clear by now, the "weak" restrictions economic theory places on tastes, resources, and technology do not imply that demand curves must slope down. Nonetheless, observations of economies suggest that in fact in most conditions demand curves do slope down. We will proceed in our analysis under this assumption. Broadly speaking, this assumption is equivalent to the assumption that substitution possibilities in both consumption and production are sufficiently available that they dominate "income" effects.

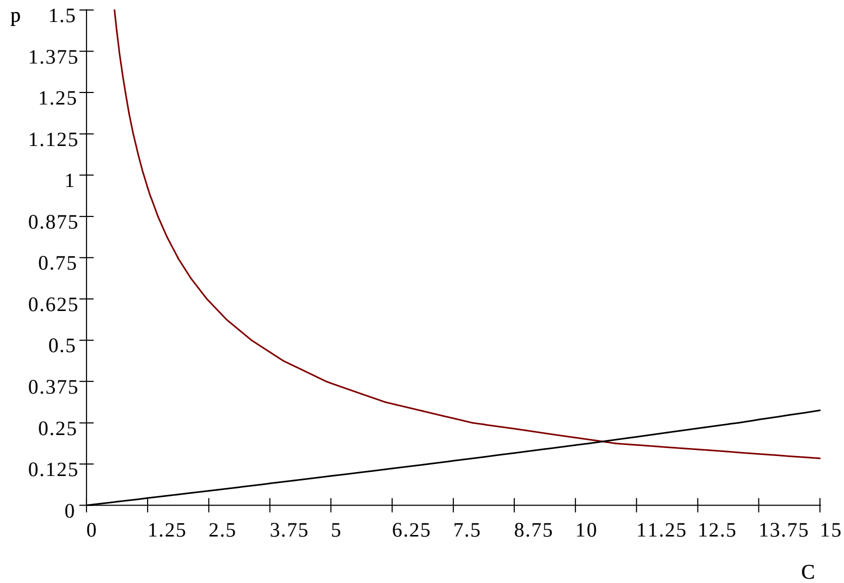
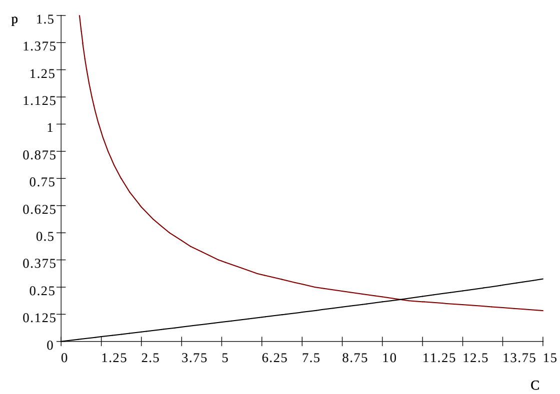


Figure 26: $p(C^s),\ p(C^d)$

5 Equilibrium: solving the model

The intersection of the demand and supply curves is a pair of numbers: the autarkic equilibrium relative price of cloth, p_a, and the quantity of cloth produced and consumed, denoted by, say, $\widehat{C}_a$. This is depicted in Figure 27:



27: $p(C^S), p(C^d)$

Once p_a is determined from the procedure of equating demand to supply, then all the other endogenous variables of the model can be determined recursively by substituting the equilibrium value of p into the individual demand curves, into the demand for labor functions, and then using this equilibrium real wage and equilibrium allocation of labor across sectors to determine the real incomes of the owners of the specific factors.

All of this is done so that we can use the model to address the two major themes of international trade: what determines the pattern of trade, and are there "gains from trade" in the usual economist's meaning that the gains to the winners are sufficiently large that they could *in principal* compensate the losers and have something left over.

To answer the "pattern of trade" question, we have to understand how the interplay of exogenous factors - tastes, resources, and technology, in this model - gives rise to a particular autarkic equilibrium price. If two countries have different autarkic equilibrium relative prices, then, with sufficiently low transport costs, trade can take place, with exports flowing from the country with the lower autarkic equilibrium relative price to the country with the higher autarkic equilibrium relative price.

We now develop a new tool to *depict* autarkic equilibrium. This depiction will let us understand why economists refer to a country that has a lower autarkic equilibrium relative price vis a vis some other country as having a **comparative advantage** vis a vis that other country.

6 Depicting autarkic equilibrium

6.1 The production possibilities frontier

Given technology and resources, the possible combinations of goods that could possibly be produced are described as the production possibilities frontier (hereafter "PPF"). In our two-good economy, a PPF is thus a collection of ordered pairs (C, V). Such a collection of combinations can be determined for any economy: its existence does not rely on the market mechanism.

For our S and M model, we can conceptually "construct" a PPF by carrying out the following thought experiment: imagine all of the mobile factor is allocated to production of one of the goods, say wine. The amount of wine produced in such an economy would be determined form the production function for wine, and could be denoted as $V_{\max}$. A point on the PPF is thus the point $(0, V_{\max})$.

Now imagine moving the "smallest possible unit" of labor from wine production to cloth production. Wine production would fall by the amount of the marginal product of labor in wine production associated with a level of employment in the wine industry of $\overline{L}$, while cloth production would increase by the marginal product of labor in cloth production associated with one "smallest possible unit" of labor (hereafter "unit" of labor). Thus, with this allocation of labor across the two sectors, the point on the PPF would be $(MPL_C(1), V_{\max} - MPL_V(\overline{L})$.

Now imagine moving one more unit of labor from wine to cloth production. Wine production would fall by the amount of the MPL_V associated with a labor force of $\overline{L} - 2$, and cloth production would increase by an amount equal to the MPL_C associated with a labor force of two (2) units of labor. Note that the reduction in wine output associated with this second hypothetical reduction in labor by one unit is more than the reduction associated with the first hypothetical reduction in labor because the $MPL_V(L_V)$ *increases* as L_V *decreases.* By analogous reasoning, the increase in cloth production due to the addition of a second unit of labor is less than the increase due to the addition of the first unit.

We could continue imagining a shift, one unit at a time, of labor from wine to cloth production until all labor was in the cloth sector. The point on the PPF with that allocation of labor would be the ordered pair $(C_{\max}, 0)$. If we plotted the associated pairs of wine and cloth produced at each of the different allocations of labor across the two sectors that fell between "all labor in the wine sector" and "all labor in the cloth sector," we would have plotted the PPF for this economy. It would be a downward-sloping curve: more cloth can only be produced by shifting labor and reducing wine production.

What would the slope of this curve be? Remember that when one unit of labor is shifted from wine to cloth production, the *change* in wine production is just a *decrease* equal to the marginal product of labor in wine production at that point. Symbolically we would represent this as

$$\Delta V = -MPL_V(L_V).$$

By the same token, the *increase* in cloth production from the shift of one unit of labor from wine to cloth production would the marginal product of labor in cloth production for that size labor force. Symbolically, we would represent this as:

$$\Delta C = MPL_C(L_C).$$

The slope of the PPF is just $\frac{\Delta V}{\Delta C}$, and so can be written as

$$\frac{\Delta V}{\Delta C} = -\frac{MPL_V(L_V)}{MPL_C(L_C)}.$$

As we move along the PPF from the point $(0, V_{\max})$ towards the point $(C_{\max}, 0)$, the slope would get steeper and steeper because the MPL_V would be rising as L_V decreases and the MPL_C would be falling as L_C increases. The PPF, then, would be "bowed out" from the origin and would look like the diagram in Figure 28.

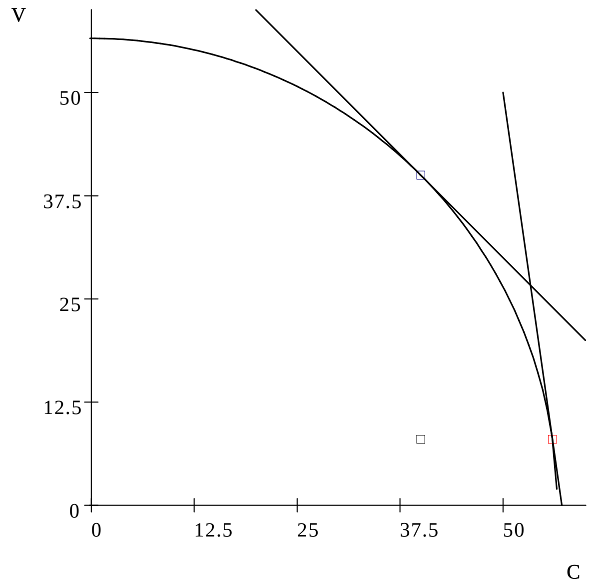
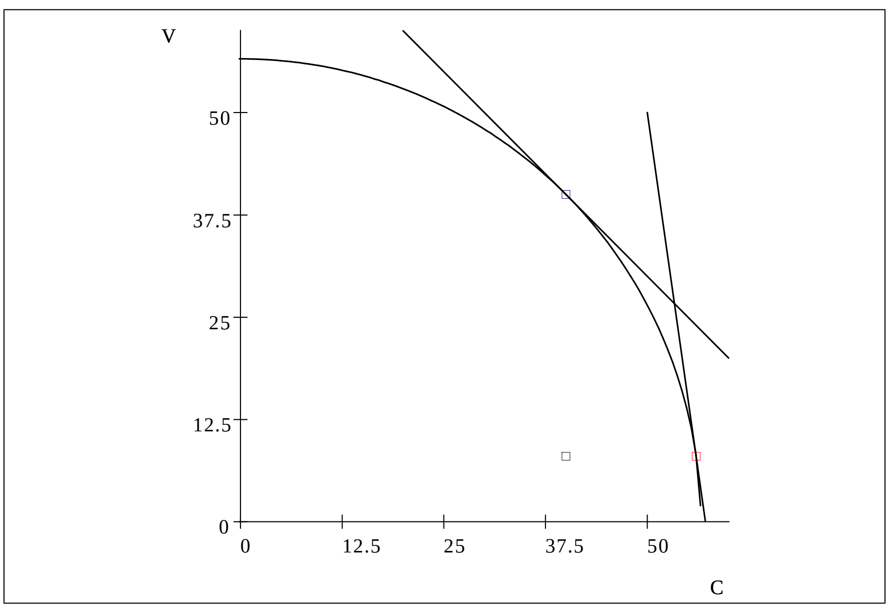


Figure 27: PPF



28: PPF

In this diagram, two distinct points on the PPF are identified, along with their associated slopes (tangent lines).

The slope of the PPF also has an interpretation as the **marginal opportu-**

nity cost for this economy of producing an extra unit of cloth. That is, at any point on the PPF, if we contemplate increasing the amount of cloth produced by one "smallest possible" unit, the slope of the PPF at that point measures how much wine would be given up, i.e., not produced.

6.2 Perfect competition and the point of production on the PPF

With this background on the PPF, we can now depict how the point of production that takes place in a perfectly competitive economy is determined by the relative price that prevails in the economy.

Recall the profit-maximizing rules for each (perfectly competitive) sector in the economy:

$$MPL_V(L_V) = \frac{w}{P_V} \tag{6.7}$$

$$p \times MPL_C(L_C) = \frac{w}{P_V}. \tag{6.10}$$

where, again, p is the relative price of cloth, $\frac{P_C}{P_V}$. In labor market equilibrium, the same real wage must be paid to workers in both sectors (otherwise it wouldn't be a "rest point" because workers in the lower-paying sector would want to move). This means

$$p \times MPL_C(L_C) = MPL_V(L_V)$$

or, upon rearrangement,

$$p = \frac{MPL_V(L_V)}{MPL_C(L_C)}$$

Now, $\frac{MPL_V(L_V)}{MPL_C(L_C)}$ is minus the slope of the PPF. Hence, in a perfectly competitive economy, production of wine and cloth must take place at the point on the PPF at which the slope of the PPF equals minus the relative price that prevails in the economy.

Put another way, this gives us a procedure for depicting the production equilibrium in the economy: find the point on the PPF at which the slope of the PPF just equals minus the equilibrium price. From the PPF we can then "read off" the amounts of cloth and wine produced.

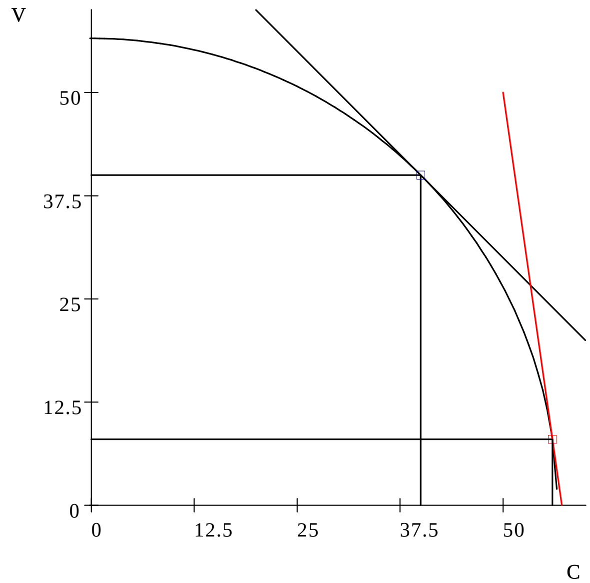
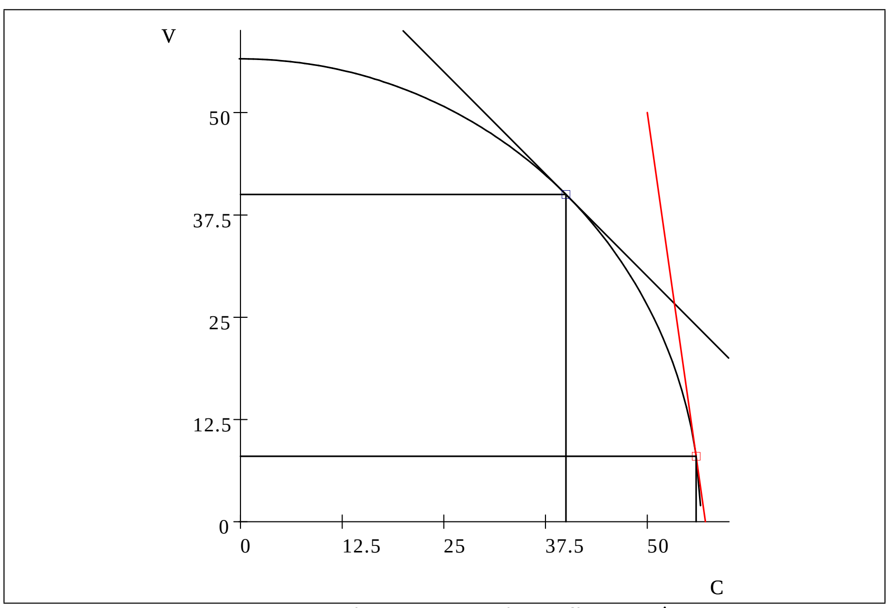


Figure 28: Points of production for different $p's$



29: Points of production for different $p's$

We are now ready to interpret what economists mean by the concept of comparative advantage. Consider a country that exports cloth. This means that in autarkic equilibrium this country's autarkic equilibrium relative price was lower than that of its trading partners. This means that the point of autarkic production was at a point where the slope of the home-country PPF was flatter than the slope of the PPF at the autarkic point of production in the foreign country. This means, in autarky, the home country could produce one more unit of cloth at lower opportunity cost than could the foreign country. This is precisely what is meant by a "comparative advantage:" in autarky, a country has a comparative advantage over another country in, say, production of cloth vis-a-vis wine if it can produce more cloth at a lower opportunity cost (of wine) than the other country.

Note the logical impossibility of a country *not* having a comparative advantage in production of any good: because the concept is opportunity cost measured in units of other goods foregone, if a country is at a comparative disadvantage in, say production of wine vis-a-vis cloth, it must have a comparative advantage in production of cloth vis-a-vis wine.

6.3 Depicting consumption

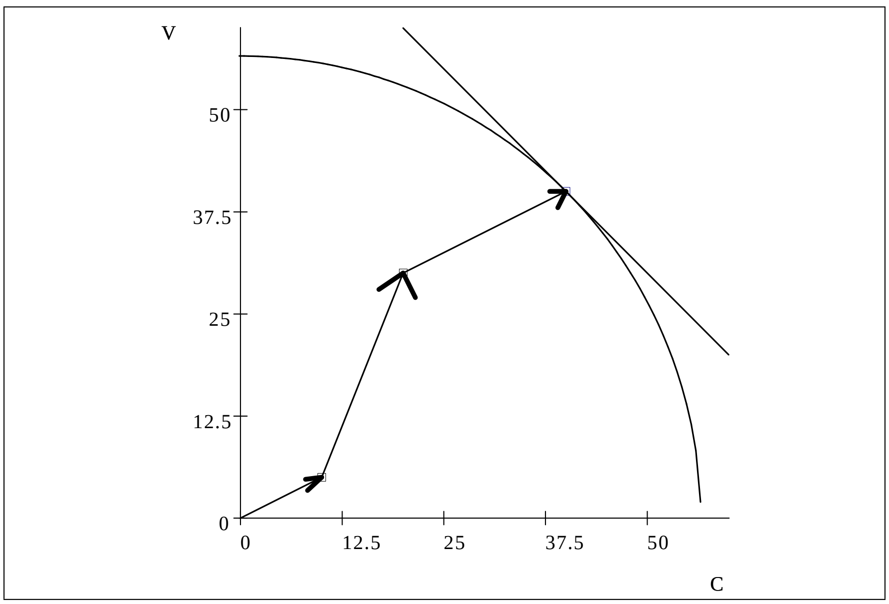
Knowing the autarkic equilibrium relative price p_a, one can calculate the autarkic equilibrium consumption of each individual in the economy by substituting p_a into each individual's general equilibrium demand curve. This generates an ordered pair of numbers for each individual; his or her consumption of cloth, $\widehat{C}_i^d$,and his or her consumption of wine, $\widehat{V}_i^d$. We use a "hat" over the variable to indicate it is an *equilibrium* value, and the "i" subscript to indicate it refers to a particular individual (the i^{th} individual). Such a pair of numbers can be represented geometrically as a **vector**: a directed line segment with horizontal distance equal to $\widehat{C}_i^d$ and vertical distance equal to $\widehat{V}_i^d$. For instance, if $\widehat{C}_i^d = 5$ and $\widehat{V}_i^d = 10$, the consumption vector for individual i would be a line segment connecting any two points in the (C, V) plane for which the horizontal distance between the two points would be five (5) and the vertical distance would be ten (10). This is a "directed" line segment because we identify the "terminal" point of the vector as the endpoint of the segment with the larger values of C and V, and the "base" of the vector as the endpoint with the smaller values. When depicting vectors in the Cartesian plane, we usually indicate the terminal point by attaching an arrowhead.

For example, if E&J's autarkic equilibrium consumption values of cloth and wine were, say, the ordered pair $(5, 10)$ - that is, five (5) units of cloth and ten (10) units of wine, E&L's consumption vector could be described as the line segment connecting the two points $(0, 0)$ and $(5, 10)$, or connecting the two points $(5, 10)$ and $(10, 20)$, or the points $(2, 12)$ and $(7, 22)$. Notice that these different depictions of the same vector can be viewed as displacing any one of them in a parallel manner in the (C, V) plane.

The advantage of thinking of equilibrium consumption pairs as vectors is that we can add vectors geometrically by locating the base of one vector at the terminal point of another vector, and thinking of the line segment that connects the base of the first vector with the terminal point of the second vector as the sum of these two vectors. For example, if E&J's autarkic equilibrium consumption vector is $(5, 10)$, and LW's autarkic equilibrium consumption vector is $(2, 17)$, The sum of E&J's and LW's consumptions is just $(7, 27)$.

With this background, we can now depict consumption equilibrium along with production equilibrium. Within the space in the first quadrant of the (C, V) plane that lies within the confines of the PPF, we can simply add the consumption vectors of all the individuals in the economy by placing them terminal point to base. The result of these additions is a vector that must start at the origin and end at the point of production on the PPF.

For example, suppose the autarkic equilibrium price is one (1), and the slope of the PPF equals one (1) at the point $(40, 40)$, i.e., at production of 40 units of cloth and 40 units of wine. At this autarkic equilibrium price, suppose the consumption values of cloth and wine for E&J and LW are $(10, 5)$ and $(10, 25)$, respectively, and $(20, 10)$ for the *aggregate* of the $\overline{L}$ number of laborers. We would depict this equilibrium as in Figure 30:



30:Depiction of production and consumption

7 Free Trade Equilibrium

Imagine now that along with the domestic economy we analyzed above there exists a foreign economy that also produces just the two products cloth and wine. The interplay of tastes, resources and technology in the foreign economy gives rise to a foreign autarkic equilibrium relative price that differs from the home-country autarkic equilibrium relative price. If transport costs are sufficiently low, then trade will take place between the two countries. To keep things as clear as possible, we will assume zero transport costs, so that trade equalizes

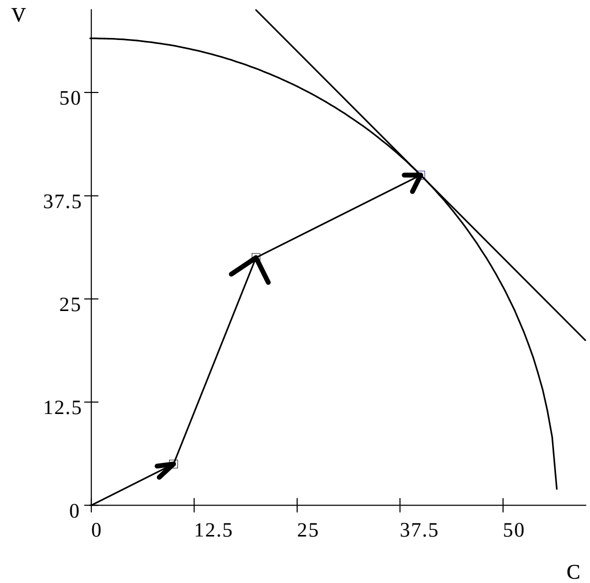


Figure 29: Depiction of consumption and production

domestic and foreign prices. The free trade equilibrium conditions are thus:

$$ES = ED^*$$

$$p = p^* \equiv p_{FT}.$$

The country with the *comparative advantage* in the production of cloth will export cloth and import wine, and the equilibrium free trade relative price of cloth will be higher than the autarkic equilibrium relative price for this country.

To be specific, assume the home country has the comparative advantage in cloth production. An analysis of the comparison between autarky and free trade for this country is just an analysis of the effects of a higher relative price of cloth on the various agents in the economy.

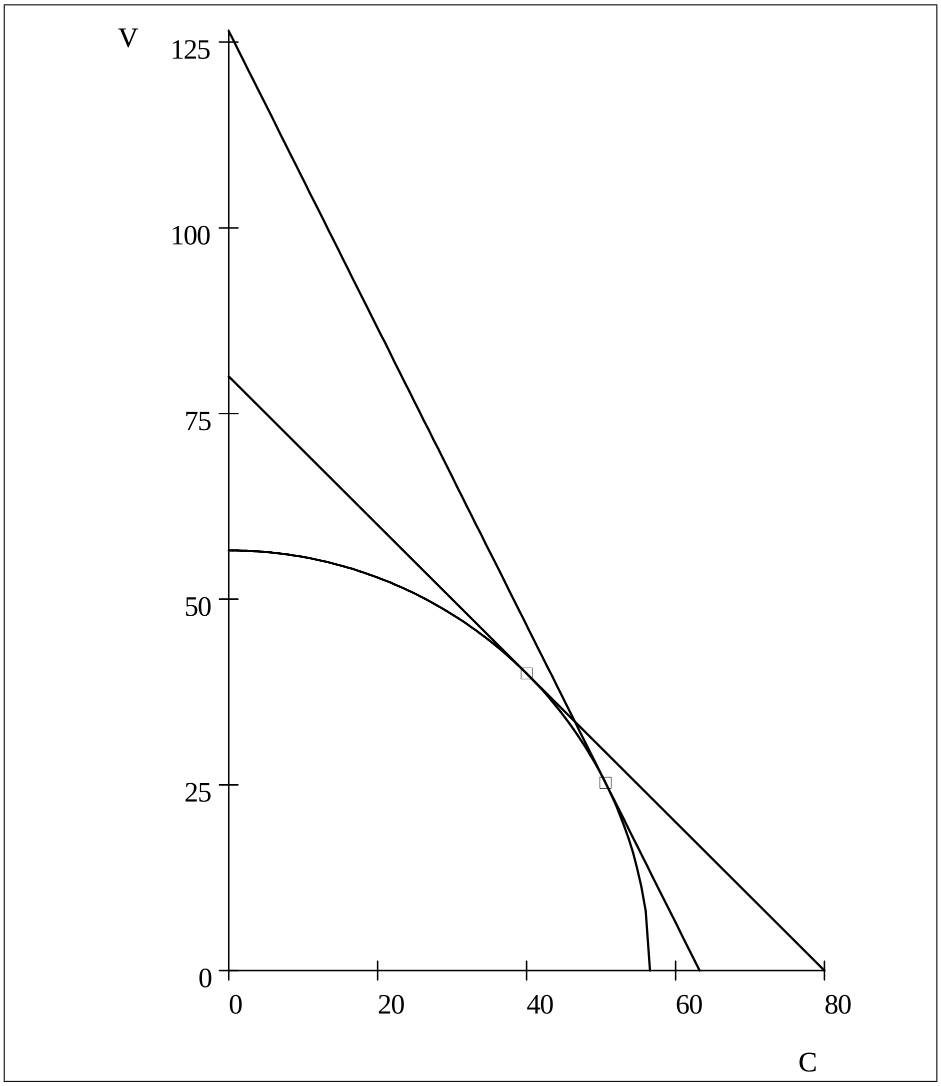
7.1 Production and consumption effects

A higher relative price of cloth leads means that production takes place at a point on the PPF where the slope is steeper than it was in autarky. This implies an increase in cloth production and a decrease in wine production, as depicted in the earlier figure. Consumption effects are in general ambiguous, but in aggregate are predictable as long as the aggregate demand curve is downward-sloping. In this case, a higher relative price of cloth reduces cloth consumption *in aggregate* by domestic residents.

7.2 Depicting and comparing autarky and free trade equilibria: Behold!

We can now depict in one diagram a comparison of the effects of trade on production and consumption within a country. Such a comparison requires a comparison between the autarkic equilibrium values of the endogenous variables and the values evaluated at the free-trade equilibrium price. Without loss of generality, assume the autarkic equilibrium relative price of cloth is less than the free-trade equilibrium price. We start by depicting separately the effects on production and the effects on the consumption of each affected individual economic agent. We then superimpose all these diagrams in one display.

The effect of a higher value of p on production is depicted by a change in the production point on the PPF to where the slope of the PPF equals the new higher price:



31: Depicting autarkic and free trade production

The effects on E&J, the owner of the factor specific to production of wine, is displayed in the following figure:

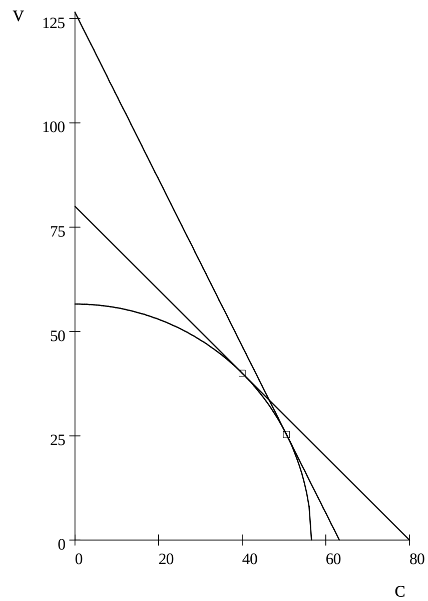
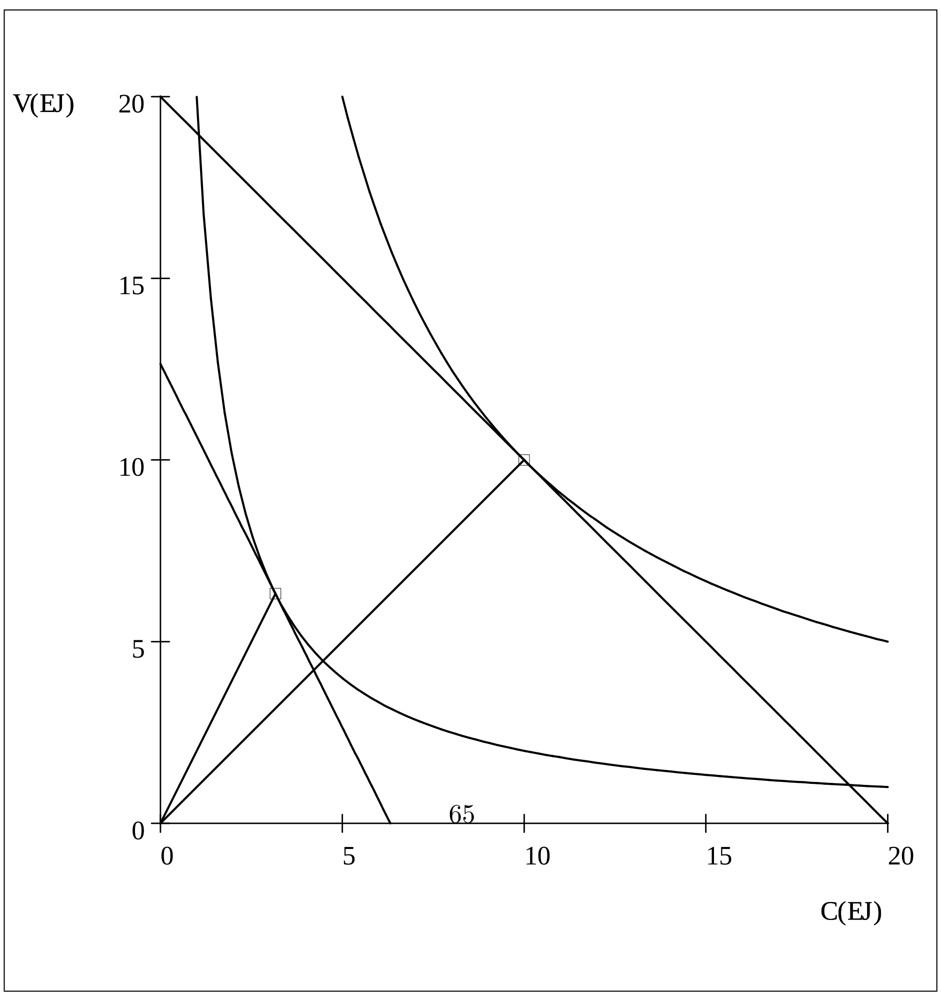


Figure 30: Depicting autarkic and free trade production



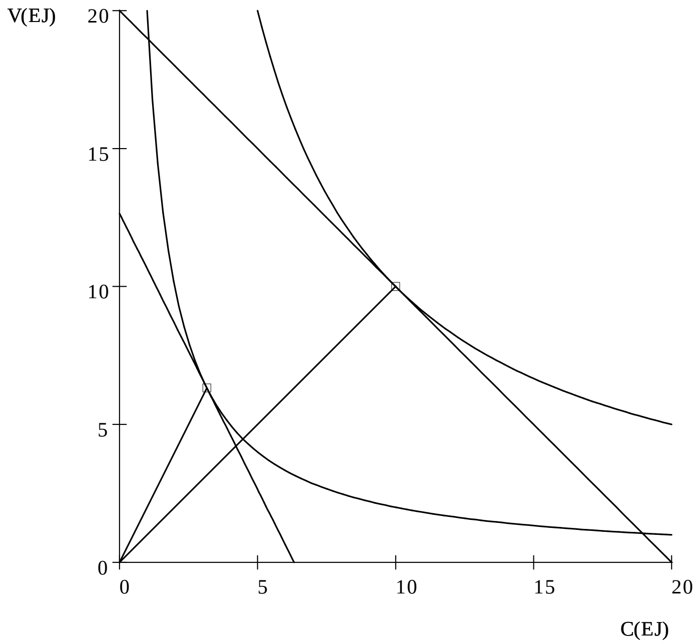
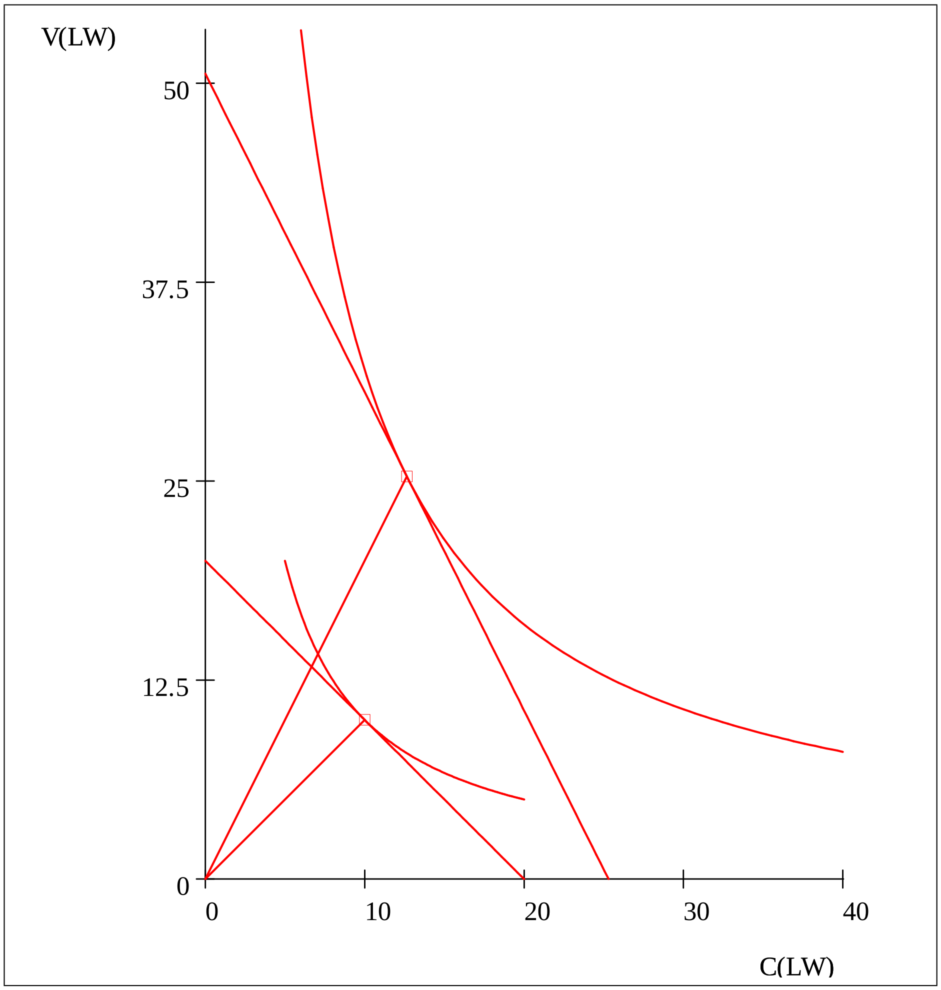


Figure 31: E&J consumption in autarky and free trade

32: EJ in autarky and free trade: $p_a < p_{FT}$

As depicted, E&J consume less of both cloth and wine in the free-trade equilibrium compared to their consumptions in autarky. In general, of course, all we can say is that E&J's budget constraint has "shifted in."

The effects on LW are displayed in Figure 33:



33: LW in autarky and free trade: $p_a < p_{FT}$

As depicted, LW consumes more of both in free trade than in autarky. Again, in general all we know for sure is that LW's budget constraint has shifted out.

Now consider the effects on workers. Assuming all workers are identical, the effects are displayed in Figure 34:

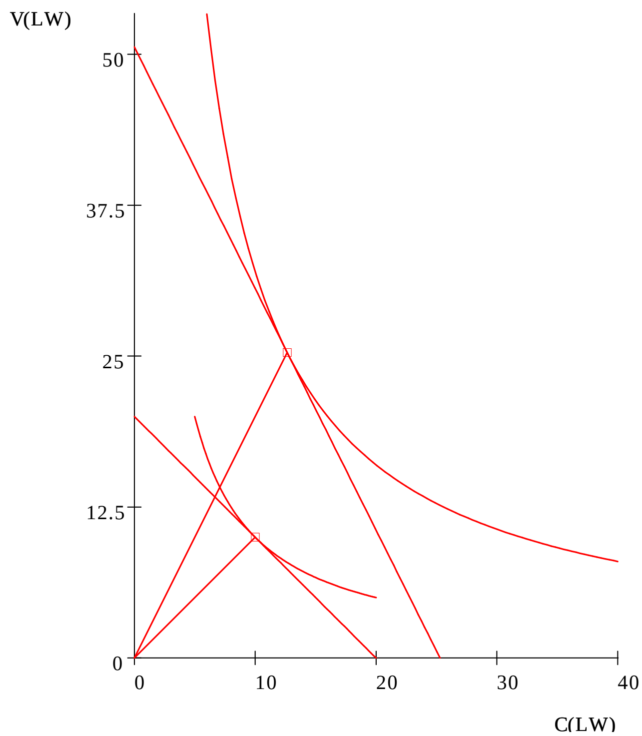
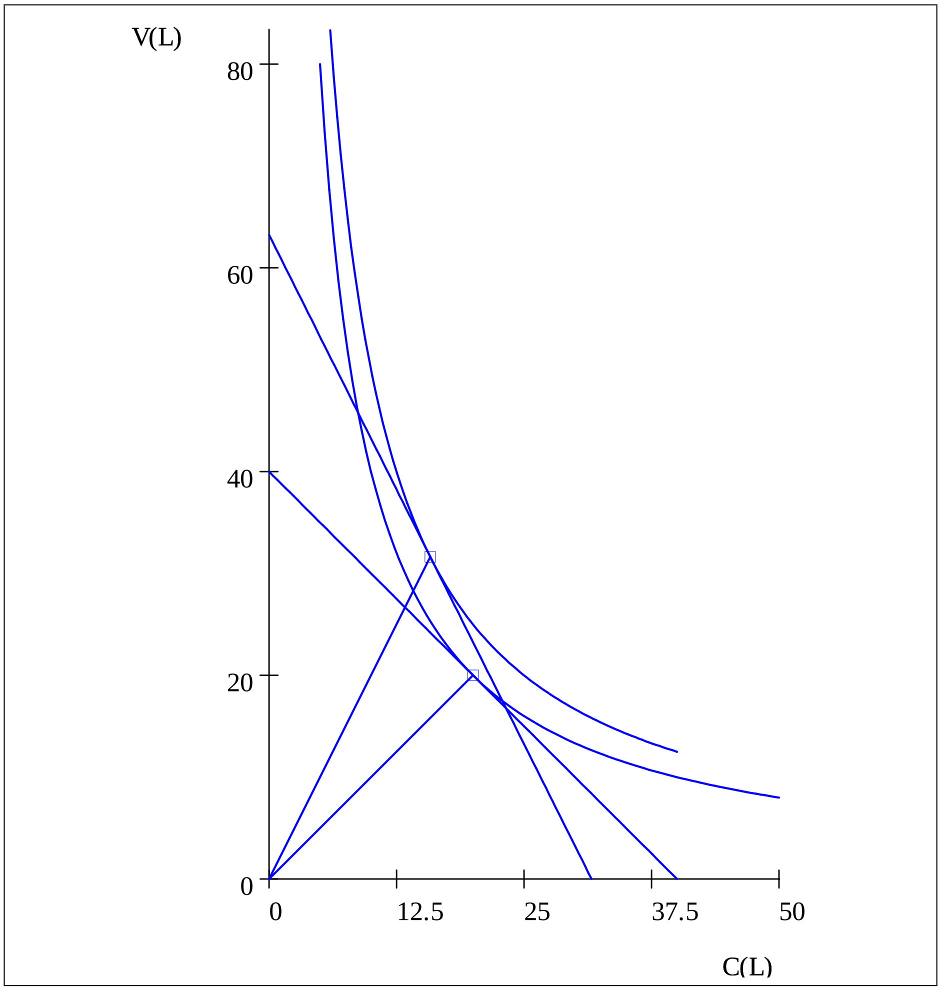


Figure 32: LW consumption in autarky and free trade



34: Workers in autarky and free trade: $p_a < p_{FT}$

As we know, all that can be said in general is that the increase in p will "twist" the worker's budget constraint, with the vertical intercept increasing and the horizontal intercept decreasing.

We can now depict all of these consumption points, along with production effects, in one diagram that also shows how the value of exports equals the value of imports. We do this by superimposing the above diagrams of the consumption vectors for each of the three distinct factors of production within the PPF, where the superimposition is done by "adding up" the consumption vectors in the autarkic case and the free trade case:

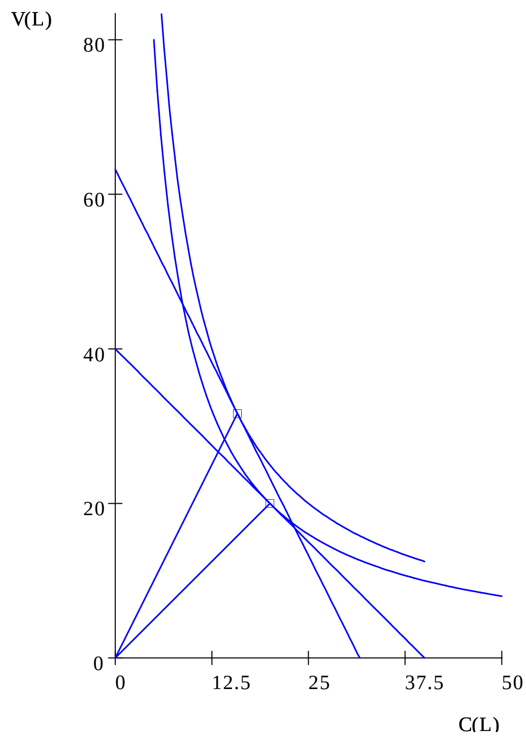
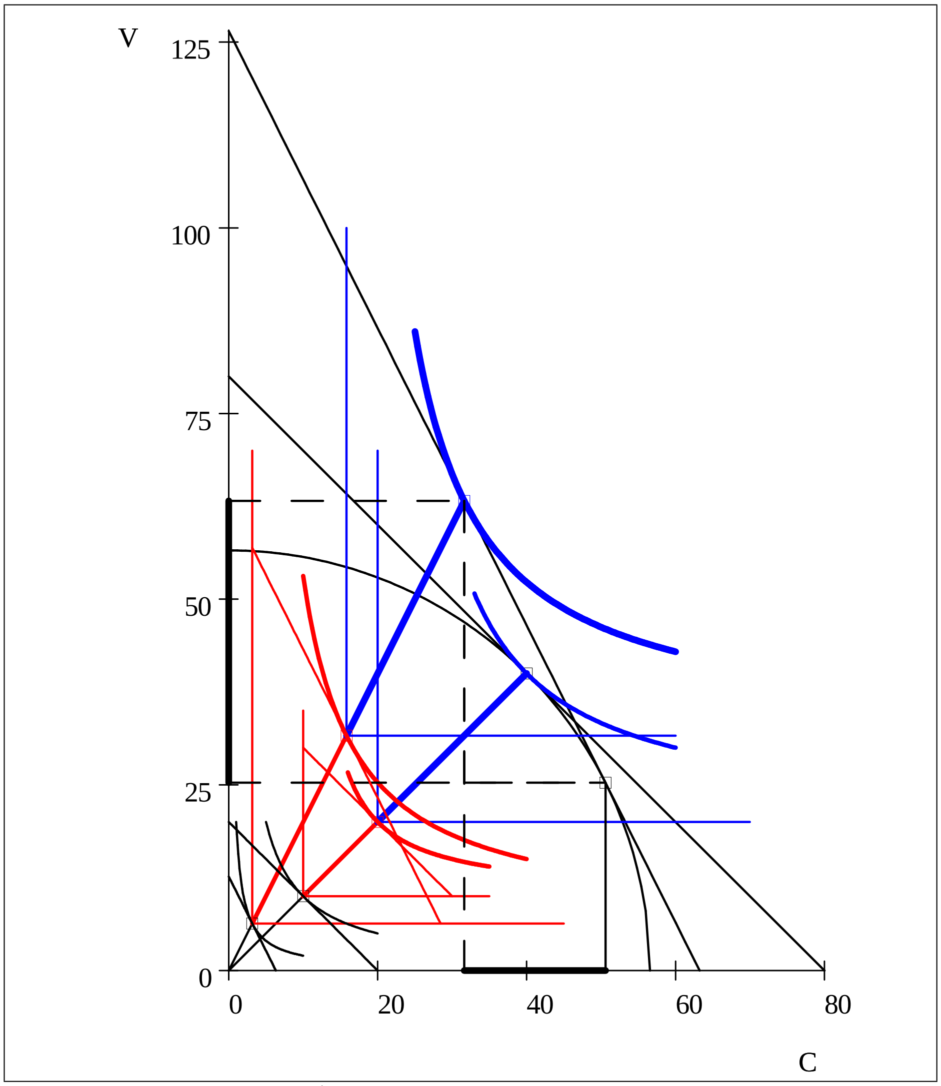


Figure 33: Workers' consumption in aut.and free trade



35:Behold! Depicting autarky and free trade

Figure 35, while appearing complex, does two things. First, it simply collects in one place the information in the preceding four diagrams, thus reducing the "visual memory" one needs to keep track of the information. Second, by adding up consumption vectors in the free trade case, it allows a depiction of economy-wide excess demand for wine and excess supply of coffee. The excess demand for wine equals the gap between consumption and production, and is depicted by the distance of the heavy vertical line segment on the vertical axis. The excess supply of coffee is depicted by the distance of the heavy horizontal line segment along the horizontal axis.

Note that the ratio of imports to exports equals the slope of the free trade

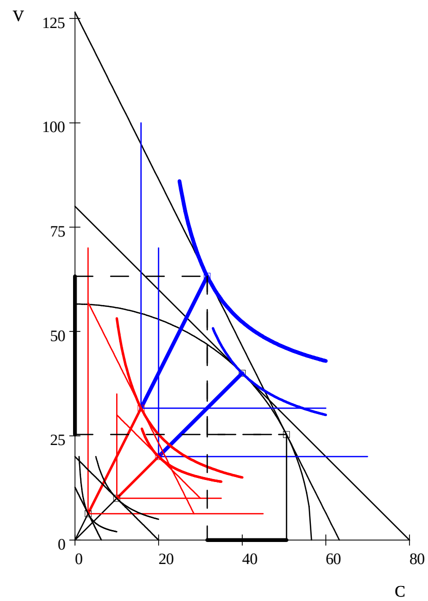


Figure 34: Depicting autarky and free trade

budget constraint, namely p_{FT},:

$$p_{FT} = \frac{\overbrace{V^d - V^S}^{imports}}{\underbrace{C^S - C^d}_{\exp orts}}$$

which implies

$$p_{FT} \times \left(C^S - C^d\right) = V^d - V^S.$$

That is, the value of exports (measured in units of wine) equal the value of imports (also measured in units of wine). In models in which the only trade among individuals of different nations is in goods and services, this equality of the value of exports and imports is a general implication.

At the risk of beating a dead horse, let us reiterate what are all the components in this diagram. Consumption vectors are drawn for the three distinct classes of people: black (E&J), red (LW), and blue (Labor). In autarky, these three vectors must add up to the amounts of cloth and wine produced. Through the equilibrium consumption points are drawn both budget constraints and the indifference curves that go through these most-preferred pairs.

Because we are adding consumption vectors, the autarkic equilibrium consumption vector for LW has its base at $(10, 10)$ and its tip at $(20, 20)$. Hence, we depict the vertical and horizontal axes for LW starting at $(10, 10)$, and we depict LW's autarkic equilibrium budget constraint, most-preferred pair, and the indifference curve through this pair drawn as if the origin for LW's cloth-wine plane had its origin at $(10, 10)$.

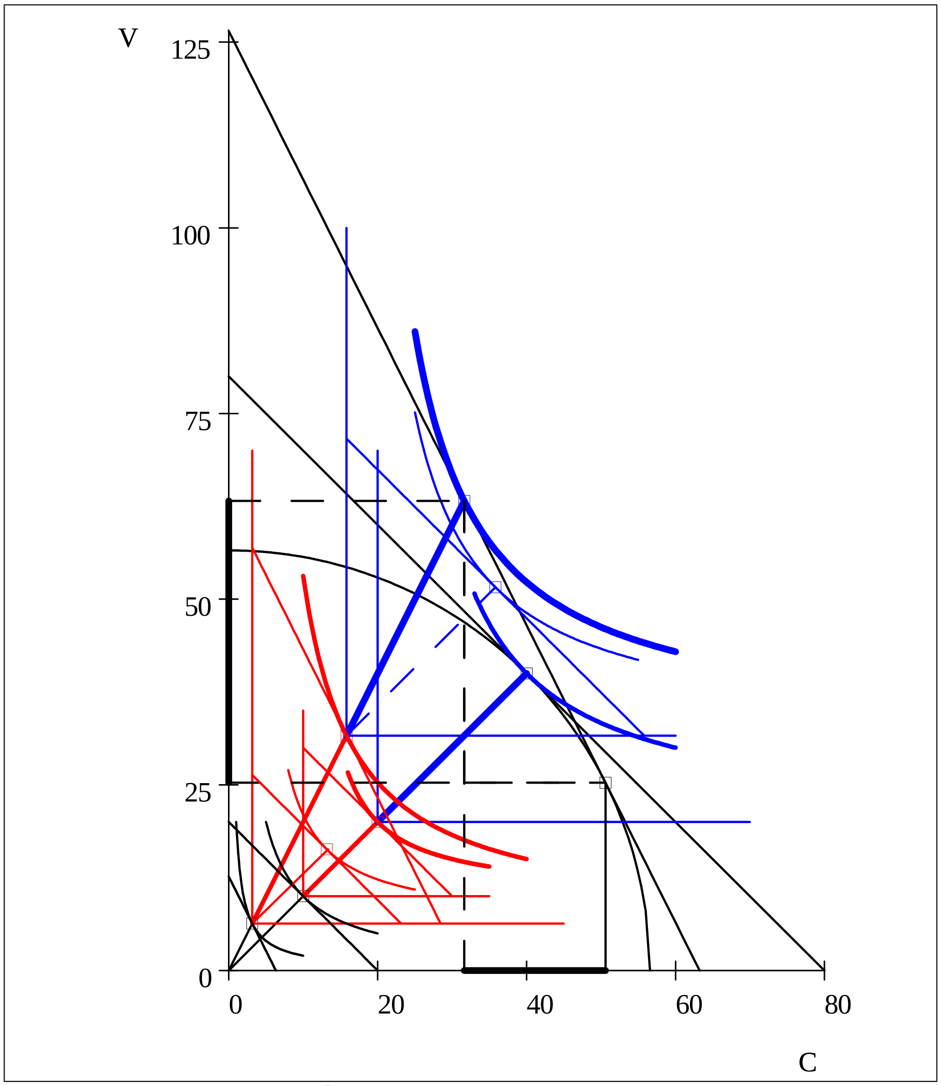
Likewise, the autarkic equilibrium consumption vector for Labor has its base at $(20, 20)$ and its tip at $(40, 40)$. Hence, Labor consumes twenty (20) units of both cloth and wine. Labor's autarkic equilibrium budget constraint, most-preferred pair, and the indifference curve through this pair are also displayed, all drawn as if the origin for Labor's cloth-wine plane had its origin at $(20, 20)$. Note that because, in autarkic equilibrium, total consumption must equal total production, the depiction of Labor's budget constraint overlays the economy-wide budget constraint.

There is nothing generic about the consumption vectors all having the same direction, that is, having the same slope: in general, all that our restrictions on tastes, technologies, and resources require of the consumption vectors is that they add up to the production vector, i.e., the point on the production possibilities frontier at which autarkic equilibrium production takes place. In this diagram, this is at the point $(40, 40)$,where the slope is minus one (-1).

Now consider the depiction of the free trade equilibrium. Starting at the origin, i.e., the point $(0, 0)$, another consumption vector for E&J is drawn. This is the equilibrium consumption vector for E&J in the free trade equilibrium with $p = 2$. This vector runs from $(0, 0)$ to $(3.162, 6.324)$. The relevant budget constraints and associated indifference curves are also displayed.

LW's free-trade consumption vector is added to E&J's by putting the base of LW's at the tip of EJ's. Vertical and horizontal axes for LW are then drawn as if the origin is $(3.162, 6.324)$, and the budget constraint, most-preferred pair, and indifference curve through this pair are also depicted in relation to this origin.

What this diagram cannot do is depict the changes in well-being for each individual. To do this, the autarkic consumption vectors, autarkic budget constraints, autarkic most-preferred pairs, and autarkic indifference curves through these pairs are all displaced in parallel fashion so as to have the same origin as the free-trade consumption vectors. Thus, the indifference curves through the autarkic and free-trade most-preferred points are depicted in the same cloth-wine planes, as in the original pre-superimposition diagrams, in Figure 36:



36: Behold! Depicting autarky and free trade

From analyzing this diagram. you should be able to tell which vectors are the autarkic ones (they add up to a point on the PPF); which "color" represents labor (the budget constraints "cross"); which color represents LW (unambiguously better off); and which color represents E&J (unambiguously worse off).

8 Gains from trade

Much as in the endowment economy model, we ask of this model: Is there a redistribution of income among the domestic residents such that, if this redistribution took place (without using up any resources in the process), would no

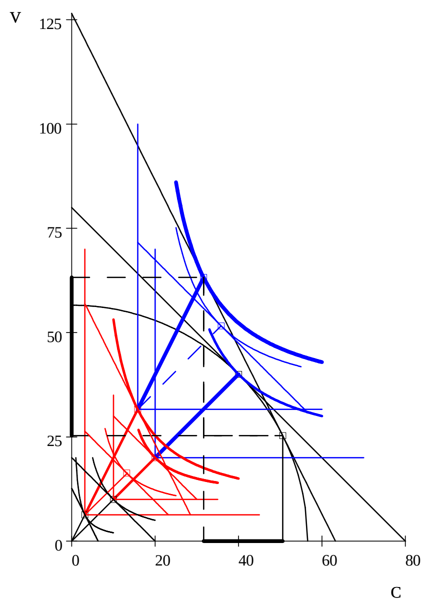


Figure 35: Behold!:

one be worse off and would at least some one be better off? The answer to this question is: yes. To demonstrate that this is so, we construct a redistribution along the same lines as we did in the endowment economy model.

Recall that to demonstrate "gains from trade" in the endowment economy model, we pointed out that a **feasible allocation** of the total supplies of commodities supplied as endowments to an economy is an allocation that would replicate the autarkic consumption choices of the individuals in the economy. With such a reallocation of endowments, every member in the economy would be better off if they could trade at a non-autarkic relative price.

In contrast to the endowment economy, in this model the amount of wine and cloth produced varies with the relative price that prevails. Consequently, at a non-autarkic relative price, the total amounts of wine and cloth produced are changed from the autarkic quantities. Hence, an allocation of wine and cloth to individuals that replicated their autarkic consumption levels would not be feasible.

The autarkic consumption levels of wine and cloth, though, do provide a useful benchmark for our analysis of "gains from trade." What we do is show how to construct a **feasible** reallocation of free-trade domestic outputs of wine and cloth that leads to budget constraints for every individual that pass to the northeast of the individual's autarkic consumption point.

As a first step in this process, we make note of key features of the changes in production that occur in a change from autarky to free trade. Imagine, without loss of generality, that the change from autarky to free trade leads to a higher relative price of cloth. The autarkic and free trade points of production on the PPF are depicted in Figure 37, with free trade production taking place with higher output of cloth and lower output of wine vis a vis their autarkic output levels. The change in wine production brought about by this change in relative prices, which we will denote by the symbol ΔV (read as "delta V"), is negative with a magnitude depicted by the vertical edge of the right-angle triangle whose vertices connect the autarkic and free-trade production points. The change in cloth production, which we denote by the symbol ΔC (read as "delta C") is positive and has magnitude depicted by the length of the horizontal edge of the aforementioned triangle.

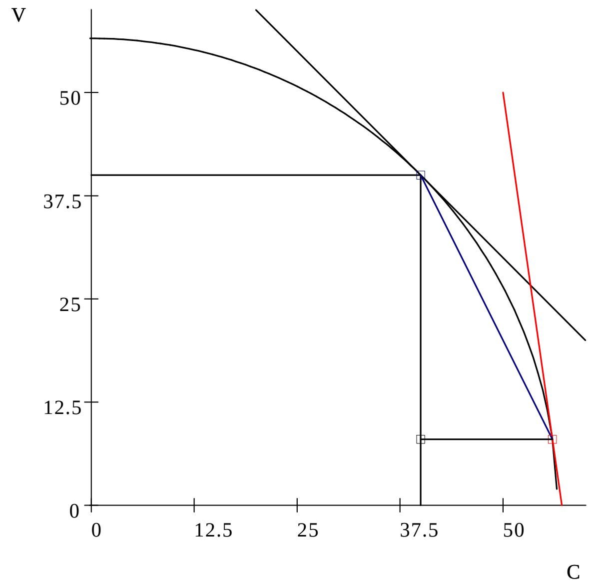
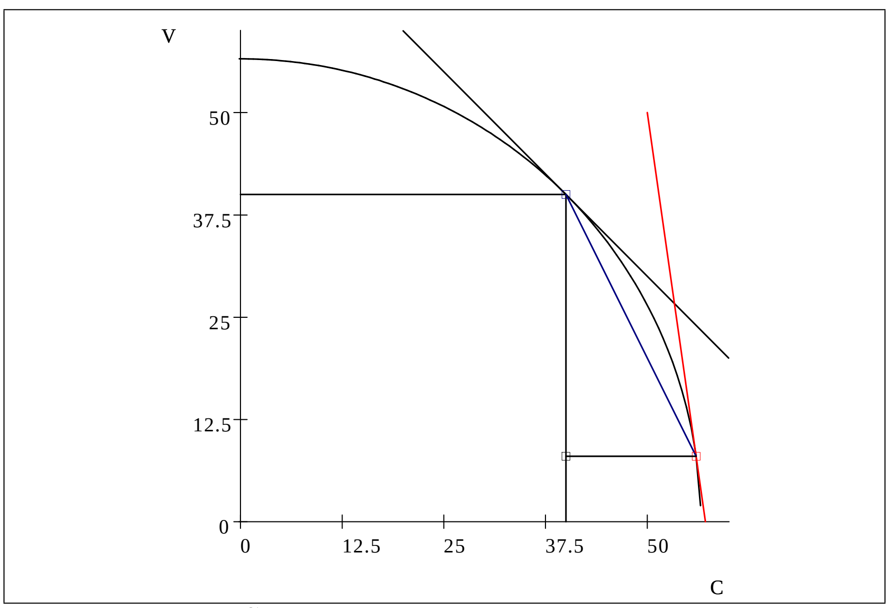


Figure 36: Production changes along PPF



37:Changes in production along the PPF

For our purposes, the key feature depicted by this diagram is that the absolute value of the ratio of the aggregate changes in production of wine and cloth, which we will denote by $\left|\frac{\Delta V}{\Delta C}\right|$,is greater than the autarkic price, p_a, and less than the free trade equilibrium price, p_{FT}:

$$p_a < \frac{\Delta V}{\Delta C} < p_{FT}.$$

This is apparent from comparing the slope of the straight line segment that connects the two points of production on the PPF with the slope of the PPF at those points of production.

Note that in the free trade equilibrium, all produced output of wine and cloth is either distributed to workers in the each of the sectors as wages or kept by the owners of the specific factors. Our task is to find at least one redistribution of these outputs of cloth and wine among the various members of the country such that, with this redistribution, they are all better off in free trade than they were in autarky.

To construct one feasible allocation that makes everyone better off under free trade, let's look for allocations for individuals all of which are southeast (in a particular fashion) of an individual's autarkic consumption point. In particular, subtract off from an individual's autarkic consumption bundle an amount $\alpha_i \times \Delta V$, where α_i is a fraction, and add an amount $\alpha_i \times \Delta C$. Such an allocation would be described as the point $(C_{i,a}+\alpha_i\Delta C, V_{i,a}+\alpha_i\Delta V)$, where $C_{i,a}$ and $V_{i,a}$ denote the autarkic equilibrium consumption bundles of any individual i. We will call such a point a *compensated endowment point*.

Our argument is that any such compensated endowment point creates a budget constraint which, at the free trade equilibrium price, ensures that an individual will be better off than he or she would have been in autarky. If we can then find such an allocation that is feasible, then we have constructed a redistribution of income such that everyone is better off under free trade than under autarky. The argument is most clearly seen when we start with an example.

Suppose, for example, that $\Delta V = -32$ and $\Delta C = 16$. For L (our aggregated "labor" individual), for example, let $\alpha_i = \frac{1}{2}$, so $\alpha_i \times \Delta V = -16$ and $\alpha_i \times \Delta C = 8$. Suppose $L\prime s$ autarkic consumption of wine was twenty (20) units, so $L\prime s$ hypothetical amount of wine under our contemplated redistribution scheme would be four (4). Suppose also that $L\prime s$ autarkic consumption of cloth was also twenty (20) units, so his/her hypothetical amount of cloth under our contemplated redistribution scheme would be twenty-eight (28) units of cloth. We depict this hypothetical "compensated endowment point" in Figure 38. The key point about this point is that the angle of the ray that connects the autarkic consumption point $(C_{i,a}, V_{i,a})$ with this new point is $\frac{\Delta V}{\Delta C}$, which is steeper than $-p_a$, the autarkic equilibrium price, and flatter than $-p_{FT}$, the equilibrium free trade price.

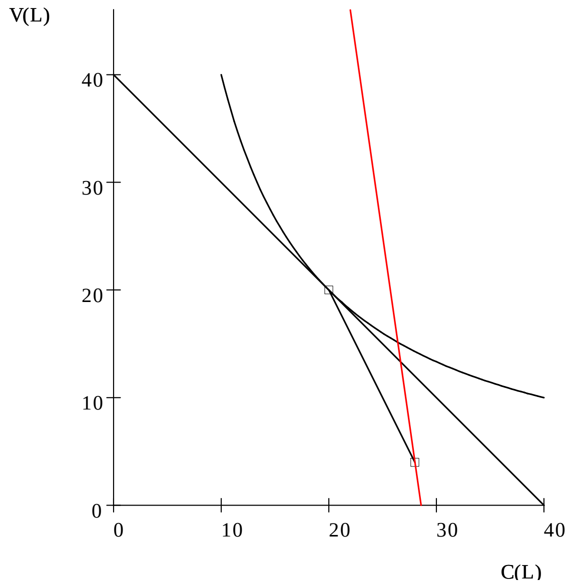
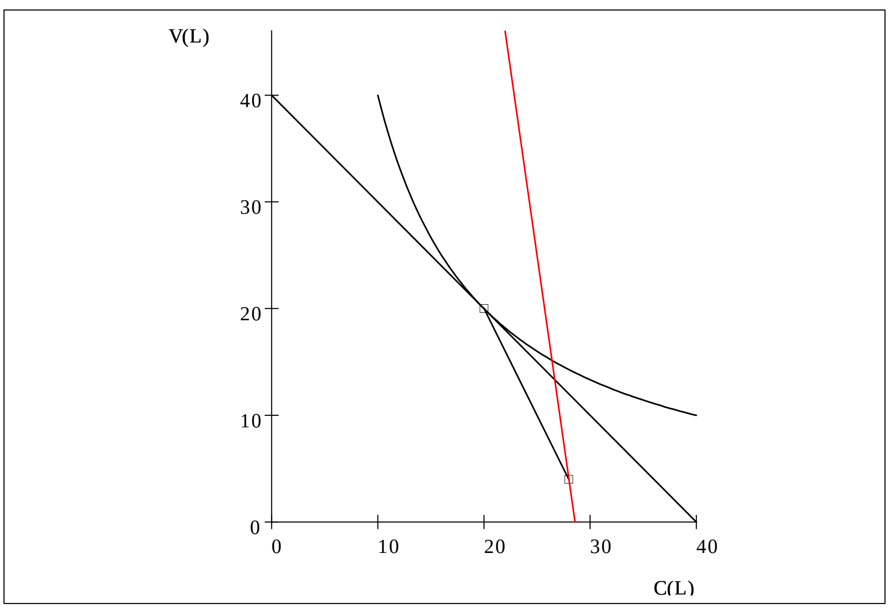


Figure 37: Labor's budget constraint at compensated endowment



38: Labor's budget constraint at compensated endowment point

In this figure we also depict as a red line through the point $(28, 4)$ what would be $L\prime s$ budget constraint at the free trade equilibrium price p_{FT} if he were to have as income this compensated endowment point. Note that this budget constraint must cross the autarkic equilibrium indifference curve of L, because $p_{FT} > -\frac{\Delta V}{\Delta C}$.

Now, some experimentation with the diagram should make clear that any "compensated endowment point" to the southeast of the autarkic consumption point for which the angle created by the ray connecting these two points is less steep than p_{FT} generates a budget constraint (with relative price p_{FT}) that intersects the individual's autarkic indifference curve. Hence, if every individual in the economy could be given such a compensated endowment point, and if the sum over all individuals of these compensated endowments of wine and of cloth added up to the free trade production levels of wine and of cloth, respectively, then we would have constructed a feasible reallocation of wine and cloth production such that, with this reallocation, everyone would be better off in free trade than they would be under autarky. That is, we would have demonstrated that what economists call "gains from trade" exist for this model.

That is, we require for feasibility that

$$\sum_i \alpha_i \Delta C = \Delta C$$

and

$$\sum_i \alpha_i \Delta V = \Delta V$$

which just means we require

$$\sum_i \alpha_i = 1.$$

(We also require that every compensated endowment point have non-negative values for C_i and V_i, which just means that no individual α_i be "too big". We put "too big" in quotes to indicate that this will be example-specific.).

For example, suppose, to continue with our example, that $\alpha_L = \frac{1}{2}$, so that $\alpha_L \times \Delta C = 8$ and $\alpha_L \times \Delta V = -16$. What are some values of α_{EJ} and α_{LW} that would lead to feasible allocations?

A perhaps obvious choice might be $\alpha_{LW} = \alpha_{EJ} = \frac{1}{4}$. This satisfies the constraint that $\sum_i = 1$, and is "obvious" because EJ and LW have equal autarkic consumptions. With this choice, $\alpha_{LW} \times \Delta C = 4$, $\alpha_{EJ} \times \Delta C = 4$, $\alpha_{LW} \times \Delta V = 8$, and $\alpha_{EJ} \times \Delta V = 8$. Hence, the compensated endowment points for LW and EJ are:

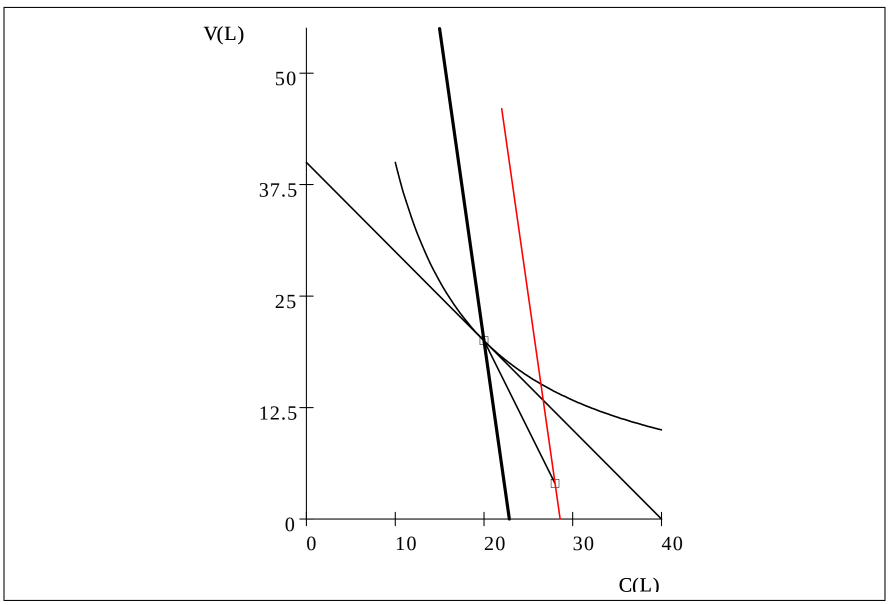
$$(14, 2).$$

Total endowments of cloth, namely $14 + 14 + 28$, equal total production (56), and total endowments of wine, namely $2 + 2 + 4$, equal total production of wine (8). This is a feasible allocation, and, by construction, everyone is better off with this allocation of the output produced at the free trade price than they were with their consumptions in autarky.

Other such feasible allocations exist, much as with endowment economy. The point here is only that there exists such a feasible allocation.

9 Comparison with the endowment economy

What does our diagram look like for an endowment economy? In an endowment economy, the amount "produced" doesn't change in response to different relative prices. This means the PPF is a point. As demonstrated in the treatment of the endowment economy, one feasible reallocation just gives each individual their autarkic consumption bundles as their compensated endowment point. The "gains from trade" for individuals in such an economy are demonstrated by the budget constraint under free trade going through the autarkic consumption point (now the compensated endowment point) but with a different slope (now $-p_{FT}$ instead of $-p_a$). This is demonstrated in the Figure 39 by the thick black line. Notice that, for the same free-trade price, the "compensated endowment point" budget constraint for this individual in the production economy is everywhere above the endowment economy compensated endowment point budget constraint. In this sense, production possibilities expand the possibilities for gains form trade.



39: Endowment comparison

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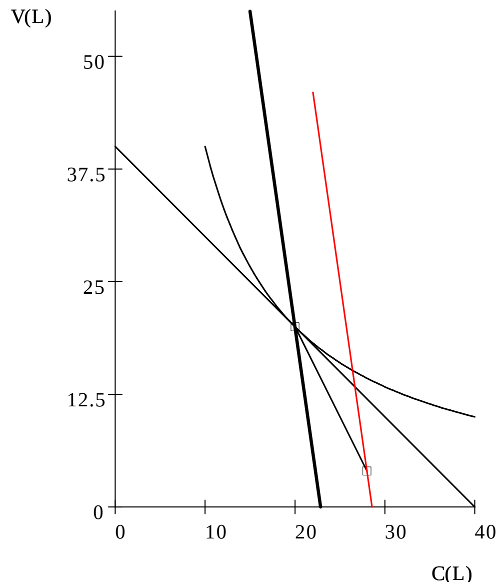

Figure 38: Endowment comparison