Overview

The market for foreign exchange can be usefully thought of as a market where demand and supply are derived from more fundamental demands and supplies–for goods and services, and for assets.

We assume a world with two traded goods: $(1)\ X$, which is exported from the home country and imported into the foreign country, and (2), M, which is produced and exported from the foreign country to the home country. Each country also produces (and consumes) its own non-traded good.

We first look at demand and supply for FX arising from underlying demands for imports and exports of goods and services. We then append to these demands demands and supplies arising from underlying demands for assets.

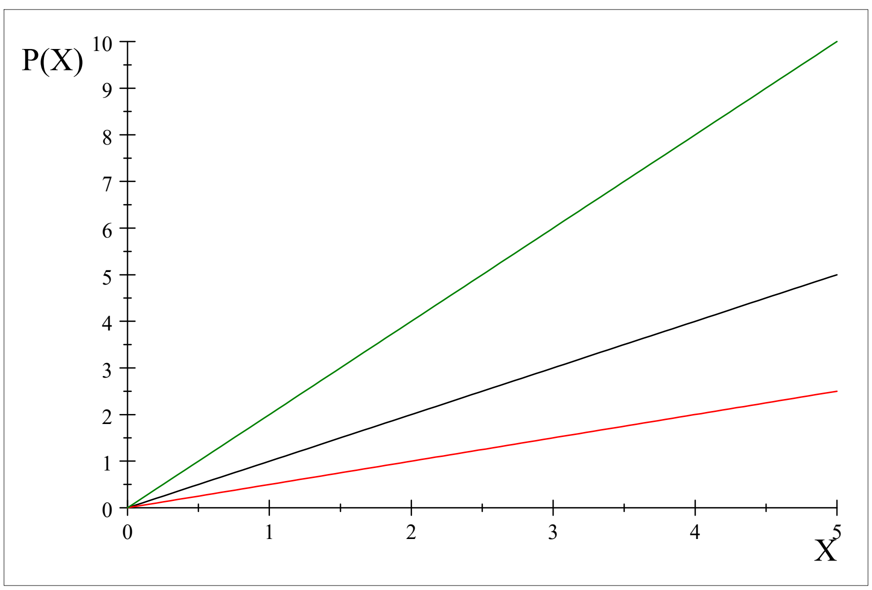
Market for X

Home country supply

$$X^S = A_X^S \cdot \frac{P_X}{P_N};\ A_X^S > 0.$$ supply fnctn

$$P_X = \frac{X^S \cdot P_N}{A_X^S}.$$ Invrs Sup.

$y = x\ (A_X^S = 1)$:



$P_N = .5, (red), 1, (Bl), 2(Gr)$

1. To keep things simple, let's focus on the *short run* by which I mean P_N is constant. Let's make it constant at $P_N = 1$. Our inverse supply function is thus

$$P_X = \frac{X^S}{A_X^S}.$$

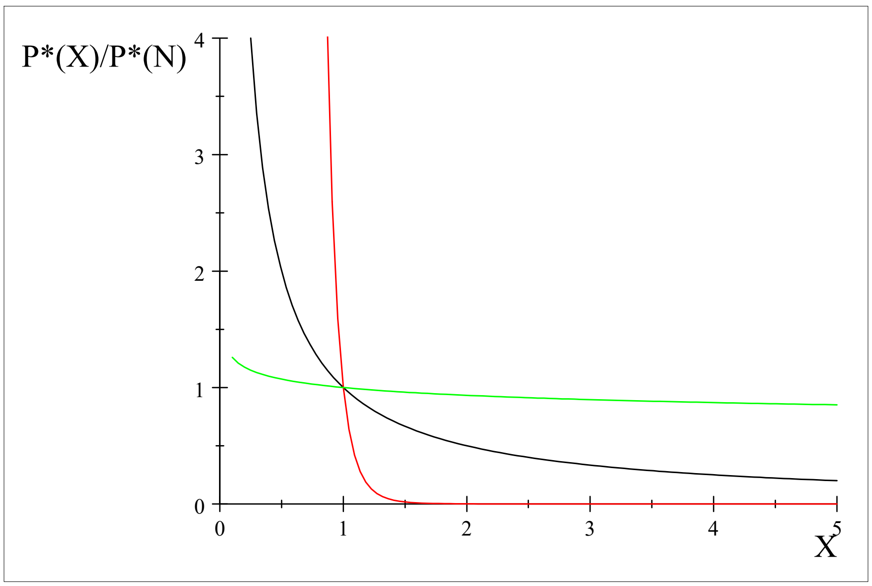
2. Our goal is to derive the (inverse) supply function for FX. To do this, we need to determine the equilibrium quantity and price in the market for X. Hence, we need a demand function from the foreign country. Let's look at a few candidates:

$$X^d = \frac{A_X^d}{\left(\frac{P_X^*}{P_N^*}\right)^{.1}}; \left(\frac{P_X^*}{P_N^*}\right)^{.1} = \frac{A_X^d}{X^d}; \frac{P_X^*}{P_N^*} = \left(\frac{A_X^d}{X^d}\right)^{10} \quad \text{red}$$

$$X^d = \frac{A_X^d}{\left(\frac{P_X^*}{P_N^*}\right)}; \left(\frac{P_X^*}{P_N^*}\right) = \frac{A_X^d}{X^d}; \frac{P_X^*}{P_N^*} = \left(\frac{A_X^d}{X^d}\right) \quad \text{black}$$

$$X^d = \frac{A_X^d}{\left(\frac{P_X^*}{P_N^*}\right)^{10}}; \left(\frac{P_X^*}{P_N^*}\right)^{10} = \frac{A_X^d}{X^d}; \frac{P_X^*}{P_N^*} = \left(\frac{A_X^d}{X^d}\right)^{\frac{1}{10}} \quad \text{green}$$

We look at these three to see the difference more elastic and less elastic demand makes for the analysis. They look like this (with $A_X^d = 1$, the colors correspond the three equations above).



Again, let's set $P_N^* = 1$, but remembering that each demand curve would shift "out" if P_N^* were to increase.

How can we link these demand and supply curves? We invoke arbitrage–the limiting case of zero transport costs–which implies:

$$P_X = EP_X^*.$$

Hence,

$$\overbrace{E \cdot P_X^*}^{P_X} = \frac{X^S \cdot P_N}{A_X^S};$$

$$P_X^* = \frac{X^S \cdot P_N}{E \cdot A_X^S}$$

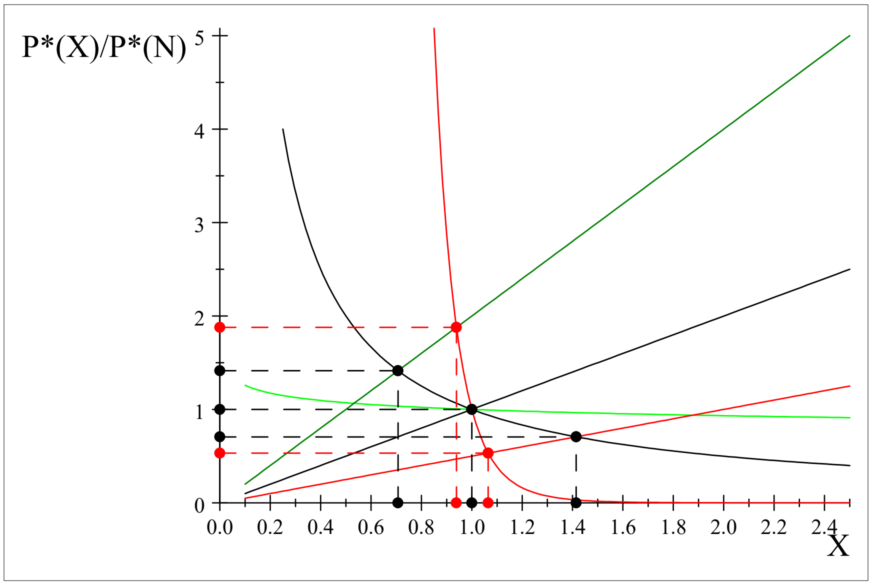
We have a "family" of inverse supply curves, each member associated with a different value of E. For example, if $E = .5$, $E = 1$, and $E = 2$ (again, keeping $P_N = 1$) the inverse supply functions are

$$P_X^* = \frac{X^S \cdot \overbrace{P_N}^{1}}{\underbrace{.5}_{E} \cdot A_X^S};$$

$$P_X^* = \frac{X^S \cdot \overbrace{P_N}^{1}}{\underbrace{1}_{E} \cdot A_X^S};$$

$$P_X^* = \frac{X^S \cdot \overbrace{P_N}^{1}}{\underbrace{2}_{E} \cdot A_X^S}$$

We superimpose three members of this family–$E = .5$, $E = 1$, $E = 2$–on the inverse demand curves ($A_X^d = A_X^S = 1$) :



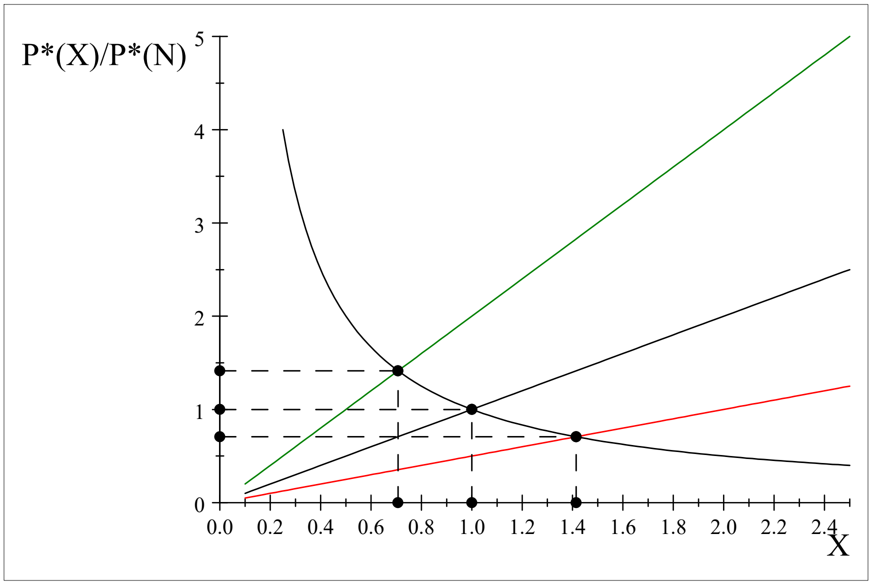
Wow, this looks complex! Let’s keep things a little more simple for the moment by focusing on the case:

$$X^d = \frac{A_X^d}{\left(\frac{P_X^*}{P_N^*}\right)};\ \left(\frac{P_X^*}{P_N^*}\right) = \frac{A_X^d}{X^d};\ \frac{P_X^*}{P_N^*} = \left(\frac{A_X^d}{X^d}\right).$$

We'll make it even simpler by keeping

$$A_X^d = A_X^s = 1.$$

Our diagram now looks like this:



Of the straight lines (which reperesent the family of supply curves) ,the steepest line is the one for which $E = \frac{1}{2}$, the next-steepest is the one for $E = 1$, and the least steep is for $E = 2$.

What is the earnings of foreign exchange from our, i.e., home-country, exports? It's price times quantity, or the area in the rectangles, each associated with a particular E. Here's the math:

$$X^s = EP_x^*;\ X^d = \frac{1}{P_x^*};$$

$$X^s = X^d;$$

$$EP_x^* = \frac{1}{P_x^*};\ \widehat{P_X^*} = \frac{1}{\sqrt{E}};$$

$$\widehat{X^s} = \widehat{X^d} = \hat{X} = E\,\widehat{P_X^*} = \frac{E}{\sqrt{E}};$$

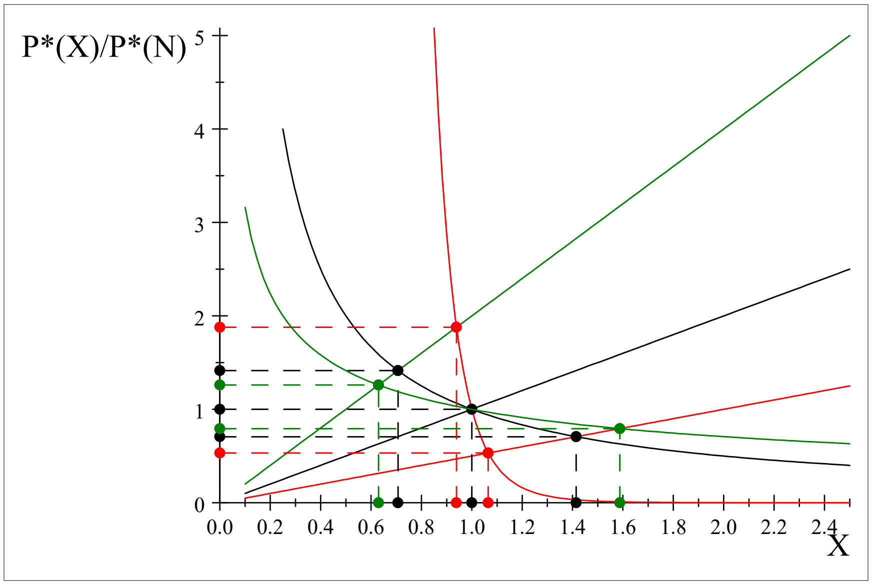
$$\widehat{P_X^*} \bullet \hat{X} = \frac{1}{\sqrt{E}} \bullet \frac{E}{\sqrt{E}} = 1.$$

For this case, we have a perfectly inelastic supply of foreign exchange wrt E. This is not the general case, but will suffice for our purposes. Perhaps of interest (to illustrate that the supply curve for FX can slope up), consider slightly different inverse demand curves:

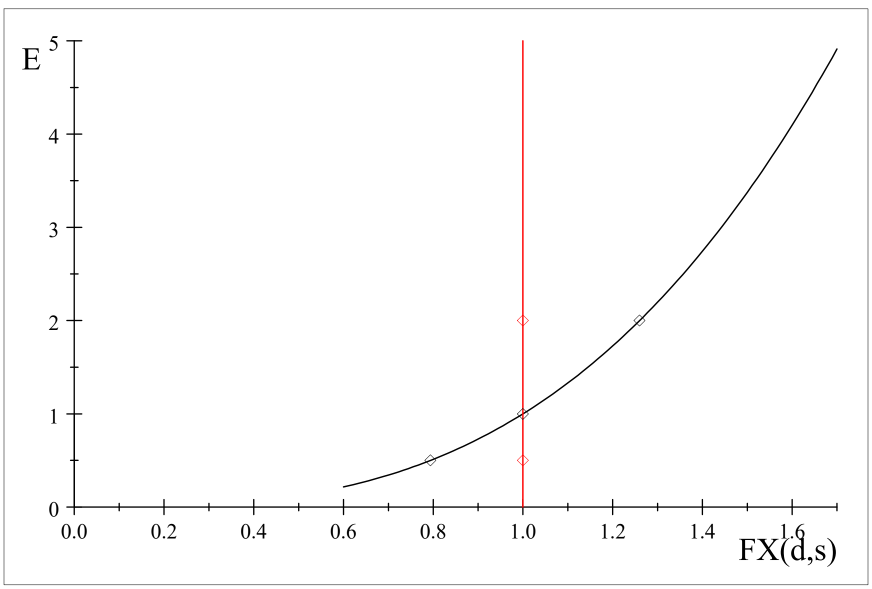
$$P_X^* = \frac{1}{X^{\frac{1}{2}}} = \frac{1}{\sqrt{X}};$$ green

$$P_X^* = \frac{1}{X^2}$$ red

The picture of the market for good X now looks like this:



The supply of FX looks like this:



$$\varepsilon = 2(b);\ \varepsilon = 1(r)$$

Can you see that a very inelastic demand curve leads to a downward-sloping supply curve? We ignore this possibility.

Market for M

The simplest case:

$$M^d = \frac{1}{P_M};\; M^s = P_M^*;$$
$$P_M = \frac{1}{M^d};\; P_M^* = M^s.$$

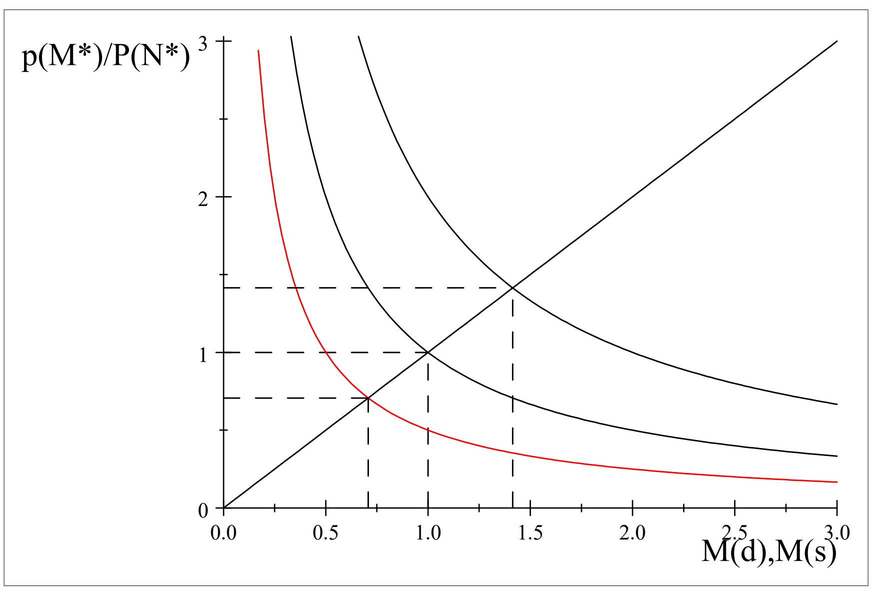
The LOOP implies

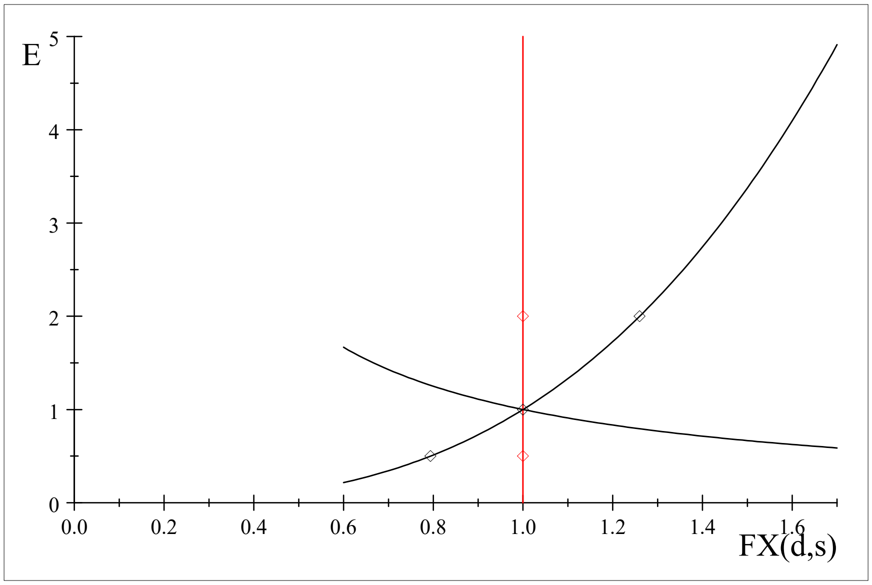
$$P_M = EP_M^*$$

so

$$EP_M^* = \frac{1}{M^d};$$
$$P_M^* = \frac{1}{EM^d}.$$

Bi-directional logic: for any value of P_M^*, if E goes up, M^d must go down. So the family of inverse demand curves displayed below has the red innermost curve associated with the largest value of E. Hence, as E gets smaller, we move to inverse demand curves farther away from the origin, and equilibrium in the market for M at higher values of M and P_M^*. So, as E falls, the rectangular area that represents the demand for foreign exchange ($P_M^* \cdot M$) gets bigger: there is a downward-sloping demand curve for FX.





$\varepsilon = 2(b);\ \varepsilon = 1(r)$