Solution theory for nonlinear partial differential delay equations

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The object of study are evolutionary processes for which the time rate-of-change depends not only on the actual state but also on the history of the process. Typical examples are diffusive population models with temporal averages over the past, such as

$$\begin{cases} \dfrac{du}{dt}(t) - \Delta u(t) = au(t)\left[1 - bu(t) - \displaystyle\int_{-R}^{0} u(t+s)d\eta(s)\right], \quad t \geq 0 \\ u_{|(-R,0]} = \varphi \end{cases}$$

(production of red blood cells), as well as corresponding models with the Laplacian being replaced by more general, possibly nonlinear, diffusion/absorption operators.

In abstract form, such models lead to the following partial differential delay equations

$$\begin{cases} \dot{x}(t) + Bx(t) \ni F(x_t), \quad t \geq 0 \\ x_{|I} = \varphi \in \hat{E}, \end{cases}$$

with $B \subset X \times X$ a (generally) nonlinear and multivalued differential expression in a Banach space X, and for given $I = [-R, 0]$, $R > 0$ (finite delay), or $I = \mathbf{R}^-$ (infinite delay), and $t \geq 0$, $x_t : I \to X$ the history of x up to $t : x_t(s) = x(t+s)$, $s \in I$.

The following basic problems will be addressed: existence, flow-invariance, and regularity of mild solutions.