Properties of extreme values of infinity-harmonic functions and a local estimate

In this talk we will discuss some local properties of infinity-harmonic functions defined on domains in $I\!R^n$, that is, viscosity solutions to

$$\sum_{i,j=1}^{n} D_i u D_j u D_{ij} u = \Delta_\infty u = 0.$$

Let $B_1(o)$ be the unit ball, centered at o and lying in $I\!R^n$. Take $u(o) = 0$, for $0 < r < 1$ set $m(r) = \inf_{B_r(o)} u$ and $M(r) = \sup_{B_r(o)} u$. We show that

$$|m(r)| \geq \left(\sqrt{rM'(r)} - \sqrt{rM'(r) - M(r)}\right)^2, \quad 0 < r < 1,$$
$$M(r) \geq \left(\sqrt{r|m'(r)|} - \sqrt{r|m'(r)| - |m(r)|}\right)^2, \quad 0 < r < 1.$$

Moreover, if u is infinity-harmonic in $I\!R^n$ and equality holds in both inequalities for every r then u is affine. Using these inequalities, we derive that if $M(r) \leq r$ in $B_1(o)$ then either $|m(r)| \leq r$, $\forall\ 0 < r < 1$ or there is an $0 < a < 1$ such that

$$|m(r)| \geq r(1 + k \log(r/a)), \quad a < r < 1,$$

for an appropriate $k > 0$. We will also discuss some related issues.