

## The Fredholm Alternative and Semilinear Elliptic Equations

We give necessary and sufficient conditions for the existence of real solutions of

$$\begin{aligned} -\Delta u + \alpha(u) &= f && \text{in } \Omega, \\ b(x) \frac{\partial u}{\partial n} + \beta(u) - qb(x) \Delta_{LB} u &= g && \text{on } \partial\Omega, \end{aligned}$$

where  $0 < b \in C(\partial\Omega)$ ,  $\alpha$  and  $\beta$  are continuous, nondecreasing functions which vanish at the origin,  $q \in [0, \infty)$ , and  $\Delta_{LB}$  is the Laplace-Beltrami operator on the boundary. Here  $f \in L^2(\Omega)$  and  $g \in L^2(\partial\Omega)$ . The proof uses the general Wentzell boundary conditions and the space  $L^2(\Omega, dx) \oplus L^2\left(\partial\Omega, \frac{dS}{b(x)}\right)$ . This work significantly extends the classical results of E. Landesman and A. Lazer.

This is joint work with C. Gal, J. A. Goldstein and S. Romanelli.