Some properties of infinity-harmonic functions

In this talk we will discuss some properties of infinity-harmonic functions on a domain $\Omega \subset I\!R^n$. A function $u : \Omega \to I\!R$ is infinity-harmonic in Ω if it solves

$$\Delta_\infty u = \sum_{i,j=1}^{n} \frac{\partial u}{\partial x_i}\frac{\partial u}{\partial x_j}\frac{\partial^2 u}{\partial x_i \partial x_j} = 0$$

in the viscosity sense. This pde arises in the study of minimal Lipschitz extensions, viscoelasticity(an old work with Emmanuele DiBenedetto and Juan Manfredi), Monge-Kantorovich mass transfer problem and in some aspects of game theory. This pde has very intimate connections with the well known p-Laplacian $div(|Du|^{p-2}Du)$. It is well known that such infinity-functions have cone comparisons which lead to Lipschitz continuity, Harnack inequality and the convexity of oscillation. We will discuss a boundary Harnack principle for C^2 boundaries and some its consequences. We will also discuss the nature of solutions of this pde in some exterior domains. The techniques are all based on the comparison principle. We will also point out some open questions.