Early Math Trajectories: Low-Income Children's Mathematics Knowledge from Age 4 to 11

Bethany Rittle-Johnson

Emily R. Fyfe

Kerry G. Hofer

Dale C. Farran

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Author Note

Bethany Rittle-Johnson, Emily Fyfe, Kerry Hofer, and Dale Farran, Vanderbilt University. Fyfe is now at the University of Wisconsin Madison and Hofer is now at Abt Associates.

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Address correspondence to Bethany Rittle-Johnson, Department of Psychology and Human Development, 230 Appleton Place, Peabody #552, Vanderbilt University, Nashville TN 37203, USA. Email: bethany.rittle-johnson@vanderbilt.edu

Abstract

Early mathematics knowledge is a strong predictor of later academic achievement, but children from low-income families enter school with weak mathematics knowledge. An Early Math Trajectories model is proposed and evaluated within a longitudinal study of 517 low-income American children from age 4 to 11. This model includes a broad range of math topics, as well as potential pathways from preschool to middle-grades mathematics achievement. In preschool, nonsymbolic quantity, counting and patterning knowledge predicted fifth-grade mathematics achievement. By the end of first grade, symbolic mapping, calculation and patterning knowledge were the important predictors. Further, the first-grade predictors mediated the relation between preschool math knowledge and fifth-grade mathematics achievement. Findings support the Early Math Trajectories model among low-income children.

KEYWORDS: mathematics development, mathematics achievement, developmental trajectory. number knowledge, patterning knowledge, longitudinal studies

Early Math Trajectories: Low-Income Children’s Mathematics Knowledge from Age 4 to 11

Proficiency in mathematics is critical to academic, economic, and life success. Greater success in mathematics is related to college completion, higher earnings, and better health decisions (Reyna, Nelson, Han, & Dieckmann, 2009; Ritchie & Bates, 2013). Mathematics knowledge begins to develop at a young age, and this early knowledge matters. General math knowledge before school entry predicted mathematics and reading outcomes across primary and secondary school (Duncan et al., 2007; Jordan, Kaplan, Ramineni, & Locuniak, 2009; Watts, Duncan, Siegler, & Davis-Kean, 2014). Unfortunately, children from low-income families enter school with weaker mathematics knowledge than children from more advantaged backgrounds, and this weak early math knowledge at school entry helps explain their weak math knowledge later in elementary school (Jordan et al., 2009).

The goal of the current study is to elucidate specific early math knowledge that is predictive of later mathematics achievement for children from low-income backgrounds. We propose an Early Math Trajectories model of specific early math knowledge that influences later mathematics achievement, integrating a broader range of mathematics topics than has been considered in past studies. We evaluated the Early Math Trajectories model within a longitudinal study of over 500 low-income children from age 4 to 11. The current study helps advance theories of mathematics development and identifies specific early math knowledge that might merit particular attention for low-income children.

Early Mathematics Development

Proficiency in mathematics requires developing knowledge of multiple topics and their interrelations (Common Core State Standards, 2010). Longitudinal research in psychology has documented that early math knowledge predicts later math knowledge, but has primarily relied on global measures of math knowledge (Duncan et al., 2007; Watts et al., 2014). Other longitudinal research has documented the importance of a particular early math topic for mathematics achievement a few years later, but without considering a broad range of early math topics or following children into the middle grades (reviewed below). Based on a synthesis of past research, we propose an Early Math Trajectory Model that encompasses a set of six early math topics that should be of particular importance for supporting mathematics achievement in the middle grades (ages 10 - 14).

Components of Early Mathematics Knowledge

Central to early math knowledge is numeracy knowledge, children's knowledge of the meaning of whole numbers and number relations (Jordan, Kaplan, Nabors Olah, & Locuniak, 2006; National Research Council, 2009). This includes *nonsymbolic quantity knowledge*, which does not require knowledge of verbal or symbolic number names. Numeracy knowledge that involves linking to verbal or symbolic number names is separated into three related, but distinct, topics: *counting* (counting objects and verbal counting, also called numbering), *symbolic mapping* (also called numerical relations), and *calculation* (also called arithmetic operations), in line with confirmatory factor analyses by Purpura and Lonigan (2013).

Early mathematics knowledge extends beyond numeracy knowledge, though there is less consensus on which additional math topics are important. Commonly highlighted topics are shape, patterning and measurement knowledge (National Research Council, 2009). We considered *patterning*, as there is longitudinal evidence for its importance in math, as well as *shape knowledge* given widespread beliefs about its importance. We did not include measurement knowledge given the dearth of assessments and research on measurement knowledge before school entry. We briefly review evidence on the development of each of the six topics, including evidence for its predictive relation to later mathematics achievement. Each topic encompasses conceptual knowledge of underlying principles as well as procedural knowledge for solving problems within the topic.

Nonsymbolic quantity knowledge. Nonsymbolic quantity knowledge is knowledge of the magnitude of sets, without the need to use verbal or symbolic number names. This knowledge begins to develop in infancy, including the ability to discriminate between small set sizes (Starkey & Cooper, 1980) as well as large set sizes (Xu, Spelke, & Goddard, 2005). Nonsymbolic quantity knowledge provides a foundation for mapping between magnitudes and verbal and symbolic numbers (Gilmore, McCarthy, & Spelke, 2010; Piazza, 2010; van Marle, Chu, Li, & Geary, 2014) and an intuitive understanding of simple arithmetic (Barth, La Mont, Lipton, & Spelke, 2005). Beginning in preschool, individual differences in the speed and precision of nonsymbolic quantity knowledge of both small and large set sizes are related to mathematics knowledge six-months to two years later (Chen & Li, 2014; Desoete & Gregoire, 2006; LeFevre et al., 2010; Libertus, Feigenson, & Halberda, 2013). The relation is strongest before age six (Fazio, Bailey, Thompson, & Siegler, 2014). Although there is some controversy over the strength of this relation with appropriate controls (De Smedt, Noël, Gilmore, & Ansari, 2013), we expected nonsymbolic quantity knowledge in preschool to predict later mathematics achievement even with a range of control variables.

Counting. Counting, also termed number or numbering knowledge, includes knowledge of the number-word sequence, the ability to make a one-to-one correspondence between objects and count words, and the use of the largest count word to identify the cardinality of the set (Purpura & Lonigan, 2013). Many children begin to learn to count between the ages of two and three, and individual differences in four- and five-year-old children's counting predicts their mathematics achievement in elementary school (Aunola, Leskinen, Lerkkanen, & Nurmi, 2004; Muldoon, Towse, Simms, Perra, & Menzies, 2013). Thus, we expected counting in preschool to predict later mathematics achievement, though we expected its importance to dissipate by the end of first grade given that counting receives less attention after kindergarten and the lack of longitudinal evidence for the predictive power of counting knowledge past kindergarten.

Symbolic mapping knowledge. Symbolic mapping knowledge is knowledge of the mapping between symbolic numerals, their number names and their magnitudes, including their relative magnitudes (Jordan et al., 2006). In mathematics education, this is often termed numerical relations, but with the inclusion of non-symbolic quantity knowledge in this category (Purpura & Lonigan, 2013). Symbolic mapping knowledge begins to develop by age three for small set sizes and gradually extends to larger set sizes and more difficult tasks (Benoit, Lehalle, & Jouen, 2004). Differences in this knowledge in the primary grades are consistently related to a range of mathematical outcomes (Fazio et al., 2014; Fuchs, Geary, Fuchs, Compton, & Hamlett, 2014; Geary, 2011; Sasanguie, Van den Bussche, & Reynvoet, 2012), with some evidence of a relation beginning at age 5 in a middle-SES sample (Kolkman, Kroesbergen, & Leseman, 2013). Recent evidence and theory suggests that symbolic mapping knowledge is more important than nonsymbolic number knowledge in predicting achievement (De Smedt et al., 2013). In particular, symbolic mapping knowledge is often more highly correlated with concurrent mathematics achievement than is nonsymbolic quantity knowledge (Fazio et al., 2014; Holloway & Ansari, 2009). Further, symbolic mapping, but not nonsymbolic quantity knowledge, at ages 5 and 6 predicted mathematics achievement the following year when a range of measures were included in the model (Kolkman et al., 2013; Sasanguie et al., 2012; Vanbinst, Ghesquière, & De Smedt, 2015). Thus, we predicted that in early primary school, symbolic mapping but not nonsymbolic quantity knowledge would be predictive of later achievement.

Calculation. Calculation, or arithmetic operations, is the ability to calculate the combination or separation of sets (Purpura & Lonigan, 2013). Toddlers can calculate when set sizes are very small and verbal number names are not needed (Huttenlocher, Jordan, & Levine, 1994), and preschool children can verbally name the result of simple problems demonstrated with objects or embedded in stories (Purpura & Lonigan, 2013). Early in primary school, children can calculate without the support of objects with a range of numbers (Jordan et al., 2006). Object-supported calculation at ages 5 and 6 is correlated with later mathematics achievement (LeFevre et al., 2010), and general calculation knowledge in first and second grade is predictive of mathematics achievement the following year (Cowan et al., 2011) as well as in the middle grades (Geary, 2011). Thus, we predicted that calculation knowledge by the early primary grades would predict later mathematics achievement.

Patterning. Patterning is finding a predictable sequence, and the first patterns children interact with are repeating patterns (i.e., linear patterns with a core unit, like the colors blue-red, that repeats, such as blue-red-blue-red). Preschool children are often asked to copy or extend a pattern, and preschool and kindergarten children are also able to make the same kind of pattern using new colors or shapes and to identify the core unit that repeats (Clements & Sarama, 2009; Rittle-Johnson, Fyfe, Hofer, & Farran, 2015; Rittle-Johnson, Fyfe, McLean, & McEldoon, 2013). Intervention research indicates that patterning knowledge is causally related to early mathematics achievement. A preschool patterning intervention led to greater numeracy knowledge at the end of kindergarten than typical preschool instruction (Papic, Mulligan, & Mitchelmore, 2011). A patterning intervention for struggling first-grade students led to comparable or better numeration, calculation, algebra and measurement knowledge at the end of the school year than a numeracy intervention (Kidd et al., 2013; Kidd et al., 2014). Longitudinal evidence is sparse, though pattern items were part of a cluster of diverse items from a kindergarten assessment that best predicted eighth-grade achievement, providing some indirect evidence for its long-term importance (Claessens & Engel, 2013). We predicted that patterning knowledge in preschool and primary school would predict later mathematics achievement.

Shape. Knowledge of shapes and their properties is a component of early math standards and considered foundational to later geometric thinking (Common Core State Standards, 2010; National Research Council, 2009). Preschool children can identify typical shapes, but struggle to classify atypical and non-valid instances of shapes (Satlow & Newcombe, 1998). In elementary school, children learn definitional properties of two- and three-dimensional shapes and learn to compose geometric shapes out of other shapes (Clements & Sarama, 2009; Satlow & Newcombe, 1998). We have not identified evidence that shape knowledge is correlated with general math measures or predictive of later mathematics knowledge. Logically, children's knowledge of shape properties seems most likely to predict later achievement. We explored whether shape knowledge in preschool or primary school would predict later mathematics achievement.

Early Math Trajectories Model

Based on existing theory and evidence, we propose the *Early Math Trajectories Model* (see Figure 1). It includes six early math topics and proposes a trajectory from preschool (ages 4–5) to early primary grades (ages 7–8) to middle grades (ages 10–14, with a focus on ages 10–11 in the current study). Although each type of knowledge begins to develop before ages 4 and 5, this model focuses on time points when individual differences in knowledge of a particular topic are sufficient to predict later mathematics knowledge, as indicated by prior research.

In preschool, nonsymbolic quantity and counting knowledge are fairly well developed and are widely believed to support the development of symbolic mapping and calculation knowledge. For example, children have many experiences seeing a set of objects and having its numerosity labeled by adults or determining its cardinality through counting. These experiences help children learn the mapping between particular quantities and verbal and alphanumeric number names (Benoit et al., 2004; Krajewski & Schneider, 2009). Further, children often count in order to calculate (Geary, 2011). Nonsymbolic quantity knowledge also supports precise calculation knowledge (Park & Brannon, 2013), perhaps because mentally tracking the approximate number of items when objects are combined or separated helps ground precise calculations. Nonsymbolic quantity and counting knowledge could also support pattern knowledge. Counting objects is useful to reproduce the exact number of each item in the core pattern unit and to iterate the core unit an appropriate number of times. Nonsymbolic quantity knowledge of equivalent sets may also support copying a pattern.

In preschool, pattern knowledge develops rapidly. Improving patterning knowledge in preschool supports the development of symbolic mapping and calculation knowledge (Papic et al., 2011). Working with patterns may provide early opportunities to deduce underlying rules and/or promote spatial skills. Sophisticated numeracy knowledge requires deducing underlying rules from examples, such as the successor principle for symbol-quantity mappings (e.g., the next number name means adding one). Repeating pattern knowledge also involves spatial relations and spatial short-term memory (Collins & Laski, 2015), and spatial skills support numeracy development (Mix & Cheng, 2012).

In turn, individual differences in symbolic mapping and calculation knowledge in the early primary grades are strong predictors of later mathematics achievement (e.g., Geary, 2011).

This knowledge is necessary for working with middle-grades mathematics topics, such as advanced number, algebra and geometry. Patterning knowledge in the primary grades supports mathematics achievement across a range of topics at the end of the same school year (Kidd et al., 2013; Kidd et al., 2014), so may also predict later mathematics achievement. Identifying, extending, and describing patterns is important in middle-grades mathematics, such as determining functional relations and generalizing place value knowledge. Finally, learning properties of shapes in the primary grades may support geometry knowledge directly and may support broader middle-grades math achievement because it provides opportunities to develop the mathematical practice of generalizing principles from examples.

Overall, the Early Math Trajectories model suggests that nonsymbolic quantity, counting and patterning knowledge in preschool may support symbolic mapping, calculation and patterning knowledge in the primary grades, which in turn may support middle-grades mathematics achievement. Shape knowledge may also predict later mathematics achievement. Although previous research provides support for individual components of the model, it has not considered such a range of early math topics nor charted their impact over a long period of time.

Current Study

In the current study, we evaluated the Early Math Trajectories model within a longitudinal study of over 500 low-income American children from age 4 to 11. In the early years, children were assessed at four time points: the beginning of the pre-k school year (the year before entry into formal schooling in the U.S. when children are four), the end of pre-k, the end of kindergarten, and the end of first grade. Their mathematics achievement was assessed four years later when most children were in fifth grade (age 11) because we were particularly interested in how early math knowledge predicted success learning the more challenging and diverse mathematical content of the middle grades. Research comparing low-income children to their more advantaged peers indicates that their mathematics development is delayed, but does not follow a different trajectory (Claessens & Engel, 2013; Jordan et al., 2006). Because the Early Math Trajectories model is based on research that included children from more advantaged backgrounds, we expected that children in the current study might develop specific mathematics knowledge at a slower rate, but follow the same trajectory as proposed by the model.

We evaluated three primary hypotheses based on the Early Math Trajectories Model. First, we hypothesized that in preschool, nonsymbolic quantity, counting and patterning knowledge would predict age 11 mathematics achievement. Second, we hypothesized that at the end of first grade, symbolic mapping, calculation, patterning, and possibly shape knowledge would predict age 11 mathematics achievement, and that nonsymbolic quantity and counting knowledge would no longer be predictive with a range of other measures in the model. Third, we hypothesized that knowledge of symbolic mapping, calculation and patterning in first grade would mediate the relation between preschool math knowledge and age 11 achievement. We explored the role of shape knowledge.

Method

Participants

Participants were drawn from a longitudinal follow-up study of 517 children originally recruited at the beginning of the prekindergarten year in 2006 for a three-year longitudinal study. Children had been recruited from 57 pre-k classes at 20 public schools and 4 Head Start sites in a large urban city in Tennessee, all of which served children who qualified for free or reduced priced lunch (family income less than 1.85 times the U.S. Federal income poverty guideline). Of the 771 children in the original sample, we were able to locate and re-consent 519 children in

2013 for a four-year follow-up study in middle school, all within the same large, urban school district. Outcome data is from the first year of the follow-up study, and two children were not available for re-testing in this year, so were not included in the current analytic sample. The final re-consented sample was 56% female, 79% black, 9% white, 8% Hispanic and 4% other races, 9% English Learners, and all continued to qualify for free or reduced price lunch. Forty-percent had attended preschool at public schools and the other 60% had attended preschool at Head Start centers. Based on maternal report when the children were in pre-k, 43% of families had an annual income under \$10,000, 38% had an income between \$10,000 and \$25,000, and the remaining 19% had an income over \$25,000; 25% of mothers had less than a high-school diploma, 33% had a high-school diploma or GED, and 42% had some post-secondary education.

At the beginning of pre-k, children had a mean age of 4.44 years (*SD* = 0.32), and at the time of the most recent assessment, the average age was 11.1 years (*SD* = 0.33). Most students were in fifth grade; but,14% had been retained a grade and were still in fourth grade. Students were distributed across 76 schools. We will continue to assess these students each year throughout middle school. In pre-k, children were drawn from classrooms that had been randomly assigned to implement a specific pre-kindergarten mathematics curriculum (Building Blocks, Sarama & Clements, 2004) or to continue with business as usual instruction (316 [61%] were part of the treatment group in pre-k, and 201 were part of the control group). There were differences in mathematics knowledge based on instructional condition at the end of pre-k, but not at the end of kindergarten or first grade in this sample (Hofer, Lipsey, Dong, & Farran, 2013).

Outcome Measures

Four standardized mathematics assessments were individually administered. The first was the *quantitative concepts* subtest from the Woodcock Johnson Achievement Battery III, which assesses students' knowledge of basic mathematical concepts, symbols, and vocabulary, including numbers, shapes, and sequences. Standard scores were used. The other three were taken from the KeyMath 3 Diagnostic Assessment (Connolly, 2007), a norm-referenced measure of essential mathematical concepts. The *numeration subtest* measures students' understanding of whole and rational numbers, covering topics such as identifying, comparing, and rounding one-, two-, and three-digit numbers as well as fractions, decimal values, and percentages. The *algebra subtest* measures students' understanding of pre-algebraic and algebraic concepts, including recognizing and describing patterns and functions, working with operational properties, variables, and equations, and representing mathematical relations. The *geometry subtest* measures students' spatial reasoning and ability to analyze, describe, compare, and classify two- and three-dimensional shapes. Other measures of basic cognitive and numeracy skills were assessed as part of the larger project, but are beyond the scope of the current paper.

Predictor Measures

Early Math Knowledge. Mathematics knowledge in pre-k through first grade was assessed using the Research-Based Early Mathematics Assessment (REMA; Clements, Sarama, & Liu, 2008). It focuses on numeracy knowledge as well as geometry, patterning and measurement knowledge. Based on our review of the literature, we broke the numeracy items into four subscales. Purpura and Lonigan (2013) categorized REMA items similarly, except they did not have a separate category for nonsymbolic quantity items. See Table S1 in the supplemental materials for a list of each REMA item number that corresponds to each subscale, and see Table S2 for the number of items and Cronbach's alpha by time point for each subscale.

Nonsymbolic quantity items involved working with quantities without the need for verbal number labels or symbols. Easy items included deciding which set of objects had more (e.g., in pictures of 3 grapes vs. 4 grapes). Harder items included ordering different numbers of objects from smallest to largest (with 6 to 12 objects). There was no time limit imposed on these items, so children could count to help them complete the task, although counting was not encouraged. Thus, the nonsymbolic items did not directly assess the Approximate Number System or subitizing, but rather tapped a more general construct of quantity knowledge that did not require symbolic mapping knowledge. Internal consistency of the measure was adequate ($\alpha = 0.52$ to 0.70; $\omega_t = .62 - .86$).

Counting items involved knowledge of the number-word sequence in forward and backward order, counting sets of objects in ordered and haphazard arrays, identifying the cardinality of a set, and detecting violations of the one-to-one correspondence principle. Easy items included counting or giving 4-8 objects. More difficult items included counting 30 pennies or seven packs of 10 batteries. Internal consistency was high ($\alpha = 0.78$ to 0.87).

Symbolic mapping items involved mapping between verbal number names or symbolic numerals to quantities. Easier items required mapping number names to objects (e.g., matching the numeral 2 to the picture of 2 grapes) and harder items required comparing numerical values (“Which is biggest, 7 or 9 or 5?” and “Which is smaller, 27 or 32?”). Internal consistency was good ($\alpha = 0.77$ to 0.88).

Calculation items involved combining or separating sets. Easier items involved calculations when groups of objects were presented along with number names, but some objects were not visible (e.g., figuring out how many pennies if added 6 more pennies to 3 pennies under a cover). Harder items involved calculations when objects were not included, with increasingly larger numbers (e.g., adding 3 to 69 or figuring out what number is 4 less than 60). Calculation items were not given at the beginning of pre-k, and although some calculations items were given at the end of pre-k, performance was very low (*M* = 0.2 out of 10), so we did not use the measure at this time point. Internal consistency on the calculation items at the end of kindergarten and first grade was strong (α =.88 and .91).

Patterning items involved working with repeating patterns made out of colored shapes or unifix cubes. Easier items involved copying and extending an AB pattern (e.g., making the same red-blue pattern with their own set of red and blue shapes or continuing the current pattern by adding at least 2 colored shapes to the existing items). Harder items involved identifying the part that repeats in an AAB pattern or making the same kind of pattern with different objects (e.g., for a red-yellow-yellow pattern, making the same kind of pattern using green and blue blocks). Only a few patterning items were given at the beginning of pre-k, with very low performance on the items (*M* = 0.6 out of 4), so we did not include a patterning measure at the beginning of pre-k. At the remaining time points, internal consistency was mediocre to adequate (α = .56 to .63; ω_t = .71 - .81). The lower reliability of the patterning measure indicates a higher rate of random error in the measure, so we interpret findings with this measure cautiously.

Shape items focused on knowledge of shapes. Easier items focused on identifying or making a square, triangle or rectangle, including discriminating atypical and non-examples of each shape. Medium difficulty items included identifying the number of sides in a particular shape and composing new shapes (e.g., hexagon) out of other shapes. Hard items focused on identifying same size angles and describing properties of triangles and squares. Internal consistency was high (α = 0.89 to 0.96).

A few measurement items were given on the assessment, but could not be used in the analyses. It was not possible to create a reliable measurement subscale because few items were given, few children answered the items correctly and alpha reliability was consistently below 0.5.

General Cognitive and Academic Skills

Mathematics achievement is also supported by general cognitive and academic skills. We included four measures that assessed a range of skills.

Self-regulated behavior. Self-regulated behavior includes the ability to plan and finish tasks, and self-regulation in the prekindergarten year is related to basic math knowledge at the end of prekindergarten and kindergarten (Blair & Razza, 2007; McClelland et al., 2007). Teachers rated children's self-regulated behavior using the Instrumental Competence Scale for Young Children-Short Form (Lange & Adler, 1997). Teachers rated behavior as displayed in the classroom on a 4-point Likert scale, and we used the four items focused on self-regulated behavior, such as "finishes tasks and activities" and "has difficulty planning and carrying out activities that have several steps (reverse)." Scores could range from 1 to 4, with 4 reflecting the greatest regulation and 1 the least (α = .85 to .89).

Work-Related Skills. Attentive behavior, especially in the classroom, should increase children's opportunities to engage in and learn from instruction, and teacher-ratings of attentive behavior is predictive of later mathematics achievement (Duncan et al., 2007; Fuchs et al., 2014). Teacher rating of attention is moderately correlated with direct measures of working memory, and teacher ratings can be as strong or a stronger predictor of future mathematics achievement than direct measure of working memory (Fuchs et al., 2012; Fuchs et al., 2014). In part, teacher ratings are situated in the content in which much mathematics learning occurs and may better capture fluctuations across the day and from day-to-day in working-memory capacity and attention than a single, direct assessment. Attentive behavior was measured via teacher ratings of children's work-related skills items on the Cooper-Farran Behavioral Rating Scale (Cooper &

Farran, 1991). These 16 items assess children's attentiveness, ability to follow directions, and task persistence in the classroom on a 7-point behavior-anchored scale (α = .95 to .96).

Narrative Recall Skill. Language skill, intelligence, and working memory capacity are also related to mathematics achievement (Duncan et al., 2007; Geary, 2011). For example, language plays a central role in learning mathematics because some mathematics knowledge is represented linguistically and mathematics is most often taught formally and informally using language (National Research Council, 2009). Narrative recall was used as a direct measure of the combination of these skills. Children listen to a narrative and then retell the story and/or answer comprehension questions. Narrative recall requires vocabulary knowledge, language skill and working memory capacity, and empirical evidence indicates that narrative recall is moderately to strongly correlated with independent measure of vocabulary, verbal IQ and working memory capacity (Florit, Roch, Altoè, & Levorato, 2009; Strasser & Río, 2013). The measure differed by time point. At the beginning and end of pre-k, narrative recall skill was measured using the Renfrew Bus Story – North American Edition (Glasgow & Cowley, 1994). We used the information score, which captures the accuracy and completeness of children's retelling. At the end of kindergarten and first grade, the Story Recall subtest from the Woodcock Johnson III Tests of Achievement was used. Story Recall assesses children's narrative recall by requiring them to answer questions about stories the assessor reads aloud. W-scores were used.

Reading Skill. Reading skill is also predictive of mathematics achievement (Duncan et al., 2007; Watts et al., 2014). As a measure of reading skill, the Letter-Word Identification subtest of the Woodcock Johnson III Tests of Achievement was used. It assesses children's letter and word identification ability. W-scores were used.

Procedure

All direct measures of children's knowledge were collected one-on-one in a quiet room at the children's school. For all direct measures (except the Renfrew Bus Story), there was a stop criterion such that after children failed a specified number of items (depending on the measure, as specified by the implementation manual), the assessment was stopped. Except for the beginning of pre-k assessment time point, all data were collected near the end of the school year.

Data Analysis

Some children were missing data at one or more of the early time points. For early math predictors, missing data were rare and ranged from 0 to 7%. For early non-math predictors, the percent of missing cases for any given variable ranged from 0 to 22%, with most missing data involving teacher ratings of self-regulation and work-related skills. We used multiple imputation in SPSS to impute all missing values. Every analytic variable was included in the imputation model. We imputed 30 datasets and then aggregated them to run analyses.

Students were nested in their school at fifth grade. We used multi-level models to account for this nesting. Specifically, we used multi-level regression models with two levels: (1) the individual level and (2) the school level. We ran a separate model at each predictive time point.

Results

Descriptive Statistics

Reflective of the disadvantaged nature of the sample, average age-equivalent scores on the KeyMath assessment when children were an average of 11.1 years indicated that children were approximately two years behind in mathematics (Numeration: *M* = 9.2 years, *SD* = 2.0; Algebra: *M* = 9.2, *SD* = 1.9; Geometry: *M* = 8.6, *SD* = 2.0). Similarly, children's standard scores on the Quantitative Concepts assessment indicated that they were about two-thirds of a standard deviation below the national average. Descriptive statistics at each early time point are presented in the supplemental material. Tables S3 through S6 show descriptive statistics and correlations among key predictor variables within a single time point and the outcome variables. Patterns of correlations were similar across time and all correlations were statistically significant at $p < .05$.

The four math outcomes in fifth grade were strongly to very strongly correlated with each other, *r*s(515) = .53 – .83, *p*s < .001. Further, early math knowledge had highly consistent correlation patterns across the four outcomes. Thus, specific early math topics did not seem to influence one math outcome but not another. For conciseness, we combined the four outcomes into one composite “mathematics achievement” measure. Specifically, we summed z-scores across the four measures and then standardized the composite variable for interpretation purposes.

At each early time point, knowledge of each math topic was moderately correlated with math outcomes in fifth grade (see final row in Tables S3 – S6). For example, Table S3 shows that correlations between math skills at the beginning of pre-k and mathematics achievement in fifth grade ranged from .34 to .44, *p*s < .001. Table S6 shows that correlations were even stronger for relating math skills at the end of first grade to mathematics achievement in fifth grade, ranging from .42 to .66, *p*s < .001. Indeed, for math topics measured in both pre-k and first grade, the correlations with fifth-grade math achievement were statistically stronger in first-grade than at the beginning of pre-k, *p*s < .05. Shape and patterning skills had slightly weaker correlations with later achievement than early numeracy measures. However, all early math predictors tended to have similar or stronger correlations with fifth-grade mathematics achievement than non-math predictors, including reading, narrative recall, work-related skills, and self-regulation.

Because many correlations among predictors were moderate to high, we tested for multicollinearity by estimating variance inflation factors (VIF) for all independent variables. A

VIF score of 10 is conventionally the threshold for testing whether multicollinearity is biasing the results. All VIF scores for independent variables were < 5, with two exceptions. At the end of kindergarten, attentive behavior and self-regulation had VIF scores of 5.25 and 5.03 respectively.

Predicting Fifth-Grade Mathematics Achievement

Our primary goal was to evaluate the Early Math Trajectories model by testing whether knowledge of specific math topics at early time points predicted mathematics achievement in fifth grade, after controlling for other math and non-math skills as well as family and student demographics. We were interested in *which* math topics mattered and *when* they mattered. In each nested regression model, all continuous variables were standardized so that parameter estimates represented standardized regression coefficients. Table 1 presents regression results for key variables from our fully controlled models. See Table S7 in supplemental materials for more details. Each model predicted fifth-grade mathematics achievement from skills at an earlier time point (beginning of pre-k, end of pre-k, end of kindergarten, and end of first grade).

Several early math topics had substantial associations with mathematics achievement in fifth grade. Some of these associations were consistent from pre-k to first grade, while others changed over time. Early non-math skills were often less strongly associated with fifth-grade mathematics achievement, and, with the exception of narrative recall skill, these non-math skills were inconsistently predictive of fifth-grade mathematics achievement.

Results were consistent with our hypotheses based on the Early Math Trajectories model (see Table 1). First, we hypothesized that in pre-k, nonsymbolic quantity, counting, and patterning knowledge would predict fifth-grade mathematics achievement. At the beginning of pre-k, nonsymbolic quantity and counting knowledge were unique predictors, with a one standard deviation increase in either one associated with about an eighth of a standard deviation increase in fifth-grade mathematics achievement (βs = .13, *SE*s = .05 and .06, *p*s < .05). At the end of pre-k, nonsymbolic quantity and patterning knowledge were unique predictors, with a one standard deviation increase in either one associated with about a fifth of a standard deviation increase in later achievement (βs = .19 and .18, *SE*s = .05 and .06, *p*s < .05). As expected, nonsymbolic quantity and counting lost their predictive power by the end of first grade.

Second, we hypothesized that at the end of first grade, symbolic mapping, calculation, patterning, and possibly shape knowledge would predict fifth-grade mathematics achievement. Indeed, at the end of first grade, symbolic mapping (β = .26, *SE* = .05, *p* < .05) and calculation (β = .24, *SE* = .04, *p* < .05) were the strongest predictors of fifth-grade mathematics achievement. Patterning was also a unique predictor of mathematics achievement (β = .08, *SE* = .04, *p* < .05), but shape knowledge was not a unique predictor of mathematics achievement in fifth grade.

We also explored the role of different math topics at the end of kindergarten. Similar to the end of pre-k, nonsymbolic quantity and patterning knowledge at the end of kindergarten were unique predictors of fifth-grade mathematics achievement (β's = .08 to .12, see Table 1), as well as calculation skills at the end of kindergarten (β = .22, *SE* = .04, *p* < .001). Shape knowledge was never a unique predictor of fifth-grade mathematics achievement. However, exploratory analyses indicated that shape knowledge was a significant predictor when patterning knowledge was excluded. If patterning knowledge was excluded, then shape knowledge was a significant predictor at the end of pre-k (β = .10, *SE* = .04, *p* < .05), at the end of kindergarten (β = .08, *SE* = .03, *p* < .05), and at the end of first grade, (β = .08, *SE* = .03, *p* < .05).

We verified that the general predictive patterns were similar for children originally assigned to treatment and control conditions in pre-k. That is, the pattern of results in all three samples (i.e., whole sample, treatment children only, and control children only) largely supports the Early Math Trajectories model. See Tables S8 through S11 in supplemental materials for descriptive statistics and correlations by condition, and see Tables S12 and S13 for the regression results when the models were done separately by condition. There was one deviation from our hypotheses. Specifically, for control children only, calculation skill was not a statistically significant predictor at the end of first grade. However, it was a significant predictor at the end of kindergarten, and the coefficient at the end of first grade was not trivial ($\beta = .12$, $SE = .07$).

Third, we hypothesized that first-grade math knowledge would mediate the relation between pre-k knowledge and fifth-grade mathematics achievement. We performed two mediation analyses that focused on the predictors at the beginning and end of pre-k. Specifically, we tested whether the associations between significant predictors in pre-k and fifth-grade mathematics achievement were mediated by significant math predictors in first grade. We used nested regression models to complete the stepwise mediation approach recommended by Baron and Kenny (1986), as well as a bootstrapping technique recommended by Preacher and Hayes (2008) to obtain estimates for the indirect effects of the pre-k predictors.

As shown in Tables 2a and 3a, the relevant predictors in pre-k were associated with the hypothesized mediators in first grade. Specifically, at the beginning of pre-k, counting, nonsymbolic quantity, and narrative recall all significantly predicted symbolic mapping, patterning, and calculation in first grade (βs = .10 – .24). At the end of pre-k, patterning, nonsymbolic quantity, reading, and narrative recall all significantly predicted symbolic mapping, patterning, and calculation in first grade, with one exception for reading (βs = .03 – .32). As shown in Tables 2b and 3b, mediation results indicate that the direct associations between predictors in pre-k and mathematics achievement in fifth-grade (βs = .13 – .25) were reduced in strength and often not significant (βs = .01 – .13) after including the first-grade mediators.

The bootstrapping technique produced similar conclusions as shown in Tables S14 and S15 in the supplemental materials. For the beginning of pre-k predictors, knowledge of the first-grade math topics mediated the effect of counting and nonsymbolic quantity knowledge. For the end of pre-k predictors, knowledge of the first-grade math topics mediated the effect of nonsymbolic quantity knowledge and patterning knowledge. Further, in each model, knowledge of all three first-grade topics were significant mediators, but symbolic mapping and calculation knowledge were significantly stronger mediators than patterning in first grade.

Discussion

The current study provides strong evidence in support of the Early Math Trajectories model for low-income children in the U.S. Individual differences in nonsymbolic quantity, counting and patterning knowledge in preschool were predictive of fifth-grade mathematics achievement over and above a variety of other math and cognitive skills. By the end of first grade, individual differences in symbolic mapping, calculation and patterning knowledge were important predictors of later mathematics achievement; shape knowledge was not. Further, first-grade knowledge mediated the relation between preschool math knowledge and fifth-grade mathematics achievement, supporting our proposed trajectory.

Early Math Trajectories Model

The Early Math Trajectories model was based on past longitudinal evidence that identified knowledge of particular math topics at particular grade levels that predicted mathematics achievement one to three years later (Jordan et al., 2009; Krajewski & Schneider, 2009; LeFevre et al., 2010). The current study extends past research in four important ways.

First, the Early Math Trajectories model was based on past research with children from more advantaged backgrounds or from a range of economic backgrounds without consideration of this factor. The current results suggest that children from low-income backgrounds follow a similar trajectory. Such findings are in line with research that shows children from low-income backgrounds follow a similar trajectory in numeracy or general mathematics development as their peers from more advantaged backgrounds (Claessens & Engel, 2013; Jordan et al., 2006). The current research suggests that children from low-income families follow a similar trajectory when considering a wider range of knowledge topics over a wider age range. Children from low-income backgrounds experience persistent difficulties in mathematics, and the current study highlights specific early math knowledge that might merit particular attention.

Second, we evaluated the role of two non-numeracy math topics – patterning and shape – that are rarely considered in longitudinal research. We found that early patterning but not early shape knowledge was a unique predictor of later mathematics achievement, over and above other math and non-math skills. This is the first evidence that early knowledge of repeating patterns is predictive of mathematics achievement many years later. This converges with recent experimental evidence that improving children's patterning skill in preschool or first grade improves their mathematics knowledge (Kidd et al., 2013; Kidd et al., 2014; Papic et al., 2011). These results suggest that patterning knowledge merits increased attention in theories of mathematical development, although the conclusions remain tentative given the low reliability of the patterning measure in the current study. Working with patterns may provide early opportunities to deduce underlying rules and/or promote spatial skills.

In contrast, shape knowledge was tentatively included in our model because it is a component of early mathematics standards in the U.S. and considered foundational to later geometric thinking (Common Core State Standards, 2010; National Research Council, 2009). Early shape knowledge was moderately correlated with later mathematics achievement, including geometry knowledge. However, early shape knowledge was never a unique predictor of mathematics achievement after controlling for other math and non-math knowledge. Exploratory analyses indicated that shape knowledge was a significant predictor when patterning knowledge was excluded from the models. Given the paucity of research on the topic, future research needs to consider alternative ways to measure shape knowledge, how shape and patterning knowledge are related, and how the two contribute to mathematics development.

Third, the current study provides new evidence on the long-term longitudinal relations between nonsymbolic quantity, symbolic mapping knowledge, and mathematics achievement. Researchers have theorized that with age, symbolic mapping knowledge replaces nonsymbolic quantity knowledge in predicting future mathematics achievement (De Smedt et al., 2013). Past research has reported that symbolic mapping, but not approximate nonsymbolic quantity knowledge, at ages 5 and 6 predicted mathematics achievement the following year when other measures were included in the model (Kolkman et al., 2013; Sasanguie et al., 2012; Vanbinst et al., 2015). This prior research was conducted with Dutch-speaking children from middle- to upper-middle-income families and focused on approximate nonsymbolic knowledge of large sets. In the current study, we worked with a large sample of low-income U.S. children, our measure of nonsymbolic quantity knowledge tapped precise knowledge of nonsymbolic set sizes between 2 and 12, and math achievement was assessed many years later. By age 7, we found a similar result that symbolic mapping knowledge, but not nonsymbolic quantity knowledge, predicted later mathematics achievement. However, the reverse was true at younger ages, in which our measure of nonsymbolic quantity knowledge, but not symbolic mapping knowledge, was predictive of later mathematics achievement (perhaps reflecting a delay in our low-income sample). Further, symbolic mapping knowledge at age 7 helped mediate the relation between nonsymbolic quantity knowledge in preschool and middle grades math achievement. This provides the most direct evidence to date in support of the hypothesis that with age, symbolic mapping knowledge replaces nonsymbolic quantity knowledge in predicting future mathematics achievement. The current findings lend support from a new type of population, a new measure and a longer time scale. However, these results must be interpreted with caution given the restricted sample, the modest predictive power of our measures and the limitations in our nonsymbolic quantity knowledge measure (e.g., there was no time limit, so children could have counted to complete the items, and the reliability estimates were low at some early time points).

Fourth, the current findings suggest that nonsymbolic quantity knowledge may support more than symbolic mapping knowledge. Nonsymbolic quantity knowledge in preschool was also predictive of calculation and patterning knowledge. Prior research with adults indicates that improving nonsymbolic quantity knowledge for large sets leads to improved precise calculation (Park & Brannon, 2013). Better facility with nonsymbolic quantities may help ground calculation skill and promote knowledge of calculation principles (e.g., the n + 1 principle for arithmetic). The link to patterning knowledge is more speculative. For example, nonsymbolic quantity knowledge of equivalent sets may support copying repeating patterns. Overall, the current findings highlight the need to attend to the range of math knowledge that early nonsymbolic quantity knowledge may support, as well as the decreasing importance of nonsymbolic quantity knowledge as those other skills develop. Future research is needed to evaluate whether our findings generalize to different populations and measures of nonsymbolic quantity knowledge.

Educational Implications

Longitudinal evidence cannot indicate the causal role of different early math topics in mathematics development. However, the current findings suggest that some early math topics may merit additional attention in preschool and the primary grades than they currently receive, at least for low-income children. Counting, calculation and symbolic mapping knowledge already receive considerable attention from teachers and parents (Skwarchuk, Sowinski, & LeFevre, 2014). However, nonsymbolic quantity knowledge may merit more attention in preschool, and patterning knowledge may merit more attention across preschool and the early primary grades. For example, recent U.S. content standards for school mathematics include shape, but not patterning knowledge, and give limited mention of nonsymbolic quantity knowledge (Common Core State Standards, 2010). The current findings do not align with this choice of topics. Causal evidence is needed before making policy recommendations, although causal evidence for the value of patterning in the early grades is already in place (Kidd et al., 2014; Papic et al., 2011).

Limitations and Future Directions

The current study was the first evaluation of our Early Math Trajectories model, and it focused on a sample of low-income American children. The proposed knowledge components could be measured in a variety of ways, so it will be important to test whether the model holds up when topics are operationalized in other ways (e.g., nonsymbolic quantity knowledge measured via Approximate Number System acuity), further subdivided (e.g., separating verbal counting and counting objects) or expanded (e.g., to include numeral knowledge, Ryoo et al., 2015). This is particularly true for shape knowledge since past research has not evaluated its predictive value. A more reliable and appropriate measure of patterning knowledge for younger children also needs to be included in longitudinal research. The model should also be evaluated when additional measures of early cognitive skills are available, such as spatial or executive function skills. For example, patterning tasks require spatial skills, such as noticing spatial relations between objects, and spatial skills are correlated with mathematics achievement. It is also possible that our patterning measure taps early spatial skills more so than our shape measure, and thus that spatial skills, rather than pattern knowledge per se, explains the current results. In addition, inclusion of direct, specific measures of executive function skills, such as inhibitory control, and nonverbal reasoning ability may reduce the associations between early math knowledge and later mathematics achievement (Mix & Cheng, 2012). In addition, the model should be evaluated with children from a broader range of backgrounds and at later grade-levels. Children from more advantaged backgrounds often develop mathematics knowledge at a faster rate and it is possible that their developmental trajectories are not the same as for low-income children. Further, the unique influence of specific early math topics on specific later math topics (e.g., shape knowledge predicting geometry but not numeration knowledge) may emerge in later grades as mathematics instruction and knowledge becomes more specialized. Finally, future research needs to better specify the theoretical underpinnings of the model, such as how and why one type of knowledge might support another type of knowledge.

In conclusion, findings supported the Early Math Trajectories model. Nonsymbolic quantity, counting and patterning knowledge in preschool predicted fifth-grade mathematics achievement. By the end of first grade, symbolic mapping, calculation and patterning knowledge became important predictors, and first-grade skills mediated the relation between preschool skills and fifth-grade achievement. Both theory and practice must attend more to trajectories from knowledge of specific early math topics to middle-grades mathematics achievement.

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Table 1

Regression estimates predicting mathematics in fifth grade from early math and non-math skills

	***M* Age = 4.4** **Beginning of pre-k**	***M* Age = 5.0** **End of pre-k**	***M* Age = 6.1** **End of kindergarten**	***M* Age = 7** **End of first grade**
Measure	**1**	**2**	**3**	**4**
Math Predictors				
Nonsymbolic Quantity	.13**	.19***	.12*	.03
	(.05)	(.05)	(.06)	(.05)
Counting	.13*	-.02	.07	-.01
	(.06)	(.06)	(.06)	(.04)
Symbolic Mapping	.06	.11	.09	.26***
	(.05)	(.06)	(.06)	(.05)
Shape	.05	.03	.05	.03
	(.04)	(.05)	(.04)	(.04)
Patterning	--	.18***	.08*	.08*
	--	(.05)	(.04)	(.04)
Calculation	--	--	.22***	.24***
	--	--	(.04)	(.04)
Non-Math Predictors				
Reading	.04	.10*	.07	.11**
	(.05)	(.04)	(.04)	(.04)
Narrative Recall	.11*	.17***	.10**	.09**
	(.04)	(.04)	(.03)	(.03)
Work-Related Skills	.10	.08	-.04	.18**
	(.07)	(.07)	(.06)	(.05)
Self-regulation	.05	.01	.21**	.01
	(.07)	(.06)	(.06)	(.05)
Controls	Inc.	Inc.	Inc.	Inc.

Note. Standard errors are in parentheses. Inc. = all models presented include the full list of control variables: age in fifth grade, current grade level (fourth or fifth), gender, ELL status in pre-k, ethnicity, pre-k school type (public or Head Start), and socio-economic status (respondent's education level and income level). *$p < .05$. **$p < .01$. ***$p < .001$.

Table 2

First-grade skills as mediators of the association between beginning of pre-k skills and fifth-grade mathematics achievement

a) Regression estimates for predicting first grade mediators from beginning of pre-k skills

	Hypothesized mediators in first grade		
	Symbolic Mapping	**Patterning**	**Calculation**
Beginning of pre-k			
Nonsymbolic Quantity	.17***	.19***	.23***
Counting	.16**	.18**	.24***
Narrative Recall	.10*	.11*	.10*
Controls	Inc.	Inc.	Inc.

b) Regression estimates for predicting fifth-grade mathematics achievement with and without hypothesized mediators in first grade

	Predicting fifth-grade mathematics achievement	
	Without mediators	**With mediators**
Beginning of pre-k predictors		
Nonsymbolic Quantity	.16***	.01
Counting	.23***	.09*
Narrative Recall	.13**	.06
End of first-grade mediators		
Symbolic Mapping	--	.36***
Patterning	--	.10**
Calculation	--	.29***
Controls	Inc.	Inc.

Note. Inc. = all models presented include the full list of control variables, which include age in fifth grade, current grade level (fourth or fifth), gender, ELL status in pre-k, ethnicity, pre-k school type (public or Head Start), and socio-economic status (respondent's education level and income level). *$p < .05$. **$p < .01$. ***$p < .001$.

Table 3

First-grade skills as mediators of the association between end of pre-k skills and fifth-grade mathematics achievement

a) Regression estimates for predicting first grade mediators from end of pre-k skills

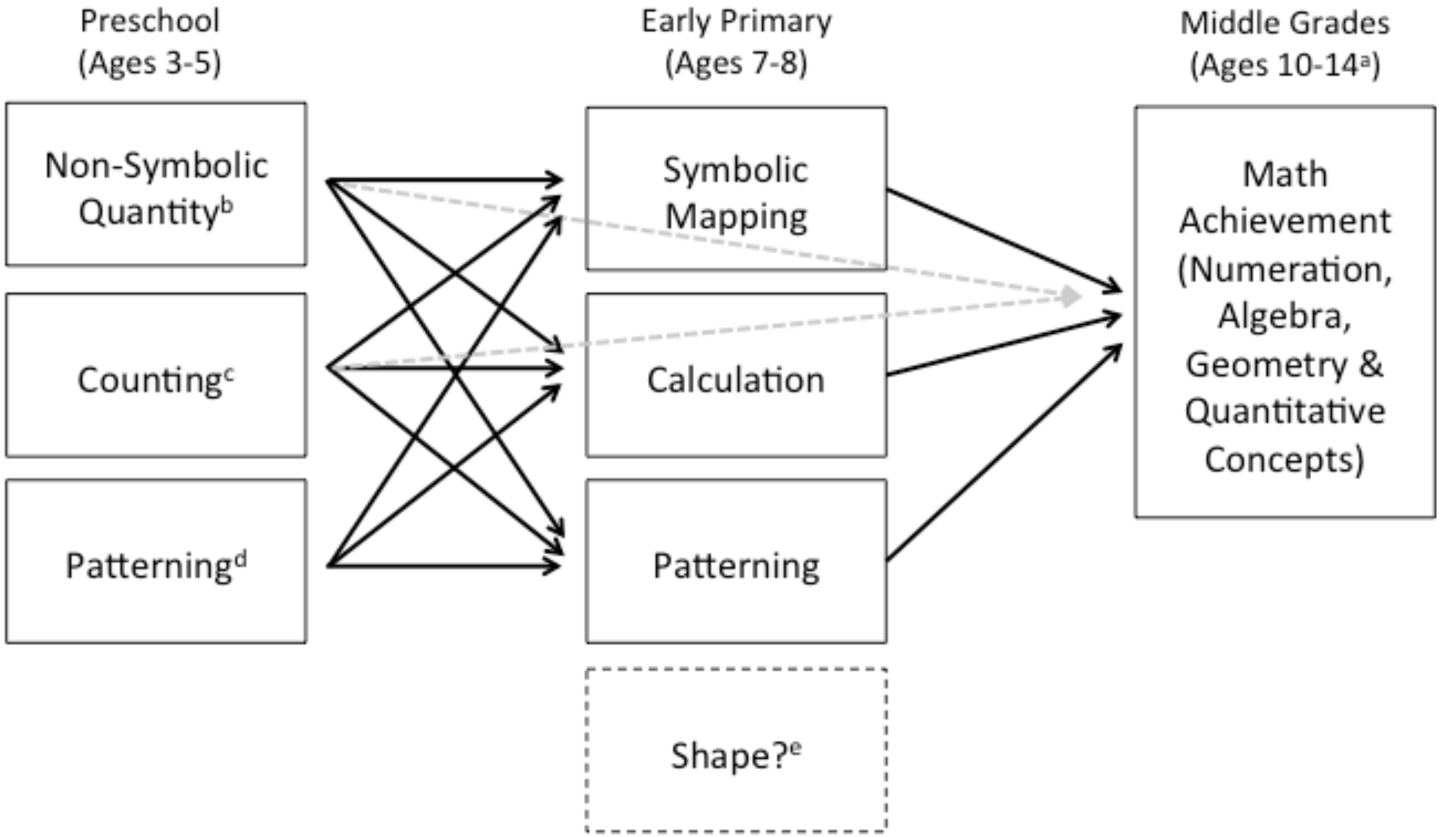
	Hypothesized mediators in first grade		
	Symbolic Magnitude	**Patterning**	**Calculation**
End of pre-k			
Nonsymbolic quantity	.20***	.18***	.20***
Patterning	.23***	.14**	.32***
Reading	.18***	.03	.11**
Narrative recall	.09*	.19***	.10**
Controls	Inc.	Inc.	Inc.

b) Regression estimates for predicting fifth-grade mathematics achievement with and without hypothesized mediators in first grade

	Predicting fifth-grade mathematics achievement	
	Without mediators	**With mediators**
End of pre-k predictors		
Nonsymbolic	.25***	.13***
Patterning	.23***	.07
Reading	.15***	.06
Narrative Recall	.19***	.11**
End of first-grade mediators		
Symbolic Mapping	--	.30***
Patterning	--	.08*
Calculation	--	.26***
Controls	Inc.	Inc.

*Note. Inc. = all models presented include the full list of control variables, which include age in fifth grade, current grade level (fourth or fifth), gender, ELL status in pre-k, ethnicity, pre-k school type (Head Start or public), and socio-economic status (respondent's education level and income level). *p < .05. **p < .01. ***p < .001.*

Figure 1. Early Math Trajectories Model, Including Current Evidence



Note. [a]Current evidence is for ages 10-11 [b]Nonsymbolic quantity knowledge was a reliable predictor of middle-grade math achievement at the beginning and end of pre-k. At the end of pre-k, non-symbolic quantity knowledge continued to predict achievement with first-grade mediators in the model, suggesting there may be a direct effect of nonsymbolic quantity knowledge (as indicated by the dashed line) as well as an indirect effect (through the first-grade predictors). [c]Counting was a reliable predictor of achievement at the beginning of pre-k, but not the end of pre-k. At the beginning of pre-k, counting knowledge continued to predict achievement with first-grade mediators in the model, suggesting there may be a direct effect of early counting knowledge (as indicated by the dashed line) as well as an indirect effect (through the first-grade predictors). [d]Patterning was not adequately assessed at the beginning of pre-k, but it was a reliable predictor of achievement at the end of pre-k, mediated through first-grade predictors. [e]Shape knowledge was not a unique predictor.