Early Math Trajectories Predicting Math Knowledge from Ages 11-15: A Longitudinal Investigation with Urban Youth

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ABSTRACT

What is “early math” and what types of early math knowledge are most predictive of future math knowledge? The current study addressed this question in the context of a longitudinal study of 457 marginalized students from ages 4 to 15. Counting, nonsymbolic quantity and shape knowledge at age 4 were predictive of different components of children’s math knowledge at age 11. Change in symbolic mapping, nonsymbolic quantity and shape knowledge from age 4 to 7 predicted multiple components of age 11 math knowledge and/or rate of growth in this knowledge from age 11 to 15. The current findings help refine an Early Math Trajectories model for understanding math development, with implications for early math content standards.

1. Objectives

What is “early math” and what types of early math knowledge are most predictive of future math knowledge? Past research has primarily relied on global measures of early and later math knowledge, without considering specific types of math knowledge or their growth (Duncan et al., 2007). The current study fills this gap by elucidating specific early math skills that are predictive of knowledge and rate of growth in four components of middle-grades math. It focuses on children from low-income backgrounds to inform efforts to improve their math knowledge, as these children face a range of gaps in experience, including an education gap and opportunity gap (Ford, 2016; Ladson-Billings, 2006).

2. Theoretical framework

Mathematics is a broad domain encompassing a variety of topics and skills (National Research Council, 2009). The *Early Math Trajectories Model* identified early math topics that should be of particular importance for supporting mathematics achievement in the middle grades, based on a synthesis of past research (Authors, 2016). The model proposes a trajectory from preschool (ages 4–5) to early primary grades (ages 7–8) to middle grades (ages 10–15). The model is based on previous longitudinal, correlational evidence, and thus indicates predictors of later achievement, although may not reflect causal relations.

In the current study, we focus on components of the Early Math Trajectories model that were measured in a longitudinal study of math development among low-income American children from ages 4 to 15. Knowledge of four math topics was assessed at age 4 and age 7: *counting* (knowledge of the number-word sequence, counting objects, and cardinality (Purpura & Lonigan, 2013)), *nonsymbolic magnitude* (knowledge of the magnitude of sets without the need to use verbal or symbolic number names (Starkey & Cooper, 1980; Xu, Spelke, & Goddard, 2005)), *symbolic mapping* (knowledge of the mapping between symbolic numerals, their number names and their magnitudes, including their relative magnitudes (Jordan, Kaplan, Nabors Olah, & Locuniak, 2006)), and *shape* (the ability to classify two- and three-dimensional shapes, supported by learning of their definitional properties (Clements & Sarama, 2009)). According to the model, counting and nonsymbolic magnitude knowledge at age 4 and growth in symbolic mapping knowledge from age 4 to 7 should predict middle-grades math knowledge. The predictive value of shape knowledge is less clear given limited research and mixed findings. Change in general math knowledge from 4.5 years to Grade 1 is a positive predictor of age 15 general math knowledge (Watts, Duncan, Siegler, & Davis-Kean, 2014), but past research has not considered growth in specific aspects of early math knowledge.

Further, predictive relations could vary with the target math outcome. Prior evaluation of the Early Math Trajectories model has found similar predictive relations for knowledge of later math topics (e.g., algebra, geometry, and numeration) at age 11 and 12 (Authors, 2016; 2019). However, knowledge of different math topics becomes more distinct in the upper grades, so predictive relations may become more specific.

Past research has also not considered predictors of rate of growth in middle-grades math knowledge. The limited available research on growth in elementary-school math knowledge suggests that predictors of achievement are not be the same as the predictors of the rate of growth in achievement (Geary, 2011). Thus, this study provides important initial evidence for predictors of *rate of growth* in middle-grades math knowledge and whether this varies by math topic.

In the current study, we used data from a longitudinal study of math development of over 450 low-income American children from age 4 to 15. We address two research questions:

(1) Knowledge of which early math topics at age 4 are unique predictors of knowledge and rate of growth in math knowledge from ages 11 to 15 for four math topics (quantitative reasoning, algebra, geometry, and numeration)?

(2) Growth in knowledge of which early math topics from ages 4 to 7 are unique predictors of knowledge and rate of growth in math knowledge from ages 11 to 15 for four math topics?

3. Methods

Participants were drawn from a longitudinal follow-up study of 519 children originally recruited at the beginning of their prekindergarten year from 57 pre-k classes in a large urban city, all of which served children who qualified for free or reduced priced lunch. The final sample was 56% female, 79% black, 9% white, 8% Hispanic and 4% other races, 9% English Learners. Based on maternal report when the children were in pre-k, 43% of families had an annual income under $10,000, 38% had an income between $10,000 and $25,000, and the remaining 19% had an income over $25,000; 25% of mothers had less than a high-school diploma, 33% had a high-school diploma or GED, and 42% had some post-secondary education. The current study reports on the 457 children who did not have missing predictor variables.

Children were first individually assessed at the beginning of Pre-K and were re-assessed at the end of first grade. The outcome measures were individually administered every year from ages 11 to 15 near the end of the school year, which was Grades 5-9 for most children, but 21% of children had been retained a grade by age 15.

4. Data

The outcome measures were four standardized mathematics assessments. The first was the *quantitative concepts* subtest from the Woodcock Johnson Achievement Battery III, which assesses students' knowledge of basic mathematical concepts, symbols, and vocabulary. The other three were *algebra, geometry* and *numeration (*i.e., understanding of whole and rational numbers) concept subtests from the KeyMath 3 Diagnostic Assessment (Connolly, 2007).

Math knowledge at age 4 and 7 was assessed using the Research-based Elementary Math Assessment (REMA) (Clements, Sarama, & Liu, 2008). It focuses on numeracy knowledge as well as shape, patterning and measurement knowledge. We broke the numeracy items into four subscales: nonsymbolic quantity, counting, symbolic mapping, and calculation. Only 4 topics had a sufficient number of items administered at age 4 - shape, nonsymbolic quantity, counting and symbolic mapping – so we focus on these 4 measures. Four early general skills were assessed and included to use as controls for general academic and cognitive skills: reading (letter-word), narrative recall (an indicator of general cognitive skill), and teacher ratings of work-related skills (e.g., attentive behavior) and self-regulated behavior.

Data Analysis: Students were nested in their school, using the school attended for the longest from age 11-15 for each student. Latent growth curves were used to model the rate of growth in math domains, using full information maximum likelihood (FIML), meaning no cases were dropped for missing outcome values. The full model was comprised of separate, simultaneous growth curves for quantitative concepts, algebra, numeration, and geometry at five time points from age 11 to 15. Growth curves were first evaluated separately without covariates where models with linear and quadratic growth factors were compared. Although quadratic models demonstrated improved fit relative to linear models for all variables, variances of the quadratic factors were negligibly small and were fixed to zero for subsequent analyses. A multivariate growth curve (i.e., parallel process) model with growth curves for each math outcome was then evaluated. Growth factors were allowed to covary within and across processes, and residual variances for each age were allowed to covary across processes. The model demonstrated good fit across indices ($\chi^2(172) = 377.65$, $p < .001$, RMSEA = .048 [90% CI = .041 - .055], CFI = .985, TLI = .983).

To address research question 1, the growth factors of the multivariate growth curve model (i.e., intercepts and slopes) were regressed on age 4 math knowledge, general academic and cognitive skills, and other demographic covariates. To address research question 2, changes in math knowledge from age 4 to age 7, defined as the difference in math knowledge between age 4 and age 7, were added as predictors to the previous model. All analyses were performed using M*plus*, Version 8 (Muthén & Muthén, 1998-2017).

5. Results

Investigating Research Question 1 and as shown in Table 1, shape, counting and nonsymbolic quantity knowledge at age 4 predicted some components of math knowledge at age 11, but age 4 math knowledge rarely predicted rate of growth in math knowledge. In particular, shape knowledge at age 4 predicted geometry knowledge at age 11, whereas early counting knowledge predicted knowledge of quantitative concepts, algebra and numeration. Nonsymbolic quantity knowledge at age 4 was predictive of geometry and numeration knowledge. For example, a one standard deviation increase in age 4 nonsymbolic quantity knowledge was associated with a .12 standard deviation increase in numeration knowledge at age 11. Finally, shape knowledge at age 4 predicted rate of growth in algebra knowledge from age 11 to 15; the only academic predictor of growth. Non-math skills were rarely predictive.

Next, for Research Question 2, consider how *change* in the four early math skills between ages 4 and 7 related to knowledge of 4 math topics at age 11 and rate of growth in that knowledge from age 11 to 15, as shown in Table 2. Controlling for age 4 knowledge, change in symbolic mapping knowledge was the most consistent predictor of age 11 math knowledge, predicting knowledge of all four topics, and also predicting rate of growth in algebra knowledge. In contrast, change in counting knowledge from ages 4-7 did not predict knowledge or rate of growth in knowledge of any of the topics. Change in nonsymbolic quantity knowledge was also important, predicting quantitative concept and algebra knowledge at age 11 and rate of growth in geometry and numeration knowledge. Finally, change in shape knowledge predicted geometry, algebra and numeration knowledge at age 11; it did not predict rate of growth. Again, non-math skills were rarely predictive.

6. Significance

The current findings help refine an Early Math Trajectories model for understanding math development. First, while counting knowledge at age 4 was a useful indicator of a variety of types of later math knowledge, change in counting knowledge through age 7 was not a unique predictor. Growth in counting knowledge seems less predictive than growth in other types of math knowledge, but we may need to distinguish between basic and advanced counting knowledge (Nguyen et al., 2016). Second, rate of growth in nonsymbolic quantity knowledge through age 7 was a useful predictor of multiple types of later math knowledge. This does not align with theories that symbolic mapping knowledge replaces nonsymbolic quantity knowledge in predicting future math achievement (De Smedt, Noël, Gilmore, & Ansari, 2013). The continued role of nonsymbolic quantity knowledge, beyond symbolic mapping knowledge, in primary school requires additional study. Third, rate of growth in symbolic mapping knowledge from age 4 to 7 was an important predictor, even though age 4 symbolic mapping knowledge was not a predictor on its own. Symbolic mapping knowledge in the early primary grades is a consistent predictor of later math knowledge (Geary, 2011); the current study begins to elucidate potential pathways, and brings attention to considering change in this knowledge from preschool to primary school.

Finally, the current study provides much needed evidence for the predictive role of shape knowledge. Few studies have evaluated the predictive value of shape knowledge, and the limited evidence that exists is mixed (Authors, 2016, 2019; Nguyen et al., 2016). In this study, shape knowledge at age 4 was predictive of geometry knowledge at age 11. Previous research has also found it to be predictive of a general math measure with an emphasis on geometry at age 11 (Nguyen et al., 2016). The current study suggests potential pathways through which early shape knowledge and growth in this knowledge may influence a variety of types of later math knowledge and its growth.

Practically, the current study lends support to calls for attention to a variety of types of early math knowledge and the need to broaden attention in Kindergarten to topics beyond basic counting and numeral knowledge (Engel, Claessens, Watts, & Farkas, 2016). Knowledge of quantities, their mappings to symbolic labels, magnitude comparison, and definitional properties of shapes are all important early math topics.

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Table 1: Regression Estimates for the Association Between Age 4 Academic Skills, Knowledge of 4 Math Topics at Age 11 and Rate of growth in that Knowledge from Age 11-15

	Intercept (Age 11)				Slope (Age 11 to Age 15)			
	QC	Algebra	Geometry	Numeration	QC	Algebra	Geometry	Numeration
	β (SE)	β (SE)	β (SE)	β (SE)	β (SE)	β (SE)	β (SE)	β (SE)
Math Predictors								
Shape	0.07	0.03	0.15**	0.07	0.05	0.16	0.02	0.01
Symbolic Map	-0.01	0.09	0.10	0.07	0.12	0.06	0.1	0.04
Counting	0.14*	0.17*	0.10	0.18**	0.04	-0.02	0.06	0.09
Nonsymbolic	0.07	0.10	0.15*	0.12*	0.02	0 .0	0.06	-0.12
Non-math Pred.								
Reading	0.07 (0.05)	0.0 (0.05)	0.01 (0.06)	0.02 (0.05)	0.0 (0.09)	0.09 (0.08)	0.06 (0.1)	0.12 (0.08)
Narrative Rec.	0.1 (0.05)	0.12 (0.05)*	0.1 (0.05)	0.04 (0.05)	-0.04 (0.09)	0.01 (0.07)	-0.04 (0.1)	0.06 (0.08)
Self-regulation	0.05 (0.08)	-0.06 (0.08)	0.05 (0.08)	0.05 (0.08)	-0.12 (0.13)	0.21 (0.11)	-0.04 (0.1)	0.08 (0.07)
Work-related	0.09 (0.08)	0.14 (0.08)	0.06 (0.09)	0.05 (0.08)	0.04 (0.14)	-0.21 (0.12)	-0.05 (0.15)	-0.02 (0.12)
Controls	Inc.	Inc.	Inc.	Inc.	Inc.	Inc.	Inc.	Inc.

Notes: *$p < .05$, **$p < .01$. Standardized coefficients are presented. Control variables were: Age at beginning of study, whether student had been retained a grade level at any point, gender, english-learner status in preK, ethnicity, SES composite.

Table 2: Regression Estimates for the Association Between Change in Math Skills from Age 4 - 7, Knowledge of 4 Math Topics at Age 11 and Rate of growth in that Knowledge from Age 11-15, Controlling for Age 4 Academic Skills.

	Intercept (Age 11)				Slope (Age 11 to Age 15)			
	QC	Algebra	Geometry	Numeration	QC	Algebra	Geometry	Numeration
	β (SE)	β (SE)	β (SE)	β (SE)	β (SE)	β (SE)	β (SE)	β (SE)
Math Predictors								
Δ Shape	0.1 (0.05)	0.12 (0.06)*	0.23 (0.06)***	0.18 (0.05)**	0.05 (0.11)	0.1 (0.09)	0.17 (0.12)	0.1 (0.09)
Δ Symbolic	0.34 (0.07)***	0.3 (0.07)***	0.42 (0.07)***	0.38 (0.07)***	0.13 (0.13)	0.44 (0.11)***	-0.11 (0.14)	0.19 (0.11)
Δ Counting	0.07 (0.08)	0.02 (0.08)	0.02 (0.08)	-0.01 (0.08)	-0.03 (0.15)	-0.2 (0.13)	-0.26 (0.16)	-0.13 (0.13)
Δ Nonsymb	0.2 (0.06)**	0.19 (0.06)**	0.06 (0.07)	0.08 (0.06)	-0.03 (0.12)	0.09 (0.1)	0.36 (0.13)**	0.25 (0.1)*
Shape	0.12 (0.06)	0.11 (0.07)	0.3 (0.07)***	0.18 (0.06)**	0.1 (0.13)	0.2 (0.11)	0.27 (0.14)	0.11 (0.11)
Symbolic Map	0.25 (0.08)**	0.32 (0.08)***	0.43 (0.08)***	0.36 (0.08)***	0.22 (0.15)	0.43 (0.13)***	-0.01 (0.17)	0.25 (0.13)
Counting	0.19 (0.1)	0.16 (0.1)	0.05 (0.11)	0.14 (0.1)	-0.02 (0.19)	-0.26 (0.16)	-0.24 (0.21)	-0.28 (0.16)
Nonsymbolic	0.14 (0.06)*	0.2 (0.07)**	0.13 (0.07)	0.13 (0.06)*	-0.01 (0.13)	-0.02 (0.11)	0.31 (0.14)*	0.24 (0.11)*
Non-math Pred.								
Reading	0.04 (0.04)	-0.02 (0.05)	0.0 (0.05)	0.0 (0.04)	0.0 (0.09)	0.08 (0.07)	-0.11 (0.1)	0.03 (0.08)
Narrative Rec.	0.06 (0.04)	0.1 (0.04)*	0.07 (0.05)	0.03 (0.04)	-0.04 (0.08)	-0.02 (0.07)	-0.06 (0.1)	0.04 (0.08)
Self-regulation	0.02 (0.07)	-0.07 (0.07)	0.06 (0.08)	0.07 (0.07)	-0.11 (0.13)	0.2 (0.11)	-0.1 (0.15)	-0.08 (0.12)
Work-related	0 0 (0.07)	0.06 (0.08)	-0.03 (0.08)	-0.06 (0.07)	0.02 (0.14)	-0.28 (0.12)*	-0.01 (0.16)	-0.05 (0.12)

Controls	Inc.	Inc.	Inc.	Inc.	Inc.	Inc.	Inc.	Inc.

Notes: *p < .05, **p < .01, ***p < .001. Standardized coefficients are presented. Control variables were: Age at beginning of study, whether student had been retained a grade level at any point, gender, english-learner status in preK, ethnicity, SES composite.