LONGITUDINAL PREDICTIONS OF SIXTH-GRADE GEOMETRY KNOWLEDGE

Emily R. Fyfe
University of Wisconsin-Madison
efyfe@wisc.edu

Bethany Rittle-Johnson
Vanderbilt University
bethany.rittle-johnson@vanderbilt.edu

In the current study, we examined the contributions of shape and pattern knowledge at four early time points to formal geometry knowledge in sixth grade within a longitudinal sample of over 500 low-income children. Shape knowledge at the beginning of pre-k predicted geometry in sixth grade, over and above general math and cognitive skills. However, at the end of pre-k, kindergarten and first grade, pattern knowledge was a unique predictor of later geometry, and shape knowledge was not. Results highlight the important roles of two important, but understudied components of math knowledge – early shape and pattern skills – and their contributions to the development of formal geometry knowledge.

Keywords: Geometry and Geometrical and Spatial Thinking; Learning Trajectories; Standards

Introduction

Mathematics knowledge begins to develop at a young age, and this early math knowledge matters. General math skill in pre-k and kindergarten predicts math achievement across primary and secondary school (Watts et al., 2014). Research is now needed to identify *specific* early math skills that help predict and improve later achievement in specific math areas. Many theories of mathematics focus on the development of numeracy knowledge, that is, skills necessary for understanding numbers and number relations (see Sarama & Clements, 2004). However, math knowledge extends beyond numeracy knowledge. The goal of this research is to focus on two important, but understudied, components of math – early shape and pattern knowledge – and test their contributions to formal geometry achievement in middle school.

Exploring shapes and patterns is a common mathematical activity for young children (Ginsburg, Lin, Ness, & Seo, 2003) and may be an important contributor to general math development. Knowledge of shapes and their properties is considered foundational to later geometric thinking (National Research Council, 2009) and is included in the Common Core State standards as early as kindergarten. Children first learn to classify typical shapes and then to describe the definitional features of both two- and three-dimensional shapes. According to the learning trajectory theory, these shape skills form the building blocks for later geometry achievement (Clements, Wilson, & Sarama, 2004). However, no evidence to date links early shape knowledge to later math outcomes, including knowledge of geometry.

Pattern knowledge includes the ability to identify, extend, and describe predictable sequences in objects or numbers, and it has been recognized by math education researchers as a core skill for mathematical thinking (Papic et al., 2011; Warren & Cooper, 2007). The first type of pattern young children engage with are repeating patterns, such as the colors red-blue-red-blue-red-blue. Children's knowledge of repeating patterns becomes systematically more sophisticated from pre-k to kindergarten (Rittle-Johnson et al., 2015), and several school-based interventions have shown that instruction on repeating patterns supports general math achievement at the end of the school year (e.g., Kidd et al., 2014). However, patterns are not included in the Common Core State Standards at any grade level, and thus receive little attention.

Method

Participants were drawn from a longitudinal study. The sample included 513 low-income children originally recruited at the beginning of their pre-kindergarten year in the U.S. (56% female, 80% black). Children were initially assessed at four early time points: beginning of pre-k (*M* age = 4.4 years), end of pre-k (*M* age = 5.0 years), end of kindergarten (*M* age = 6.1 years), and end of first grade (*M* age = 7.0 years). These children were re-assessed five years later when most students were near the end of sixth grade (*M* age = 12.1 years, 17% had been retained a grade and were in fifth grade). Students were distributed across 51 middle schools.

The outcome measure of interest (administered in sixth grade) was the *Geometry* subtest from the KeyMath 3 Diagnostic Assessment (Connolly, 2007), a standardized math test. The geometry subtest measures a student's spatial reasoning as well as his ability to analyze, describe, compare, and classify two- and three-dimensional shapes.

For early math predictors (administered in pre-k, kindergarten, and first grade), we assessed children's early *shape and pattern knowledge*, which were measured using items from the Research-based Early Math Assessment (Clements, Sarama, & Liu, 2008). Table 1 provides example items. Shape items (n = 14 – 23 depending on time point) focused on identifying, creating, and defining shapes. Pattern items (n = 4 – 7) focused on copying, extending, or identifying patterns made out of colored shapes or cubes. We also assessed children's general math achievement using the *quantitative concepts* and *applied problems* subtests from the Woodcock Johnson Achievement Battery III (Woodcock, McGrew, & Mather, 2001). Quantitative concepts assesses the knowledge of basic math concepts, symbols, and vocabulary. Applied problems assesses the ability to analyze and solve various math problems.

Table 1. Example items from the early shape and pattern knowledge subscales.

Knowledge Subscale	REMA Item #	Item Description
Shape Knowledge	N05	Given a mat with 26 different shapes on it, the child is asked, "Can you point to all the triangles?"
	NG6	"Is this shape a square? How do you know?"
Pattern Knowledge	G04	The child is shown an ABA_AB pattern and asked, "Can you find the missing piece in this pattern?"
	G30	The child is shown an ABBABB shape pattern. "Make the same kind of pattern here, using these blocks."

We also assessed four non-math predictors to control for general cognitive and academic skills. These included a measure of early *reading* skill (The Woodcock Johnson Letter-Word Identification), a measure of *narrative recall* that varied by time point (the Renfrew Bus Story or the Woodcock Johnson Story Recall), and teacher ratings of *work-related skills* (Cooper-Farran Behavioral Rating Scale;), and *self-regulation* (using the Instrumental Competence Scale).

Results

Reflective of the disadvantaged nature of the sample, age- and grade-equivalent scores on the KeyMath geometry assessment indicated that students were approximately two years behind in geometry. Recall, the assessment was administered near the end of sixth grade when students were an average of 12.1 years old. However, the average grade-equivalent score in this sample was 4.8 (*SD* = 2.1) and the average age-equivalent score was 9.5 (*SD* = 2.0).

In the early years, children's shape and pattern knowledge increased from pre-k to first grade. For example, at the beginning of pre-k, children solved an average of 3.3 shape items and 0.6 pattern items correctly. By the end of first grade, children solved an average of 9.4 shape items and 3.7 pattern items correctly. At each of the four early time points, scores on the shape and pattern subscales were moderately correlated with geometry knowledge in sixth grade (for shape scores, *r*s = .34 – .45, *p*s < .001; for pattern scores, *r*s = .25 – .49, *p*s < .001).

The primary goal was to test whether shape and pattern knowledge at early time points predicted formal geometry knowledge in sixth grade, after controlling for other general math and non-math skills. We ran multi-level regression models at each early time point with students nested in their sixth-grade schools. The results are presented in Table 2.

Table 2: Longitudinal predictions of geometry knowledge in sixth grade

	Predicting Geometry in Sixth Grade From Four Early Time Points			
	Beginning of Pre-K	**End of Pre-K**	**End of Kindergarten**	**End of First Grade**
Math Predictors				
Shape Knowledge	.16 (.05)**	-.02 (.05)	-.03 (.04)	.06 (.05)
Pattern Knowledge	-.01 (.05)	.25 (.05)***	.14 (.04)***	.11 (.05)*
Quantitative Concepts	.27 (.05)***	.30 (.06)***	.20 (.05)***	.18 (.05)***
Applied Problems	.16 (.05)**	.13 (.05)*	.24 (.04)***	.23 (.05)***
Non-Math Predictors				
Reading	-.08 (.05)	-.05 (.05)	.09 (.04)*	.09 (.05)
Narrative Recall	.04 (.05)	.17 (.04)***	.10 (.03)**	.09 (.04)*
Work-Related Skills	.02 (.08)	.03 (.07)	.05 (.06)	.21 (.06)**
Self-Regulation	.05 (.07)	-.06 (.07)	.11 (.07)	-.03 (.06)
Control Variables	Included	Included	Included	Included

Note. All models include these control variables: age in sixth grade, current grade level, gender, ELL status in pre-k, ethnicity, pre-k school type (public or Head Start), and socio-economic status (respondent's education level and income level). *$p < .05$. **$p < .01$. ***$p < .001$.

First, we showed that past findings on the importance of early math to later achievement generalize to later geometry. Specifically, across all early time points, children's scores on the quantitative concepts and applied problems tests predicted their sixth-grade geometry knowledge (βs = .13 – .30). Second, the contributions of general math knowledge to sixth-grade geometry were often substantially stronger than the contributions of general cognitive/academic skills, including reading, narrative recall, work-related skills, and self-regulation.

Third, the contributions of early shape and pattern knowledge differed. At the beginning of pre-k, shape knowledge was a significant predictor of sixth-grade geometry knowledge ($\beta = .16$, $SE = .05$, $p < .05$). However, at the three remaining time points (end of pre-k, end of kindergarten, and end of first grade), pattern knowledge was a significant predictor (βs = .11 – .25), and shape knowledge was not. For example, at the end of pre-k, a one standard deviation increase in pattern knowledge was associated with a quarter of a standard deviation increase in sixth-grade geometry knowledge ($\beta = .25$, $SE = .05$, $p < .05$), over and above controls. As a note, if we excluded patterning from the model, shape was still not a significant predictor at the end of pre-k or kindergarten, but it was a significant predictor in first grade ($\beta = .13$, $SE = .04$, $p < .05$).

Conclusion

We evaluated the role of two non-numeracy math skills – shape and pattern knowledge – in the development of geometry achievement. We found that shape knowledge was only predictive prior to formal schooling (at the beginning of pre-k). However, early pattern knowledge from the end of pre-k to first grade consistently predicted middle school geometry knowledge over and above general math and cognitive skills. These results are consistent with recent research foregrounding the importance of early patterning skills, including intervention work that found a preschool patterning intervention led to greater numeracy knowledge at the end of kindergarten than typical preschool instruction (Papic et al., 2011).

These findings provide some limited support for the learning trajectory theory that suggests early shape knowledge is foundational to later geometry knowledge (Clements et al., 2004). However, they also suggest a more prominent role for patterning. Indeed, the current Common Core State Standards include shape knowledge as a key component as early as kindergarten, yet fail to include pattern knowledge at any grade level. Contrary to these recommendations, the current results suggest that math standards should include repeating pattern knowledge in kindergarten and first grade (i.e., copying, extending, and identifying predictable sequences), and that more research is needed on the importance of shape knowledge.

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