September 17, 2020 (Thursday), 4:10 pm

Quantitative measure equivalence

Romain Tessera, Institut de Mathématiques de Jussieu-Paris
Virtual Talk via Zoom
Zoom Meeting ID: 998 6775 5871
Email mathcolloquium@vanderbilt.edu to request pass code
Measure equivalence is an equivalence relation between countable groups that has been introduced by Gromov. A fundamental instance are lattices in a same locally compact group. According to a famous result of Ornstein Weiss, all countable amenable groups are measure equivalent, meaning that geometry is completely rubbed out by this equivalence relation. Recently a more restrictive notion has been investigated called integrable measure equivalence, where the associated cocycles are assumed to be integrable. By contrast, a lot of surprising rigidity results have been proved: for instance Bowen has shown that the volume growth is invariant under integrable measure equivalence, and Austin proved that nilpotent groups that are integrable measure equivalent have bi-Lipschitz asymptotic cones. I will present a work whose goal is to understand more systematically how the geometry survives through measure equivalence when some (possibly very weak) integrability condition is imposed on the cocycles. We shall put the emphasis on amenable groups, for which we will present new rigidity results, and the first flexibility results known in this context. (Contact Person: Dietmar Bisch)

September 24, 2020 (Thursday), 4:10 pm

Equivariant homotopy commutativity, trees and chicken feet

Constanze Roitheim, University of Kent
Zoom Meeting ID: 998 6775 5871
Email mathcolloquium@vanderbilt.edu to request pass code
Commutativity up to homotopy can be daunting, and it becomes even more difficult to track when group actions get introduced. In the case of a finite group, however, the options for equivariant homotopy commutativity can be encoded using simple combinatorics, and we will show some examples. (Contact Person: Jocelyne Ishak)
Talk will be offered live at 11:10a and replayed at 4:10pm

October 1, 2020 (Thursday), 4:10 pm

Ramanujan: A Century Of Inspiration

Bruce C. Berndt, University of Illinois
Zoom Meeting ID: 998 6775 5871
Email mathcolloquium@vanderbilt.edu to request pass code
Srinivasa Ramanujan is perhaps the most enigmatic mathematician in the history of our subject. First, an account of Ramanujan's life will be given. Second, the history of Ramanujan's (earlier) notebooks and "lost" notebook will be provided. Third, the speaker will describe how he became fascinated with Ramanujan's work, beginning with proving a few claims from his notebooks in February, 1974, and then since May, 1977, devoting all of his research efforts to proving the claims in Ramanujan's earlier notebooks, lost notebook, and published papers. Fourth, some examples from Ramanujan's notebooks and lost notebook will be given. This lecture will be aimed at a general audience. (Contact Person: Larry Rolen)

October 8, 2020 (Thursday), 4:10 pm

The Navier-Stokes, Euler and Other Related Equations

Edriss S. Titi, Texas A&M University, University of Cambridge, Weizmann Institute of Science
Zoom Meeting ID: 998 6775 5871
Email mathcolloquium@vanderbilt.edu to request pass code
In this talk I will present the most recent advances concerning the questions of global regularity
of solutions to the three-dimensional Navier-Stokes and Euler equations of incompressible fluids. Furthermore, I will also present recent global regularity (and finite time blow-up) results concerning certain three-dimensional geophysical flows, including the three-dimensional viscous (non-viscous) "primitive equations" of oceanic and atmospheric dynamics. (Contact Person: Gieri Simonett)

November 12, 2020 (Thursday), 4:10 pm

There are 160,839 + 160,650 3-planes in a 7-dimensional cubic hypersurface

Kirsten Wickelgren, Duke University
Zoom Meeting ID: 998 6775 5871
Zoom Meeting link: https://vanderbilt.zoom.us/j/99867755871
Email mathcolloquium@vanderbilt.edu to request pass code
Given a generic choice of polynomials with complex coefficients, one can compute the dimension of the straight lines, planes, or d-dimensional planes contained in the common zeros of the polynomials. When this dimension is 0, there is some finite number of d-planes. For example, there are 321,489 3-dimensional planes in the zero locus of a degree 3 homogeneous polynomial in 9 variables over the complex numbers. This number can be identified with the topological Euler number of a certain vector bundle. However, it only corresponds to the count of d-planes over an algebraically closed field like the complex numbers. We can get information over other fields like the real numbers, the rational numbers, or finite fields, by using an Euler number from A1-homotopy theory instead. This Euler number is no longer an integer; instead it is a bilinear form, and invariants of bilinear forms record information about the arithmetic and geometry of the planes. In this talk, we will introduce these enumerative problems and A1-Euler numbers. We establish integrality results for the A1-Euler class, and use this to compute the Euler numbers associated to arithmetic counts of d-planes on complete intersections in terms of topological Euler numbers over the real and complex numbers. The example in the title then follows from work of Finashin–Kharlamov. The new work in this talk is joint with Tom Bachmann.
(Contact Person: Anna Marie Bohmann)

December 3, 2020 (Thursday), 4:10 pm

Open problems within the N-body problem

Richard Montgomery, University of California, Santa Cruz
Zoom Meeting ID: 998 6775 5871
Zoom Meeting link: https://vanderbilt.zoom.us/j/99867755871
Email mathcolloquium@vanderbilt.edu to request pass code
The 333 year old N-body problem is alive and well. We survey four open problems within the problem and recent progress on them, beginning with a survey of some solution curves.
(Contact Person: Marcelo Disconzi)

February 4, 2021 (Thursday), 4:10 pm

Counting problems: open questions in number theory, from the perspective of moments

Lillian Pierce, Duke University
Email Math Colloquium to request Zoom ID and pass code
Many questions in number theory can be phrased as counting problems. How many number fields are there? How many elliptic curves are there? How many integral solutions to this system of Diophantine equations are there? If the answer is “infinitely many,” we want to understand the order of growth for the number of objects we are counting in the “family.” But in many settings we are also interested in finer-grained questions, like: how many number fields are there, with fixed degree and fixed discriminant? We know the answer is “finitely many,” but it would have important consequences if we could show the answer is always “very few indeed.” In this accessible talk, we will describe a way that these finer-grained questions can be related to the bigger infinite-family questions. Then we will use this perspective to survey interconnections between several big open conjectures in number theory, related in particular to class groups and number fields. (Contact Person: Larry Rolen)

February 11, 2021 (Thursday), 4:10 pm

Euler characteristics of spaces of graphs

Karen Vogtmann, University of Warwick
Email Math Colloquium to request Zoom Meeting ID and passcode
Many phenomena in mathematics and science can be modeled by drawing finite graphs whose nodes are connected by edges with specified lengths. The set of all such models forms a geometric space, with one point for each possible model. The topological structure of this space is still quite mysterious and is the focus of current activity in several different fields, including perturbative quantum field theory, number theory and tropical algebraic geometry. In this talk I will describe spaces of graphs, explain why they are of particular interest in geometric group theory, then highlight some recent results (joint with M. Borinsky) on their Euler characteristics (Contact: Spencer Dowdall)

February 18, 2021 (Thursday), 4:10 pm

Mathematics of Evolution: mutations, selection, and random environments

Natalia Komarova, UC Irvine
Email mathcolloquium@vanderbilt.edu to request Zoom Meeting ID and passcode
Evolutionary dynamics permeates life and life-like systems. Mathematical methods can be used to study evolutionary processes, such as selection, mutation, and drift, and to make sense of many phenomena in life sciences. I will present two very general types of evolutionary patterns, loss-of-function and gain-of-function mutations, and discuss scenarios of population dynamics — including stochastic tunneling and calculating the rate of evolution. I will also talk about evolution in random environments. The presence of temporal or spatial randomness significantly affects the competition dynamics in populations and gives rise to some counterintuitive observations. Applications include origins of cancer, passenger and driver mutations, and how aspirin might help prevent cancer. (Contact**:** Glenn Webb)

March 11, 2021 (Thursday), 4:10 pm

Graphs, network motifs, and threshold-linear algebra in the brain

Carina Curto, Pennsylvania State University
Email Math Colloquium to request Zoom Meeting ID and passcode
Threshold-linear networks (TLNs) are commonly-used rate models for modeling neural networks in the brain. Although the nonlinearity is quite simple, it leads to rich dynamics that can capture a variety of phenomena observed in neural activity: persistent activity, multistability, sequences, oscillations, etc. Here we study competitive threshold-linear networks, which exhibit both static and dynamic attractors. These networks have corresponding hyperplane arrangements whose oriented matroids encode important features of the dynamics. We will show how the graph associated to such a network yields constraints on the set of (stable and unstable) fixed points, and how these constraints affect the dynamics. In the special case of combinatorial threshold-linear networks (CTLNs), we find an even stronger set of "graph rules" that allow us to predict emergent sequences and to engineer networks with prescribed dynamic attractors.
(Contact person: Ioana Suvaina)

March 25, 2021 (Thursday), 4:10 pm

A Scale Invariant Approach for Signal and Image Recovery

Yifei Lou, University of Texas, Dallas

Email Math Colloquium to request Zoom Meeting ID and passcode

I will talk about the ratio of the L_1 and L_2 norms, denoted as L_1/L_2, to promote sparsity. Due to the non-convexity and non-linearity, there has been little attention to this scale-invariant model. Compared to popular models in the literature such as the L_p model for $p\in(0,1)$ and the transformed L_1 (TL1), this ratio model is parameter free. Theoretically, we present a strong null space property (sNSP) and prove that any sparse vector is a local minimizer of the L_1/L_2 model provided with this sNSP condition. We then focus on a variant of the L_1/L_2 model to apply on the gradient. This gradient model is analogous to total variation, which is the L_1 norm on the gradient. We discuss an iteratively reweighed algorithm to minimize the proposed model with guaranteed convergence. Experiments on the MRI reconstruction and limited-angle CT reconstruction show that our approach outperforms the state-of-the-art methods.

April 8, 2021 (Thursday), 4:10 pm

Microlocal Analysis in Compton Tomography: Joint Compton/X-ray-CT Reconstruction

Todd Quinto, Tufts University
Math Colloquium to request Zoom Meeting ID and passcode
We present a new joint reconstruction method for thesimultaneous reconstruction of attenuation coefficient and electron density from X-ray transmission and Compton-scattered backscattered data. After introducing some ideas from microlocal analysis, we use them to describe what features are visible in both limited data sets. We show that the Compton and X-ray CT data set are complementary–this ensures that most features (wavefront directions) of the object are visible from the combined data. However, both limited-data reconstruction methods introduce streak artifacts in standard reconstruction methods.
In addition, we introduce a new joint reconstruction scheme for low effective atomic number imaging. Motivated by our microlocal results, we develop a Lambda tomography penalty term in the regularization procedure in the algorithm to integrate the two data sets together. We evaluate our reconstruction method on the “parallel line segment” acquisition geometry of [Webber, J. and Miller, E., Inverse Problems 36(2020), 025007] which is motivated by a specific architecture for airport security screening. Finally, we show the effectiveness of our method in combating the noise and image artifacts on simulated phantoms.