

A Broadly Based Analysis of Mathematical Giftedness

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This article addresses several questions raised by contemporary research on mathematical giftedness. Most issues are confronted empirically, based on a stratified random sample of 95,650 tenth-grade students and a highly select subsample of mathematically gifted individuals (boys $N = 497$, girls $N = 508$) drawn from this larger pool. Psychological profiles of the mathematically gifted were compared (by gender) to those of their normative cohorts. Typical gender differentiating attributes (e.g., interest patterns) were less stereotyped in gifted boys and girls; and students' homes covered a broad socioeconomic spectrum. Mathematically gifted students were found to be intellectually superior across a wide range of cognitive abilities; however, evidence for somewhat more mathematical specificity in the gifted than in the general population was also detected. The hypothesis that spatial visualization interacts synergistically with mathematical ability in the prediction of sophisticated levels of advanced mathematics was tested with negative results. "Classic" male/female differences were observed on measures of mathematical ability with the former generating larger means and variances. We suggest that gender differences reflected by these two statistics may have distinct antecedents. The social implications for not attending to group differences in ability-dispersion are discussed in the context of ability assessment in general and meta-analytic reviews in particular. Longitudinal data (13 years) revealed that 8% of gifted males and 19% of gifted females in the follow-up samples did not obtain college degrees. For the era of the 60s this difference is not surprising, but the proportion of both sexes who did not make full use of their abilities is shocking. Many of our results correspond to other longitudinal findings, such as Terman's classic studies as well as ongoing contemporary investigations on mathematical giftedness.

The publication by Benbow and Stanley (1980) that reported substantial male superiority among seventh- and eighth-grade students at extremely high levels of mathematical talent and the accompanying interpretative article by Kolata (1980) was followed by several letters to the editor (*Science*, vol. 212, April 1981).

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Most of the letters were highly critical of the genetic hypothesis advanced for the observed male superiority, but others questioned the scientific significance of the findings at the phenotypic level. In the subsequent years, the controversy has remained alive, with numerous articles appearing in a wide variety of prestigious journals and scholarly volumes (Chipman, Brush, & Wilson, 1985; Ethington & Wolfle, 1984; Freed, 1983; Holden, 1987; Meece, Parsons, Kaczala, Goff, & Futterman, 1982; Mura, 1987; Paulsen & Johnson, 1983; and Stanley & Benbow, 1983). The data reported by Benbow and Stanley (1980, 1983b) were derived from an ongoing project beginning in the early 1970s, the Study of Mathematically Precocious Youth (SMPY). This project continues to generate interesting and important data on mathematically gifted seventh and eighth graders, including biographical information, educational experiences, and longitudinal tracking of subjects' career paths (Benbow & Stanley, 1983a). Readers unaware of this outstanding research program can be brought quickly up-to-date by Benbow's (1988) recent target article for *Behavioral and Brain Sciences*, followed by subsequent commentary and Benbow's rejoinders.

Using scores on the College Board's Scholastic Aptitude Test SAT-M to index mathematical giftedness, SMPY researchers have consistently observed gender differences in *two statistics*: Not only do males reliably score higher on the SAT-M (mean difference approximately .5 standard deviation), they also display greater variability on such measures. There has never been an exception in the studies conducted at SMPY (Benbow, 1988; Benbow & Minor, 1986; Stanley & Benbow, 1983, 1986). Other workers observed similar results, both within more homogeneous subsets of students [e.g., ethnic groups living in the U.S. (Jones, 1987; Moore & Smith, 1987)] and cross-culturally [e.g., using German and Chinese translated versions of SAT-M (Benbow, 1988; Stanley, Huang, & Zu, 1986)]. Gender differences in *level* and *variability* appear to be robust with respect to this parameter of cognitive functioning (as measured by the SAT-M, and most other mathematical tests). Therefore, as one moves to more exceptional levels of mathematical talent, the proportion of males to females *increases* for two reasons; and it becomes critical to devote particular attention to gender differences observed in the upper tail of any mathematics distribution.

For example, using a cutting score of SAT-M ≥ 500 the ratio of seventh and eighth-grade boys to girls is 2 to 1; however, with a cutting score of SAT-M ≥ 700 (a score indicative of the top 5% of male college-bound high school seniors), the proportion of boys to girls is roughly 12 to 1; this extreme difference is generated by the collective effect of group differences in both level and variation. In her recent review, Benbow (1988) discusses evidence for this phenomenon and the research concerning its correlates. Causal hypotheses derived from the correlates are also evaluated.

Unfortunately, many of the samples in the literature reviewed by Benbow (1988) are small and are not from well-defined populations. Also, the number of measures entering a typical study is small, thus providing few correlates per

study and little opportunity to observe the overlap among the correlates. Even the size of the ratio of boys to girls at a known point above the mean is uncertain because the populations sampled differ in various ways from a nationwide one. Examinees who are volunteers, for example, are atypical from students in general.

The data in Project Talent (Flanagan, Dailey, Shaycoft, Gorham, Orr & Goldberg, 1962) avoid these problems because a great deal of information is available for a nationwide probability sample of more than 900 high schools. The number of students in the project was also quite large, with about 100,000 students being tested in each of the four high school grades. An added dimension to research on gifted students available in this Project is the information about many individuals in the sample obtained 11 years after high school graduation. This places our subjects' follow-up within the same time frame as the initial talent searches conducted by the SMPY investigators. These longitudinal data, among other things, provide a sketch of the educational achievements of mathematically gifted students at the time the SMPY studies were conceived.

We had available tapes for 10th-grade students in Project Talent. The tapes contain data from the 1960 test administrations for 48,474 boys and 47,176 girls. There are more than 50 cognitive tests, numerous cognitive composites, 27 interest and personality scores, and 17 scores based on background information available for analysis.

That the initial data were gathered in 1960 is a problem to some, but it is our hypothesis, for which there is a great deal of empirical support, that structural relationships are not as sensitive to social change or cultural differences as mean levels of performance. There is a good deal of data that can be used to document this conclusion, but it is beyond the scope of this paper to review the pertinent literature. Consider, however, that relationships between variables reflect *how* people function. Basic principles of human behavior do not change from one generation to another. An impression to the contrary is almost certainly the result of the same mechanism that produced a widely accepted myth among personnel psychologists: namely, that predictive validities were highly sensitive to minor differences among jobs, people, conditions of work, and so forth. The principle mechanism was their failure to give adequate weight to the large sampling errors of correlations based on small samples.

If Project Talent were repeated today, we would expect more changes in means than in structural relations among the tests or between the tests and external criteria. Means on ("Q-data") *self-report questionnaires* of interests and personality would, in turn, be expected to show more change than means on ("T-data") *tests* of information, knowledge, and skills. In addition, the quality of the data collected by Project Talent, as well as the sample's comprehensiveness, is unparalleled by contemporary standards. We felt this massive data bank was capable of shedding light on the following questions raised by contemporary research on mathematical giftedness (e.g., Benbow, 1988). Questions:

1. Are the causes for gender differences in the overall *level* of mathematical ability the same causes responsible for generating sex differences in *variability*?
2. How *generalized* is exceptional mathematical talent (i.e., to what extent are mathematically gifted students able to excel in nonquantitative but intellectually demanding areas)?
3. Is there something unique about mathematical talent that sets the possessors apart from their fellows in other ways?
4. Where in our population are students found who have exceptionally high levels of mathematical talent?
5. Are they given adequate educational support (e.g., guidance for planning their higher education)?
6. How large are the sex differences on other measures (e.g., abilities & interests) for the gifteds compared to the controls?
7. Are both boys and girls who have high levels of mathematical talent more masculine as the masculine-identification hypothesis would predict?
8. Is there evidence for a special relationship between a high level of math talent and spatial visualization?

We conclude our analysis with 13-year longitudinal data related to the foregoing questions and suggest some possible causal mechanisms for our results.

PROCEDURE AND MEASURES

Our first step was the selection and weighting of components for a mathematics score to be used in the selection of the gifted groups. Talent has four mathematical tests: Mathematics Information (23 items, involving the vocabulary of mathematical notation and definitions), Arithmetic Reasoning (16 items, involving the reasoning required to solve arithmetic problems), Introductory Mathematics (24 items, consisting of all forms of math taught through the 9th grade), and Advanced Mathematics (14 items, of questions on algebra, plane and solid geometry, probability logic, logarithms, and introductory calculus).

For our selection composite, we chose Mathematics Information, Arithmetic Reasoning, and Introductory Mathematics; to avoid overweighting of formal mathematics, Arithmetic Reasoning was given slightly more weight (raw scores on the three tests were multiplied by the following constants, contained in parentheses): Mathematics Composite = Mathematics Information (.55) + Arithmetic Reasoning (1.0) + Introductory Mathematics (.55). We did not use Advanced Mathematics (as a component), because not all gifted students in the 10th grade would have had access to that content domain. However, this scale was employed as a correlate. We selected, roughly, the top 1% of mathematically gifted students of each gender based on male ($N = 48,142$) and female ($N = 47,127$)

distributions of the Mathematics Composite. This gave us 497 boys and 508 girls drawn from the total number of boys and girls, respectively.

[Our gifted samples were selected in separate frequency distributions to avoid an inordinate sex ratio favoring males of the sort reported by Benbow and Stanley (1980, 1983a). Our decision to select sexes separately does not mean that sex differences in mathematical giftedness (as indexed by conventional assessment devices) are not psychologically important or scientifically significant. Rather, we selected sexes separately to ascertain gender differences in mathematical ability that would be more difficult to interpret if a single cutting score producing a disparate ratio of boys to girls were used on the total sample. When a single standard for giftedness is used, girls are more highly selected than boys relative to their respective sexes. It also follows that girls are more highly selected than boys on all measures correlated with the Mathematics Composite. We also wanted a representative number of each sex for the longitudinal follow up. Given this, our cutting scores for selecting gifted students for either gender were not as stringent as those imposed by the SMPY investigators. However, we also performed analyses using two different cutting scores on the joint distribution. Cutting scores were 2.4 and 2.7 standard score units above the mean of the combined sample. These selection criteria produced "classic" sex differences, with the former generating a sample of $N = 974$ males, $N = 349$ females and the latter resulting in $N = 340$ males and $N = 76$ females. The results of these analyses were in accord with what follows in all important respects, but we were not satisfied with our attempts to correct for the different amounts of selection that the methodology made inevitable.]

Project Talent tests were short because the aim was to obtain the maximum amount of information from the examinees in the limited time made available by the schools. As a result, most of the tests were only modestly reliable with most in the range of .50 to .80. The reliabilities are satisfactory for research involving central tendencies, but short tests have a more important limitation from our point of view. Tests with a small number of items are likely to exhibit floor and ceiling effects for talented groups. These potential scale defects were enhanced by the decision to measure the four high school classes with a single form of each test. Therefore, like our selection composite, our intellectual correlates of giftedness, with one exception, Advanced Mathematics, are weighted composites of individual Talent tests. These composites have for the most part, adequate floor and ceiling properties for the gifted groups and estimated reliabilities of about .90 (Humphreys, in press). To ascertain the specificity of mathematical giftedness, we chose to construct composites of other parameters of cognitive functioning involving intellectual content distinct from, but complementary to, our selection composite. A Spatial Reasoning Composite and English Language Composite were assembled based on several individual tests relevant to each ability. These two composites were highly diverse in content and their

components did not overlap with those of the mathematics composite. They were chosen for analysis based on previous factor-analytic studies indicating that both are salient markers of *general intelligence* ("g") and separate group factors. They were composed of the following Talent tests (with number of items and raw score constants given in parentheses, followed by a scale description):

- Spatial Composite = [2-d Spatial visualization (24 × 1.0), ability to visualize two-dimensional figures turned around on a flat surface and, when turned over on a flat surface] + [3-d Spatial visualization (16 × 3.0), ability to visualize two-dimensional figures after they have been folded into three-dimensional figures] + [Mechanical reasoning (20 × 1.5), measures deductions based on primitive kinds of mechanisms like gears, pulleys, and springs, and knowledge of the effects of common physical forces (e.g., gravity)] + [Abstract reasoning (15 × 2.0), a nonverbal test of logical relationships in complex patterns];
- English Composite = [Word Functions in Sentences (24 × 1.0), sensitivity of grammatical structure] + [Talent's English Composite (113 × 1.0), which consists of tests measuring spelling, capitalization, punctuation, usage, and affective expression].

We also assembled a Verbal Composite with tests distinct from those used for our English Composite. Substantively, the English Composite focused on the mechanics and syntax of the English language, whereas our Verbal Composite measured skills such as reading comprehension, vocabulary, and literary knowledge. Verbal Composite = [Literary Information (24 × 1.0) knowledge of literature typically assigned in high school] + [Reading Comprehension (48 × 1.0) comprehension of written text across a broad range of topics] + [Vocabulary (30 × 1.0) general knowledge of word meanings].

Other composites assembled by Project Talent were also employed as criteria, but only two of these, the Intelligence and Scientific Composites, have overlapping components with the Mathematics Composite (again, number of items and multiplicative constants are in parentheses):

- IQ = [Reading Comprehension (48 × 3), described above] + [Abstract Reasoning (15 × 5), described above] + [Arithmetic Reasoning (16 × 4), described above];
- Technical Information = [67 items, tests of physical science, aeronautics/space, electricity and electronics, and mechanics];
- General Information = [143 items, a broad range of tests involving: art, law, health, engineering, architecture, journalism, foreign travel, military, accounting, business, sales, practical knowledge, clerical, the Bible, colors, etiquette, hunting, fishing, outdoor activities, photography, games, theater & ballet, food, and vocabulary];

- Scientific Composite = [1063 items, a large number of scales including those of the IQ Composite, Math Composite, Advanced Mathematics, Technical Composite, and Creativity (one's ability to solve a number of practical problems in novel ways)].

SELF-REPORT MEASURES

The interest and background scales are relatively homogeneous and will not be described individually in detail. Raw scores of the former were multiplied by various constants to achieve a comparable range metric (viz., maximum score = 40); the latter were also multiplied by constants, but not targeted toward a common range. Interest measures include: Physical Science, Biological/Medical Science, Public Service, Social Service, Literature, Art, Music, Office Work, Business/Management, Sales, Computation, Mechanical/Technical, Skilled Trades, Farming, Labor, Sports, and Hunting/Fishing. All of these measures fairly directly index the behaviors depicted by their labels. One scale was modified, however: "Skilled trades" initially contained 13 stereotypically male trades plus 3 stereotypically female trades; the latter three items were culled from this measure to convert it to a purely "masculine index" (our rationale for this modification will become evident in subsequent discussion). Our background measures included: Socioeconomic Status (SES), High School Grades, Academic Courses (taken in high school), High School Guidance, Guidance Elsewhere, Study Habits, Self-perception of Writing Skill, Self-perception of Reading Skill, Extra Reading, Variety of Hobbies, Participation in Sports, and Leadership Roles. For more detailed descriptions of these measures, as well as other measures in Project Talent, readers are referred to Flanagan et al. (1962) and Wise, McLaughlin, and Steel (1979).

RESULTS

We analyzed more data than can be condensed for a research report. The data that we consider most critical for understanding high levels of mathematical talent and how such talent relates to questions enumerated earlier will be presented. In our presentation we rarely pay explicit attention to standard errors and *p*-values, but some general guidelines are useful. The standard error of a standardized mean based on approximately 50,000 cases is .0045. The standard error of a difference in mean standardized scores for two independent samples of 50,000 each is .0064. For the same sample size, these are also the standard errors of the *z*-transforms of correlations and of differences in *z*-transforms in independent samples. Differences of trivial size are thus statistically significant. For the gifted samples of approximately 500 cases, the same standard errors are larger by a factor of 10. Even so, fairly small sex differences of .13 and .17 are significant with *p*-values less than .05 and .01, respectively.

Obtaining definitive answers to our research questions is ultimately related to the nature of the units of measurement of the measuring instruments. Units of measurement on any given psychological test may not be equal in all parts of the score distribution. Too little floor or ceiling on a test is a common cause of inequality of units. Our rule of thumb was that a gifted mean more than two gifted standard deviations from the floor or ceiling represents a reasonably safe basis for making comparisons. Departures from homoscedasticity in bivariate distributions are produced by unequal units. Heteroscedasticity in the upper tail of the bivariate distribution is revealed by estimating the variance of a measure correlated with the mathematics composite in the gifted groups. If the obtained variance is not reasonably close to $S_y^2 (1 - r_{xy}^2)$, where x is the composite and y is the correlate, a regression methodology is not dependable. Hoping to find consensus among several ways of making comparisons for numerous measures associated with giftedness represents the approach in which most confidence can be placed. With this in mind, the following three tables were assembled to address the aforementioned questions empirically and in multiple ways.

Table 1 contains the raw score means and standard deviations for selected tests for each gender separately in the unselected samples as well as correlations between the Mathematics Composite and these measures. Table 2 (pp. 336–337) reports data on our gifted samples. For each gender, the first column contains the raw score means while the third column contains corresponding standard deviations of selected tests. Column four is the standard score distance between the observed mean for the gifted groups and the maximum score possible (using the gifted groups SD for standardization); as discussed above, when < 2 , this statistic indicates that the interpretation of our regression estimates is compromised. The second column consists of the regression estimates, where the Mathematics Composite is used as a predictor to estimate observed scores on the selected tests. These mean estimates (“ $M_{est.}$ ”) were computed: $M_{est.} = (SD_y \times r_{yx} \times Z_x) + M_y$ (where SD_y = standard deviation of the select test, r_{yx} = correlation between the Mathematics Composite and the select test, Z_x = standard score differences between the gifted and unselected subjects on the Mathematics Composite, and M_y = the mean of the unselect sample on the selected test). Finally, Table 3 (p. 338) contains the standardized differences between and within the sexes, for both the gifted and normative groups.

(1) Level and Variability of Mathematical Talent

Because the units of measurement are not comparable, only a limited amount of information can be obtained from raw score means in Tables 1 and 2. Quick inspection does reveal that there are many gender differences in both directions in the unselected sample (Table 1) and that these differences persist, for the most part, in both size and sign in the gifted sample (Table 2). There are no surprises in the unselected sample in the results for either the cognitive or self-report tests, and mathematically gifted boys and girls conform to much the same pattern of

TABLE 1
Distribution Statistics and Correlations with the Mathematics Composite
in the Unselected Sample

	Unselected Males			Unselected Females		
	M	S	r_{xy}	M	S	r_{xy}
Cognitive Tests						
Advanced math (14)	2.98	1.93	.51	2.59	1.72	.43
Composites						
IQ composite (283)	156.40	54.27	.83	157.35	52.69	.82
Space composite (132)	74.45	22.77	.62	63.42	20.80	.63
Math composite (41.85)	18.05	7.65	—	16.65	7.08	—
English composite (137)	84.37	18.12	.77	92.29	17.43	.77
Verbal composite (102)	55.57	20.32	.75	55.85	19.30	.75
Technical information (67)	33.21	11.71	.67	22.64	7.91	.63
General information composite (143)	65.68	21.05	.72	63.45	18.88	.71
Scientific composite (1063)	506.45	166.70	.88	444.11	144.38	.88
Background						
SES (135)	97.65	10.37	.42	97.33	10.15	.41
H.S. grades (50)	24.15	10.15	.34	26.25	10.11	.44
Academic courses (90)	28.06	11.26	.33	26.00	10.15	.45
H.S. guidance (80)	21.43	17.36	-.04	19.17	15.71	.02
Background						
Guidance elsewhere (30)	12.36	7.10	.16	14.58	6.77	.19
Study habits (40)	22.96	6.03	.37	25.95	5.64	.36
Self-perception writing skill (40)	20.57	8.51	.21	23.85	8.72	.23
Self-perception reading skill (40)	23.03	7.98	.30	23.55	7.87	.27
Extra reading (90)	30.39	21.59	.16	33.32	21.22	.19
Variety of hobbies (100)	60.72	23.52	-.03	51.36	17.23	.08
Participation in sports (24)	15.56	5.71	.14	12.34	5.32	.13
Leadership roles (50)	17.04	14.70	.04	19.71	14.21	.08
Interests						
Physical science	20.40	8.62	.34	11.84	7.65	.28
Biological science/Medicine	18.42	9.74	.18	16.25	10.35	.20
Public service	17.14	11.72	.07	11.44	10.90	.07
Social service	15.42	7.34	-.02	23.78	7.37	.09
Literature	15.21	8.19	.11	20.39	8.88	.22
Art	16.16	9.06	.04	19.98	9.84	.19
Music	13.43	10.60	-.03	17.58	11.34	.12
Office work	12.42	8.13	-.09	23.78	9.64	-.19
Business/Management	19.10	7.63	-.01	15.57	7.75	.02
Sales	16.28	8.84	-.04	13.33	8.73	.00
Computation	15.31	8.06	.10	15.34	8.10	.03
Mechanical/Technical	19.83	8.04	-.08	7.92	6.27	.04
Skilled trades	18.44	6.31	-.19	11.87	4.48	-.03
Farming	20.28	10.00	-.11	13.35	9.18	.09
Labor	12.07	6.79	-.28	7.47	6.05	-.07
Sports	25.97	9.49	.03	19.30	9.65	.11
Hunting/Fishing	27.13	10.53	-.05	14.37	11.51	.07

Note. Bracketed numbers next to the scale labels represent highest possible score.

TABLE 2
Distribution Statistics, Regressed Estimates of Means, and Floor and Ceiling Effects in the Gifted Sample

	Gifted Males				Gifted Females			
	M	$M_{est.}$	S	C.I.	M	$M_{est.}$	S	C.I.
Cognitive Tests								
Advanced math	7.70	5.60	2.32	2.72	6.24	4.63	2.04	3.08
Composites								
IQ composite	253.95	—	14.77	1.97	251.06	—	14.06	2.27
Space composite	106.11	112.00	14.55	1.78	95.49	99.32	15.69	2.33
Math composite	38.39	—	1.13	3.06	36.08	—	1.45	3.98
English composite	118.05	121.48	8.60	2.20	121.73	129.06	6.89	2.22
Verbal composite	88.32	96.11	8.97	1.53	88.93	95.51	8.94	1.54
Technical information	52.54	54.08	8.46	1.71	37.04	36.29	8.18	3.66
General information composite	100.50	105.99	14.14	3.01	96.73	100.18	11.75	3.94
Scientific composite	864.20	—	67.37	2.95	774.09	—	69.86	4.14
Background								
SES	108.78	109.24	8.31	3.16	108.40	108.73	8.54	3.11
H.S. grades	39.02	33.33	8.91	1.23	41.20	38.44	8.05	1.09
Academic courses	37.75	37.94	6.76	7.73	37.15	38.51	5.55	9.52
H.S. guidance	21.03	19.58	15.77	1.33 ^a	19.61	20.03	14.66	4.14
Guidance elsewhere	14.52	15.38	6.65	2.33	16.71	18.10	5.75	2.31
Study habits	29.51	28.89	5.17	2.22	30.94	31.51	4.58	1.98
Self-perception writing skill	27.56	25.32	7.63	1.63	29.73	29.35	7.44	1.38
Self-perception reading skill	30.33	29.40	6.69	1.45	29.62	29.37	7.18	1.45

Extra reading	46.50	39.58	23.90	1.82	48.08	44.37	23.02	1.82
Variety of hobbies	60.32	58.84	20.67	2.92 ^a	55.58	55.14	14.14	3.14
Participation in sports	15.62	17.69	5.47	1.53	12.70	14.23	4.74	2.38
Leadership roles	17.67	18.60	13.11	2.47	21.48	22.82	13.49	2.11
Interests (#0)								
Physical science	30.64	28.20	6.35	1.47	21.59	17.71	7.98	2.31
Biological science/Medicine	25.01	23.08	8.81	1.70	23.18	21.92	9.72	1.73
Public service	20.81	19.32	11.08	1.73	16.29	13.53	11.36	2.09
Social service	16.77	15.03	7.29	3.19	24.77	25.60	7.49	2.03
Literature	21.94	17.61	8.12	2.22	27.22	25.74	7.67	1.66
Art	19.66	17.12	8.47	2.40	25.39	25.10	9.50	1.54
Music	18.58	12.58	11.57	1.85	23.05	21.31	11.21	1.51
Office work	12.38	10.47	7.61	1.63 ^a	17.03	18.76	9.84	1.82 ^a
Business/Management	19.59	18.90	6.65	3.07	16.14	15.99	7.36	3.24
Sales	16.07	15.34	8.32	1.93 ^a	12.93	13.33	8.53	1.52 ^a
Computation	19.50	17.45	8.24	2.49	17.05	16.01	9.03	2.54
Mechanical/Technical	18.64	18.12	7.64	2.44 ^a	10.44	8.61	6.56	4.51
Skilled trades	15.29	15.25	5.41	2.83 ^a	12.25	11.50	4.82	5.76
Farming	15.55	17.35	8.76	1.78 ^a	15.18	15.61	9.36	2.65
Labor	8.05	7.01	5.79	1.39 ^a	6.77	6.31	5.63	1.20 ^a
Sports	24.52	26.73	9.54	2.57 ^a	22.35	22.21	8.74	2.02
Hunting/Fishing	22.60	25.73	10.28	2.20 ^a	16.01	16.58	10.70	2.24

Note. For each gender, column one includes the raw score means on selected tests; column two contains the regressed estimates of these means computed: $M_{Est.} = (SD_y \times r_{yz} \times Z_x) + M_y$ (see text). These estimates are omitted for tests sharing components with the Mathematics Composite. *C.I.* = the ceiling index, or the range from the mean of the gifted group to the maximum score possible, standardized by the gifted group's standard deviation. For *C.I.* values with "a" superscripts, these values represent the distance from the gifted mean to the floor; the lower bound range is given because these gifted means were below the mean of the unselected group.

TABLE 3
Standardized Mean Differences Between and Within Sexes

	Between Sexes (Males-Females)		Within Sexes (Gifted-Unselected)			
	Unselected	Gifted	Observed		Regressed	
			M	F	M	F
Cognitive Tests						
Advanced math	.21	.80	2.45	2.12	1.27	1.03
Composites						
IQ composite	-.02	.05	1.80	1.78	—	—
Space composite	.51	.49	1.39	1.54	-.33	-.24
Math composite	.19	.31	2.66	2.74	—	—
English composite	-.45	-.21	1.86	1.69	-.30	-.66
Verbal composite	-.01	-.03	1.61	1.71	-.58	-.52
Technical information	1.06	1.58	1.65	1.82	-.18	.12
General information composite	.11	.19	1.65	1.76	-.38	-.26
Scientific composite	.40	.58	2.15	2.29	—	—
Background						
SES	.03	.04	1.07	1.09	-.05	-.04
H.S. grades	-.21	-.22	1.47	1.48	.60	.30
Academic courses	.19	.06	.86	1.10	-.02	-.15
H.S. guidance	.14	.09	-.02	.03	.08	-.03
Guidance elsewhere	-.32	-.32	.30	.31	-.12	-.21
Study habits	-.51	-.24	1.09	.88	.11	-.11
Self-perception writing skill	-.38	-.25	.82	.67	.27	.04
Self-perception reading skill	-.07	.09	.91	.77	.13	.03
Extra reading	-.14	-.07	.75	.70	.33	.18
Variety of hobbies	.45	.23	-.02	.24	.06	.03
Participation in sports	.58	.53	.01	.07	-.37	-.29
Leadership roles	-.18	-.26	.04	.12	-.06	-.10
Interests						
Physical science	1.05	1.11	1.19	1.27	.30	.53
Biological science/Medicine	.22	.18	.68	.67	.20	.12
Public service	.50	.40	.31	.44	.13	.25
Social service	-1.14	-1.09	.18	.13	.24	-.11
Literature	-.61	-.62	.82	.77	.53	.17
Art	-.40	-.61	.39	.55	.28	.03
Music	-.38	-.41	.49	.48	.57	.15
Office work	-1.27	-.52	.00	-.70	.24	-.18
Business/Management	.46	.45	.06	.07	.09	.02
Sales	.34	.36	-.02	-.05	.08	-.05
Computation	.00	.30	.52	.21	.26	.13
Mechanical/Technical	1.65	1.15	-.15	.40	.07	.29
Skilled trades	1.22	.56	-.50	.08	-.01	.17
Farming	.72	.04	-.47	.20	-.18	-.05
Labor	.72	.20	-.59	-.12	.16	.08
Sports	.70	.23	-.15	.32	-.23	.01
Hunting/Fishing	1.16	.60	-.43	.14	-.30	-.05

Between sex differences were computed by males-females divided by the square root of a pooled variance estimate for the total sample, namely: $[\text{Var}(\text{males}) + \text{Var}(\text{females})/2]^{1/2}$. Within sexes observed differences were computed by: Gifted group-Unselected group divided by the standard deviation, for each gender. Within sexes regressed differences were based on the values found in Table 2. $M_x - M_{est}$, divided by the standard error of measurement: $SD_y[(1-r^2_{yx})^{1/2}]$ where "x" is the mathematics composite and "y" is the correlate.

test scores albeit at higher levels on the cognitive tests. The general intellectual superiority of the mathematically gifted students is evidenced on Talent's IQ Composite, constructed as much like the Stanford-Binet (Terman & Merrill, 1960) and the various Wechsler (1974) tests of general intelligence as could be achieved with the components available. It includes verbal, quantitative, and figural reasoning content, and shares a different component with each of the Mathematics, Spatial, and Verbal Composites.

Table 1 also contains important information concerning the ratio of boys to girls at exceptional levels of mathematical giftedness when giftedness is defined as beyond the same point on the combined distribution. In the unselected sample, boys not only have a higher mean on the Mathematics Composite but a larger standard deviation as well, a result comparable to the findings of several earlier studies (Benbow, 1988; Benbow & Minor, 1986; Benbow & Stanley, 1980; Jones, 1987; Moore & Smith, 1987; Stanley & Benbow, 1983, 1986; Stanley et al., 1986). Going beyond this, however, boys are also more variable on *all* nine cognitive tests, including three on which girls have higher means (IQ, English, & Verbal Composites). This suggests that there are probably different causes for sex differences in means and in variabilities. There are probably at least two determinants or sets of determinants of male superiority at the high end of the distribution of mathematical talent.

(2) How Narrow is Exceptional Mathematical Talent?

Correlations on Table 1 also reveal that mathematical talent is associated highly with general intellectual ability. The correlations with Advanced Mathematics are not as high as one might expect, but they are depressed by the inappropriate content for average 10th-grade students (a detailed analysis of this measure is given below). Measures of ability in mathematics are positively correlated with every cognitive measure, including those not shown (ranging from farming & home economics to literature & sports). There is little basis in these correlations for narrow mathematical talent.

The standardized (within sexes) cognitive test means of the gifted students on Table 3 also document generality in ability. General Information, which is a composite of a large number of short tests narrow in scope, such as information about hunting, fishing, art, theater, the military, engineering, health, the Bible, and so forth, indicates the breadth of superiority of the gifted groups and leaves only a little room for hypothetical uniqueness.

(3) Uniqueness of Mathematical Talent

Although the gifted are well above average on every cognitive measure, they are not as far above average as they are on the composite on which they were selected. That there is specific, nonerror variance in scores on mathematics tests is an expected finding. This is true for mathematics and all other specialized tests in the unselected population. The problem for the gifted groups is to determine

whether their overall level of performance is more or less general than in the unselected population. That is, is mathematical talent more or less unique than one would expect on the basis of the construct of general intelligence? Or, put another way, is there a discontinuity in the distribution of mathematical talent, at the upper tail, that goes beyond systematic sources of individual differences?

For the most part, mathematical talent appears intimately related to general intelligence as indexed by conventional measures, however mathematical giftedness also appears to be somewhat specific. This uniqueness is manifested in Table 2, by contrasting regression estimates for the cognitive composites (Column 2) with the observed means for the gifted groups (Column 1). (Standardized differences of these contrasts are found in Table 3). For both genders, across all comparisons but one (Technical Information for the females), regression estimates exceed the observed means. This nonlinearity of regressions at the high end of the distribution of the Mathematics Composite represents evidence for a small amount of increased uniqueness of mathematical talent.¹

(4) Where Are Mathematically Gifted Students Found?

Among the background self-report scores, the index of socioeconomic status is of special interest. Pooling information from Tables 1, 2, and 3, our highly gifted students are, on average, from families about one standard deviation above the national 10th-grade mean. There is also a great deal of variability about their mean. Because the SES index is approximately normal, about 16% of the gifted boys and girls come from families that are *below* the national mean. A search for talent must cast a wide net. And this fact is not emphasized nearly enough in contemporary investigations.

The socioeconomic differences tell us that mathematically gifted students are in families that are above average in SES ("privileged") only to the extent

¹Anastasi (1974) has commented that the foregoing analysis of estimated regressions (using selection measures employed to isolate gifted groups to, in turn, predict their scores on other distinct cognitive tests) is useful. Her discussion was aimed at indexing how much regression toward the mean to expect from mathematically gifted subjects, using math ability tests (on which they were selected) to estimate other cognitive measures (e.g., verbal ability). Anastasi's (1974) suggested methodology is useful for revealing *nonlinear trends* in the upper tail of selection measures and, in the present analysis, indicates some degree of uniqueness of mathematical giftedness. To test whether this finding was a product of the units of measurement, we conducted the same analysis on three different groups of gifted students (selected from the entire 10th-grade sample): The top 1% of each gender, selected by the IQ, English, and Space Composites. (Other data on these three groups of gifted subjects is reported subsequently in Table 4, p. 343.) For each gifted group (i.e., high IQ, high English, and high Space), the corresponding selection composite was used as a predictor, by gender, to estimate scores on the remaining cognitive tests, including the Mathematics Composite. The clear-cut trend was *under* prediction, for all three selection composites. The one exception to this general trend was spatial ability as a predictor of the remaining cognitive composites for spatially gifted females; for these subjects, spatial ability over predicted General Information, and the Verbal and English Composites. This finding suggests that some specificity is associated with exceptional levels of spatial ability in females.

expected from the general intellectual level of the students. Mathematical talent is not unique or special with respect to family status, but there is a sharing of antecedents. Neither does being gifted set these students apart from the unselected sample in the guidance they received, or failed to receive, in their high schools. We will return to the importance of SES for educational achievement below.

(5) Guidance and Interests

These gifted students are also above average in grades, study habits, and academic courses taken, that is, classes aimed at college preparation (foreign languages, math, and science) in contrast to subjects like bookkeeping, commercial arithmetic, and shop. None of this is surprising. It is surprising or even shocking that they are about average with respect to guidance received in high school. Expressed interests in physical science are high and in music well above average. The former seems a natural accompaniment of high levels of mathematical ability, and music and mathematics are traditionally associated with each other (with the first scientific studies conducted at Wundt's laboratory in Leipzig, cf. Shuter, 1965; Shuter-Dyson, 1981). Regardless of gender, these interest domains appear to attract mathematically gifted students and represent profitable avenues for academic/career exploration. There are, however, gender differences in mathematically gifted students, similar to those observed in the unselected groups, across both ability *and* interest levels, that warrant careful attention.

(6) Gender Differences in Interests and Abilities for the Mathematically Gifted

Table 3 reveals that several gender-differentiating ability/preference constellations remain pronounced at high levels of mathematical giftedness. The critical comparison for these configurations is the standardized between sex differences, for unselected and gifted groups. Beginning with the females, superior abilities on the English Composite are revealed, and in the preference domain, females are more interested in social service, literary, artistic, and musical interests than males. A quite different ability/preference configuration characterizes gifted males: Gender differentiating abilities favoring males are centered around Advanced Math, and the Mathematics, Space, Technical Information, and Scientific Composites, while preference for Physical Science remains intense. Some noteworthy gender differences are also observed in Business Management, Sales, and Computation. (Gender differences in other nonacademic stereotypic interests are discussed below.)

This analysis may actually underestimate gender differences in academic/vocational preferences, given that, for the gifted males, physical science is encumbered by a ceiling effect and, for the gifted females, literary, artistic, and musical interests are somewhat encumbered. Except for the Verbal Composite

(and possibly the Technical Composite for the males), the ability measures exhibit adequate ceilings.

(7) The "Masculine-Identification Hypothesis"

Observe that, on the most stereotyped interest measures (e.g., Mechanical/Technical, Skilled Trades, Farming, Sports, Hunting/Fishing), gifted girls have interests tending toward the typical male, while the gifted boys have interests tending toward the typical female (see the between sex and observed within sex differences in Table 3). It has long been known that increasing amounts of education accompany a decrease in size of sex differences on many self-report tests (Campbell, 1971; Strong, 1955), particularly for the better students (Faunce & Loper, 1972; Terman & Miles, 1936), but here the correlate is level of ability. Level of general intelligence or of socioeconomic status may be the primary variate rather than mathematical ability per se.

These findings appear to bear on the masculine-identification hypothesis (Fox, Tobin, & Brody, 1979; Nash, 1979; Parsons, 1983), a view derived from writings across multiple psychological frameworks (Carlsmith, 1964; Kohlberg, 1966; Maccoby, 1966; Mills, 1981; and Plank & Plank, 1954), suggesting a mechanism responsible for gender differences in cognitive functioning. Presumably, through early learning experiences, children developed an internalized sex role standard; typically, males develop a masculine identity while females cultivate a feminine identification, although individuals can acquire a "crossed" sex role identity (i.e., one typical of the opposite sex). Once developed, an individual's sex role functions in a dissonance reduction mode to maintain consistency between behavior and one's (masculine/feminine) standard. This is accomplished by only engaging and excelling in behaviors perceived as consistent with one's internalized sex role and shunning activities considered discordant to, say, masculinity.

According to this view, to the extent that one's sense of self is masculine, early learning experiences gravitate toward interests in the outdoors, sports, math, and science, and a high degree of proficiency in these areas ensues from concentrated exposure. Individuals strongly identified with the masculine role, however, tend to avoid social arenas involving interpersonal warmth and interests in art, literature, and the humanities; and therefore skills characterizing these areas lag behind and appear developmentally inchoate, if manifested at all. Since boys and girls are considered comparable in terms of overall intellectual ability, sex differences on general global measures of intelligence should not appear, but gender differences in specific components (e.g., verbal vs. spatial ability) would be expected.

Given the above considerations (coupled with our findings on masculine interest patterns in mathematically gifted students), we conducted the following analysis. Three additional groups of gifted students were selected from the entire 10th-grade tape, the top 1% of each gender based on separate frequency distribu-

tions of the Spatial and English Composites, and IQ. We then inspected their interest patterns on scales traditionally considered nonacademic but "stereotypic" in character. Each of the three gifted groups was then compared to its gender equivalent normative cohort (which, of course, are the same reference values found in Table 1). Results of this analysis are given in Table 4. Regardless of the dimension used for selecting intellectually gifted students, high ability students display *less* stereotypic interests: They diverge from the norm of their gender toward the direction of the opposite sex. It is especially noteworthy that this generalization holds for the most *stereotypically feminine/masculine abilities*, English and spatial visualization, respectively. Table 4 also includes a stereotypic "feminine" measure, office work. The findings reveal that gifted females, even in the 10th grade and in 1960, reject traditional career paths and probably find certain components of these occupations aversive.

Researchers investigating stereotypic nonintellectual correlates of mathematical giftedness, or intellectual giftedness more broadly defined (e.g., Signorella & Jamison, 1986), should take the following into consideration (from our analysis of other samples of gifted students selected with nonmathematical parameters of cognitive functioning): Intellectually precocious males *and* females are less stereotyped regardless of whether they are verbally gifted or intellectually gifted in spatial visualization. Less stereotypic behavior appears to be "simply" a corollary of intellectual giftedness in general.

(8) Spatial Ability Assessed as a Moderator Variable

The standard scores for Advanced Mathematics seem to show that performance in mathematics at *sophisticated levels* is a more specific cognitive attribute, but

TABLE 4
Standard Scores for Four Gifted Groups (Selected on Math, Space, English, & IQ Composites) by Gender on Stereotyped Interests

Stereotyped Interests	Gifted Groups							
	Math		Space		English		IQ	
	M	F	M	F	M	F	M	F
Office work	.00	-.70	-.10	-.45	-.03	-.62	-.16	-.76
Mechanical/Technical	-.15	.40	.21	.54	-.38	.15	-.19	.43
Skilled trades	-.50	.08	-.18	.30	-.58	.00	-.48	.12
Farming	-.47	.20	-.19	.41	-.48	.12	-.37	.36
Sports	-.15	.32	-.18	.36	-.17	.18	-.14	.22
Hunting/Fishing	-.43	.14	-.10	.35	-.42	.09	-.29	.25
Sample N	497	508	512	476	535	514	539	499

Note. Means were computed: Gifted Sample–Unselect Sample. Cutting scores for the gifted groups follow: Mathematics Composite (boys = 36.9, girls = 34.35), Spacial Composite (boys = 118.5, girls = 110), English Composite (boys = 123, girls = 126), and IQ Composite (boys = 259, girls = 257). The number of gifted students above the cutting score on *only one* of the selection composites follows: Mathematics (males = 192, females = 210), Space (males = 327, females = 310), English (males = 267, females = 296), IQ (males = 204, females = 203).

this evidence of specificity is largely due to the inadequacy of the test for typical 10th-grade students. The raw score variances of the gifted groups were seen in Table 2 to be substantially larger than the variances of the unselected subjects (Table 1). This is not a possible outcome for an adequate measurement scale. The attenuated correlations between Advanced Mathematics and the selection composite constitute evidence leading to the same conclusion; this contributes to the drastic underestimation of Advanced Mathematics by the Mathematics Composite (see Table 2).

Nevertheless, to test for the posited synergistic relationship between spatial visualization (S) and general mathematical ability (M) with respect to skill in advanced mathematics (C), we conducted a hierarchical multiple regression analysis with interaction ($M \times S$) and squared (M^2 and S^2) terms. We were interested in ascertaining whether spatial ability (as indexed by our Spatial Composite) functions as a moderator variable in interaction with general mathematical ability (the Mathematics Composite) to enhance the prediction of Advanced Mathematics. If so, perhaps this could explain our earlier findings indicating some degree of uniqueness associated with mathematical giftedness.

We were also interested in determining if the squared components of the main effects (M and S) carried incremental validity beyond their linear relationship with C . Our analysis involved the following equation:

$$C = B_1(M) + B_2(S) + \begin{array}{l} \text{Step 1} \\ B_3(M^2) + B_4(S^2) + B_5(M \times S) \\ \text{Step 2} \end{array}$$

where Step 1 consisted of entering the M and S main effects in an incremental stepwise fashion; and Step 2 followed with the same incremental approach after the variance associated with the linear effects of M and S was removed. This methodology was applied to both genders separately and produced the following results.

For females, conjoint M and S main effects generated R^2 of .19. However, from Table 2, recall that the r^2 between M and Advanced Mathematics was .18. With rounding error considerations taken into account, the precise contribution or incremental validity added to this value by S is $R^2 = .005$. While this finding is statistically significant because of our sample size, the contribution of S to the prediction of Advanced Mathematics is inconsequential substantively, and may be due largely, if not entirely, to measurement error in M . The first variable entered in Step 2 was M^2 , which accounted for an r^2 increment of .02. None of the remaining terms in Step 2 increased R^2 significantly. M carried a negative beta weight as a function of its overlap with M^2 , while positive weights were assigned to S and M^2 (interpretations of these signs are provided below). The application of our two-step regression equation to the male sample generated the same pattern of results: Joint M and S main effects produced an $R^2 = .27$ (with S adding only an increment to $R^2 = .006$); and M^2 accounted for an additional 3%

of criterion variance. None of the remaining terms increased R^2 significantly. Similarly, the signs for all three variables corresponded to those for the females.

The finding of a positive B-weight for M^2 when added to the regression equation can be interpreted as follows: Mathematical ability is a predictor of Advanced Mathematics but the strength of this relationship increases with higher levels of talent in mathematical ability. Thus for a large segment of the lower ability students on the Mathematics Composite, the relationship between individual differences in general quantitative ability and advanced mathematics is negligible, because few subjects below, say, 50% on this normative sample possess the necessary antecedent skills to acquire even rudimentary concepts for advanced mathematics. Subjects with quantitative abilities in the upper range of their cohort, on the other hand, not only possess the requisites for advanced math in various degrees, but they are much more likely to develop such skills independent of formal instruction. This may explain the nonlinear regression for the advanced test.

To highlight the nature of this higher-order relationship, we computed correlations between Advanced Mathematics and the Mathematics Composite for high and low ability male/female samples, defined as those scoring above or below the mean of their respective gender on the Mathematics Composite: For the lower range of talent, correlations were small ($r = .12$ for males and $r = .10$ for females); however, the upper range of talent generated male/female correlations of $.52$ and $.45$, respectively.

Finally, the hierarchical regression analysis, as it relates to spatial visualization, brings into question the hypothesis that this facet of cognitive functioning is of special importance for advanced levels of mathematical skill either synergistically in interaction with general quantitative ability or as an independent main effect.² Our findings can, however, accommodate the hypothesis that mathematical reasoning and spatial visualization have common antecedents (cf. Benbow & Benbow, 1984).

LONGITUDINAL FOLLOW UP

Our longitudinal data represent 38% of the females ($N = 191$) and 43% of the males ($N = 216$) in our original sample. The longitudinal data from Project Talent are biased because follow-up responders tended to be above the norm on IQ and SES. Our gifted sample, due to their high standing on both variables, represent a higher proportion of respondents than the unselected, but there is some degree of overestimation of proportions obtaining credentials above the high school diploma. Table 5 (p. 346) presents proportions of the male and

²These findings have been replicated with cohorts 9, 11, and 12 of Project Talent (Lubinski & Humphreys, 1990). This article also contains methodological refinements of certain data-analytic techniques for assessing moderator variables and other trait-interaction concepts.

TABLE 5
Standard Scores for the Mathematically Gifted Across Four Levels
of Educational Achievement

	Ph.D.‡		M.A.		B.A.		H.S.	
	M	F	M	F	M	F	M	F
Mathematics composite	2.67	2.81	2.65	2.75	2.65	2.73	2.61	2.73
Space composite	1.37	1.95	1.44	1.56	1.49	1.55	1.32	1.46
IQ composite	1.82	1.94	1.82	1.83	1.78	1.81	1.86	1.62
English composite	1.89	1.42	1.93	1.83	1.81	1.76	1.85	1.55
SES	1.32	1.63	1.03	1.38	.98	1.01	1.11	.76
Sample N	64	10	59	40	60	87	17	37

‡Includes professional degrees in medicine and law.

female gifted samples attaining each of four levels of educational credentials. Also included are standard scores on SES and selected cognitive composites for each attainment group, by gender. It is particularly disheartening to see that only 5.0% of the females went on to achieve doctoral degrees in contrast to 30% of the males. In keeping with this trend, only 8.0% of the males ended their formal education with high school, whereas 19% of the females stopped at this level. Sample percentages for educational credentials no higher than a high school diploma represent underestimates of population values. Although males are a more select group than the females with respect to mathematical talent, inspection of the females' status on cognitive measures suggests that the primary factors operating to generate gender discrepancies are not intellectual in nature. Clearly, the present sample of males and females possessed the intellectual capacity to secure advanced degrees in the most difficult disciplines. This suggests that, concomitant with the inception of SMPY, an inordinate number of mathematically talented individuals never approached their full intellectual capabilities. This is especially true for high ability females, although such gender discrepancies between talent and achievement are less pronounced today.³

One factor that appears related to educational achievement is socioeconomic

³It is informative to scrutinize the specific degrees earned by these students: For students stopping at the B.A., the most frequently earned degrees by males were in quantitative areas (44%), social science (17%), natural science (12%), humanities (10%), and accounting-business-commerce (10%). In contrast, earned degrees by the females were in the humanities (34%), quantitative areas (17%), social science (14%), education (10%), natural science (10%), and nursing (6%). The most frequently obtained masters degrees for males were quantitative areas (38%), humanities (19%), accounting-business-commerce (17%), natural science (3%), and social science (3%); females at this level more often graduated in education (28%), humanities (23%), quantitative areas (20%), social science (10%) and natural science (8%). At the doctoral level, almost half of the males took degrees in either law (21%) or medicine (21%), others were in quantitative areas (24%), natural science (10%), social science (9%) humanities (7%); three of the ten females earned M.D.s, and another three-subject cluster was formed by doctorates in natural science.

status. Table 5 reveals that gifted students are more likely to achieve their academic potential *if* they are raised in more affluent homes. This suggests that interventions (e.g., educational/vocational guidance) need to be directed toward gifted children from lower socioeconomic levels. The data also indicate that such attention is especially relevant for young girls. The type of accelerated intervention offered by SMPY may be what is needed for maximizing the achievement opportunities for gifted youth (Stanley & Benbow, 1983, 1986). However, the SMPY investigators focus their energies on an extremely select sample; such accelerated programs could be profitably extended to less gifted, although normatively superior, youth.

For example, our 13-year follow-up also included data for the initial sample of 95,650 students. We selected from this sample those students who subsequently earned doctorates or professional degrees in medicine or law. We then computed from their 10th-grade test performance their scores on the English language, Mathematical, and Spatial Composites, as well as IQ and SES. Table 6 shows that the average score obtained by this highly select group of individuals was appreciably *below* the mean of our gifted sample, regardless of gender, across all four of these highly important dimensions of cognitive functioning. Further, to select a sample of gifted students comparable in talent to those typically studied by SMPY, our cutting score on the Mathematical Composite would have to extend an *additional* .30 standard deviations on the male distribution and .69 on the female distribution (i.e., approximately 1.5 standard deviations of mathematical ability above that normally displayed by individuals earning Ph.Ds)! This gives one a feel for the remarkable range of individual differences between what is considered mathematically "gifted" in SMPY and what is typically needed to achieve a Ph.D. or professional equivalent. These data further illustrate the broad scope of intellectual functioning characterizing the present sam-

TABLE 6
Standardized Differences Between Mathematically
Gifted Sample and Doctoral Recipients From the
Entire Project Talent Sample on Four Cognitive
Composites and SES

	Males	Females
Mathematical composite	1.10	.82
IQ composite	.49	.28
Space composite	.56	.23
English composite	.55	.31
SES	-0.05	-0.31

Doctoral group includes professional degrees in medicine and law (male $n = 515$; female $n = 45$). The difference scores were computed: gifted group—doctoral group.

ple. Their giftedness is not restricted to quantitative areas; they possess the capacity to excel intellectually in multiple academic and applied domains.

Notice also the higher socioeconomic status of the doctoral group in contrast to the gifted students. The reversal in the direction of the difference from measured abilities to the status of the family indicates the importance of privilege in the prediction of higher education credentials. It required a very high level of talent in the gifted groups to overcome partially the general trend. Because mathematical giftedness covers a wide range of SES, studies aimed at selecting mathematically precocious youth should attend to this fact. Like Terman (1954; Terman & Oden, 1959), we found that one of the major determinants of whether gifted subjects develop their intellectual potential is family background; gifted students are more likely to achieve advanced educational credentials to the extent that their family is socioeconomically privileged, especially if the gifted student is female.

DISCUSSION

Causal attribution based on data not controlled experimentally is typically equivocal, but these data seem to narrow the range of possibilities.

Uniqueness of Mathematical Giftedness

If there is a specific basis for high levels of mathematical talent in each sex considered separately, as our regression estimates indicate, the contribution is small relative to the person's level of general intelligence. It appears, however, that the mechanism responsible for this uniqueness is *not* a synergistic relationship between mathematical ability and spatial visualization; this conclusion is based on the negative results, for the linear-by-linear product term ($M \times S$), obtained in our hierarchical regression analysis (cf. Lubinski & Humphreys, 1990). Perhaps taxometric methodologies, like these currently being developed for isolating psychopathological taxa (Meehl & Golden, 1982), could be profitably extended to analyzing mathematical giftedness. Such methods may help in the determination of whether mathematical giftedness is a taxonic entity or, a "real type."

Spearman's (1904) original formulation of ("g") general intelligence (viz., that all systematic sources of individual differences in cognitive functioning emanate from a common source) is in relatively good accord with these findings. In Sir Frances Galton's (1869) pioneering work on *Hereditary Genius*, he argued that intellectual distinction (whether achieved in statesmanship, generalship, literature, science, poetry, or art) does not stem from unique or, "purely special powers." Rather, he stressed that high achievements in specialized areas are best understood as resulting from concentrated efforts made by individuals who are *widely gifted*. Hence the present data correspond to more original observations, as well as current findings indicating high achievement, across a variety of

intellectually demanding occupational domains, typically stems from a superior level of general intelligence (cf., *Journal of Vocational Behavior*, 1986, 29, whole volume), as opposed to more specific abilities that uniquely correspond to an individual's area of excellence.

In Terman's (1954) classic longitudinal studies of intellectual giftedness, his subjects were selected solely on the bases of one psychological dimension, *general intelligence*; yet, different subgroups of these individuals excelled in science, literature, the military, art, and business and commerce. This lends credence to the idea that exceptional levels of general intelligence provides the necessary condition for exceptional achievement, but other factors (e.g., energy, interests, needs, values, and family background) function to channel the particulars of vocational development (Dawis & Lofquist, 1984).

Genetic Factors

Regarding the antecedents to general intelligence, there is a great deal of debate about the precise contribution of genetics, but there is wide, though not universal, agreement that there is a substantial contribution (Bouchard, 1983, 1984; Bouchard & McGue, 1981). In fact, Meehl (1971, 1972, 1986) has argued that genetic factors are psychologically significant in other behavioral domains as well: "Parental intelligence, personality, and temperament factors are transmitted to the child in part genetically (no informed and unbiased person today could dispute this, but many social scientists are both uninformed and prejudiced against behavior genetics) and partly through social learning" (Meehl, 1971, p. 81). This suggests that, like general intelligence, the *nonintellectual* personal attributes that function to structure the specifics of vocational development, may stem from biological predispositions to behavioral tendencies as well. Eysenck (1988) has recently stressed the importance of conducting genetic analyses on subjects such as those studied by SMPY, and Benbow (1988) concurred with his assessment. In addition to the intellectual antecedents to mathematical giftedness, we suggest that such analyses should be extended to relevant nonintellectual attributes. That such an extension might be profitable is provided by recent findings in other contexts (cf. Arvy, Bouchard, Segal, & Abraham, 1989; Pedersen, Plomin, McClearn, & Friberg, 1988; Tellegen, Lykken, Bouchard, Wilcox, Segal, & Rich, 1988; see also, Nichols, 1978).

Environmental Factors

The environmental model of specialized achievement is also congruent with the sex difference in means on Mathematics Composite. Highly intelligent children could be shaped by environmental forces operating over the entire period of development toward high levels of specialized achievement. In the present context, this means that bright children presumably had early exposure to quantitative experiences, were rewarded by those experiences, and continued to seek such experiences. Data on the performance of the sexes on the Project Talent

Intelligence Composite are relevant. The 10th-grade girls were less than .02 of a combined standard deviation *higher* than the boys on the Intelligence Composite, but there were presumably fewer environmental forces pushing them in the direction of achievement in mathematics.

Academic/Vocational Development

That more males than females choose to enter highly quantitative academic/vocational domains is, to be sure, causally related to multiple factors (Benbow, 1988). Some of these factors can be analyzed in the context of well-established formulations of vocational adjustment. For example, according to Dawis and Lofquist (1984), academic/vocational adjustment is a function of two broad dimensions of correspondence, *satisfaction* and *satisfactoriness*. The former is a motivational parameter indexed by the extent to which *preferences* (e.g., occupational needs and interests) correspond to the reinforcers offered by a particular academic or vocational arena; whereas the latter is indexed by the extent to which *abilities* correspond to the ability requirements of a given academic/occupational path. The model is important because it stresses the need to look at constellations of distinct classes of personal attributes, namely, abilities *and* preferences for purposes of predicting academic/vocational adjustment (cf., Lubinski & Thompson, 1986).

Satisfactoriness (or ability/ability-requirement correspondence) determines how receptive various educators or employers will be toward a given individual; whereas satisfaction (needs/reinforcers offered correspondence) determines the motivation of a given individual to approach and maintain contact with a given academic/vocational domain.

The data on gender-differentiating ability/preference constellations suggest that one possible source for the profound gender difference in quantitative careers is more intense "competing" interests in other areas. Chipman (1988) recently referred to this possibility in the same context, noting that females *tend to be* more interested in people rather than "things." Across the unselected and gifted samples, we found similar gender differences in abilities: Females tend to excel in English language, whereas males excel in scientific/technical areas, mathematics, and spatial visualization. Further, gifted subjects are also similar to the norm with respect to gender differences in interest pattern: physical science, public service, and business (favoring males) and artistic, literature, music, and social service (favoring females). The gifted girls in our sample *clearly* possess the intellectual requisites to excel in the most quantitatively sophisticated areas, but they appear to have more intense interests in other areas. Some of the work by the SMPY group lends generality to this conclusion.

Using the Allport, Vernon, and Lindzey (1970) inventory of values, Fox and Denham (1974) found that values of mathematically gifted females tend to be distributed across theoretical, aesthetic, and social domains, whereas mathematically gifted males are much more focused on theoretical pursuits (the value

most highly associated with mathematical and scientific activities). Similarly, using Holland's themes of vocational interests, Fox, Pasternak, and Peiser (1976) found that females appear less focused on the *investigative theme* than males (especially on the science and mathematics components of this multifaceted index); this dimension of vocational interests stresses mathematical/medical/scientific course work and careers). The same investigators also found gifted females scoring much higher on the social and artistic themes than their male counterparts. Gender differences in "people versus things" has a long history in psychology (Thorndike, 1911); females tend to be more interested in the former, males the latter (Benbow & Stanley, 1983a). Subsequent work on mathematical giftedness should take this possibility into consideration, inasmuch as it suggests a causal factor for the development of gender differences in abilities (and, if so, achievement) in mathematical areas.

Gender Differences in Variability

None of the above findings can be related directly to the problem of greater male variability. Like gender differences in "people versus things," that males are more variable than females on psychological measures of cognitive functioning has a long history in individual differences research (cf. Anastasi, 1958; Lehrke, 1978; McNemar & Terman, 1936; Rhinehart, 1947; Scottish Council for Research in Education, 1933; Tyler, 1965). But gender differences in ability dispersion have been conspicuously absent from many contemporary reviews (Deaux, 1985; Feingold, 1988; Hyde, Fennema, & Lamon, 1990; Hyde & Linn, 1988; Linn & Hyde, 1989; Meece, Parsons, Kaczala, Goff, & Futterman, 1982) and conceptual schemes (Kimball, 1989). In our study, for example, the 10th-grade boys were somewhat more variable on Intelligence Composite (.035 of a standard deviation) than the girls in spite of a slightly smaller mean. The result is again a ratio of more than 1:1 of boys to girls in the extreme tails of the distribution. Jensen (1988) has recently reported similar findings. We have no hypothesis to offer, but we do recommend that sex differences in variance should receive attention equal to that accorded means (cf. Becker & Hedges, 1988; Humphreys, 1988).

A recent meta-analytic review by Hyde and Linn (1988) is relevant to this discussion. The authors report a decrease over the years in gender differences on several measures of verbal ability (the average effect size of studies published prior to 1973 was $d = .23$, while studies published subsequently show an attenuated difference, $d = .10$). The authors conclude that little evidence exists today for gender differences in global measures of verbal ability (Hyde & Linn, 1988). But the review contains evidence suggesting that males display more dispersion on verbal measures. If the foregoing findings on ability-dispersion are both valid and stable, *and* the advice of the authors of the review is implemented, namely, that college entrance examiners can use these measures with alacrity, a disproportionate number of males to females will be accepted by more

prestigious institutions of higher learning. The social implications of consistent gender differences in dispersion can be profound; and the causes for greater male variability are not near final adjudication. We stress that future reviews of gender differences in cognitive abilities (and group differences in general) report statistical findings on *variability* as well as overall *effect size*.

Temporal Changes in Ability Statistics

Early in our discussion we mentioned the possibility that means in Project Talent tests might have changed since the tests were administered in 1960. If means change, they can also change differently for boys and girls, thus change the size of sex differences. This possibility has recently received empirical support (Feingold, 1988) for Mechanical Reasoning and Space Relations, as indexed by the Differential Aptitude Tests, across four points in time starting in 1947 and ending in 1980. This has also been corroborated for the Mathematics score of the Preliminary Scholastic Aptitude Test of the College Board for four points in time starting in 1960 and ending in 1983. (Data for SAT-M are ambiguous because the gender mix of the applicant population changed markedly between 1967 and 1983.) Gender differences *decreased monotonically* over the years studied in each of the unambiguous comparisons. Today, boys and girls probably display less stereotyped behavior on many dimensions than students did in the 1960s, but it is our confident expectation that the *relationships* of mathematically talented boys and girls to each other and to their unselected counterparts have not structurally changed. We therefore suggest that gifted students today are probably less stereotyped than gifted students of the 1960s.

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