Key for Quiz 14 RA

QUESTION 1

Consider the relation R(A, B, C, D) with functional dependencies (FDs)

A —> B, C

A, C —> D

C, D —> A, B

Which of the following sets of FDs is a minimal set that is informationally equivalent to the set of FDs given above.

The key characteristics are "minimal set" and "informationally equivalent"(or just equivalent), where two sets of FDs are equivalent if the closures of their FDs are the same. For example, {A→B, B→ C ,C→A} has A→C, B→A, and C→B in its FD closure { A→B, B→ C ,C→A, A→C, B→A, C→B, ... AC→B, ... ABC→A}. {A→B, B→ C ,C→A} is also a minimal set, since no proper subset of these FDs has the same FD closure.

Clearly choice 3 (below) is equivalent (it's the same as the given FD set), but its not minimal.

What is a minimal equivalent set for the given FDs? Start with the set I gave. In this illustration I use the ordering in which I gave the FDs.

A —> B, C (or A→ B and A → C)

A, C —> D

C, D —> A, B (or C,D → A and C,D → B)

Step 1: Are the left hand sides of any FD redundant?

Yes, A, C → D can be simplified to A → D, since A→C (if A, C determines D, but A determines C, then A all by itself determines D).

So, **A,C → can be replaced by A → D**, and this can be combined with the first FD in the list, yielding A → B, C, D

The left-hand side of C,D → A,B **cannot** be simplified.

The energy monitoring FDs example in class (and online), where "Temp" was eliminated, would have helped you here

Can any FD be eliminated?

A —> B, C, D

C, D —> A, B

Go through them in given order – it can be in any order, but different orderings of the FDs can lead to different minimal sets. Choice 2 below, which was the only correct choice, results from considering the FDs in **RIGHT-TO-LEFT** order:

A→B; A→C; A→D; C,D → A; C,D→ B, so that C,D → B is the first to be considered.

Pretend C,D → B doesn't exist and take the attribute closure of {C, D}.

C,D → A allows us to add A to the closure: {A, C, D}.
A→ B allows us to add B to the closure: {A,B,C,D}.
So, the attribute closure of {C,D} includes B, even without explicitly giving C,D → B. **Eliminate C,D → B as redundant.**

So, a minimal set of the FDs that is informationally equivalent to those given at the start is

A —> B, C, D

C, D —> A

or choice 2 below.

A different minimal set results if we consider the FDs in **LEFT-TO-RIGHT** order as given above, so that A→B is the first FD to be considered. Verify that a different minimal set results in this case (but it was not an option among those given).

An aside*: relation R in this problem has two minimal keys: {A} and {C,D}. And note the heuristic of starting with attributes that were not on the right hand side of any FD wouldn't have bought you anything in this case (all attributes are on the right hand side of at least one FD)*

1.
A —> B, C, D

C, D —> B

This choice follows if C,D→A is redundant in the list above, but if you take the attribute closure of {C,D} without the C,D→A, you will NOT find A in the closure. Not informationally equivalent

2.
A —> B, C, D

C, D —> A

3.
A —> B, C

A, C —> D

C, D —> A, B
not minimal, as noted above

4.
A —> B, C

A, C —> D

C, D —> A
not minimal; the energy monitoring FD example in class (and online), where "Temp" was eliminated, would have helped you here

5.
A —> B, C

A, C —> D

C, D —> B
not minimal and not equivalent

QUESTION 2

Consider the relation R(A, B, C, D) with functional dependencies (FDs)

B, C —> A

A, D —> C

Select all true statements

After glancing at the choices, you would (ideally) see that you need to know the keys of R to answer some of these.

B and D must be part of any key (they aren't on the right hand side of any FD), but the attribute closure of {B,D} is {B,D}. But if you add either A or C to {B,D}, you have **minimal keys: {A,B,D} and {B,C,D}**

Also you should verify that the given FDs are a minimal set of FDs. It is harder to verify that this is the only minimal set (but it is). *Aside: in this class, I may ask you to identify when there is more than one minimal set (by construction), but I won't ask you to prove that only one exists.*

1.
R(A, B, C, D) is in BCNF
no, both FDs that were given have left hand sides that are NOT keys of R; both violate the BCNF condition

2.
The decomposition of R(A, B, C, D) into relations R1(A, B, C) and R2(B, C, D) is dependency preserving
no, A,D → C is not assignable to either R1 or R2

3.
Each of R1(A, B, C) and R2(B, C, D) is in BCNF, where R1 and R2 are a decomposition of R
yes, B,C→ is assignable to R1 and B,C is a key of R1; no FD is assignable to R2. This no FD violates BCNF condition for either relation

4.
Relation R has exactly one minimal key
no, R has two minimal keys

5.

The decomposition of R into R3(A, B, C) and R4(A, C, D) is lossless

no, A,C is a basis for natural join, but not without losing information (A,C determines nothing else, and under-constrains join). A lossless decomposition of a relation R, with tuples T, is a set of smaller relations for which a natural join of those smaller relations gives back R (both the relational schema and its tuples Consider R =

A B C D with two tuples (and consistent with the given FDs)
1 3 2 4
1 5 2 6

Represent these as R1 and R2

A B C and A C D
1 3 2 1 2 4
1 5 2 1 2 6

Do a natural join (on A and C), and get back

A B C D
1 3 2 4
1 3 2 6
1 5 2 4
1 5 2 6

6.

The decomposition of R into R5(A, B, C), R6(B, C, D), and R7(A, C, D) is dependency preserving

yes, R5 and R6 result from the same strategy as option 2 above, where B,C→A is assignable to R5, but A,D → C is not assignable to either R5 or R6. A common "trick" to obtain a dependency preserving decomposition is to simply add a relation to the decomposition that includes all the attributes of an unassignable FD – R7(A,C,D) in this case to cover A,D→C

7.

Each of R5(A, B, C), R6(B, C, D), and R7(A, C, D) is in BCNF, where R5, R6, and R7 are a decomposition of R

yes, you can verify