Constructive Functions 2025

in conjunction with the

37th Shanks Lecture

May 19-22, 2025, Vanderbilt University

ABSTRACTS OF TALKS

On a Multiple Subset Sum Problem with Connections to Best Packing

Austin Anderson*, Steven Damelin
Florida Polytechnic University
aanderson@floridapoly.edu
https://floridapoly.edu/directory/faculty/austin-anderson.php

We discuss an infinite max-min multiple subset sum problem (MSSP) with connections to the asymptotics of best packing on linear sets of Lebesgue measure zero. It is known that if $\Gamma = I \setminus \cup_{j=1}^{\infty} I_j$ is a monotone cut-out set with gap sequence satisfying the power law $l_j = m_1(I_j) \sim Lj^{-1/d}$, as $j \to \infty$ for some $d \in (0,1)$, and $0 < L < \infty$, then the packing function $N(\Gamma, \varepsilon)$ satisfies $N(\Gamma, \varepsilon)\varepsilon^d \sim L^d A_d$, as $\epsilon \to 0^+$ for a positive constant A_d.

Motivated by this fact, we discuss the problem of estimating the largest number of subsets $\gamma_1, \gamma_2, \ldots \gamma_N$ in a partition of the naturals such that $\sum_{j\in\gamma_i} l_j \geq \varepsilon$ for each $i = 1, 2, \ldots, N$, where $\mathcal{L} = (l_j)_{j=1}^{\infty}$ is any non-increasing positive sequence satisfying the power law stated above. We call this quantity the *mass distribution function* for the sequence $\mathcal{L}$ and denote it by $N(\mathcal{L}, \varepsilon)$. In previous work, we conjectured that $N(\mathcal{L}, \varepsilon)\varepsilon^d \sim C_d L^d$, as $\varepsilon \to 0^+$ for a positive constant C_d depending only on d. In this talk, we lay the background for this question, as well as ongoing attempts to resolve it.

Constructing exponential bases on polygons by approximation

Oleg Asipchuk*, Laura De Carli
Florida International University
asip001@fiu.edu

The problem of proving (or disproving) the existence of exponential bases on measurable sets of the plane is still largely unsolved. An exponential basis is a set of exponential functions in the form of $\{e^{2\pi i\langle\lambda,\vec{x}\rangle}\}_{\lambda\in\Lambda}$, where Λ is a discrete set of $\mathbb{R}^2$. The main examples of domains where exponential bases exist are shapes that tile the plane by translations, polygons with central symmetry, and unions of rectangles with sides parallel to the axes. We still know nothing about the existence of exponential bases on triangles, irregular polygons, and disks.

In my talk, I will introduce an approach for constructing exponential bases on specific types of polygons using approximation by dyadic squares. I will also discuss examples and explore generalizations of the results.

A Runge Type Theorem for meromorphic k-differentials

Nadya Askaripour
University of Toronto Mississuaga
Nadya.askaripour@utoronto.ca

The original Runge's theorem concerns the approximation of holomorphic functions on open subsets of a Riemann surface by entire rational functions. In this presentation, we will discuss a Runge-type theorem for k-differentials (automorphic forms of weight k) on an open subset of a compact Riemann surface. We will also explore some of the key tools used in proving the main theorem, such as Poincaré series.

Multiscale scattered data interpolation in samplet coordinates

Sara Avesani*, Rüdiger Kempf, Michael Multerer, Holger Wendland
Università della Svizzera Italiana
sara.avesani@usi.ch

We study multiscale scattered data interpolation schemes for globally supported radial basis functions with focus on the Matérn class. The multiscale approximation is constructed through a sequence of residual corrections, where radial basis functions with different lengthscale parameters are combined to capture varying levels of detail. We prove that the condition numbers of the diagonal blocks of the corresponding multiscale system remain bounded independently of the particular level, allowing us to use an iterative solver with a bounded number of iterations for the numerical solution. To apply the multiscale approach to large data sets, we suggest to represent each level of the multiscale system in samplet coordinates. Samplets are localized, discrete signed measures exhibiting vanishing moments and allow for the sparse approximation of generalized Vandermonde matrices issuing from a vast class of radial basis functions. Given a quasi-uniform set of N data sites, and local approximation spaces with exponentially decreasing dimension, the overall cost of the proposed approach is $\mathcal{O}(N \log^2 N)$.

On inverse source problems in divergence form

Laurent Baratchart*
Centre Inria de l'Université de Nice
Laurent.Baratchart@inria.fr

We shall discuss the issue of recovering a vector source μ from knowledge of the solution u to a strictly elliptic equation of the form $\operatorname{div}(\Sigma\nabla u) = \operatorname{div}\mu$. Such inverse problems arise in inverse magnetization problems, geosciences, medical imaging and material science, and they correspond mathematically to finding the Helmholtz decomposition of the vector field μ. From early works on inverse magnetization problems involving spherical harmonic expansions and Riesz transforms, to more recent developments dealing with data extension, moment estimation, consistency and sparse recovery, we shall discuss some known results and pending issues.

The talk will elaborate on results by many people including E. Andrade Lima, S. Chevillard, C. Gerhards, D. Hardin, J. Leblond, J.P. Marmorat, M. Nemaire, M. Northington, D. Ponomarev, Ed Saff, C. Villalobos-Guillén and B. Weiss.

THE q^{Volume} Lozenge Tiling Model via Non-Hermitian Orthogonal Polynomials

Ahmad Barhoumi*, Maurice Duits
KTH Royal Institute of Technology
ahmadba@kth.se
www.AhmadBarhoumi.com

Lozenge tilings of a regular hexagon are in bijection with boxed plane partitions and can therefore be assigned a volume; a fact that is best illustrated by staring at a picture of one such tiling (of which there will be plenty in the talk). The q^{Volume} tiling model is a measure on the space of tilings which assigns to each tiling a probability proportional to q^{Volume}, where q is a real parameter. In this talk, I will recall the model and basic result about it and propose an approach to studying its statistical properties by analyzing a family of non-Hermitian orthogonal polynomials. This talk is based on joint work with Maurice Duits.

Modified Bessel functions, Stieltjes transforms and infinite divisibility

Árpád Baricz*
Department of Economics, Babeş-Bolyai University, 400591 Cluj-Napoca, Romania
Institute of Applied Mathematics, Óbuda University, 1034 Budapest, Hungary
bariczocsi@yahoo.com
https://sites.google.com/site/bariczocsi/

In a large number of real life situations some concrete models require a random effect to be the sum of several independent random components with the same distribution. In this kind of situations a very convenient way is to suppose the infinite divisibility of the distribution of these random effects. Similar situations may occur in biology, physics, economics and insurance.

In this talk I will focus on continuous univariate probability distributions, like McKay distribution, K-distribution, generalized inverse Gaussian distribution and generalised McKay distribution with support $[0, \infty)$, which are related to modified Bessel functions of the first and second kinds and in most cases I will show that they belong to the class of infinitely divisible distributions, self-decomposable distributions, generalized gamma convolutions and hyperbolically completely monotone densities. Integral representations of quotients of Tricomi hypergeometric functions as well as of quotients of Gaussian hypergeometric functions, or modified Bessel functions of the second kind play an important role in this study. In addition, I also obtain a new infinitely divisible modified Bessel distribution with Laplace transform related to modified Bessel functions of the first and second kind.

The talk is based on the following paper: Á. Baricz, D.K. Prabhu, S. Singh, V.A. Vijesh, Infinitely divisible modified Bessel distributions, https://arxiv.org/abs/2406.17721.

On the Lebesgue constant of the Morrow-Patterson points

Tomasz Beberok*, Leokadia Bialas-Ciez, Stefano De Marchi
University of Agriculture in Krakow, Poland
tomasz.beberok@urk.edu.pl
https://repo.ur.krakow.pl/info/author/2193

The study of interpolation nodes and their associated Lebesgue constants are central to numerical analysis, impacting the stability and accuracy of polynomial approximations. In this talk, we will explore the Morrow-Patterson points, a set of interpolation nodes introduced to construct cubature formulas of a minimum number of points in the square for a fixed degree n. We prove that their Lebesgue constant growth is $\mathcal{O}(n^2)$ as was conjectured based on numerical evidence about twenty years ago in the paper by Caliari, M., De Marchi, S., Vianello, M., *Bivariate polynomial interpolation on the square at new nodal sets*, Appl. Math. Comput. 165(2) (2005), 261–274.

Best rational approximants of Markov functions

Bernhard Beckermann*
Université de Lille, France
Bernhard.Beckermann@univ-lille.fr

We give an upper bound for the relative error of (best) rational approximants of a Markov function, and show that this inequality becomes equality for a particular worst-case measure.

Orthogonality and last-passage percolation in layered medium

Sergey Berezin*
Katholieke Universiteit Leuven
sergey.berezin@kuleuven.be

Biorthogonal and multiple orthogonal polynomials play crucial role in integrable probability. In this talk, I will introduce a particular last-passage percolation model, in which the environment is comprised of layers with different characteristics. The last-passage time in this model can be described by a Fredholm determinant with respect to a kernel that possesses both biorthogonal and multiple orthogonal structure. This kernel is intimately related to the product-matrix ensemble associated to the random truncated unitary matrices. Scaling the problem yields an interesting limit, which can be viewed as a generalization of the critical scaling limit in random matrix theory. The key ingredient of the proof is the double contour-integral representation of the kernel obtained by Borodin, Gorin, and Strahov, who extended the result of Kuijlaars and Zhang for the case of Ginibre matrices. The talk is based on joint work with Eugene Strahov (HUJI).

Minimizing Energies with More than Two Inputs

Dmitriy Bilyk
University of Minnesota
dbilyk@umn.edu

We shall discuss a topic that is rather novel in the energy minimization literature: three-input discrete energies $\sum_{i,j,k=1}^{N} K(x_i, x_j, x_k)$, where $x_1, \ldots, x_N$ is a point configuration in the domain Ω and energy integrals $\int_\Omega \int_\Omega \int_\Omega K(x, y, z) d\mu(x) d\mu(y) d\mu(z)$ where μ is aprobability measure on Ω, as well as multi-input analogues of such energies. Particularly interesting examples in the case $\Omega = \mathbb{S}^{d-1}$ include $K(x, y, z) = A^s(x, y, z)$ and $K(x, y, z) = V^s(x, y, z)$, where $A(x, y, z)$ is the area of the triangle spanned by the points x, y, $z \in \mathbb{S}^{d-1}$ and $V(x, y, z)$ is the volume of the parallelepiped spanned by three vectors x, y, z. These energies can be interpreted as three-input analogues of the classical two-input Riesz energies on the sphere and the projective spaces, respectively. We shall explain the main differences between the classical two-input and multi-input energy minimization, present some known results and conjectures, as well as results of numerical experiments and relations to other problems. The talk is based on joint work with A. Glazyrin, R. Matzke, J. Park, O. Vlasiuk, C.-J. Yeh.

Discrete Minimizers of p-frame Energies and other Energy Integrals

Dmitriy Bilyk
University of Minnesota
dbilyk@umn.edu

We shall discuss a curious phenomenon: some energy integrals of the form $\int_\Omega \int_\Omega f(x, y) d\mu(x) d\mu(y)$, where μ is a probability measure on Ω, are minimized by discrete measures, i.e. minimizers tend to concentrate on discrete sets rather than spread out. One of the interesting examples of such energies is the p-frame energy which corresponds to the interaction $f(x, y) = |\langle x, y\rangle|^p$ with $p > 0$ and $\Omega = \mathbb{S}^{d-1}$. It is conjectured that all the minimizers of this energy are discrete whenever $p > 0$ is not an even integer. We shall discuss known results: optimality of tight designs, smallness of support of optimal measures, spherical codes arising as minimizers in numerical experiments for the p-frame and other energies, as well as connections to other problems of discrete geometry. The talk is based on joint work with A. Glazyrin, R. Matzke, J. Park, O. Vlasiuk.

Approximating the Solutions to PDEs with Uncertain Boundary Information

Peter Binev*
University of South Carolina
binev@math.sc.edu
https://people.math.sc.edu/binev/

We consider the problem of numerically approximating the solutions to a partial differential equation (PDE) when there is insufficient information to determine a unique solution. Our main example is the Poisson boundary value problem, when the boundary data is unknown and instead one observes finitely many linear measurements of the solution. We view this setting as an optimal recovery problem and develop theory and numerical algorithms for its solution. The main vehicle employed is the derivation and approximation of the Riesz representers of these functionals with respect to relevant Hilbert spaces of harmonic functions. The talk is based on the paper P. Binev, A. Bonito, A. Cohen, W. Dahmen, R. DeVore, G. Petrova. *Solving PDEs with Incomplete Information*, SIAM J. on Numerical Analysis 62, (2024), 1278-1312.

Min-Max Polarization Property of Sharp Spherical Codes that are non-Tight Designs

Sergiy Borodachov*, Peter Boyvalenkov, Peter Dragnev, Douglas Hardin, Edward Saff, Maya Stoyanova
Towson University
sborodachov@towson.edu
https://www.towson.edu/fcsm/departments/mathematics/facultystaff/sborodachov.html

We study the problem of minimizing the absolute maximum over the d-dimensional sphere of the potential of an N-point spherical code defined by an absolutely monotone function of the dot product. This problem, known as the min-max polarization, and its dual, the max-min polarization problem, were first considered in the papers by Ohtsuka (1967) and Stolarsky (1975). We prove that any m-distance sharp code C that is not a tight design is still a solution to the min-max polarization problem provided that its $2m$-frame potential is maximized at points of C. This result can be applied to six different sharp codes on Euclidean spheres.

We also establish an analog of this result in the real projective space and show that certain ten sharp codes there (that are not tight designs) are solutions. We use the linear programming approach developed in the works by Delsarte, Goethals, Seidel, Levenshtein, Cohn, and Kumar utilizing properties of spherical harmonics, orthogonal polynomials, and polynomial interpolation.

An analogue of Gonchar's theorem

Methawee Wajasat, Nattapong Bosuwan*
Department of Mathematics, Faculty of Science, Mahidol University,
Rama VI Road, Ratchathewi District, Bangkok, 10400, Thailand
nattapong.bos@mahidol.ac.th

Padé approximations constructed from orthogonal and Faber polynomials on some compact set E serve as tools to detect poles of an approximated function around the set E. The goal of this work is to study the rate of pole detection using the indicators introduced by A.A. Gonchar in 1981.

On universal optimality of distance avoiding spherical codes

Peter Boyvalenkov*, Danila Cherkashin, Peter Dragnev
Institute of Mathematics and Informatics, Bulgarian Academy of Sciences
peter@math.bas.bg

Given an open set (a union of open intervals), $T \subset [-1, 1]$ we introduce the concepts of T-avoiding spherical codes and designs, that is, spherical codes that have no inner products in the set T. We show that certain codes found in the Leech lattices as well as codes derived from strongly regular graphs are universally optimal in the class of T-avoiding codes. We also extend a result of Delsarte–Goethals–Seidel about codes with three inner products α, β, γ (in our terminology (α, β)-avoiding γ-codes). Parallel to the notion of tight spherical designs, we also derive that these codes are minimal (tight) T-avoiding spherical designs of fixed dimension and strength. In some cases, we also find that codes under consideration have maximal cardinality in their T-avoiding class for given dimension and minimum distance.

On the Lower Bounds for the Spherical Cap Discrepancy

Dmitriy Bilyk, Johann Brauchart*
Graz University of Technology
j.brauchart@tugraz.at

The spherical cap $\mathbb{L}_2$-discrepancy measures the irregularity of a distribution of N points on the unit d-sphere in $\mathbb{R}^{d+1}$ in terms of spherical caps as test sets in the $\mathbb{L}_2$ sense. We give a very simple and elementary new proof of the classical bound due to J. Beck. Our approach leads to many further new results: estimates of the discrepancy in terms of various geometric quantities, an easy proof of (point-independent) upper estimates for the sum of positive powers of Euclidean distances between points on the sphere, and lower bounds for the discrepancy of rectifiable curves and sets of arbitrary Hausdorff dimension.

A refinement of the proof yields asymptotic constants that are surprisingly close to conjectured optimal constants in the four cases $d = 2, 4, 8, 16$.

Logarithmic and Riesz Energy on the Sphere: better Bounds via elementary Methods

Johann S. Brauchart*
Graz University of Technology
j.brauchart@tugraz.at

Using elementary methods and some asymptotic analysis, we reprove known and prove new bounds (that are surprisingly close to conjectured bounds) for the minimal logarithmic and Riesz s-energy of point sets on the unit sphere in $\mathbb{R}^{d+1}$ for $d \geq 2$. The novel approach works in the continuous case $-s < s < 0$ as well as in the logarithmic and, in particular, in the singular case $0 < s \leq d$. We give an outlook of further applications of our approach.

On a very large concept of Lagrange interpolation

Ludovico Bruni Bruno
Istituto Nazionale di Alta Matematica and Università di Padova

bruni@math.unipd.it
https://sites.google.com/view/ludovicobruni

In this talk we address the problem of interpolating and approximating objects that produce diffused data, generally defined by integration. We show that many aspects of nodal interpolation can naturally be carried to this more general framework; in contrast, some of them require the introduction of geometric and measure theoretic hypotheses. After characterizing the norms of the operators involved, we review the main recent results on the topic and explore the related techniques. An application to imaging is presented.

Approximation results of Sampling Kantorovich operators in Besov spaces

Marco Cantarini*, Gianluca Vinti
Department of Mathematics and Computer Science, University of Perugia
marco.cantarini@unipg.it

We continue our analysis of approximation properties of sampling Kantorovich operators in fractional functional spaces. In particular, we prove a convergence theorem of the sampling Kantorovich operators for functions belonging to general Besov spaces $B^s_{pq}(\mathbb{R})$. As a consequence, we are able to prove that such operators converges also in the setting of other important functional spaces, like the Triebel-Lizorkin spaces $F^s_{pq}(\mathbb{R})$ and some significant fractional Sobolev spaces, that is the Bessel potential spaces $H^{s,p}(\mathbb{R})$ and the Gagliardo fractional Sobolev spaces $\widehat{W}^{s,p}(\mathbb{R})$.

Fast-Decaying Polynomial Reproduction

Stefano De Marchi, Giacomo Cappellazzo*
University of Padua
giacomo.cappellazzo@math.unipd.it

Polynomial reproduction plays a relevant role in deriving error estimates for various approximation schemes. Local reproduction in a quasi-uniform setting is a significant factor in the estimation of error and the assessment of stability but for some computationally relevant schemes, such as Rescaled Localized Radial Basis Functions (RL-RBF), it becomes a limitation. To facilitate the study of a greater variety of approximation methods in a unified and efficient manner, this talk proposes a framework based on fast decaying polynomial reproduction: we do not restrict to compactly supported basis functions, but we allow the basis function decay to infinity as a function of the *separation distance*. Implementing fast decaying polynomial reproduction provides stable and convergent methods, that can be smooth when approximating by moving least squares otherwise, it is very efficient in the case of linear programming problems. All the results presented in this talk concerning the rate of convergence, the Lebesgue constant, the smoothness of the approximant, and the compactness of the support have been verified numerically, even in the multivariate setting.

Coulomb and Riesz gases

Djalil Chafaï*
Paris-Dauphine and École normale supérieure - PSL
djalil@chafai.net
https://djalil.chafai.net/

Coulomb and Riesz gases are natural Boltzmann-Gibbs measures arising in statistical physics and connected to many other areas, including potential theory, approximation theory, harmonic analysis, numerical simulation, random matrices, statistics, number theory, quantum physics, partial differential equations, stochastic processes, and interarting particle systems. They are the subject of intense mathematical research. This introductory talk will present selected historical and structural aspects of these models, and relate them to the recent advances discussed in the minisymposium.

Redundancy of Probabilistic Frames and Approximately Probabilistic Dual Frames

Dongwei Chen*, Emily J. King, Clayton Shonkwiler
Colorado State University
dongwei.chen@colostate.edu

This talk is about the redundancy of probabilistic frames and approximately probabilistic dual frames. The redundancy of a probabilistic frame is defined as the dimension of the kernel of associated synthesis operator. For a given non-redundant probabilistic frame, we claim that the canonical probabilistic dual frame is the only dual frame of pushforward type. Furthermore, we show that probabilistic frames with finite redundancy are atomic. In the second part, we will introduce the approximately probabilistic dual frames and characterize all the approximate duals of pushforward type. We also study the approximately dual frame of perturbed probabilistic frames, and show that if a probability measure is close to a probabilistic dual pair in some sense, then this probability measure is an approximately probabilistic dual frame.

Sampling expansions and zeros of hypergeometric ${}_1F_2$ functions

Yong-Kum Cho*, Seok-Young Chung, Young Woong Park
Chung-Ang University, Seoul
ykcho@cau.ac.kr

As for the hypergeometric ${}_1F_2$ function of the form

$$\Phi(z) = {}_1F_2\left[\begin{array}{c} a \\ b,\ c \end{array} \middle| -\frac{z^2}{4}\right] \qquad (z \in \mathbb{C}),$$

we establish a set of sampling expansions in terms of normalized Bessel functions and investigate how the sign of samples changes. As an application, we obtain an extensive range of parameters a, b, c for which $\Phi(z)$ has only real zeros and hence it belongs to the Laguerre-Pólya class.

Weighted Chebyshev Polynomials and Widom Factors

Jacob S. Christiansen*, Benjamin Eichinger, Olof Rubin, Maxim Zinchenko
Lund University
jacob_stordal.christiansen@math.lth.se

In this talk, I will discuss recent advances in weighted uniform approximation. Weights naturally arise when transforming a given set into a more convenient setting, but they also hold intrinsic mathematical interest. I will start with a brief overview of the classical case on an interval, highlighting key results dating back to Bernstein, before extending the discussion to more general sets, with a particular focus on circular arcs. The main emphasis will be on the asymptotic behavior of weighted Chebyshev polynomials on arcs and the so-called Widom factors, which correspond to properly scaled minimal weighted norms. Notably, a certain reproducing kernel emerges in these asymptotic formulas, offering further insight into the structure of the problem.

Complex zeros of Bessel function derivatives and associated orthogonal polynomials

Seok-Young Chung*, Sujin Lee, Young Woong Park
University of Central Florida
seok-young.chung@ucf.edu
https://sites.google.com/view/seok-young-chung

We introduce a sequence of orthogonal polynomials whose associated moments are the Rayleigh-type sums, involving the zeros of the Bessel function derivative J'_ν of order ν. We also discuss the fundamental properties of those polynomials such as recurrence, orthogonality, etc. Consequently, we obtain a formula for the Hankel determinant, elements of which are chosen as the aforementioned Rayleigh-type sums. As an application, we complete the Hurwitz-type theorem for J'_ν, which deals with the number of complex zeros of J'_ν depending on the range of ν.

Stable Separation of Orbits for Finite Abelian Group Actions

Jameson Cahill, Andres Contreras, Andres Contreras Hip*
University of Chicago
acontreraship@uchicago.edu

In this talk, we present two families of invariant maps that effectively separate the orbits of a finite Abelian group acting on a finite-dimensional complex vector space. The first family is Lipschitz continuous with respect to the quotient metric on the space of orbits but involves computing large powers of vector components. The second family avoids this by applying powers only to the phase of the components, but it is discontinuous. However, we establish that these phase-based maps are Lipschitz continuous on subsets of vectors with fixed support, including all vectors with nonzero entries. Notably, both families achieve separation with a target dimension that grows linearly with the original.

Approximation Properties and Applications of Sampling-type Operators

Danilo Costarelli*
Department of Mathematics and Computer Science, University of Perugia
danilo.costarelli@unipg.it
https://www.unipg.it/personale/danilo.costarelli

In this talk, I will present an historical overview concerning the main approximation results related to the classical sampling theory, starting from the celebrated *Sampling Theorem* of Whittaker, Kotel'nikov and Shannon, and its various generalizations. We analyze the main reasons that brought back the introduction of the sampling-type operators, showing both classical results and new developments of such a theory.

In particular, I will also present some extensions of the mentioned families of operators, including the very recent *Steklov sampling operators* and some results related to the so-called Kantorovich sampling series. In conclusion, some real world applications are presented, in which the application of the multivariate form of the above operators plays a central role.

Acknowledgment
The author has been supported by the project PRIN 2022 "AIDA" funded by the European Union under the Italian National Recovery and Resilience Plan (NRRP) of NextGenerationEU, under the Italian Ministry of University and Research (Project Code: 20229FX3B9, CUP: J53D23000660001).

On energy, discrepancy, group invariant measures, alignment of neural data and Whitney extensions.

Steven Damelin*, Fred Hickernell, David Ragozin, Xen Zeng
zbMATH Open, European Mathematical Society
steve.damelin@gmail.com
stevendamelin.com

Given X, some measurable subset of Euclidean space, one sometimes wants to construct a design, $P \subset X$ with a small energy or discrepancy. Here it is shown that these two measures of design quality are equivalent when they are defined via positive definite kernels $K : X^2 \to \mathbb{R}$. The error of approximating the integral $\int_X f(x)d\mu(x)$ by the sample average of f over P has a tight upper bound in terms of the energy or discrepancy of P. The tightness of this error bound follows by requiring f to lie in the Hilbert space with reproducing kernel K. The theory presented here provides an interpretation of the best design for numerical integration as one with minimum energy, provided that the measure defining the integration problem is the minimizer measure corresponding to the energy kernel, K. If X is the orbit of a compact, possibly non-Abelian group, G, acting as measurable transformations of X and the kernel K is invariant under the group action, then it is shown that the minimizer measure is the normalized measure on X induced by Haar measure on G. This allows us to calculate explicit representations of minimizer measures. We will explain how this problem relates to interactions of neural data on the flat torus, Whitney extensions and manifold learning alignment .

On the conjugate functions

Arthur Danielyan*
University of South Florida
adaniely@usf.edu
https://www.usf.edu/arts-sciences/departments/mathematics-statistics/people/faculty/arthur-danielyan.aspx

The paper is dedicated to the solution of Problem 5.62 from a well-known collection of research problems by D. M. Campbell, J. G. Clunie, and W. K. Hayman, published in 1980.

Solving Roughly Forced Nonlinear PDEs via Misspecified Kernel Methods and Neural Networks

Ricardo Baptista, Edoardo Calvello, Matthieu Darcy*, Houman Owhadi, Andrew M. Stuart, Xianjin Yang
Computing and Mathematical Sciences, California Institute of Technology
mdarcy@caltech.edu
www.matthieudarcy.com

We consider the use of Gaussian Processes (GPs) or Neural Networks (NNs) to numerically approximate the solutions to nonlinear partial differential equations (PDEs) with rough forcing or source terms, which commonly arise as pathwise solutions to stochastic PDEs. Kernel methods have recently been generalized to solve nonlinear PDEs by approximating their solutions as the maximum a posteriori estimator of GPs that are conditioned to satisfy the PDE at a finite set of collocation points. The convergence and error guarantees of these methods, however, rely on the PDE being defined in a classical sense and its solution possessing sufficient regularity to belong to the associated reproducing kernel Hilbert space. We propose a generalization of these methods to handle roughly forced nonlinear PDEs while preserving convergence guarantees with an oversmoothing GP kernel that is misspecified relative to the true solution's regularity. This is achieved by conditioning a regular GP to satisfy the PDE with a modified source term in a weak sense (when integrated against a finite number of test functions). This is equivalent to replacing the empirical L^2-loss on the PDE constraint by an empirical negative-Sobolev norm. We further show that this loss function can be used to extend physics-informed neural networks (PINNs) to stochastic equations, thereby resulting in a new NN-based variant termed Negative Sobolev Norm-PINN (NeS-PINN).

Weighting Inputs by Sensitivity in Random Feature Expansions

Laura Weidensager, John Darges*
Emory University
jdarges@emory.edu
https://jedarges.github.io/

This talk proposes random feature expansions (RFEs) with probability distributions that place weights on inputs based on sensitivity. RFEs approximate functions as an expansion of basis functions selected randomly from a particular class. Recent work has touted using RFEs for approximating multivariable functions. We put forth an advancement in approximation with RFEs, leveraging input

sensitivity to increase the likelihood that we sample basis functions that reflect the input importance and interaction structure of the target function. This procedure is equivalent to approximation using a weighted ANOVA kernel. For classes of multivariable functions dominated by lower order interactions, or by just a few of the inputs, approximation by RFEs with input weighting is fast and efficient. Moreover, show that we can find the correct weights to use through iterative approximations. Theoretical results are accompanied by numerical experiments.

Meshless Moment-Free Quadrature on Domains and Manifolds

Oleg Davydov*
University of Giessen
oleg.davydov@math.uni-giessen.de
oleg-davydov.de

I will present a new method of generating high order quadrature formulas for integrals over Lipschitz domains in manifolds of any dimension developed jointly with Bruno Degli Esposti, arXiv:2409.03567. Given an arbitrary set of nodes in the closure of a domain Ω, we compute the quadrature weights as a minimum norm solution of a sparse underdetermined linear system arising from numerical differentiation formulas that discretize a boundary value problem of Neumann type on Ω. The method requires neither meshing nor pre-computation of any moments, such as integrals of monomilas or other basis functions. Numerical differention formulas may be obtained by kernel methods or by ambient polynomials or splines. Apart from the efficient integration on complicated domains, potential applications include meshless Galerkin methods for partial differential equations on manifolds, conservative meshless finite difference methods and mimetic methods.

Numerical Experiments with Potential Flow and Vortices in Multiply Connected Domains in the Plane

Thomas DeLillo*
Wichita State University
thomas.delillo@wichita.edu
http://www.math.wichita.edu/∼delillo/

We will give an overview of recent numerical experiments computing plane potential flow and vortices in multiply connected domains with circular boundaries in the complex plane. The velocity potentials determining the motion of both the interacting vortices and the circular cylinders are represented by Laurent series. The Blasius formula is used to compute the force on the cyinders.

Nonlinear Carleson conjecture for OPUC: new development

Sergey A. Denisov*
University of Wisconsin - Madison
denissov@wisc.edu

https://people.math.wisc.edu/∼denissov/

The Nonlinear Carleson conjecture for OPUC (advocated by T. Tao and C. Thiele for Dirac operators) asks to prove asymptotics for monic orthogonal polynomials $\Phi_n(z, \sigma)$ when $n \to \infty$ for a.e. z on the unit circle in the case when the measure of orthogonality σ belongs to the Szegő class. In this talk, we will discuss some recent developments and relate the problem to a fine behavior of roots on Φ_n. This work is based on a joint project with Roman Bessonov (SPDMI and Ljubljana University).

Perfect state transfer and orthogonal polynomials

Maxim Derevyagin*
University of Connecticut
maksym.derevyagin@uconn.edu
https://derevyagin.math.uconn.edu

I will begin by presenting the interplay between orthogonal polynomials, spectral theory of Jacobi matrices, and the theory of perfect quantum state transfer in 1D quantum wires. Then, I will discuss some new and old results. After that, I will show how linear combinations of Chebyshev polynomials helped to answer a question regarding speeding up the transfer time in quantum wires.

My Journey Through Mathematics with Ed Saff

Peter Dragnev*
Purdue University Fort Wayne
dragnevp@pfw.edu
https://users.pfw.edu/dragnevp/

In this talk I will survey close to 30 years of collaboration with Ed Saff on minimal energy problems. Four themes encompassing the interplay between Approximation and Potential theories and as of late Coding theory, will be at the center of presentation - from my early days in constrained equilibrium problems and zero asymptotics of discrete orthogonal polynomials, and in particular Krawtchouk polynomials; to external field problems on the sphere with applications to separation of minimal Riesz energy points; and to the more recent investigation on universal lower and upper bounds on energy, and on polarization. While the focus of the latter will be on spherical codes, a full circle leading to Krawtchouk polynomials will occur for codes in Hamming spaces.

Iterated balayage techniques for minimal energy problems

Peter Dragnev*
Purdue University Fort Wayne
dragnevp@pfw.edu
https://users.pfw.edu/dragnevp/

The balayage (or "sweeping out") of a measure is a central notion in potential theory and its applications to constructive approximation. For example, extremal (equilibrium) measures solving minimal energy problems often govern weak and strong asymptotics of orthogonal polynomials. In this introductory tutorial presentation we shall re-visit some iterative balayage techniques, such as *the iterated balayage algorithm* or *the balayage ping-pong*, allowing us to conclude properties of equilibrium measures. An in-depth use of such iterative techniques will be found in R. Orive's talk.

Minimal Error Functions on Irregular Subsets of the Real Line

Robert Dukes*, Maxim Zinchenko
University of New Mexico
dukesr@unm.edu
https://www.robertmdukes.com/

Chebyshev Polynomials, those that minimize the maximal error on a compact set, are one of the most practical tools for approximating smooth functions. The classical results are on the set [-1, 1], and much has been written about other regular sets. We introduce the notion of a "semi-regular set," in which the regular part is closed, and the Regularity Coefficient, formed by summing the Green's Function at irregular points. We conclude that, on semi-regular Parreau-Widom sets, the Chebyshev Norms are bounded if and only if the Regularity Coefficient is finite. We will also show some related results involving Residual Polynomials.

Dynamics of Iteration of the Newton Map of Complex Trigonometric Functions

Kasey Bray, Jerry Dwyer*, Brock Williams
University of Mississippi
jfdwyer@olemiss.edu

The dynamical systems of trigonometric functions are explored, with a focus on and The fractal images are created by iterating the Newton maps, of and of. The basins of attraction created from iterating and are analyzed and some bounds are determined for the primary basins of attraction. We further prove and -axis symmetry of the functions and explore the infinite nature of the fractal images.

Minimal Riesz energy on the Grassmannian II

Ujué Etayo*, Pedro R. López-Gómez
CUNEF Universidad
ujue.etayo@cunef.edu
https://sites.google.com/view/ujuetayo/home

In this talk, we study Riesz and logarithmic energies in the $\mathrm{Gr}_{2,4}$ Grassmannian of two-dimensional subspaces of $\mathbb{R}^4$. This space differs from other classical spaces where such potentials are studied by

the fact that it is not 2-point homogeneous. Therefore, we develop new techniques to show that the continuous Riesz and logarithmic energies are uniquely minimized by the uniform measure, and we obtain asymptotic upper and lower bounds on the discrete minimum energies, with coinciding orders for the higher-order terms.
This is joint work with Pedro R. López-Gómez.

Distributing points on manifolds via a norm inequality

Ujué Etayo*
CUNEF Universidad
ujue.etayo@cunef.edu
https://sites.google.com/view/ujuetayo/home

There are many ways to distribute points on manifolds so that they represent the aforementioned space. One can look for interpolation nodes, repulsive potential minimizers, discrepancy minimizers, etc. Inspired by a formula presented in 2011 by Armentano, Beltran and Shub relating elliptic Fekete points, the condition number of polynomials and a quotient relating polynomial norms, we propose a new way to distribute points on the sphere, which can be generalized to any Riemann surface.
Part of these results are joint work with Haakan Hedenmalm and Joaquim Ortega-Cerdà.

Symplectic Zauner's Conjecture is the Skew Hadamard Conjecture

Kean Fallon*
Iowa State University
keanpf@iastate.edu

Consider the problem of placing lines through the origin in $\mathbb{F}^d$ so that each of the pairwise angles between them is the same. When $\mathbb{F} = \mathbb{C}$, Gerzon's bound states that you can have no more than d^2 of these equiangular lines. Zauner's conjecture, a significant open problem that has garnered considerable interest, speculates that this bound is saturated for every d.

In this talk, we introduce an analogous notion of equiangular lines and, subsequently, equiangular tight frames, over real symplectic space. In particular, when porting Zauner's conjecture to this symplectic setting, we find that the symplectic Zauner's conjecture is equivalent to the skew Hadamard conjecture.

On Type I Higher-Order Three-Term Recurrences

U. Fidalgo*, E. Huertas
Lorain County Community College
ufidalgo@lorainccc.edu

We consider a sequence of type I multiple orthogonal polynomials constructed via a higher-order three-term recurrence relation with fixed coefficients. We prove that the components of these polynomial vectors vanish on starlike sets, and that the zeros of these polynomials exhibit specific interlacing properties. These algebraic results enable us to derive an expression characterizing the sequence's strong asymptotic behavior.

Approximations with Non-symmetric Green's Kernels and their Application to Fractional Differential Equations

Nick Fisher*
Portland State University
nicholfi@pdx.edu

Several kernel-based methods for the numerical solution of fractional differential equations have been developed in the recent past. However, these techniques exclusively relied on the use of radial basis function approximations. In the present work, we consider the non-symmetric Green's kernel perspective on fractional order spline interpolation and its application to a kernel Galerkin method for the numerical solution of certain fractional order differential equations. The resulting kernel interpolants obtain provably optimal order convergence rates in a reproducing kernel Banach space and these theoretical results are confirmed through rigorous numerical experiments.

Locally Supported Bases for the Approximation of Functions on Graphs

Edward Fuselier*, John Paul Ward
High Point University
efuselie@highpoint.edu
https://math.highpoint.edu/∼efuselier/

In approximation settings where the underlying domain is a graph, often a basis for the approximation space is not available analytically and must be computed. We consider perturbations of Lagrange bases on graphs, where the Lagrange functions come from a class of functions closely related to kernels and variational splines on graphs. The basis functions have local support, with each basis function obtained by solving a small energy minimization problem related to a differential operator on the graph. We present error estimates between the local basis and the corresponding interpolatory Lagrange basis functions in cases where the underlying graph satisfies an assumption on the connections of vertices where the function is not known, and the theoretical bounds are examined further in numerical experiments.

COMBINATORICS OF EVEN-VALENT GRAPHS ON RIEMANN SURFACES

Roozbeh Gharakhloo*, Tomas Lasic Latimer
University of California Santa Cruz
roozbeh@ucsc.edu
https://roozbehgharakhloo.wordpress.com

I will discuss a method for determining the topological expansion of the recurrence coefficients of orthogonal polynomials with weights $e^{-NV(z)}$, where $V(z) = z^2/2 + uz^{2\nu}/2\nu$, and $\nu \in \mathbb{N}$. This enables the derivation of explicit formulas for $\mathcal{N}_g(2\nu, j)$ − the number of 2ν-valent connected and labeled graphs with j vertices on a compact Riemann surface of genus g − and their two-legged counterparts for general ν, fixed g, and j.

Additionally, I will outline the steps required to extend the results from a prior collaboration with P. Bleher and K. McLaughlin (arXiv:2112.09412) to the case of hexic weights, where $V(z) = z^2/2+uz^6/6$. This extension facilitates the explicit calculation of $\mathcal{N}_g(6, j)$ for any number of vertices j and fixed g. This talk is based on joint work with Tomas Lasic Latimer (UCSC).

Approximating Discrete Averaging Operators

Christina Giannitsi*
Vanderbilt University
christina.giannitsi@vanderbilt.edu
https://sites.google.com/view/christina-giannitsi/home

Bounding averaging operators has been an area of particular interest in discrete harmonic analysis, especially over the recent years. In a series of results for averaging operators with arithmetic weights obtaining what we call improving and sparse bounds has heavily relied on approximating the kernel of the corresponding Fourier multiplier with the standard usual averages and carefully controlling the error of that approximation. In this talk, we will go over examples of some of the more "standard" arithmetic weights, and compare their behavior with the usual averages, and see how doing so allows us to obtain interesting results. We shall conclude with one of the more complicated examples, and explore its interesting applications.

Minimizing Measures for Locally Polynomial Potentials on Spheres

Alexey Glazyrin*
The University of Texas Rio Grande Valley
Alexey.Glazyrin@utrgv.edu
https://faculty.utrgv.edu/alexey.glazyrin/

We study the energy minimization problem for measures on spheres. Given a potential $f : [-1, 1] \to \mathbb{R}$, we want to find all Borel probability measures μ minimizing the energy

$$I_f(\mu) = \int\limits_{\mathbb{S}^{d-1}} \int\limits_{\mathbb{S}^{d-1}} f(\langle x, y\rangle) d\mu(x) d\mu(x).$$

I will talk about the history of the problem and show several new results for polynomial and locally polynomial potentials.

This is joint work with Damir Ferizović.

From sphere packings, to interpolation, to crystalline measures

Felipe Gonçalves*
The University of Texas at Austin
goncalves@utexas.edu
w3.impa.br/∼goncalves

In this talk we shall give an overview how these three different worlds collide. We will then present an ongoing classification result (in conjunction with G. Vedana) where we answer a question of F. Dyson, and classify all pairs of sequences $[(b_n, \gamma_n)_n, (a_n, \lambda_n)_n]$ such that one has a Fourier summation formula

$$\sum_n b_n \widehat{\varphi}(\gamma_n) = \sum_n a_n \varphi(\lambda_n)$$

that holds for any compactly supported test function $\varphi \in C^\infty(\mathbb{R})$, where $\widehat{\varphi}$ is its Fourier transform.

Harmonic-measure distribution functions in various geometries

Christopher C. Green*, Arunmaran Mahenthiram, Lesley A. Ward
Wichita State University
christopher.green@wichita.edu

Consider releasing a Brownian particle from a basepoint z_0 in a planar domain Ω. What is the chance, denoted $h_{\Omega,z_0}(r)$, that the particle's first exit from Ω occurs within a fixed distance $r > 0$ of z_0? The function $h_{\Omega,z_0}(r) : [0,\infty) \to [0,1]$ is called the harmonic-measure distribution function, or h-function, of Ω with respect to z_0. It can also be formulated in terms of a Dirichlet problem on Ω with suitable boundary values. For simply connected domains Ω, the theory of h-functions is now quite well-developed, and in particular the h-function can often be explicitly computed, making use of the Riemann mapping theorem. However, until recently, for multiply connected domains the theory of h-functions has been almost entirely out of reach. In this talk, it will be shown how to construct explicit formulae for h-functions of symmetric multiply connected slit domains whose boundaries consist of an even number of colinear slits, and how these formulae can be generalized to compute h-functions for multiply connected slit domains on a spherical surface. Special function theory and conformal mapping are judiciously combined to this end.

Constructing a Computational Pipeline for Solving Real-World Boundary-value Problems with Splines

Aussie Greene*
Vanderbilt University
aussie.m.greene@vanderbilt.edu

Solving real-world boundary value problems is comprised of three stages: pre-processing (modeling), evaluation, and post-processing (visualization). Industry standard techniques such as finite element method (FEM) and isogeometric analysis (IGA) have been implemented within larger computational frameworks in order to provide engineers methods for solving boundary value problems from start to end. Recently, Schumaker [J. Sci. Comp. **80(3)** (2019), 1369–1394] introduced an alternative to existing methods which has significant advantages in both the pre-processing and evaluation stages.

In this talk we discuss the pre- and post-processing required to develop a framework incorporating the immersed penalized boundary method for solving boundary value problems on real-world geometries. Topics to be discussed include 3D modeling, point cloud processing, surface reconstruction, and normal vector approximation.

Learning Equivariant Tensor Functions with Applications to Sparse Vector Recovery

Wilson G. Gregory*, Josué Tonelli-Cueto, Nicholas F. Marshall, Andrew S. Lee, and Soledad Villar
Johns Hopkins University
`wgregor4@jhu.edu`
`https://www.cis.jhu.edu/~wgregor4/`

This work characterizes equivariant polynomial functions from tuples of tensor inputs to tensor outputs. Loosely motivated by physics, we focus on equivariant functions with respect to the diagonal action of the orthogonal group on tensors. We show how to extend this characterization to other linear algebraic groups, including the Lorentz and symplectic groups.

Our goal behind these characterizations is to define equivariant machine learning models. In particular, we focus on the sparse vector estimation problem. This problem has been broadly studied in the theoretical computer science literature, and explicit spectral methods, derived by techniques from sum-of-squares, can be shown to recover sparse vectors under certain assumptions. Our numerical results show that the proposed equivariant machine learning models can learn spectral methods that outperform the best theoretically known spectral methods in some regimes. The experiments also suggest that learned spectral methods can solve the problem in settings that have not yet been theoretically analyzed. This is an example of a promising direction in which theory can inform machine learning models and machine learning models could inform theory.

Uniqueness of Phase Retrieval in $L^2(\mathbb{R})$ for Functions with super-Gaussian Fourier Decay

Kasso Okoudjou, Shuang Guan*
Tufts University
`shuang.guan@tufts.edu`

we study injectivity of phase retrieval problems in $L^2(\mathbb{R})$ for real-valued functions with Fourier transform decays faster than Gaussian, a set of functions can be regarded as an super set of bandlimited functions. Using a specific version of Paley-Wiener theorem, it can be shown that these functions can be extended to entire functions of order 2. Using knowledge on an upper bound of sampling sequence, we show that determination of such function can be done if it is sampled at more than twice of the sampling sequence density. Finally, we prove a similar result for Short-time Fourier Transform measurement with condition on window function.

Comparative Analysis of Deterministic and Stochastic SIR Models in Modeling Measles Epidemics

Dawit Denu, Minh Ha*
Georgia Southern University
mh34830@georgiasouthern.edu

In this study, we examine measles epidemic dynamics through deterministic and stochastic SIR models. In the deterministic model, we investigate viral infection dynamics through the basic reproductive number $\mathcal{R}_0$ in order to evaluate equilibria stability for $\mathcal{R}_0 < 1$ and $\mathcal{R}_0 > 1$. In the stochastic model, we demonstrate the existence of global solutions and define the stochastic reproductive number $\mathcal{R}_0^s$ while studying disease dynamics for both $\mathcal{R}_0^s < 1$ and $\mathcal{R}_0^s > 1$ when dealing with large environmental variations. In particular, the deterministic model suggests disease persistence, but stochastic factors can cause extinction events under large environmental fluctuations, which demonstrate the impact of random elements in disease transmission. In addition, this study will present conditions for infection extinction alongside the stochastic stability of the solution. Numerical simulations are displayed at the end to demonstrate the theoretical results.

Conformal Capacity on Uncertain Domains

Harri Hakula
Aalto University
Harri.Hakula@aalto.fi

Partial differential equation related uncertainty quantification has become one of the topical research areas in applied mathematics and, in particular, engineering. Remarkably, computational function theory provides a rich set of invariants and identities that can be applied in designing model problems where the domain is random or uncertain. In this talk the focus is on conformal capacity in a simple, yet general case where the sides of a quadrilateral are assumed be random and parameterised with a suitable Karhunen-Loève expansion. The capacity (Dirichlet energy $\int_\Omega |\nabla u|^2$) is computed when two opposing sides are set to fixed potentials and the other two have zero Neumann boundary conditions.

Different approaches for stochastic finite element method (SFEM) solution of conformal capacity problems are reviewed. Simple bounds for the expected capacities are derived based on the so-called reciprocal relation of the original problem and its conjugate.

Kernel methods for PDEs on embedded manifolds

Thomas Hangelbroek*
University of Hawaiʻi – Mānoa
hangelbr@math.hawaii.edu

https://math.hawaii.edu/∼hangelbr/

This talk will introduce some popular techniques employing kernels for solving PDEs on embedded submanifolds of Euclidean space (i.e., using restrictions to the manifold of kernels defined in the ambient space.) After discussing some of the foundational challenges to convergence, stability and computational complexity, we will present a number of recent advances in moving least squares and kernel approximation geared toward treating PDEs in this setting.

In particular, we present new mesh-free approximation results for algebraic manifolds which allow for local estimation of the kernel power function as well as a streamlined moving least squares algorithm which does not require tangent plane approximation. We will also present convergence and stability results for pseudospectral methods on spheres, including a perturbation theory for eigenvalues of differentiation matrices specifically designed to address stability issues in RBF-FD methods.

This is based on joint work with Christian Rieger (Philipps-Universität Marburg) and Grady Wright (Boise State University).

A Sharp Commutator Estimate for Sub-Coulomb Riesz Modulated Energies

Elias Hess-Childs*, Matthew Rosenzweig, Sylvia Serfaty
Carnegie Mellon University
ehesschi@andrew.cmu.edu
https://www.math.cmu.edu/∼ehesschi/

The modulated energy plays a central role in deriving mean-field convergence rates for particle systems interacting via Riesz or Coulomb potentials. In particular, for Coulomb and super-Coulomb interactions ($\mathsf{d} - 2 \leq \mathsf{s} < \mathsf{d}$), the third author established a functional inequality that controls the derivative of the modulated energy along a transport by the modulated energy itself. Combined with a Grönwall-type argument, this yields quantitative rates of mean-field convergence in the modulated energy distance. This framework was later extended to the full range of Riesz (and Riesz-like) interactions by Q.H. Nguyen and the last two present authors.

Building on these developments, we prove improved functional inequalities that are sharp in their additive error terms. As a consequence, we establish the expected convergence rate of $N^{\frac{\mathsf{s}}{\mathsf{d}}-1}$ in modulated energy distance for first-order gradient flows with $\mathsf{s} < \mathsf{d} - 2$. This complements recent work of the second and third authors on the optimal rate for the Coulomb/super-Coulomb case ($\mathsf{d} - 2 \leq \mathsf{s} < \mathsf{d}$), and therefore completely resolves the Riesz case. Our estimates rely on a novel truncation scheme for the interaction potential, based on a wavelet-type integral representation of the Riesz potential, combined with Kato–Ponce-type commutator bounds.

The Marvelous Meixner Polynomials: Second Kind, First Class

Olga Holtz
University of California, Berkeley
holtz@math.berkeley.edu
https://math.berkeley.edu/ holtz/

In this talk, we will explore the fascinating world of Meixner polynomials of the second kind, tracing their journey from Meixner's groundbreaking 1934 classification to their multi-faceted role in modern mathematics. We illuminate their orthogonality measure through its integral transforms, revealing its remarkable connections to the Riemann Zeta function, and decoding its continued fraction representation. We will also glimpse into the rich structure of their generating function and the combinatorial treasure chest of their moment generating function, following on the work of Carlitz, Flajolet, Viennot, and Hetyei in order to explain their ubiquitous presence across probability theory, quantum physics, and beyond.

Multivariate Spline Interpolation with High-Order Smoothness over Triangle Mesh Surfaces

Tsung-Wei Hu*, Ming-Jun Lai
Weill Cornell Medicine
znkt492357816@gmail.com

Consider a triangle mesh surface in 3D. We introduce a class of smooth functions that interpolate the surface. In particular, by specifying a degree of smoothness $\gamma \geq 1$, these interpolating functions can be constructed as polynomials. Our algorithm is based on gluing small patches together, allowing local processing without requiring global optimization. To manage the additional degrees of freedom, we apply an energy minimization approach that reduces the need for extensive user input. Moreover, by imposing appropriate conditions, we ensure that the resulting surface is free from cusps and self-intersections, while maintaining a visually pleasing shape. In this talk, I will outline the key construction principles and explain the mathematical foundation for the high-order smoothness of these functions.

Damped Mass-Critical Nonlinear Schrödinger Equation with Stark Effect

Yi Hu*, Yongki Lee, Shijun Zheng
Georgia Southern University
yihu@georgiasouthern.edu

In this talk, we will introduce the nonlinear Schrödinger equation with a Stark potential and a linear damping. In particular, we study the problem of singularity formation in the mass-critical case, and we will characterize the threshold for global existence and finite time blow-up phenomenon. We will also present the log-log blow-up speed that the solution flow admits for certain initial data above the ground state. In addition, we will give a concentration compactness description for the limiting behavior of blow-up solutions.

Spatiospectral localization and band shape for Zernike polynomials

Christian Gerhards, Xinpeng Huang*
Central South University

xinpeng.huang@extern.tu-freiberg.de

As an orthogonal system on unit balls, Zernike polynomials serve as useful tools for processing signals defined on ball-like geometries. Consequently, they find widespread applications in optical engineering, as well as in bio- and geophysical modeling. In this work, we will address a related Slepian-type spatial-spectral concentration problem, where the bandlimit is interpreted as the span of a predetermined subset of Zernike polynomials. In particular, via numerical experiments, we demonstrate that the coupling of the radial and angular degrees of Zernike polynomials (which we conceptualize as the band shape) could play a crucial role in characterizing the bimodal eigenvalue distribution (also known as the Shannon number and the 2WT theorem) of the concentration operators.

Artificial Intelligence Methods for Numerical Solutions of Partial Differential Equations

Ionut E. Iacob
Georgia Southern University
ieiacob@georgiasouthern.edu

The advance of artificial intelligence (AI) methods has become ubiquitous and it is expected to advance at a fast pace in the near future. Given the huge interests in advancing AIs methods to solve various problems and the tremendous hardware and software support it receives, AI assisted methods for solving engineering problems represented by Partial Differential Equations (PDEs) has attracted increasing interest at present. We present some new methodologies for computing numerical solutions of PDEs using AI methods. While the current AI based methods are more computational intensive than the traditional methods, it is expected that the new methods will help solving some more complex, previously unsolved problems in the near future.

Non-symmetric SICs over finite fields

Joseph W. Iverson*, Dustin G. Mixon
Iowa State University
jwi@iastate.edu
https://www.jwiverson.com

Gerzon's bound asserts that if there are n equiangular lines in $\mathbb{C}^d$, then $n \leq d^2$; if equality holds, then the lines are said to form a (complex) *SIC*. So far, there are only finitely dimensions in which complex SICs are known to exist, and it is a major open problem to determine if they exist in every dimension. Zauner's conjecture claims (1) they do, and (2) the lines can be chosen as an orbit of a Weyl-Heisenberg group. So far, every known complex SIC arises this way.

In this talk, we consider SICs in unitary geometries over finite fields, where Gerzon's bound is still valid. Our main result is the following: if there exists a real Hadamard matrix of order d, and if $d-8$ is divisible by a prime $p = 3 \bmod 4$, then there exists a SIC in a d-dimensional unitary geometry over $\mathbb{F}_{p^2}$. Remarkably, many of the resulting SICs can **not** be chosen as an orbit of Weyl-Heisenberg group, or in fact, of any group at all. This contrasts with all previously known SICs.

Compatible Orthobiangular Tight Frames

Matthew Fickus, John Jasper*, Tyler J. Myers
Air Force Institute of Technology
jjasper30@gmail.com

The title refers to certain nice collections of tight frames that can be "melded" together to form an equiangular tight frame (ETF). This concept generalizes several known constructions of ETFs, and it produces several new infinite families of ETFs.

Realizations of equilibrium soliton condensates

Manuela Girotti, Robert Jenkins*, Ken McLaughlin
University of Central Florida
robert.jenkins@ucf.edu

We consider the primitive potential model of a soliton gas proposed by Zakharov, Zakharov and Dyachenko in 2016. Previous works have considered reductions of the full primitive potential model in which half of the data is set to zero. Such reductions describe soliton gases far from uniform. In this talk we will describe very recent work on the full primitive potential model and show that the solutions encoded are (nearly) uniform.

Hidden Structures associated with Symmetric Freud weights

Peter Clarkson, Kerstin Jordaan*, Ana Loureiro
University of South Africa
jordakh@unisa.ac.za

This talk presents a detailed study of orthogonal polynomials associated with exponential weights on the real line, in particular the symmetric Freud weight

$$\omega(x;t,\tau) = \exp\left\{-x^6 + \tau\, x^4 + t\, x^2\right\}, \qquad x \in \mathbb{R}$$

with τ, t parameters, where the recurrence coefficients β_n satisfy a nonlinear discrete equation linked to the second member of the discrete Painlevé I hierarchy. The recurrence coefficients have been studied in the context of Hermitian one-matrix models and random symmetric matrix ensembles with researchers in the 1990s observing "chaotic, pseudo-oscillatory" behaviour of the recurrence relation coefficients. More recently, this "chaotic phase" was described as a dispersive shockwave in a hydrodynamic chain. We analyse the behavior of these coefficients through discrete and differential-difference equations, asymptotic expansions, and numerical methods across parameter regimes. The moments of the weight are also examined, satisfying linear differential equations and recursive relations, with closed-form expressions available in certain special cases.

On a Welch Bound for Frames and Equiangular Lines over Finite Fields

Ian Jorquera*, Emily J. King
Colorado State University
ian.jorquera@colostate.edu
jorqueraian.github.io/

This talk concerns frames and equiangular lines over finite fields. We find a necessary and sufficient condition for systems of equiangular lines over finite fields to be equiangular tight frames (ETFs). As is the case over subfields of $\mathbb{C}$, it is necessary for the Welch bound to be saturated, but there is an additional condition required involving sums of triple products. We also prove that similar to the case over $\mathbb{C}$, collections of vectors are similar to a regular simplex essentially when the triple products of their scalar products satisfy a certain property. Finally, we investigate switching equivalence classes of frames and systems of lines focusing on systems of equiangular lines in finite orthogonal geometries with maximal incoherent sets, drawing connections to combinatorial design theory.

Exceptional Orthogonal Polynomials, KP Wave Functions, and Calogero-Moser Particles

Alex Kasman*
College of Charleston
kasmana@cofc.edu
http://kasmana.people.cofc.edu

Surprisingly, the generating functions of Exceptional Hermite Polynomials are wave functions of the KP Hierarchy. This unexpected connection between orthogonal polynomials and soliton theory can be used to answer open questions regarding the Exceptional Hermites and to write them in terms of the Lax matrices of an integrable particle system. This talk will review these previously published results and also briefly address ongoing research into *why* they are true and to what extent they can be generalized. (Joint work with Rob Milson and Luke Paluso.)

Cauchy Integral Formula for Fuchsian Groups

Alexander Kheifets*
University of Massachusetts Lowell
Alexander_Kheifets@uml.edu

The famous Hasumi's "Direct Cauchy Theorem" property of a Fuchsian group comes from the uniformization of the classical Cauchy Integral Formula for multiply connected domains. In this talk we discuss the formula that comes form the uniformization of the Cauchy Integral Formula for the multiply connected domains differentiated several times.

Some Trigonometric Identities Associated to the Roots of Unity

Michael K.-H. Kiessling*
Rutgers University
miki@math.rutgers.edu

Some years ago, based on earlier work by Brauchart, Hardin, and Saff, it was shown by Brauchart that when $s = 2m$ with m any natural number, then the total Riesz s-energy of the N-th roots of unity equals N times a polynomial in N of degree $2m$. In this talk it is proved that these polynomials factor into $N^2 - 1$ times a convex polynomial in N of degree $2m - 2$. As a spin-off, when $s = 2m$ the pair-averaged Riesz s-energy of the N-th roots of unity is a polynomial in N of degree $2m - 1$ that is strictly convex in N when $m > 1$. N.B.: Due to a scheduling conflict for M.K.-H.K., Johann Brauchart has kindly agreed to present this talk.

Introduction to Symmetric Subspace Configurations

Emily J. King*
Colorado State University
emily.king@colostate.edu
https://www.math.colostate.edu/~king/

In this introductory talk for the Minisymposium on Symmetric Subspace Configurations, we will explore the theory and applications of symmetric subspace configurations. Group actions may be leveraged to create subspace configurations which are optimal in frame theory and quantum information theory, with equiangular Fourier frames and symmetric, informationally complete, positive operator-valued measures (SIC-POVMs) being two of the most well-known examples. In machine learning, highly symmetric vectors emerge when training neural networks under certain regimes (neural collapse), while orbits of group actions may be leveraged to perform classification tasks (group-invariant max filtering, invariant/equivariant neural networks).

A constructive approach to Zauner's conjecture via the Stark conjectures

Marcus Appleby, Steven T. Flammia, Gene S. Kopp*
Louisiana State University
kopp@math.lsu.edu
gskopp.com

We propose a construction of d^2 complex equiangular lines in $\mathbb{C}^d$, also known as SICs or SIC-POVMs, which were conjectured by Zauner to exist for all d. The construction gives a putatively complete list of SICs with Weyl–Heisenberg symmetry in all dimensions $d > 3$. Specifically, we give an explicit expression for an object that we call a ghost SIC, which is constructed from the real multiplication values of a special function and which is Galois conjugate to a SIC. The special function, the Shintani–Faddeev modular cocycle, is more precisely a tuple of meromorphic functions indexed by a congruence subgroup of $\mathrm{SL}_2(\mathbb{Z})$. We prove that our construction gives a valid SIC in every case assuming two conjectures: the order-1 abelian Stark conjecture for real quadratic fields and a special value identity for the Shintani–Faddeev modular cocycle. The former allows us to prove that the ghost and the SIC are Galois conjugate over an extension of $\mathbb{Q}(\sqrt{\Delta})$ where $\Delta = (d+1)(d-3)$, while the latter allows us to prove idempotency of the presumptive fiducial projector. We provide computational tests of our SIC construction by cross-validating it with known solutions, particularly the extensive work of Scott and Grassl, and by constructing four numerical examples of nonequivalent SICs in $d = 100$, three of which are new. We further consider rank-r generalizations called r-SICs given by maximal equichordal configurations of r-dimensional complex subspaces. We give similar conditional constructions for r-SICs for all r, d such that $r(d-r)$ divides (d^2-1). Finally, we study the structure of the field extensions conjecturally generated by the r-SICs. If K is any real quadratic field, then either every abelian Galois extension of K, or else every abelian extension for which 2 is unramified, is generated by our construction; the former holds for a positive density of field discriminants.

Switchable Constraints: A New Approach to Minimizing Error in GPS

Nikolas Koutroulakis*
Georgia Southern University
nk03707@georgiasouthern.edu

Switchable constraints are a technique for minimizing measurement error. They were originally implemented in factor graphs for SLAM algorithms in the field of robotics. During optimization, switch nodes assign weights to factors in a factor graph based on how "trustworthy" they are. Factors producing larger residuals are filtered out as their weight approaches 0, facilitating a more resolute convergence. This allows for a precise estimation in the face of sparse data or large measurement faults. Factor graphs are also used for estimating the position of a receiver using satellite data. It is therefore natural to ask whether switchable constraints are as effective for GPS as they are for robotics.

The current default algorithm for detecting and minimizing measurement faults in satellites is known as Random Autonomous Integrity monitoring (RAIM). While RAIM is useful in specific circumstances, it cannot handle large volumes of error well. Even small measurement faults can have a large negative impact on its performance when introduced by spoofing or jamming. Switchable constraints minimizes faults *during* optimization. This results in more accurate estimations, even in the face of adversarial interference. We present results from multiple simulations performed during a research internship at the Air Force Institute of Technology. These simulations were generated by extracting data from TLE files for STARLINK satellites, and using that data to imitate realistic scenario where satellites are using to estimate the position of a receiver. The factor graphs were constructed using the GTSAM package (Georgia Tech Smoothing and Mapping) which specializes in the creation of factor graphs. These results demonstrate the potential for switchable constraints to be a powerful tool for minimizing large volumes of measurement faults in satellite data.

Equilibrium Measures on a Riemann Surface and Random Tilings

Arno Kuijlaars
Katholieke Universiteit Leuven

arno.kuijlaars@kuleuven.be
https://perswww.kuleuven.be/∼u0017946/index.html

Ed Saff is well known for his many contributions to equilibrium measures in external fields in the complex plane. Many results carry over to a Riemann surface provided one replaces the logarithmic kernel by the so-called bipolar Green's kernel.

The motivation to develop this more general theory comes from random tiling models with periodic weightings. An equilbrium problem with external field plays a crucial role in the asymptotic analysis of random lozenge tilings of a hexagon.

Orthogonal polynomials in the spherical ensemble with two insertions

Sung-Soo Byun, Peter Forrester, Arno Kuijlaars and Sampad Lahiry*,
KU Leuven and University of Melbourne
sampad.lahiry@kuleuven.be

We consider planar orthogonal polynomials $P_{n,N}$ (where $\deg P_{n,N} = n$) with respect to the weight

$$\frac{|z-w|^{2NQ_1}}{(1+|z|^2)^{N(1+Q_0+Q_1)+1}}, \quad (Q_0, Q_1 > 0)$$

in the whole complex plane. With $n, N \to \infty$ and $N - n$ fixed, we obtain the strong asymptotics of the polynomials, and asymptotics for the weighted L^2 norm and the limiting zero counting measure. These results apply to the pre-critical phase of the underlying Coulomb gas system, when the support of the equilibrium measure is simply connected. Our method relies on specifying the mother body of the two-dimensional potential problem. It relies too on the fact that the planar orthogonality can be rewritten as a non-Hermitian contour orthogonality. This allows us to perform the Deift-Zhou steepest descent analysis of the associated 2×2 Riemann-Hilbert problem.

Evolution of time-fractional stochastic hyperbolic diffusion equations on the unit sphere

Tareq Alodat, Quoc Thong Le Gia*
University of New South Wales, Sydney, Australia
qlegia@unsw.edu.au
https://research.unsw.edu.au/people/dr-quoc-thong-le-gia

In this work, we examine the temporal evolution of a two-stage stochastic model for spherical random fields. The model is based on a time-fractional stochastic hyperbolic diffusion equation, which describes the evolution of spherical random fields on the two-dimensional unit sphere over time. The diffusion operator incorporates a time-fractional derivative in the Caputo sense.

In the first stage, we consider a homogeneous problem, where an isotropic Gaussian random field on the unit sphere serves as the initial condition. In the second stage, the model transitions to an inhomogeneous problem driven by a time-delayed Brownian motion on the sphere. The solution is expressed as a truncated series of real spherical harmonics up to a certain degree.

An analysis of truncation errors reveals their convergence behavior, showing that convergence rates depend on the decay of the angular power spectra of both the driving noise and the initial condition.

Polynomial-to-Polynomial Mapping of Weighted Szegő Projections on Ellipses

Alan Legg*
Purdue University Fort Wayne
leggar01@pfw.edu

We explain how on planar ellipses there is a weighted Szegő projection taking polynomials to polynomials without increasing degree. This is reminiscent of the Khavinson-Shapiro conjecture (that only ellipses have the property that the solution to the Dirichlet problem for the Laplacian with polynomial boundary data is polynomial). In earlier work it was shown that the conjecture is related to polynomial-to-polynomial mapping of the Bergman projection.

On the single layer potential ansatz for the system of thermoelasticity with microtemperatures

Vita Leonessa*, Angelica Malaspina
Department of Engineering, University of Basilicata
vita.leonessa@unibas.it
http://docenti.unibas.it/site/home/docente.html?m=004131

In this talk we consider the Dirichlet problem in the context of the linear equilibrium theory of thermoelasticity with microtemperatures. In particular, we deal with the problem of the representability of the solutions by means of a single-layer potential. The main result concerns the solvability of a boundary integral equation of the first kind. Such a result is obtained by using the theories of differential forms and reducible operators.

All the results presented are taken from a recently submitted paper entitled "On the Dirichlet problem for the system of thermoelasticity with microtemperatures", written in collaboration with Prof. A. Malaspina of the University of Basilicata.

Applications of equal area sphere partitions to the mathematics of planet Earth

Paul Leopardi*
ACCESS-NRI, The Australian National University
paul.leopardi@anu.edu.au
https://sites.google.com/site/paulleopardi

Equal area sphere partitions have proved useful to the approximate solution of some problems on the unit sphere. Various authors have adapted sphere partitioning and related techniques to problems arising in application areas such as climate science, numerical weather prediction, geology, geophysics, and the visualization of geographical data. This talk examines some of these problems and the ways in which the techniques have been adapted to solve them.

The Askey-Wilson Operator on Bernstein Spaces of Entire Functions

Xin Li*
University of Central Florida
xin.li@ucf.edu

This talk presents joint work with Rajitha Ranasinghe on Askey-Wilson operators acting on Bernstein spaces—spaces of entire functions of exponential type whose restrictions to the real line are bounded. Since Askey-Wilson operators serve as discrete analogues of differentiation, we extend several classical results concerning polynomials and their derivatives to these spaces. In particular, we discuss recent advances, including a Riesz-type interpolation formula, a Bernstein-type inequality, and an equivalence result to sampling theorems tailored to this setting.

Inverse Problems in Magnetic Microscopy Analysis of Geological Samples

Eduardo A. Lima*
Massachusetts Institute of Technology
limaea@mit.edu

Igneous rocks become magnetized when they are formed and cool down in the presence of a magnetic field. Such records of ancient fields that may have existed on planets, moons, and other planetary bodies could potentially be preserved over geologic time scales, providing invaluable information on the formation and evolution of the solar system. However, magnetization is a physical property that cannot be measured directly except at a very thin surface layer. To overcome this difficulty, the magnetic field produced by a magnetized geological sample is mapped on a planar grid above the sample using magnetic microscopes. An ill-posed inverse problem is then solved to obtain either the magnetization distribution within the sample or its integral over the sample volume (the net magnetic moment). We discuss different approaches for estimating magnetization distributions and net moments from magnetic microscopy measurements considering the limitations introduced by instrumentation and experimental constraints.

Generalization Analysis of Transformers in Distribution Regression

Peilin Liu*, Ding-Xuan Zhou
University of Sydney
peilin.liu@sydney.edu.au

To investigate the foundational mechanisms of transformers, we introduce a transformer learning framework inspired by distribution regression, where distributions serve as inputs. This framework integrates a two-stage sampling process with natural language processing and offers a mathematical formulation of the attention mechanism, termed the attention operator. We demonstrate that, due to the advantages of the attention operator, transformers provide a robust approximation to functionals with more intricate structures than those managed by convolutional neural networks and fully connected networks. Furthermore, we derive a generalization bound within the distribution regression framework, accounting for both approximation error and sampling error. Through our theoretical findings, we also explore successful techniques emerging with large language models (LLMs), such as prompt tuning, parameter-efficient fine-tuning, and efficient scaling.

Nikishin systems on star-like sets. The case of unbounded and touching supports

A. López García, G. López Lagomasino*, M. Wajasat
Universidad Carlos III de Madrid
lago@math.uc3m.es
https://sites.google.com/view/lagomasino/

In this talk, we present work in progress concerning algebraic and analytic properties of multiple orthogonal polynomials associated with Nikishin systems defined on star-like sets and generated by measures with possibly unbounded and touching supports. In particular, we study the location and interlacing properties of the multiple orthogonal polynomials and their associated functions of the second type. We also investigate Stieltjes type results on the convergence of the corresponding Hermite-Padé approximants.

Non-standard energy problems for Green potentials in the complex plane

Abey López-García*, Alexander Tovbis
University of Central Florida
abey.lopez-garcia@ucf.edu
https://sciences.ucf.edu/math/lopez-garcia/research/

We consider several non-standard discrete and continuous Green energy problems in the complex plane and study the asymptotic relations between their solutions. In the discrete setting, we consider two problems; one with variable particle positions (within a given compact set) and variable particle masses, the other one with variable masses but prescribed positions. The mass of a particle is allowed to take any value in the range $0 \leq m \leq R$, where $R > 0$ is a fixed parameter in the problem. The corresponding continuous energy problems are defined on the space of positive measures μ with mass $\|\mu\| \leq R$ and supported on the given compact set, with an additional upper constraint that appears as a consequence of the prescribed positions condition. It is proved that the equilibrium constant and equilibrium measure vary continuously as functions of the parameter R (the latter in the weak-star topology). In the unconstrained energy problem we present a greedy algorithm that converges to the equilibrium constant and equilibrium measure. In the discrete energy problems, it is shown that under certain conditions, the optimal values of the particle masses are uniquely determined by the optimal positions or prescribed positions of the particles, depending on the type of problem considered.

Asymptotics of the optimal values of potentials generated by greedy energy sequences on the unit circle

Abey López-García*, Erwin Miña-Díaz
University of Central Florida
abey.lopez-garcia@ucf.edu
https://sciences.ucf.edu/math/lopez-garcia/research/

For the Riesz and logarithmic potentials, we consider greedy energy sequences $(a_n)_{n=0}^{\infty}$ on the unit circle S^1, constructed in such a way that for every $n \geq 1$, the discrete potential generated by the first n points $a_0, \ldots, a_{n-1}$ of the sequence attains its minimum value (say U_n) at a_n. We obtain asymptotic formulae that describe the behavior of U_n as $n \to \infty$, in terms of certain bounded arithmetic functions with a doubling periodicity property. After properly translating and scaling U_n, one obtains a new sequence (F_n) that is bounded and divergent. We find the exact value of $\liminf F_n$ (the value of $\limsup F_n$ was already known), and show that the interval $[\liminf F_n, \limsup F_n]$ comprises all the limit points of the sequence (F_n).

Minimal Riesz energy on the Grassmannian $\mathrm{Gr}_{2,4}$

Ujué Etayo, Pedro R. López-Gómez*
Universidad de Cantabria
lopezpr@unican.es
https://sites.google.com/view/pedrorlopezgomez

Over the past decades, the problem of equidistributing points on compact Riemannian manifolds through the minimization of certain repulsive, pairwise interaction energies has garnered significant interest in the mathematical community. Prominent examples of these energies include the Riesz and logarithmic energies, which have been primarily studied on spheres and projective spaces. A natural generalization of projective spaces is the Grassmann manifold or Grassmannian $\mathrm{Gr}_{m,d}$, defined as the set of m-dimensional linear subspaces of $\mathbb{R}^d$. While the Grassmann manifold has received considerable attention in fields like information theory and computer vision, little is known about energy minimizers. In this talk, I will introduce the problem of minimizing the Riesz and logarithmic energies on the Grassmannian $\mathrm{Gr}_{2,4}$ and present, for the first time, upper and lower bounds for the minimal discrete energies on this space.

This is joint work with Ujué Etayo.

Numerical Aspects of Multiscale Approximation

Federico Lot*
Philipps-Universität Marburg
lotf@mathematik.uni-marburg.de

In many practical applications the need arises to construct high-fidelity models from discretely given data. Kernel methods are a popular choice as they can cope with unstructured point clouds. The main drawback is, however, that usually a dense linear system with high condition number has to be solved.

Kernel-based multiscale methods have proven to provide a remedy to those limitations. These methods are characterized by an appropriate scaling of a kernel and a hierarchical organization of the data. The resulting approximant is known to be accurate as well, but its computation is substantially more stable.

In this talk we will discuss both recent analytical understandings and novel approaches to the implementation of the kernel based multiscale method.

This work is funded by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) – Projektnummer 452806809.

Details can be found at the preprint: arXiv:2503.04914.

A free journey from higher order recurrence relations to the zero distribution

Ana Loureiro*
University of Kent, U.K.
a.loureiro@kent.ac.uk
https://blogs.kent.ac.uk/aloureiro/

The focus of this talk is on polynomial sequences satisfying a recurrence relation of order $r + 1 \geq 3$. These types of polynomial sequences are part of a larger collection, the Multiple Orthogonal Polynomials. This set of polynomials is orthogonal with respect to a vector of r measures, which equips them with a structure that becomes prone to applications. Several polynomial sets as these are known in the literature, but the list is far from being complete. The idea here is to describe the ratio asymptotics of these polynomials and their asymptotic limiting zero distribution. The discussion becomes more intricate and interesting when the recurrence coefficients have up to r different asymptotic behaviours, as many as the number of orthogonality measures. To enhance the analysis, we will incorporate in the party ideas and results from finite free probability.

A Selection of Ed's Saffires

Doron Lubinsky*
Georgia Institute of Technology
lubinsky@math.gatech.edu

Ed Saff has made seminal contributions to many topics, including rational approximation, orthogonal polynomials, weighted approximation, and potential theory. A small selection of his contributions will be discussed. In particular we will focus on his work on best rational and Padé approximation of the exponential function, and the Mhaskar-Saff theory and its ramifications.

Dynamics and analytic measures for higher-dimensional continued fractions

Anton Lukyanenko*
George Mason University

alukyane@gmu.edu

Regular continued fractions represent a real number as a descending fraction with integer coefficients, such as $3+1/(7+1/(15+\ldots))$, and have found applications ranging from Diophantine approximation in gear ratios to encoding geodesic flow on the modular surface. The associated Gauss map is a textbook example of an ergodic dynamical system, with a straightforward an invariant measure given by $\frac{1}{\log 2}\frac{dx}{1+x}$.

Generalizing the story to higher dimensions has been more difficult. A complex continued fraction was introduced in 1887 by Hurwitz and shown in 1976 by Nakada to be ergodic, with an invariant measure that is piecewise-Lipschitz on 12 pieces. Hensley showed in 2005 that the measure is in fact piecewise-analytic.

In this talk, I will provide an overview of higher dimensional continued fraction variants and describe some recent results on the search for ergodic continued fraction systems and associated measures.

Conditional regression for the Nonlinear Single-Variable Model

Yantao Wu, Mauro Maggioni*
Johns Hopkins University
mauromaggionijhu@icloud.edu
https://mauromaggioni.duckdns.org

Several statistical models for regression of a function F on $\mathbb{R}^d$ without the statistical and computational curse of dimensionality exist, for example by imposing and exploiting geometric assumptions on the distribution of the data (e.g., that its support is low-dimensional), or strong smoothness assumptions on F, or a special structure F. Among the latter, compositional models assume $F = f \circ g$ with g mapping to $\mathbb{R}^r$ with $r \ll d$, have been studied, and include classical single- and multi-index models and recent works on neural networks. While the case where g is linear is rather well-understood, much less is known when g is nonlinear, and in particular for which g's the curse of dimensionality in estimating F, or both f and g, may be circumvented.

We consider a model $F(X) := f(\Pi_\gamma X)$ where $\Pi_\gamma : \mathbb{R}^d \to [0, \text{len}_\gamma]$ is the closest-point projection onto the parameter of a regular curve $\gamma : [0, \text{len}_\gamma] \to \mathbb{R}^d$ and $f : [0, \text{len}_\gamma] \to \mathbb{R}^1$. The input data X is not low-dimensional and is far from γ, conditioned on $\Pi_\gamma(X)$ being well-defined. The distribution of the data, γ and f are unknown. This model is a natural nonlinear generalization of the single-index model, which corresponds to γ being a line. We propose a nonparametric estimator, based on conditional regression, and show that under suitable assumptions, the strongest of which being that f is coarsely monotone, it can achieve the *one-dimensional* optimal min-max rate for non-parametric regression, up to the level of noise in the observations, and be constructed in time $\mathcal{O}(d^2 n \log n)$. All the constants in the learning bounds, in the minimal number of samples required for our bounds to hold and in the computational complexity, are at most low-order polynomials in d.

Geometric Mean of Conformal Radii for Compact Sets Contained in a Simply conneted domain

Neranjan Marasinghe*, Alexander Solynin
Texas Tech University

nemarasi@ttu.edu

The well known results by G. Pólya and G. Szegö assert that the disk has the largest conformal radius among all simply connected domains of a fixed area and, the equilateral triangle and square have the largest conformal radii among all triangles with a given area and among all quadrilaterals with a given area, respectively. In this talk, I will discuss three theorems that extend these results to geometric mean of conformal radii over compact subsets with prescribed positive area, contained in a simply connected domain of a given area, in a triangle of a given area, and in a quadrilateral of a given area, respectively. In addition to that, the geometric mean of conformal radii of rectifiable arcs contained in a simply connected domain will be defined and few related inequalities will be introduced. This is a collaborative work with Dr. Alexander Solynin.

The Many Facets of Iterated Differentiation

Andrei Martínez-Finkelshtein*
Baylor University (Texas, USA), and University of Almería (Spain)
a_martinez-finkelshtein@baylor.edu

The behavior of critical points–zeros of derivatives–of polynomials has captivated mathematicians for centuries. In recent years, a deceptively simple problem has gained renewed attention: understanding the distribution of zeros in sequences of polynomials subjected to repeated differentiation. New results revealed surprising and profound links with diverse areas of analysis, including approximation theory, logarithmic potential theory in the complex plane, free probability, random matrix theory, and certain nonlinear partial differential equations. This talk offers a partial survey of key results and highlights some open problems that continue to drive current research.

Logarithmic energy of points on the sphere

Jordi Marzo*
Universitat de Barcelona
jmarzo@ub.edu

Let $\mathcal{E}(N)$ be the minimal discrete logarithmic energy of N points on the two-dimensional sphere. The asymptotic behavior of $\mathcal{E}(N)$ has been extensively studied, and it is known that the asymptotic expansion has a linear term cN for some constant c, for which upper and lower bounds are known. In this talk, we will show how to improve the currently known lower bound.

Vortex Shedding from Bluff Bodies: Conformal Mapping Approaches

Vijay Matheswaran*
Department of Aerospace Engineering, Wichita State University
vijay.matheswaran@wichita.edu

A Hybrid Potential Flow model for flow around bluff bodies in the sub-critical Reynolds number regime is presented. The solution uses elementary flow solutions, empirical data, and conformal mapping. By joining this with von Karman's model of a vortex wake, a complete solution is calculated. Experimental data from wind and water table results are discussed and used to validate the model. An overview of future modeling challenges of wall effects is also presented.

Solving an Equilibrium problem using Rational Approximation

Bernd Beckermann, Ana Matos*
Laboratoire Paul Painlevé, Département de Mathématiques, University of Lille, France
Ana.Matos@univ-lille.fr
https://pro.univ-lille.fr/ana-matos

We are interested in computing the unknown density of an equilibrium problem in logarithmic potential theory, where the support I of the measure is a union of m real compact and disjoint intervals $I_j = [a_j, b_j]$. More precisely, given I and $Q \in \mathcal{C}(\mathbb{R})$, find a (signed) measure μ of mass $\int d\mu = 1$ with $\text{supp}(\mu) \subset I$ and $F \in \mathbb{R}$ s.t.

$$\forall x \in I : \quad U^\mu(x) + Q(x) = F,$$

with logarithmic potential $U^\mu(x) = \int \log(\frac{1}{|x-y|})d\mu(y)$. We will show that this problem is equivalent to solving a system of singular integral equations with Cauchy kernels. After fixing the functional spaces where we search for the solution , we obtain a theorem of existence and unicity. We then develop a general framework of a spectral method to compute an approximate solution , giving a complete error analysis. We will consider polynomial and rational approximations in orthogonal basis, showing the advantage of using rational interpolation when the intervals are close. Inspired by the third Zolotareff problem, the poles and the interpolation points are chosen in such a way that we can ensure small errors. Some numerical examples showing the good approximation results will be given.

Riesz Energy with External Fields: Spheres, Balls, and Annuli as minimizers

Djalil Chafaï, Ryan Matzke*, Ed Saff, Rob Womersley, Minh Vu
Vanderbilt University
ryan.w.matzke@vanderbilt.edu
https://www.ryanmatzke.com/

We will discuss the minimization of Riesz energies with external fields

$$I_{s,V}(\mu) = \int_{\mathbb{R}^d}\int_{\mathbb{R}^d}\Big(\frac{1}{s}\|x-y\|^{-s} + V(x) + V(y)\Big)d\mu(x)d\mu(y)$$

We are interested in how the choices of s (the strength of repulsion between "electrons") and V (the external field) affect the structure of the equilibrium measure. We will focus on radially symmetric external fields, i.e. $V(x) = v(\|x\|^2)$, and discuss combinations of s and v that lead to energy minimizers being supported on balls, spheres, and annuli.

The Mellin Transform in discrete-time setting

Mamadou Mboup*
University of Reims Champagne Ardenne, CReSTIC, France
Mamadou.Mboup@univ-reims.fr

The discrete-time analog of the scale-shift operator, $x(t) \to x(\alpha t), \alpha > 0$, is defined through the action of a subgroup of the hyperbolic Blaschke group. The Mellin transform is then derived in the discrete-time setting as the abstract Fourier transform for the Blaschke group. More precisely, given a causal discrete-time signal $\{x_n\}_{n\geqslant 0} \in \ell_2$, its Mellin transform with respect to a given time-shift m, is defined as $X_m(\omega) = \rho(\omega)\overline{P_m(\omega)} \sum_{n\geqslant 0} x_n P_n(\omega), \quad \omega \in \mathbb{R}$.

The atoms of the transform, which are the analogues of the exponential functions of the classical discrete-time Fourier transform, are fully characterized: The family $\{P_n\}$ is formed by polynomials of degree n with rational coefficients, orthonormal in $L_2(\mathbb{R})$ with respect to the weight $\rho(\omega) = 2\pi/\cosh(\pi\omega)$. The presentation will outline several properties of this family of polynomials, and thereby the Mellin transform and its inverse. In addition, some computational aspects for its practical implementation will be discussed.

Dynamical Sampling, Kadec's Theorem, and Carleson Frames

Brendan Miller
Northern Illinois University
bmiller14@niu.edu

Dynamical sampling is now a well-studied subfield of frame theory, which classically seeks to determine the necessary time-space samples to reconstruct an unknown signal evolving in time. While this question has been resolved for signals in finite-dimensional vector spaces, the case of infinite-dimensional Hilbert spaces remains quite mysterious. Recently, it has been shown that the so-called Carleson frames—frames of the form $\{T^n f\}_{n=0}^{\infty}$, where T is a diagonal operator—exhibit strong properties regarding which subsystems form frames. These subsystems are conjectured to also satisfy robust perturbation results. Motivated by this conjecture, we show that Kadec's 1/4 theorem generalizes to a perturbation result about dynamical frames, and we discuss the methods used in its proof.

Asymptotics and zeros of Bergman polynomials for domains with reflection-invariant corners

Erwin Miña-Díaz*, Aron Wennman
University of Mississippi
minadiaz@olemiss.edu

We present new results on the strong asymptotic behavior and limiting zero distribution of polynomials $(p_n)_{n=0}^{\infty}$ that are orthogonal over a domain D with piecewise analytic boundary. More specifically, D is assumed to have the property that conformal maps φ of D onto the unit disk extend analytically across the boundary L of D, and that φ' has a finite number of zeros $z_1, \ldots, z_q$ on L. The boundary L is then piecewise analytic with corners at the zeros of φ'. We prove that a Carleman-type strong asymptotic formula for p_n holds outside a certain compact set K that contains each corner of L but otherwise sits entirely inside D. Near each corner, K consists of an analytic arc departing from the corner. As $n \to \infty$, the zeros of p_n accumulate on K and every boundary point of K is a zero limit point.

Universality in the Small-Dispersion Limit of the Benjamin-Ono Equation

Elliot Blackstone, Peter Miller, Matthew D. Mitchell*
University of Central Florida
matt.mit@ucf.edu

We examine the solution of the Benjamin-Ono Cauchy problem for rational initial data in three types of double-scaling limits; the dispersion is taken to zero while simultaneously the independent variables either approach a point on one of the two branches of the caustic curve of the inviscid Burgers' equation, or approach the critical point where the branches meet. Our method involves steepest descent analysis on contour integrals appearing in an explicit representation of the solution of the Cauchy problem. Our results reveal universal limiting profiles in each case that are independent of details of the initial data.

Optimal Arrangements of $2d$ Lines in $\mathbb{C}^d$

Joseph W. Iverson, John Jasper, Dustin G. Mixon*
The Ohio State University
mixon.23@osu.edu

An equiangular tight frame is a special kind of matrix, and when it exists, its columns represent an arrangement of lines through the origin that maximizes the minimum interior angle. In this talk, we pose the $d \times 2d$ conjecture: there exists a $d \times 2d$ equiangular tight frame for every dimension d. We also construct two new infinite families of such equiangular tight frames, we identify a third construction that conjecturally applies for all d, and we show that the conjecture holds whenever d is at most 162.

Asymptotics of Real Solutions of the sinh-Gordon PIII

Alexander R. Its, Kenta Miyahara*, Maxim L. Yattselev
Indiana University Indianapolis
kemiya@iu.edu
https://sites.google.com/view/kentaswebsite

We present our recent study on the asymptotic analysis of real-valued solutions of the sinh-Gordon reduction of the Painlevé III equation (sinh-Gordon PIII, in short) on the positive real line $x \to 0^+$ and $x \to +\infty$. Since the sinh-Gordon PIII is an isomonodromic equation of a Lax pair, we can parametrize real solutions using monodromy parameters. For both asymptotics at 0 and ∞, there exist smooth and singular solutions, which we obtain by asymptotically solving the associated Riemann-Hilbert problems.

In this presentation, we primarily focus on the large x asymptotics. Additionally, we introduce a double-scaling parameter involving x and a monodromy parameter to describe the transition from smooth to singular solutions at ∞.

Asymptotic analysis for a class of planar orthogonal polynomials on the unit disc

Alfredo Deaño, Ken McLaughlin, Leslie Molag* and Nick Simm
Carlos III University of Madrid
lmolag@math.uc3m.es
https://sites.google.com/view/LeslieMolag

We carry out the asymptotic analysis of a class of polynomials $p_n(z)$, $z \in \mathbb{C}$, orthogonal with respect to the planar measure

$$d\mu(z) = (1 - |z|^2)^{\alpha-1}|z - x|^{\gamma}\mathbf{1}_{|z|<1}d^2z$$

where d^2z is the two dimensional area measure, α is a parameter that can grow with n, while $\gamma > -2$ and $x > 0$ are fixed. This measure arises naturally in the study of characteristic polynomials of non-Hermitian ensembles and generalises the example of a Gaussian weight that was recently studied by several authors. We obtain asymptotics in all regions of the complex plane and via an appropriate differential identity, we obtain the asymptotic expansion of the partition function. The main approach is to convert the planar orthogonality to one defined on suitable contours in the complex plane. Then the asymptotic analysis is performed using the Deift-Zhou steepest descent method for the associated Riemann-Hilbert problem.

Algebraic interpretation of discrete families of matrix orthogonal polynomials.

Quentin Labriet, Lucia Morey*, Luc Vinet
Centre de Recherches Mathématiques, Université de Montréal
lucia.morey@umontreal.ca

The goal of this talk is to give an algebraic interpretation of matrix-valued orthogonal polynomials. The construction is based on representations of a (q-deformed) Lie algebra $\mathfrak{g}$ into the algebra $\mathrm{End}_{M_n(\mathbb{C})}(M)$ of $M_n(\mathbb{C})$-linear maps over a $M_n(\mathbb{C})$-module M. Cases corresponding to the Lie algebras $\mathfrak{su}(2)$ and $\mathfrak{su}(1,1)$ as well as to the q-deformed algebra $\mathfrak{so}_q(3)$ at q a root of unity are presented which lead to matrix analogues of Krawtchouk, Meixner and discrete Chebyshev polynomials.

On the Global Optimality of Fibonacci Lattices in the Torus

Nicolas Nagel*
Chemnitz University of Technology
nicolas.nagel@mathematik.tu-chemnitz.de
https://www-user.tu-chemnitz.de/~ninag/person.html

It has been observed for a long time that so called Fibonacci lattices perform particularly well concerning periodic L_2-discrepancy, diaphony and the performance of quasi-Monte Carlo integration. In some cases it is even known that they are the global minimizers among all point sets of a given size in the torus. Inspired by this and the similarities among the above three notions of measuring "uniformity", we introduce *tensor product energies* in the torus and pose the question of the existence of point sets which are simultaneous minimizers for a large class of potentials, a concept also refered to as *universal optimality.* By applying linear programming bounds on the torus we achieve the global optimality of the 5-point Fibonacci lattice for the worst case error of quasi-Monte Carlo integration over certain parametrized, periodic Sobolev spaces H_p^2 of mixed smoothness $s = 1$ for a whole range of parameters p. Furthermore we prove the universal optimality of the 3-point rational lattice in any dimension for a large class of potentials fulfilling natural, geometric assumptions.

This talk aims to give a selfcontained overview on the involved methods with the hopes of initiating further interest in this area to eventually obtain the global optimality of arbitrary Fibonacci lattices for a large class of potentials.

Extrapolation of Magnetic Field Using Reproducing Kernel

Takaaki Nara* and Manato Yo
The University of Tokyo
takaakinara@g.ecc.u-tokyo.ac.jp

In the inverse problem of magnetoencephalography (MEG), current dipole sources inside the human brain are reconstructed from the measured magnetic field outside the head. However, due to its ill-posedness, estimating the number of dipoles is challenging. To overcome this difficulty, in this talk, we propose a novel approach to reconstruct the magnetic field at positions closer to the current source, aiming to obtain the magnetic field distribution more localized around the source. By defining a Hilbert space of the magnetic field that satisfies the Laplace equation with an appropriate norm, we can explicitly derive its reproducing kernel. Then, from a representer theorem, the magnetic field at a position closer to the source compared to sensor positions can be computed by a linear combination of the reproducing kernel. It is numerically shown that a magnetic field pattern generated by two dipoles can be calculated from the magnetic field data at actual sensor positions as if it originated from a single dipole.

Fast implementation of Generalized Koebe's iterative method

Khiy Lee, Ali Murid, Mohamed Nasser*, Su Yeak
Wichita State University
mms.nasser@wichita.edu

Let G be a given bounded multiply connected domain of connectivity $m + 1$ bounded by smooth Jordan curves. Generalized Koebe's iterative method is an iterative method for computing the conformal mapping from the domain G onto a bounded multiply connected circular domain obtained by removing $m - 1$ disks from a circular ring. A fast numerical implementation of this iterative method will be presented in this talk. The proposed method is based on using the boundary integral equation with the generalized Neumann kernel.

Subharmonic Kernels and Energy Minimizing Measures, with Applications to the Flat Torus

S.B. Damelin, Joel Nathe*
Purdue University
jnathe@purdue.edu

We study the minimization of the energy integral $I_K(\mu) = \int_\Omega \int_\Omega K(x,y)d\mu(x)d\mu(y)$ over all Borel probability measures μ, where (Ω, ρ) is a compact connected metric space and $K : \Omega^2 \to [0, \infty]$ is continuous in the extended sense. We focus on kernels K which are subharmonic, which we define so that the potential $U_K^\mu(x) = \int_\Omega K(x,y)d\mu(y)$ satisfies a maximum principle on $\Omega \setminus \operatorname{supp} \mu$. This extends the classical electrostatics minimization problem for logarithmic energy $\int_\Omega \int_\Omega \log\left(\frac{1}{||x-y||}\right)$, which is used heavily as a tool in approximation theory. Using properties of minimizing measures, we show that if the singularities of the subharmonic kernel K are such that K is regular, then K is positive definite, and μ is a minimizing measure if and only if its potential is constant (outside of a small exceptional set).

We then apply this result to group invariant kernels on compact homogeneous manifolds, in particular the d-dimensional flat torus $\mathbb{T}^d$. We use our results to see that the Riesz kernel $K_s(x,y) = \operatorname{sign}(s)\rho(x,y)^{-s}$ is minimized by σ (and thus positive definite) when $d > s \geq d-2$. Additionally, the positive definiteness gives us a condition which implies that the multivariate Fourier series of a function $f : [0,\pi]^d \to [0,\infty]$ has nonnegative coefficients.

Silent Sources of Spaces of Vector-Valued Sobolev Distributions

Masimba Nemaire
Université Bordeaux
masimba.nemaire@ihu-liryc.fr

We characterised all non-zero vector-fields $S \in [W^{-1,p}(\Omega)]^n$, $1 < p < \infty$, $n \geq 3$ whose potential, ϕ, linked to S by the elliptic problem $\nabla \cdot (M\nabla\phi) = \nabla \cdot S$, attains a constant value on each of the finitely many connected components of $\mathbb{R}^n \setminus \Omega$, where M a symmetric positive definite matrix. Our characterisation states that such S posses a Stokes decomposition and when such S are extended by zero to $\mathbb{R}^n$ their Stokes decomposition vanishes identically outside Ω. We also showed that given $S \in [W^{-1,p}(\Omega)]^n$ there is a unique $S_{nm} \in [W^{-1,p}(\Omega)]^n$ of minimum norm among all vector-fields that generate the same potential as S on $\mathbb{R}^n \setminus \Omega$ modulo constants. We showed that when Ω admits the Gauss divergence theorem there is a unique $h^* \in W^{2,q}(\Omega)$ such that $S_{nm}\langle S, \nabla h^*\rangle\Delta_q\nabla h^*$ where $q = \frac{p-1}{p}$ and Δ_q is the vector q-Laplacian hence each vector-field $S \in [W^{-1,p}(\Omega)]^n$ can be written as $S = \Delta v + \nabla\psi - \langle S, \nabla h^*\rangle\Delta_q\nabla h^*$ for unique $v \in [W^{1,p}(\Omega)]^n$ and $\psi \in L^p(\Omega)$. Finally, we showed that when Ω is Lipschitz, under certain circumstances it is possible to determine S_{nm} from ϕ on $\partial\Omega$.

Riesz equilibrium on a ball in the external field of a point charge

P. Dragnev, R. Orive*, E. B. Saff, F. Wielonsky
University of La Laguna
rorive@ull.edu.es

The Riesz energy minimization problem on a d-dimensional ball, with $d \geq 1$, in the presence of an external field created by a point charge above the ball in R^{d+1}, is studied, considering both the case of an attractive and a repulsive charge. The notions of balayage and signed equilibrium measure are important tools in the current analysis.

The Hörmander–Bernhardsson Extremal Function

A. Bondarenko, J. Ortega-Cerdà*, D. Radchenko, K. Seip
Universitat de Barcelona
jortega@ub.edu

We characterize the function φ of minimal L^1 norm among all functions f of exponential type at most π for which $f(0) = 1$. This function, studied by Hörmander and Bernhardsson in 1993, has only real zeros $\pm\tau_n$, $n = 1, 2, \ldots$. Starting from the fact that $n + \frac{1}{2} - \tau_n$ is an ℓ^2 sequence, established in an earlier paper of ours, we identify φ in the following way. We factor $\varphi(z)$ as $\Phi(z)\Phi(-z)$, where $\Phi(z) = \prod_{n=1}^{\infty}(1 + (-1)^n \frac{z}{\tau_n})$ and show that Φ satisfies a certain second order linear differential equation along with a functional equation either of which characterizes Φ. We use these facts to establish an odd power series expansion of $n + \frac{1}{2} - \tau_n$ in terms of $(n + \frac{1}{2})^{-1}$ and a power series expansion of the Fourier transform of φ, as suggested by the numerical work of Hörmander and Bernhardsson. The dual characterization of Φ can be seen as resulting from a commutation relation that holds more generally for a two-parameter family of differential operators, a fact that is used to perform high precision numerical computations. This family of differential operators also yields, via the zeros of their eigenfunctions, an abundance of summation formulas for computing $f'(0)$ for f in PW^1.

Gradient flow solutions for porous medium equations with nonlocal Lévy-type pressure

Guy Foghem, David Padilla-Garza*, Markus Schmidtchen
Einstein Institute of Mathematics, Hebrew University of Jerusalem
david.padilla-garza@mail.huji.ac.il
https://sites.google.com/nyu.edu/davidpadilla-garza/home

We study a porous medium-type equation whose pressure is given by a nonlocal Lévy operator associated to a symmetric jump Lévy kernel. The class of nonlocal operators under consideration appears as a generalization of the classical fractional Laplace operator. For the class of Lévy-operators, we construct weak solutions using a variational minimizing movement scheme. The lack of interpolation inequalities leads to significant technical obstacles that render our setting more challenging than the one for fractional operators.

Generalized Faulhaber's Formula with an Explicit Error Bound

Yong-Kum Cho, Young Woong Park*
Chung-Ang University
ywpark1839@gmail.com

In this work, we generalize Faulhaber's formula for $\sum_{k=1}^{n} k^p$ to arbitrary complex number p. An asymptotic series that resembles Faulhaber's formula is obtained, and we give an explicit bound of its remainder terms. The methodology turns out to be applicable to the partial sums of the Hurwitz zeta function as well. Additionally, a generalized Euler-Mascheroni constant is discussed in connection with the Riemann zeta function.

Poisson behavior for Coulomb Gases

David Padilla-Garza, Luke Peilen*, Eric Thoma
Temple University
luke.peilen@temple.edu
https://sites.temple.edu/lpeilen/

We are interested in the behavior of the microscopic point process for Coulomb gases at intermediate temperature regimes $\beta \downarrow 0$, $\beta N \to \infty$. The behavior at high temperatures $\beta N \simeq 1$ is well understood (cf. Lambert '21) for general Riesz gases and at intermediate temperatures for Gaussian β-ensembles (Benaych-George, Péché '15), but the intermediate temperature behavior for Coulomb and Riesz gases is not well understood. In our work, we establish convergence of the microscopic point process for the Coulomb gas in $d = 2, 3$ to a Poisson point process for a strictly larger intermediate temperature regime than was previously known. Our approach relies on a precise asymptotic description of the correlation functions and overcrowding estimates due to Thoma '25.

I will also discuss how many of these technical challenges can be circumvented in the case that the interaction kernel has an integrable Fourier transform; there, we can establish convergence to a Poisson point process more easily and in a wider temperature regime. This is joint work with David Padilla-Garza and Eric Thoma.

Relatively bounded and relatively trace class perturbations of dissipative operators. The trace formula

Vladimir V.Peller
St.Petersburg State University
peller@msu.edu

Let L and M be maximal dissipative operators such that $(M-L)(L+iI)^{-1}$ is a bounded (trace class) operator. In this case the operator $K = M - L$ is called a *relatively bounded* (*relatively trace class*) perturbation of L. A function f analytic in the open upper half-plane and continuous in the closed upper half-plane is called *relatively operator Lipschitz* if

$$\|f(M) - f(L)\| \leq \text{const}\, \|(M-L)(L+iI)^{-1}\|$$

whenever M and L are maximal dissipative operator such that $M - L$ a relatively bounded perturbation of L.

I am going to speak about properties of relatively operator Lipschitz functions applied to maximal dissipative operators. In particular, I am going to reveal an analogue of the Lifshits–Krein trace formula for a pair of maximal dissipative operators $\{L, M\}$ and a relatively operator Lipschitz function f:

$$\operatorname{trace}(f(M) - f(L)) = \int_{\mathbb{R}} f'(t)\xi(t)dt$$

where ξ is a spectral shift function, which satisfies the condition

$$\int_{\mathbb{R}} \frac{|\xi(t)|}{1 + |t|} dt < \infty.$$

The talk is based on my joint work with A.B. Aleksandrov.

The Scattering Transform for Geometric Deep Learning

Michael Perlmutter
Boise State University
mperlmutter@boisestate.edu
https://sites.google.com/view/perlmutma/home

The scattering transform is a mathematical model of convolutional neural networks (CNNs) introduced for functions defined on Euclidean space by Stephane Mallat. It differs from traditional CNNs by using predesigned, wavelet filters rather than filters which are learned from training data. This leads to a network which provably has desirable mathematical properties such as translation invariance and diffeomorphism stability. Moreover, in situations where the wavelets can be designed in correspondence to underlying physics, it can produce numerical results which rival state of the art CNNs. However, many data sets of interest have an intrinsically non-Euclidean structure and are better modeled as graphs or manifolds. This motivates us to construct geometric versions of the scattering transform using the spectral decompositions of Laplace-Beltrami operator and Graph Laplacian. We will discuss applications of these networks to a variety of geometric deep learning tasks and show that analogously to its Euclidean predecessor, the manifold scattering transform possesses desirable invariance and stability properties with respect to the actions of the isometry and diffeomorphism groups.

Sharp bounds for Neural Network Operators in Sobolev-Orlicz and Orlicz spaces with applications

Danilo Costarelli, Michele Piconi*
University of Perugia
michele.piconi@unipg.it

In this paper, we establish sharp bounds for a family of Kantorovich-type neural network operators within the general frameworks of Sobolev-Orlicz and Orlicz spaces. We obtain both strong estimates (with respect to the Luxemburg norm) and weak estimates (in terms of the modular functional) using different approaches.

First, we carry out an asymptotic analysis in Sobolev-Orlicz spaces. For strong estimates, we use φ-functions that are N-functions or satisfy the Δ'-condition. For weak estimates, we introduce a suitable subspace $\mathcal{W}^{1,\varphi}(I)$, embedded in the Sobolev-Orlicz space $W^{1,\varphi}(I)$ and modularly dense in $L^{\varphi}(I)$, which allows us to obtain results for a broader class of φ-functions, including those that do not satisfy the Δ_2-condition. To extend the study to the whole Orlicz setting, we generalize a Sobolev-Orlicz density result in the Luxemburg norm, originally given by H. Musielak using Steklov functions, and provide a weaker (modular) counterpart. Finally, we investigate the relationship between weak and strong Orlicz Lipschitz classes, obtaining qualitative results on the convergence rates of the operators.

Moreover, also some applications of the above results will be presented.

Acknowledgements The author M. Piconi has been supported within the project: PRIN 2022 PNRR: “RETINA: REmote sensing daTa INversion with multivariate functional modeling for essential climAte variables characterization”, funded by the European Union under the Italian National Recovery and Resilience Plan (NRRP) of NextGenerationEU, under the MUR (Project Code: P20229SH29, CUP: J53D23015950001).

Wiener-Hopf Factorizations and Matrix-valued Orthogonal Polynomials

Arno B.J. Kuijlaars, Mateusz Piorkowski*
KTH Royal Institute of Technology, Stockholm
mateuszp@kth.se
https://sites.google.com/view/mateuszpiorkowski/home?pli=1

Matrix-valued orthogonal polynomials appear naturally in certain random tiling models as shown by Duits and Kuijlaars using Lindström–Gessel–Viennot theory. Similar techniques were later used by Berggren and Duits to relate Wiener–Hopf factorizations with certain random infinite-path models

As shown in a recent collaboration with Kuijlaars, both methods turn out to be equivalent in the special case of the doubly periodic Aztec diamond tiling model. The resulting matrix-valued orthogonal polynomials satisfy a contour orthogonality with respect to a rational matrix-valued weight function and can be expressed in terms of Wiener–Hopf factors. We will review the rather unusual properties that polynomials of this type satisfy. In particular, they possess apart from the usual Fokas–Its–Kitaev Riemann–Hilbert formulation a second Riemann–Hilbert formulation, of lower matrix dimension, which is directly related to Wiener–Hopf factorizations.

Mathematical Ideas in Lattice Based Public Key Cryptography

Jill Pipher
Brown University
jill_pipher@brown.edu

A lattice-based public key cryptosystem (PKC) makes use of the computationally hard problem of solving the shortest/closest vector problem in an integer lattice. NTRU is a lattice-based PKC presented at Crypto '96, published in ANTS in 1998. With NTRU, we (Hoffstein, Pipher, Silverman) introduced the concept of ideal (cyclic) lattices, enabling a fast and efficient encryption scheme which is also resistant to speed ups afforded by quantum computing. In this talk, I'll explain the algorithm and its associated lattice problems as well as discuss some of the mathematical developments in lattice based cryptography over the past 30 years. No previous knowledge of public key cryptography is assumed.

p-Electric duality and modulus

Nathan Albin, Joan Lind, Pekka Pankka, Pietro Poggi-Corradini*
Kansas State University
pietro@ksu.edu
https://www.math.ksu.edu/~pietro/

We establish p-electric duality on networks, by introducing non-linear analogs of the Dirichlet and Thomson problems. We then relate these notions to modulus of connecting families of paths on graphs. And finally we establish discrete non-linear analogs of the Cauchy-Riemann equations and use them to prove a convergence result from the discrete to the continuum.

Optimization Meets Tverberg

Alexander Polyanskii*
Emory University
apolian@emory.edu
http://polyanskii.com

In this talk, I will discuss the so-called tight colorful no-dimensional Tverberg theorem, its proof based on two optimization steps, and its connection to monotone transportation.

A special case of this result states that for any n red and n blue points in Euclidean d-space, there exists a perfect red-blue matching (a pairing of each red point with exactly one blue point) $\mathcal{M}$ such that the balls whose diameters are the segments connecting the matched pairs in $\mathcal{M}$ share a common point.

Field extrapolation issues in the inverse magnetisation problem

Dmitry Ponomarev*
Centre Inria d'Université Côte d'Azur
dmitry.ponomarev@inria.fr

We consider an inverse magnetisation problem in the paleomagnetic context. Namely, we are dealing with the situation when the remanent magnetisation is to be characterised (reconstructed under some unicity constraint) from measurements of its magnetic field available over a planar region in vicinity of a magnetised rock sample. Since the measurement region is typically very small while the impact of its size on the magnetisation reconstruction results are drastic, the field extrapolation issue is an important one. While both inverse magnetisation and field extrapolation problems are ill-posed due to lack of continuous dependence on the measured data, the latter has a unique solution whenever the field in the measured region is exactly known. We explore different approaches for stable construction of the field extrapolant and illustrate the results numerically.

D-optimal designs in sampling recovery and function discretization problems

Kateryna Pozharska
Institute of Mathematics, NAS of Ukraine; Chemnitz University of Technology, Germany
pozharska.k@gmail.com
https://www-user.tu-chemnitz.de/∼pozk/index.html#cv

We will discuss the problems of optimal function recovery and discretization. Namely, using a procedure of maximization of the determinant of a certain Gramian matrix with respect to points and weights (i.e., finding the optimal D-design), one obtains a discrete measure with at most $n^2 + 1$ atoms, which accurately subsamples the L_2-norm of complex-valued functions contained in a given n-dimensional subspace.

This approach can as well be used for the reconstruction of functions from general RKHS in L_p where one only has access to the most important eigenfunctions. The general results apply to the d-sphere or multivariate trigonometric polynomials on the torus $\mathbb{T}^d$ spectrally supported on arbitrary finite index sets $I \subset \mathbb{Z}^d$.

Joint work with Felix Bartel (UNSW Sydney), Lutz Kämmerer (TU Chemnitz), Martin Schäfer (TU Chemnitz) and Tino Ullrich (TU Chemnitz).

Real zeros of random orthogonal polynomials

Igor Pritsker*
Oklahoma State University
igor.pritsker@okstate.edu
https://math.okstate.edu/people/igor

We study the expectation and variance for the number of real zeros of random linear combinations of orthogonal polynomials, which are usually referred to as random orthogonal polynomials. The spanning polynomials are orthonormal with respect to a deterministic measure supported on the real line, while the coefficients are independent and identically distributed real random variables. In this talk, we consider orthogonality measures with compact support on the real line, and those supported on the whole real line. We discuss asymptotic results for the expectation and variance, emphasizing their dependence on the orthogonal polynomial basis.

Hard Thresholding Hyperinterpolation Over General Regions

Congpei An, Jiashu Ran*
University of Alberta
jran@ualberta.ca

This paper proposes a novel variant of hyperinterpolation, called hard thresholding hyperinterpolation. This approximation scheme of degree n leverages a hard thresholding operator to filter all hyperinterpolation coefficients, which approximate the Fourier coefficients of a continuous function by a quadrature rule with algebraic exactness $2n$. We prove that hard thresholding hyperinterpolation is the unique solution to an ℓ_0-regularized weighted discrete least squares approximation problem. Hard thresholding hyperinterpolation is not only idempotent and commutative with hyperinterpolation, but also adheres to the Pythagorean theorem in terms of the discrete (semi) inner product. By the estimate of the reciprocal of Christoffel function, we present the upper bound of the uniform norm of hard thresholding hyperinterpolation operator. Additionally, hard thresholding hyperinterpolation possesses denoising and basis selection abilities akin to Lasso hyperinterpolation. To judge the L_2 errors of both hard thresholding and Lasso hyperinterpolations, we propose a criterion that integrates the regularization parameter with the product of noise coefficients and the signs of hyperinterpolation coefficients. Numerical examples on the sphere, spherical triangle and the cube demonstrate the denoising ability of hard thresholding hyperinterpolation.

Yet another connection between Riesz and Gaussian energies

Alexander Reznikov*, Jonathan Schillinger
Florida State University
areznikov@fsu.edu

A Riesz kernel can be expressed as an infinite convex combination of Gaussian kernels, which potentially implies some similarities between minimal Riesz and Gaussian energies. We discuss one of those connections: namely, we compare the limiting case of the s-Riesz energies, when the potential becomes less and less integrable, to the limiting case of (properly scaled) Gaussian energies, where the peak becomes more and more narrow.

Kernelbased Multigrid on Manifolds

Christian Rieger*
Philipps-Universität Marburg
riegerc@mathematik.uni-marburg.de
https://www.uni-marburg.de/de/fb12/arbeitsgruppen/numerik

In this talk, we will discuss recent progress on using a geometric multigrid method on manifolds combined with a kernel-based meshless approximation scheme for elliptic partial differential equations.

We will discuss the kernel based trial spaces. Here, we will especially focus on localized Lagrange functions. This approach was for instance discussed in detail in a paper entitled *A high-order meshless*

Galerkin method for semilinear parabolic equations on spheres by J. Künemund, F.J. Narcowich, J.D. Ward and H. Wendland from 2019.

We present theoretical analysis for an efficient numerical solver of the resulting linear system of equations. The use of multigrid methods is called for, as the condition number of this linear system is growing with the meshsize converging to zero.

Our new multigrid formulation constructs an iterative solver, where the norm of the iteration matrix is provably a contraction with norm independently of the meshsize.

This is based on joint work with T. Hangelbroek (University of Hawai'i –Mānoa). Please find more information here: Kernel Multi-Grid on Manifolds, Journal of Complexity, Special Issue on Parabolic PDEs, T. Hangelbroek, C. Rieger, 2024

The Efficient Length Sixteen Parametrized Wavelets

Dr. David Roach
Murray State University
droach@murraystate.edu

In this talk, we present an efficient parametrization for the length sixteen orthogonal wavelets based on the dilation coefficients of the trigonometric polynomial which satisfies the necessary conditions for orthogonality. Our approach improves upon the method of Schneid and Pittner, who introduced a general technique for constructing finite-length orthogonal wavelet parametrizations with n free parameters. While their method was applicable to any length, it led to an exponential increase—approximately 2^n—in the number of transcendental terms, making explicit representations impractical for longer wavelets.

In contrast, the parametrization presented here eliminates redundancy, reducing the number of transcendental terms to scale linearly with wavelet length. This improvement enables more efficient computation and practical implementation of the length sixteen wavelets while maintaining orthogonality.

Connection between generalized Krawtchouk polynomials and the fifth Painlevé equation

Galina Filipuk, Juan F. Mañas–Mañas, Juan J. Moreno–Balcázar, Cristina Rodríguez–Perales*
Universidad de Almería, Almeria (Spain)
crp170@ual.es

In this work we consider a generalization of Krawtchouk polynomials and investigate connection between certain auxiliary quantities involving their recurrence coefficients and the fifth Painlevé equation. For that purpose we make use of the iterated regularization of a differential system satisfied by these quantities. This approach leads to differential systems with simpler structure enabling a straightforward connection to the Painlevé equation. Furthermore, we show how iterative regularization allows us to obtain polynomial systems which results in decompositions of certain birational transformations.

Optimal Quantization via Riesz Maximum Mean Discrepancy

Elias Hess-Childs, Matthew Rosenzweig*, Sylvia Serfaty
Carnegie Mellon University
mrosenz2@andrew.cmu.edu
https://sites.google.com/view/rosenzweig/home

The problem of optimal quantization of a target probability measure μ consists of finding N points $x_1, \ldots, x_N \in \mathbb{R}^{\mathsf{d}}$ whose associated empirical measure $\frac{1}{N}\sum_{i=1}^{N}\delta_{x_i}$ "best approximates" μ. To quantify the approximation, one considers a metric between the empirical and target measures and minimizes over all possible point configurations. The *maximum mean discrepancy (MMD)*, whose square is the energy associated to a positive definite kernel of the difference between the two measures, is one such choice. MMDs associated to *Riesz kernels* $\frac{1}{\mathsf{s}}|x-y|^{-\mathsf{s}}$ for $\mathsf{s} \in (-2, 0)$ have attracted interest in the machine learning and statistics community, though to our knowledge their quantization properties have not been rigorously studied.

We prove upper and lower bounds for the minimal MMD that are matching in order as $N \to \infty$. Our proof leverages new potential truncation techniques developed in the modulated-energy approach to Riesz gases together with local Chebyshev-type cubatures of possible independent interest. Our method is robust and applies for a large class of kernels beyond Riesz. In particular, our work identifies a new scaling relation between the regularity of the MMD kernel and the large N decay of the minimal MMD value: the more regular the kernel, the faster the decay of the minimal MMD. Time permitting, I will also discuss work on the convergence of the associated gradient flow of the MMD to a minimizer.

Energy Asymptotics of the Gaussian Kernel

Alexander Reznikov, Jonathan Schillinger*
Florida State University
jschillinger@fsu.edu
www.math.fsu.edu/∼jschilli

In this talk we will outline some recent results for the asymptotics of the continuous energy functional, $\mathcal{I}_K[\mu, A]$, using a Gaussian interaction kernel $K_a = e^{-a||x-y||^2}$ as the parameter $a \to \infty$. We will consider the asymptotics for sets such as cubes, discs, arbitrary compact subsets $A \subset \mathbb{R}^n$ with Hausdorff dimension n, as well as self-similar fractal sets. After discussing the continuous asymptotics, we will introduce an avenue of current research into the discrete energy asymptotics with a proposed computation strategy for the second-order term involving linear programming and Mercer's theorem.

The Small Dispersion Limit of KdV

Kurt Schmidt*, Matt Mitchell, Robert Buckingham
University of Central Florida
kurt.schmidt@ucf.edu

We study the small-dispersion KdV equation $u_t + 6uu_x + \varepsilon^2 u_{xxx} = 0$ with initial data $u_0(x) = \operatorname{sech}^2(x)$. When $\varepsilon = \varepsilon_N = (N(N+1))^{-1/2}$, the solution takes the form of the Kay-Moses solution, known as the

N-soliton. These solutions are numerically difficult to compute for large N. In this talk, we analyze the asymptotic behavior as $N \to \infty$ using the Deift-Zhou nonlinear steepest descent method for Riemann-Hilbert problems.

Connections Between Frames with Rational Eigensteps and Semistandard Young Tableaux

Kylie Schnoor*
Colorado State University
kylie.schnoor@colostate.edu

We explore a correspondence between frames with rational eigensteps and semistandard Young tableaux (SSYT), via the relation assigning a Gelfand-Tseltin pattern to a frame via the frame's eigensteps. We will identify how certain key structures in SSYTs correlate with particular frame properties. For example, the weight of an SSYT yields the sequence of norms of any compatible frame. Additionally, this correspondence leads to a novel way to construct the eigensteps of a frame coming solely from the tableaux. We can further employ other combinatorial techniques to generate a "complement" SSYT. On the frame side, this corresponds to a tight frame's Naimark complement as well as to a generalization of the Naimark complement for non-tight frames.

Solving Boundary-value Problems on Curved Domains in 3D with Splines

Larry Schumaker*
Vanderbilt University
larry.schumaker@vanderbilt.edu
https://math.vanderbilt.edu/schumake/

In this talk we explore the use of the immersed penalized boundary method introduced in my paper J. Sci. Comp. **80(3)** (2019), 1369–1394 to solve second order boundary-value problems defined on 3D curved domains. Examples are presented for both trivariate tensor-product splines and for polynomial splines on tetrahedral partitions. The results show that the method is a very attractive alternative to currently available methods, including the recently developed IGA methods.

The Asymptotic Distribution of Riesz Energy

Raffaello Seri*
InsIDE Lab, University of Insubria
raffaello.seri@uninsubria.it
https://rseri.me

We consider the asymptotic distribution of the Riesz s-energy for a sample of N independent, uniformly distributed points on the surface of a d-dimensional hypersphere as N diverges. We identify three asymptotic regimes. In the first regime, both the mean and variance of the energy exist. In the second regime, only the mean exists. In the third regime, no integer moments exist. We characterize the asymptotic distribution in all three regimes and identify five special cases at their boundaries. Additionally, we highlight the connections between these results and Ed Saff's work on minimal Riesz energy point configurations.

Kernel Methods for Operator Learning on Manifolds

Varun Shankar
Kahlert School of Computing, University of Utah
shankar@cs.utah.edu
https://cs.utah.edu/ shankar

Operator learning is a promising surrogate modeling technique that involves learning the map from function spaces to function spaces. In this talk, I will present a kernel-based technique for operator learning that leverages the strengths of kernel approximation to learn surrogates for numerical solvers of partial differential equations (PDEs) on manifolds.

Linear subspaces complemented by an ideal

Boris Shekhtman*
University of South Florida
shekhtma@usf.edu

In this talk I will pose a general problem: *Let A be an algebra. What linear subspaces of A are complemented by an ideal?* We present some complete and some partial answers to this question for some particular algebras (continuous functions, polynomials in several variables, matrix algebras) and particular ideals (general ideals, closed ideals, one-sided ideals).

Transformers for Learning on Noisy and Task-Level Manifolds

Zhaiming Shen*, Alex Havrilla, Rongjie Lai, Alexander Cloninger, Wenjing Liao
Georgia Institute of Technology
zshen49@gatech.edu
https://sites.google.com/view/zhaiming-shen

Transformers serve as the foundational architecture for large language and video generation models, such as GPT, BERT, SORA and their successors. Empirical studies have demonstrated that real-world data and learning tasks exhibit low-dimensional structures, along with some noise or measurement error. The performance of transformers tends to depend on the intrinsic dimension of the data/tasks, though theoretical understandings remain largely unexplored for transformers. This work establishes a theoretical foundation by analyzing the performance of transformers for regression tasks involving noisy input data on a manifold. Specifically, the input data are in a tubular neighborhood of a manifold, while the ground truth function depends on the projection of the noisy data onto the manifold. We prove approximation and generalization errors which crucially depend on the intrinsic dimension of the manifold. Our results demonstrate that transformers can leverage low-complexity structures in learning task even when the input data are perturbed by high-dimensional noise. Our novel proof technique constructs representations of basic arithmetic operations by transformers, which may hold independent interest.

Shape Preserving Approximation of Periodic Functions – Conclusion

Dany Leviatan, Oksana Motorna, Igor Shevchuk*
Taras Shevchenko National University of Kyiv
shevchukh@ukr.net

Recently we proved, that if an r times continuously differentiable 2π-periodic function changes its monotonicity $2s$ times in a period, that is, if there are $2s$ points y_i, such that $y_1 < \cdots < y_{2s} < y_1 + 2\pi$ and

$$f'(x)\prod_{i=1}^{2s}\sin\frac{x-y_i}{2} \geq 0, \quad x \in \mathbb{R},$$

then for each $n \in \mathbb{N}$ there is a trigonometric polynomial T_n of degree $< n$, such that

$$f'(x)T_n'(x) \geq 0, \quad x \in \mathbb{R}, \tag{1}$$

and

$$\|f - T_n\|_{C(\mathbb{R})} \leq \frac{c}{n^r}\omega_1\left(f^{(r)}, \frac{1}{n}\right), \tag{2}$$

where $\omega_1(f^{(r)}, \cdot)$ is the modulus of continuity of $f^{(r)}$, and $c = c(r, s)$ is a constant, depending only on r and s.

We will present all quadruplets (k, r, s, q) of natural numbers, for which the "SPA Jackson inequality" of type (1) – (2) holds, with constant $c = c(k, r, s, q)$, depending on these four numbers only. Here k is the order of the modulus of continuity (smoothness) ω_k, where ω_k replaces ω_1 in (2), r is the number of derivatives, $2s$ is the number of changes of q-monotonicity, q is the type of monotonicity. I.e, "1-monotone" means "monotone", "2-monotone" means "convex", etc. The case $r = 0$ is covered as well.

For all other quadruplets of these numbers we have contrexamples.

Extended convexity and uniqueness of minimizers for interaction energies

Ruiwen Shu*
University of Georgia
ruiwen.shu@uga.edu
https://shuruiwen.com/

Linear interpolation convexity (LIC) has served as the crucial condition for the uniqueness of interaction energy minimizers. We introduce the concept of the LIC radius which extends the LIC condition.

Uniqueness of minimizer up to translation can still be guaranteed if the LIC radius is larger than the possible support size of any minimizer. Using this approach, we obtain uniqueness of minimizer for power-law potentials $W_{a,b}(\mathbf{x}) = \frac{|\mathbf{x}|^a}{a} - \frac{|\mathbf{x}|^b}{b}$, $-d < b < 2$ with a slightly smaller than 2 or slightly larger than 4. The estimate of LIC radius for a slightly smaller than 2 is done via a Poincaré-type inequality for signed measures. To handle the case where a slightly larger than 4, we truncate the attractive part of the potential at large radius and prove that the resulting potential has positive Fourier transform. We also propose to study the logarithmic power-law potential $W_{b,\ln}(\mathbf{x}) = \frac{|\mathbf{x}|^b}{b} \ln |\mathbf{x}|$. We prove its LIC property for $b = 2$ and give the explicit formula for minimizer. We also prove the uniqueness of minimizer for b slightly less than 2 by estimating its LIC radius.

(Deformations of) orthogonal polynomials, (nonlocal) integrable systems, and RHPs

Guilherme Silva*
Universidade de São Paulo
silvag@icmc.usp.br
sites.google.com/site/guilhermesilvamath/

Connections between orthogonal polynomials and integrable systems are certainly not new. From recurrence relation's coefficients satisfying difference equations, to their continuous limits connecting to Painlevé-type equations, there is a vast literature exploring such connections.

However, a new perspective has recently arisen: deformations of exponential weights of orthogonality give rise to connections of orthogonal polynomials with non-local versions of Painlevé equations, both discrete and continuous. As we plan to explain, such deformations are motivated naturally by probabilistic constructions, and the discussed findings are interpreted under the eyes of Riemann-Hilbert Problems.

On the maximal hyperplane in ℓ_p^n

Leslaw Skrzypek*
University of South Florida
skrzypek@usf.edu

We introduce the family of projections $P_{f,\alpha} = Id - \frac{1}{\sum_{i=1}^n |f_i|^\alpha} f \otimes \langle f \rangle^{\alpha-1} : \ell_p^n \to \ker f$ and study their maximal p-norms. For fixed $\alpha > 1$, we are able to compute $\max_f ||P_{f,\alpha}||_p$ and characterize the maximizer f. This allows us to compute the orthogonal $\lambda^H_{orth}(\ell_p^n)$ and polar $\lambda^H_{pol}(\ell_p^n)$ hyperplane constants of ℓ_p^n and to confirm that $\lambda^H_{orth}(\ell_p^n) > \lambda(\ker 1, \ell_p^n)$ and $\lambda^H_{pol}(\ell_p^n) > \lambda(\ker 1, \ell_p^n)$. These estimates represents the main difficulty in the situation of finding the hyperplane constant of ℓ_p^n. A subtle argument is needed. For each p and n we need to find a unique $\alpha_0 = \alpha(p, n)$ such that $\max_f ||P_{f,\alpha_0}||_p$ is attained when $f = (1, ..., 1)$. We present here the proof for $n = 3$. This shows that the hyperplane constant of ℓ_p^n equals $\lambda^H(\ell_p^n) = \lambda(\ker 1, \ell_p^n)$, i.e., $\ker 1$ is a maximal hyperplane in ℓ_p^n, confirming the $n = 3$ case of the long standing conjecture. Joint work with G. Lewicki.

QMC designs – cubature on the sphere without polynomial exactness

Ian H Sloan
University of New South Wales, Sydney
i.sloan@unsw.edu.au

QMC designs were introduced in a 2014 paper by Brauchart, Saff, Sloan and Womersley (*Math. Comp.* **83**: 2821-2851). They represent a novel approach to cubature on the sphere $\mathbb{S}^d$, in which instead of requiring cubature rules to be exact for polynomials up to a certain degree, a sequence of cubature rules is a QMC design sequence if for some $s > d/2$ the worst-case error for functions in a Sobolev space $H^s(\mathbb{S}^d)$ is of order $O(N^{-s/d})$, where N is the number of cubature points. The original paper allowed only equal cubature weights (hence "QMC"), but here we allow general positive weights.

In rececnt joint work with Robert Womersley we have devised a new necessary and sufficient condition, and obtained new theoretical and experimental results. However, the construction of QMC designs with high "strength" (i.e. achieving large values of s) and large values of N remains elusive, especially for dimensions $d > 2$. Also missing in most cases is a rigorous proof of the QMC design property. Fresh ideas are needed!

A-Stabilization: Concept, Problems and Applications

Alexey Solyanik
Odessa Polytechnical
solyanik@op.edu.ua

The talk is devoted to studying a new method for stabilizing unstable equilibriums and unstable cycles in discrete dynamical systems. To stabilize an unstable equilibrium in complex dynamics, the control vector is chosen in such a way that the new state of the system is a linear combination of the previous states with specially selected complex coefficients. The choice of the coefficients, whose sum is equal to one, depends solely on the location of eigenvalues of the linear part of the mapping that defines the dynamics at equilibrium.

The method and its modification for stabilizing real variable dynamical systems makes it possible to stabilize equilibrium/cycle in an nonlinear dynamical system exactly to the initial equilibrium or to the initial unstable cycle, thus demonstrating its non-invasiveness. A remarkable feature of the proposed stabilization method is its ability to stabilize dynamics with any spectrum of the linear part, including those with positive real eigenvalues.

The talk presents the conceptual framework of the method and discusses several open questions. Various applications of the method will be provided, including the search for unstable cycles in real discrete dynamical systems.

Interestingly enough, the optimal complex coefficients turn out to be the coefficients of classical complex polynomials.

M-Stable Polynomials and Complex Dynamics

Alexey Solyanik*
Odessa Polytechnical

solyanik@op.edu.ua

The talk is dedicated to the study of stabilization of unstable fixed points and unstable cycles in complex dynamics. Stabilization designs are constructed using normalized complex polynomials with interesting geometric properties — the image of the unit disk excludes a given subset of the complex plane.

It turns out that the polynomials for extremal (shortest) designs are the Ruscheweyh-Varga polynomials, originally introduced to solve a different problem. These polynomials depend on a parameter, and at their minimal value, they coincide with the normalized complex Fejér polynomials.

We also demonstrate how our stabilizing algorithms can be used to find all unstable cycles of a given length in real chaotic dynamical systems.

Multivariate Biharmonic C^1 Splines

Tatyana Sorokina*
Towson University
tsorokina@towson.edu
https://tigerweb.towson.edu/tsorokin/

We develop a new approach to construct finite element methods to solve the biharmonic equation in several variables. The idea is to use a subspace of piecewise biharmonic polynomials that are globally C^1-smooth. We expand tools of Bernstein-Bézier analysis to biharmonic operators on splines, and show what types of underlying partitions are well-suited for biharmonic spline spaces. We provide numerical resluts for the case of two variables.

Upper and lower estimates for the discrete energies on the sphere

Tetiana Stepaniuk*
Institute of Mathematics of National Academy of Sciences of Ukraine
stepaniuk.tet@gmail.com
tetianastepaniuk.com

On the unit sphere of arbitrary dimensions we consider generalized Sobolev spaces, which are embedded into into the space of continuous functions and whose Legendre-Fourier coefficients decrease to zero approximately as power functions. For these classes we find upper and lower estimate for the worst-case integration error. Obtained results allow also to write upper and lower bounds for the discrete energies with the similar Legendre-Fourier coefficients.

Extremizers for the Rogosinski - Szegö estimate of the second coefficient in nonnegative sine polynomials

D. Dmitrishin, A. Stokolos*, W. Trebels

Georgia Southern University
astokolos@georgiasouthern.edu
https://sites.google.com/a/georgiasouthern.edu/astokolos/

For the class of sine polynomials $b_1 \sin t + b_2 \sin 2t + ... + b_N \sin Nt$, $(b_N \neq 0)$, which are nonnegative on $(0, \pi)$, W. Rogosinski and G. Szegö derived, among other things, exact bounds for $|b_2|$ via the Lukács presentation of nonnegative algebraic polynomials and a variational type argument for exact bounds, but they did not find the extremizers. Within this algebraic framework, we construct explicit polynomials which attain these bounds and prove their uniqueness. The proof uses the Fejér - Riesz representation of nonnegative trigonometric polynomials, a 7-band Toeplitz matrix of arbitrary finite dimension, and Chebyshev polynomials of the second kind and their derivatives.

Universal Bounds on Energy and Polarization of Weighted Spherical Codes and Designs

S. Borodachov, P. Boyvalenkov, P. Dragnev, D. Hardin, E. Saff, M. Stoyanova*
Sofia University "St. Kliment Ohridski"
stoyanova@fmi.uni-sofia.bg

For a general absolutely monotone function $h : [-1, 1) \to \mathbb{R}$ we obtain universal bounds on the h-energy of weighted spherical codes via linear programming. The universality is in the sense of Cohn-Kumar – every attaining code is optimal with respect to a large class of potential functions (absolutely monotone), in the sense of Levenshtein – there is a bound for every weighted code, and in the sense of parameters (nodes and weights) – they are independent of the potential function. We also derive a universal bound on the minimum of the discrete potential of weighted spherical designs.

The importance of spherical codes in neural collapse

James Alcala, Vladimir Kobzar, Dustin Mixon, Sanghoon Na, Shashank Sule*, Yangxinyu Xie
University of Maryland, College Park
ssule25@umd.edu
www.math.umd.edu/∼ssule25

In this talk we discuss the case of neural collapse where the number of classes exceeds the feature dimension. In particular, we show that spherical codes play a distinguished role as minimizers of cross entropy in the low temperature limit. On the way to this result, we characterize softmax codes and highlight the role of a balanced temperature in promoting certain types of spherical codes over others.

Trajectory-Based RBF Collocation Method for Surface Advection-Diffusion Equations

Xiaobin Li, Leevan Ling, Yizhong Sun*
Hong Kong Baptist University
yzsun95@hkbu.edu.hk

We introduce the Trajectory-Based RBF Collocation (TBRBF) method for solving surface advection-diffusion equations on smooth, compact manifolds. TBRBF decouples advection and diffusion by applying a characteristic treatment with a Kansa-type RBF collocation method for the diffusion PDE. By embedding the manifold into a narrow band domain, we prove the equivalence between the embedded advection-diffusion PDE and the combined system of the characteristic ODE and diffusion PDE, when the system is subsequently restricted back to the manifold. This theoretical framework ensures that our results apply to both the narrow band domain formulation and the original manifold formulation. Extensive numerical experiments confirm the robust stability and accuracy of the proposed method.

Two-Weight Multiplier Weak-Type Inequalities

David Cruz-Uribe, Kabe Moen, Brandon Sweeting*
Washington University in St. Louis
sweeting@wustl.edu
https://www.brandonsweeting.com

We discuss a class of weighted weak-type inequalities first studied by Muckenhoupt and Wheeden. In this formulation, the weight for the target space appears as a multiplier rather than as a measure, leading to fundamentally different behavior. Notably, as Muckenhoupt and Wheeden showed, the class of weights characterizing such inequalities for the maximal operator in the one-weight case is strictly larger than A_p. In joint work with David Cruz-Uribe and Kabe Moen, we extend these inequalities to the two-weight setting for both the Hardy-Littlewood maximal operator and singular integrals. For the maximal operator, we establish a necessary and sufficient Sawyer-type testing condition, and for singular integrals, we provide sufficient Pérez-type bump conditions.

Swarm-Based Gradient Descent Method for Non-Convex Optimization

Eitan Tadmor
Universit of Maryland, College Park
tadmor@umd.edu
https://www.math.umd.edu/∼tadmor/

We discuss a novel class of swarm-based gradient descent (SBGD) methods for non-convex optimization. The swarm consists of agents, each is identified with position, x, and mass, m. There are two key ingredients in the SBGD dynamics. (i) persistent transition of mass from agents at high to lower ground; and (ii) time stepping protocol which decreases with m.
The interplay between positions and masses leads to dynamic distinction between 'leaders' and 'explorers': heavier agents lead the swarm near local minima with small time steps; lighter agents use larger time steps to explore the landscape in search of improved global minimum, by reducing the overall 'loss' of the swarm. Convergence analysis and numerical simulations demonstrate the effectiveness of SBGD method as a global optimizer.

Local Clustering for Lung Cancer Image Classification via Sparse Solution Technique

Jackson Hamel, Ming-Jun Lai, Zhaiming Shen, Ye Tian*
University of Georgia
yt95681@uga.edu

We propose to use a local clustering approach based on some sparse solution techniques to study the medical image classification problem, especially the lung cancer image classification task. We view images as the vertices in a weighted graph and compute the similarity between a pair of images as the edges in the graph. The vertices within the same cluster can be assumed to share similar features and properties, thus making the applications of graph clustering techniques very useful for image classification. Recently, the approach based on the sparse solutions of linear systems for graph clustering has been found to identify clusters more efficiently than traditional clustering methods such as spectral clustering. We propose to use the two newly developed local clustering methods based on sparse solution of linear system for image classification. However, a straight-forward application of the two methods does not work. In addition, we employ a box-spline-based tight-wavelet-frame method to clean up these images and help build a better adjacency matrix before clustering. These result an excellent method in classifying images. The performance of our method is significantly more efficient and either favorable or equally effective compared with other state-of-the-art approaches, e.g. convolutional neural network approach. Finally, we shall remark by pointing out two image deformation methods that can be used to build up more artificial image data to increase the number of labeled images.

Introduction to Extremal Problems and Spectral Theory of Soliton Gases for Integrable Systems

Alexander Tovbis*
University of Central Florida
alexander.tovbis@ucf.edu

Nonlinear Dispersion Relation (NDR) is one of the central objects in spectral theory of soliton gases for integrable equations. Mathematically, they are represented by certain integral equations defining the Density of States (DOS) and the Density of Fluxes (DOF). The NDR can be treated as variational equations for certain quadratic energy functionals, which connects their analysis with potential theory. Some related minimization and minimax problems will be discussed in the minisymposium.

Chebotarev continuum problem and focusing NLS soliton condensates of minimal intensity

Marco Bertola and Alexander Tovbis*
University of Central Florida
alexander.tovbis@ucf.edu

We consider the family of polycontinua $K \subset \mathbb{C}^+$ that contain a preassigned finite anchor set E. For a given harmonic external field, we define a (weighted) Green energy functional $I(K)$ and show that within each "connectivity class" of the family, there exists a minimizing compact $\hat{K}$ consisting of critical trajectories of a quadratic differential Qdz^2. In many cases $Qdz^2 = dp^2$, where dp is the meromorphic real normalized quasimomentum differential associated with the finite gap solutions of the focusing Nonlinear Schroedinger equation (fNLS) defined by a hyperelliptic Riemann surface R branched at the points $E \cup \bar{E}$. In the latter case the minimizing compact is known as Zakharov-Shabat spectrum (for finite gap solutions on R). Jenkins interseption property for orthogonal trajectories plays a significant role in the solution.

An fNLS soliton condensate is defined by a compact spectral support set K, whereas the average intensity of the condensate is proportional to $I(K)$ (with the external field $-2\Im z$). The motivation for this work lies in the problem of soliton condensate of least average intensity with a given anchor set $E \subset K$. We prove that spectral support $\hat{K}$ indeed provides the fNLS soliton condensate of the least average intensity within a given "connectivity class".

AAA Approximation and Potential Theory

Lloyd N. Trefethen*
Harvard University
trefethen@seas.harvard.edu
https://people.maths.ox.ac.uk/trefethen/

Approximation by rational functions used to be mainly a theoretical subject, but with the introduction of the AAA algorithm in 2018, it became computationally practical and indeed easy. The implications for what we can do numerically are enormous. First, this talk will outline the algorithm and demonstrate about a dozen applications. Then we will show how AAA beautifully illustrates the potential theory associated with rational approximation, which involves positive point charges at poles and negative point charges at interpolation points.

Lower Bounds in the Kreiss Matrix Theorem

Nikolaos Chalmoukis, Georgios Tsikalas*, Dmitry Yakubovich
Vanderbilt University
georgios.tsikalas@vanderbilt.edu

A matrix A is said to be Kreiss bounded if its spectrum is contained in the closed unit disk and there exists $K \geq 1$ such that

$$||(zI - A)^{-1}|| \leq \frac{K}{|z| - 1}, \qquad |z| > 1.$$

Kreiss (1962) proved that A is power-bounded (i.e. $\sup_{n\geq 1} ||A^n|| < \infty$) if and only if it satisfies the above condition, a result of fundamental importance in applied matrix analysis. It was later shown that, if A is $N \times N$ with Kreiss constant K, then

$$\sup_{n\geq 1} ||A^n|| \leq eKN.$$

While this upper bound is asymptotically sharp as $K \to \infty$, it is not known whether requiring K to remain bounded can lead to an improvement. In fact, a (still unresolved) conjecture of Nikolski

(2013) states that, for matrices A with Kreiss constant at most K, there exists an upper bound for $\sup_{n\geq 1} ||A^n||$ that is sublinear in N.

In this talk, we will discuss the sharpness of various bounds related to the Kreiss matrix theorem.

Angelesco and AT Systems on the Unit Circle

Rostyslav Kozhan, Marcus Vaktnäs*
Uppsala University
marcus.vaktnas@math.uu.se

Angelesco systems and AT systems provide the most well-known examples of large classes of perfect systems of multiple orthogonality measures on the real line. We present unit circle analogues of these systems, and prove in what sense they are perfect. The main idea here lies in the definition of the polynomials, where we use generalized orthogonal Laurent polynomials rather than the usual multiple orthogonal polynomials. This leads to different moment matrices, which we compute to prove perfectness. This approach also leads to a natural two-point Hermite–Padé approximation problem, as well as relations for the Szegő mapping.

Asymptotics of the coefficients of polynomials versus their asymptotic zero distribution

Walter Van Assche*
KU Leuven, Belgium
walter.vanassche@kuleuven.be

The asymptotic behavior of the coefficients of a sequence of polynomials is given under the conditions that all the zeros are real and positive and the zeros have an asymptotic zero distribution on $(0, \infty)$. We show how the limit for the coefficients is related to the Stieltjes transform of the asymptotic zero distribution and make a connection with the R- and S-transform in free probability. We illustrate with some examples involving orthogonal and multiple orthogonal polynomials.

Geometric rectifiability

Vyron Vellis
University of Tennessee
vvellis@utk.edu
https://sites.google.com/site/vyronvellis/

The classical Denjoy-Riesz Theorem states that every compact totally disconnected metric space in $\mathbb{R}^2$ is contained in an arc (homeomorphic image of $[0, 1]$). In this talk, we present a geometric strengthening of the Denjoy-Riesz Theorem in two directions. First, we replace the topological regularity (arc) by geometric regularity (quasisymmetric arc, bi-Lipschitz arc) and second, beyond the Euclidean setting, we identify those metric spaces that support such rectifiability theorems. This is joint work with J. Honeycutt and S. Zimmerman.

Uniqueness for TV-Norm Regularized Inverse Problems with Source Term in Divergence Form

L. Baratchart, D. P. Hardin, C. Villalobos Guillén*
Vienna University
cristobal.villalobos.guillen@univie.ac.at
https://cristobal-villalobos.github.io/eng/

Inverse source problems in divergence form consist in finding a vector field with prescribed support S, whose divergence is the Laplacian of some observed potential. We assume the unknown vector field is a vector-valued measure, and we study the corresponding least square inversion problems, regularized by penalizing the total variation, without discretizing the criterion nor the unknown. We prove that this problem has a unique minimizer in the case where S is a "slender" set; i.e., it has zero Lebesgue measure and each connected component of its complement has infinite Lebesgue measure.

Constrained Triangulations in Boundary Representation

Alex Vlasiuk*
PTC Inc
oleksandr.vlasiuk@gmail.com
vlasiuk.com

We give a brief outline of incremental geometry construction for computer-aided design using the boundary representation (B-rep), define constrained Delaunay triangulations, and, time permitting, connect the two topics through shading of parametric surfaces.

Recovering a Group from Few Orbits

Dustin Mixon, Brantley Vose*
The Ohio State University
vose.5@osu.edu
https://prismika.github.io/

Suppose an unknown finite group G acts isometrically on a finite-dimensional Hilbert space. How many orbits must one observe before one can deduce G? We will discuss sharp bounds on the number of generic G-orbits needed to recover G up to group isomorphism, as well as the number needed to recover G as a concrete set of automorphisms.

Computation of Conformal Capacity

Matti Vuorinen*
University of Turku
vuorinen@utu.fi
https://users.utu.fi/vuorinen/

A condenser is a pair $(\mathbb{B}^2, E)$ where $\mathbb{B}^2$ is the unit disk and E is a compact subset of $\mathbb{B}^2$. The conformal capacity $\mathrm{cap}(\mathbb{B}^2, E)$ is defined as $\inf \int_{\mathbb{B}^2} |\nabla u|^2 \, dm$ where the infimum is taken over all functions of the class $C_0^\infty(\mathbb{B}^2)$. This notion has numerous applications to geometric function theory, potential theory, some topics of classical analysis.

Yet the explicit values of the capacity are known only in a few cases and this motivates numerical computation and estimation of these values. A review of recent estimates of the capacity in terms of hyperbolic geometry is given. This work is joint work with M. M.S. Nasser and H. Hakula, who have developed the boundary integral equation method and the *hp*-FEM method, resp., for the purpose.

Applications of Fourier-Padé Approximation

J.A.C. Weideman*
Stellenbosch University
weideman@sun.ac.za

A few applications of Fourier-Padé approximation will be discussed. These include the mitigation of the Gibbs phenomenon, post-processing of numerical solutions of differential equations, and singularity detection in the complex plane.

Kernel-based Methods for Manifold-valued Function and PDE Approximation

Holger Wendland*
University of Bayreuth
holger.wendland@uni-bayreuth.de
https://www.num-analysis.uni-bayreuth.de/en/index.html

The learning or approximation of manifold-valued functions has attracted a lot of attention in recent years. The nonlinear structure of the problem requires techniques and methods which significantly differ from standard approximation schemes. In this talk, I will concentrate on the approximation of functions and the solution of certain PDEs with values in the class of positive definite, symmetric matrices. The PDEs considered here arise, for example, when constructing a Riemannian contraction metric for a dynamical system given by an autonomous ODE. Though not a compact manifold, positive definite matrices allow us to globally employ the log-Euclidean approach to define a Hilbert space structure on these matrices. Combining this approach with a kernel-based approximation method for symmetric matrices leads to a complete theory, including a thorough error analysis for deterministic approximations.

This talk is based on work with Peter Giesl (Sussex University, UK) and Nir Sharon (Tel Aviv University, Israel).

Adaptive meshfree approximation with PDE-greedy kernel methods

Tizian Wenzel*, Daniel Winkle, Gabriele Santin, Bernard Haasdonk

LMU Munich
wenzel@math.lmu.de

We consider meshless approximation for solutions of PDEs both on domains as well as manifolds, in particular the sphere. We discuss the importance of the choice of the collocation points, in particular by using greedy kernel methods. We introduce a scale of PDE-greedy selection criteria that generalizes existing techniques, such as the PDE-P-greedy and the PDE-f-greedy rules for collocation point selection. For these greedy selection criteria we provide bounds on the approximation error in terms of the number of greedily selected points and analyze the corresponding convergence rates. Especially, we show that target-data dependent algorithms that make use of the right hand side functions PDE exhibit faster convergence rates than the target-data independent PDE-P-greedy.

Intermediate superconvergence in kernel-based approximation

Toni Karvonen, Gabriele Santin, Tizian Wenzel*
LMU Munich
wenzel@math.lmu.de

Results on superconvergence for kernel based approximation show that a doubling of the convergence rate can be obtained for functions in the image of the Mercer kernel integral operator. In this talk, we extend these results to the full scale of intermediate convergence orders.

This is achieved by considering general embedding operators and their adjoints, which can frequently be expressed as kernel integral operators with modified kernel. We discuss several characterizations of these spaces and elaborate on the connections between superconvergence and solutions of certain PDEs.

Classification of Rational Functions with Newton Map Möbius Conjugate to a Polynomial

Drew Macha, G. Brock Williams*
Texas Technical University
brock.williams@ttu.edu

Newton's iterative root finding method and the dynamics of the Newton maps of rational functions have been studied in recent years, resulting in the classification of rational functions whose Newton maps are Möbius conjugate to polynomials of degree 1, 2, and 3. We describe a new approach for obtaining the same results, then classify the sets of rational functions whose Newton maps are Möbius conjugate to polynomials of degree 4, polynomials of degree 5, and polynomials of any finite degree.

Good Riesz Energy Points on the Sphere $\mathbb{S}^2$: Asymptotics, Voronoi Cells and Scars

Robert S. Womersley*, Douglas P. Hardin, Edward B. Saff
School of Mathematics and Statistics, UNSW Sydney, Australia
R.Womersley@unsw.edu.au

https://web.maths.unsw.edu.au/∼rsw/

This talk explores the numerical calculation of sets of N points on the unit sphere $\mathbb{S}^d \subset \mathbb{R}^{d+1}$ for $d = 2$ with *good/low* Riesz s-energy, using a variety of optimization techniques and for N up to $27,000$. The aim is to investigate the role of the terms in the asymptotic expansion of the minimum energy involving $N^{(1+s)/d}$, after those of order N^2 and $N^{1+s/d}$ (as appropriate). Particular attention is paid to the $s = 4$, $s = 1$ (Thompson's problem) and $s = 0$ or log (Smale's 7th problem) cases. For larger values of N low energy configurations are related to the behaviour of scars, chains of pentagonal and heptagonal Voronoi cells.

Bypassing the quadrature exactness assumption of hyperinterpolation

Hao-Ning Wu*
University of Georgia, Athens, GA
hnwu@uga.edu
https://haoningwu.github.io/

Hyperinterpolation is a quadrature-based approximation scheme for continuous functions, which can be regarded as a discrete version of the L^2 orthogonal projection. This scheme boasts an elegant L^2 convergence theory and is relatively easy to implement. However, the convergence theory heavily relies on certain degrees of quadrature exactness, limiting its applicability in multivariate domains. In this talk, we will discuss our recent progress in addressing the limitation by relaxing the quadrature exactness requirement, aided by the Marcinkiewicz–Zygmund inequality. By deriving error bounds for approximation, we are then able to use sets of points that do not necessarily exhibit quadrature exactness for hyperinterpolation, while still maintaining reasonable error rates. Numerical results of the relaxed approximation scheme are also presented.

Moving Lest-Squares Methods for Solving Vector-Valued PDEs on Unknown Manifolds

Rongji Li, Qile Yan*, Shixiao W. Jiang
University of Minnesota
yan00082@umn.edu

In this paper, we extend the Generalized Moving Least-Squares (GMLS) method in two different ways to solve the vector-valued PDEs on unknown smooth 2D manifolds without boundaries embedded in $\mathbb{R}^3$, identified with randomly sampled point cloud data. The two approaches are referred to as the intrinsic method and the extrinsic method. For the intrinsic method which relies on local approximations of metric tensors, we simplify the formula of Laplacians and covariant derivatives acting on vector fields at the base point by calculating them in a local Monge coordinate system. On the other hand, the extrinsic method formulates tangential derivatives on a submanifold as the projection of the directional derivative in the ambient Euclidean space onto the tangent space of the submanifold. One challenge of this method is that the discretization of vector Laplacians yields a matrix whose size relies on the ambient dimension. To overcome this issue, we reduce the dimension of vector Laplacian matrices by employing an appropriate projection. The complexity of both methods scales well with the dimension of manifolds rather than the ambient dimension. We also present supporting numerical examples, including eigenvalue problems, linear Poisson equations, and nonlinear Burgers' equations, to examine the numerical accuracy of proposed methods on various smooth manifolds.

Uniformity of Strong Asymptotics in Angelesco Systems

Maxim Yattselev*
Indiana University Indianapolis
maxyatts@iu.edu
https://math.indianapolis.iu.edu/~maxyatts/

Let μ_1 and μ_2 be two, in general complex-valued, Borel measures on the real line such that $\operatorname{supp}\mu_1 = [\alpha_1, \beta_1] < \operatorname{supp}\mu_2 = [\alpha_2, \beta_2]$ and $d\mu_i(x) = -\rho_i(x)dx/2\pi\mathrm{i}$, where $\rho_i(x)$ is the restriction to $[\alpha_i, \beta_i]$ of a function non-vanishing and holomorphic in some neighborhood of $[\alpha_i, \beta_i]$. Strong asymptotics of multiple orthogonal polynomials will be discussed as their multi-indices (n_1, n_2) tend to infinity in both coordinates. The emphasis is placed on the uniformity of the error terms in the asymptotic formulae with respect to $\min\{n_1, n_2\}$.

Learning sparsity priors using Bilevel method

Jiangyu Yu*
Syracuse University
jyu121@syr.edu

Analysis-based sparsity priors have been popular to perform denoising. In this talk, we consider a bilevel optimization framework to learn the analysis operator under a sparsity-promoting prior. After investigating the analytical properties of the proposed model, we present a smoothing-based optimization scheme to effectively solve it. We also analyze the relationship between the smoothed and original models and establish convergence of the proposed algorithm. Applications to 1-D signal denoising are reported.

Fast Branch-Free Algorithms for High-Precision Computer Arithmetic

David K. Zhang*
Stanford University
dkzhang@stanford.edu
dkzhang.com

Many scientific and mathematical problems demand extremely precise calculations that exceed the limits of IEEE binary64 (double precision) floating-point arithmetic. However, existing multiprecision software libraries, such as GNU MP and MPFR, are considered unsuitable for high-performance applications because they are hundreds to thousands of times slower than native machine arithmetic.

In this talk, I introduce a new class of algorithms for high-precision floating-point arithmetic that are nearly as fast as native machine arithmetic. These algorithms have provably optimal error bounds and are completely branch-free, making them highly performant on data-parallel processors, including vector CPUs and GPUs. To prove their correctness, I present a novel technique that leverages automatic theorem provers to rigorously compute worst-case inputs and tight error bounds via reduction to quantifier-free Presburger arithmetic.

Asymptotics of weighted Chebyshev and residual polynomials on a C^{1+} region

Maxim Zinchenko*
University of New Mexico
maxim@math.unm.edu

In this talk I will discuss an application of orthogonal polynomials to an asymptotics problem for L^∞-extremal polynomials. In particular, I will show how Szegő's asymptotics for Christoffel function of OPUC can be used to derive the strong asymptotics of weighted Chebyshev and residual polynomials for weights supported on a C^{1+} Jordan region.

The talk is based on a joint work with Benedikt Buchecker and Benjamin Eichinger.

Hypergeometric Series and Divided Differences

Fatma Zürnacı-Yetiş*, Nazile Buğurcan Dişibüyük
Istanbul Technical University, Dokuz Eylül University
fzurnaci@itu.edu.tr

Divided differences and hypergeometric series are intimately related. In this work, divided differences provide new proofs for some well-known hypergeometric series identities in addition to the existing proofs in the literature. The proofs of Chu-Vandermonde, Pfaff-Saalschütz, and one of Thomae's ${}_3F_2$ transformation formulas are established by a method based on divided differences. The proofs of the q-versions of these formulas, q-Chu-Vandermonde, q-Pfaff-Saalschütz, and Sear's Transformation formulas, are also derived by using divided differences.