Abstracts - Jan. 2023 January 16, 2023

- M. Czubak
  Title: The formulation of the Navier-Stokes equations on the Riemannian manifolds
  Abstract: In their 1970 seminal article, David Ebin and Jerrold Marsden presented the first in-depth study of the Navier-Stokes equations on the Riemannian manifolds. There are several inequivalent formulations of the Navier-Stokes equations on the Riemannian manifolds. This is due to the different possibilities for the Laplacian operator acting on vector fields. In their article, Ebin and Marsden indicated what form the Navier-Stokes equations should take when considered on a Riemannian manifold. In this talk, we discuss the operator proposed by Ebin and Marsden, and present the recent developments in the study of the formulation of the Navier-Stokes equations in the setting of Riemannian manifolds.

- S. Friedlander
  Title: In Search of Euler Equilibria via the MR Equations
  Abstract: The subject of "geometric" fluid dynamics flourished following the seminal work of V.I. Arnold in the 1960s. A famous paper was published in 1970 by David Ebin and Jerrold Marsden who used the manifold structure of certain groups of diffeomorphisms to obtain sharp existence and uniqueness results for the classical equations of fluid dynamics. Of particular importance are the fixed points of the underlying dynamical system and the "accessibility" of these Euler equilibria. In 1985 Keith Moffatt introduced a mechanism for reaching these equilibria not through the Euler vortex dynamics itself but via a topology preserving diffusion process called "Magnetic Relaxation". In this talk we will discuss some recent results for Moffatt's MR equations which are mathematically challenging not only because they are active vector equations but also because they have a cubic nonlinearity. This is joint work with Rajendra Beckie, Adam Larios and Vlad Vicol .

- D. Holm
  Title: Emergent Singular Solutions of Nonlinear Wave Equations

Abstract: We discuss emergent singular solutions in nonlinear fluid wave PDEs.
(1) Start with asymptotic expansion for nonlinear shallow water waves.
(2) Identify the b-equation in n dimensions, H-Staley [2003]
In 1D, b=2 Camassa-H [1993], b=3 Degasperis-H-Hone [2002]
(3) Q: Why is b=2 special?
A: Singular solution is a momentum map, H-Marsden [2005]
(4) Other geodesic equations with b=2 in 1D? Fringer-H [2001]
(5) Stochastic CH eqn, Crisan-H [2019] & Bendall-Cotter-H [2022]
(6) Cotter-H-Pryer [2023] r-CH equation for $W^{1,r}$ norm
(7) Higher dimensional emergent singular solutions H-Staley [2004]

- M. Ignatova
  Title: Voigt Boussinesq Equations
  Abstract: The Boussinesq equations are a member of a family of models of incompressible fluid equations, including the 3D Euler equations, for which the problem of global existence of solutions is open. The Boussinesq equations arise in fluid mechanics, in connection to thermal convection a nd they are extensively studied in that context. Formation of finite time singularities from smooth initial data in ideal (conservative) 2D Boussinesq equations is an important open problem, related to the blow up of solutions in 3D Euler equations. The Voigt Boussinesq is a conservative approximation of the Boussinesq equations which has certain attractive features, including sharing the same steady solutions with the Boussinesq equations. In this talk, after giving a brief description of issues of local and global existence, well-posedness and approximation in the incompressible fluids equations, I will present a global regularity result for critical Voigt Boussinesq equations.

- B. Keyfitz
  Title: Can a scalar conservation law imitate a diffeomorphism?
  Abstract: This is a report on some recent work of John Holmes, Feride Tiglay and myself. Following a program developed by Vladimir Arnold, we looked for suitable "Lagrangian" coordinates to represent a scalar convex conservation law (for example, the inviscal Burgers' equation) as a path in a space of diffeomorphisms. For weak solutions, this path becomes a "weak diffeomorphism" - a bi-Lipschitz invertible mapping - which is itself the solution to a system of conservation laws.

- B. Khesin
  Title: Geodesic framework for vortex sheets and generalized fluid flows
  Abstract: We discuss ramifications of Arnold's group-theoretic approach to ideal hydrodynamics as the geodesic flow for a right-invariant metric on the group of volume-preserving diffeomorphisms. It turns out that many equations of mathematical physics, such as the motion of vortex sheets or fluids with moving boundary, have Lie groupoid, rather than Lie group, symmetries. We present their geodesic setting, which also allows one to describe multiphase fluids, homogenized vortex sheets and Brenier's generalized flows. This is a joint work with Anton Izosimov.

- L. Lichtenfelz
  Title: Conjugate points and the MC criterion
  Abstract: We will discuss several results about conjugate points on the group of volume-preserving diffeomorphisms of a compact Riemannian manifold equipped with the L^2 metric. In particular, we will describe new examples of conjugates points found using the so-called MC criterion, a curvature-like functional first introduced by Misiolek which has recently attracted attention again.

- H. Q. Nguyen
  Title: Traveling wave solutions to the one-phase Muskat problem: existence and stability
  Abstract: We study the Muskat problem for one fluid in arbitrary dimension, bounded below by a flat bed and above by a free boundary given as a graph. In addition to a fixed uniform gravitational field, the fluid is acted upon by a generic force field in the bulk and an external pressure on the free boundary, both of which are posited to be in traveling wave form. We prove that for sufficiently small force and pressure data in Sobolev spaces, there exists a locally unique traveling wave solution in Sobolev-type spaces. The free boundary of the traveling wave solutions is either periodic or asymptotically flat at spatial infinity. Moreover, we prove that small periodic traveling wave solutions induced by external pressure only are asymptotically stable. These results provide the first class of nontrivial stable solutions for the problem. This is joint work with I. Tice (CMU).

- S. Shkoller

Title: Geometry of shock formation and maximal development for Euler
Abstract: I will describe a new geometric framework for proving the maximal development of Cauchy data for shock formation for the multidimensional Euler equations.

- V. Sverak
  Title: On the evolution of an idealized vortex filament of circular shape in low-viscosity fluids
  Abstract: We consider the Cauchy problem for the 3d Navier-Stokes equations when the initial vorticity field is a "current" supported on a circle. (In terms of the magnetostatic analogy, we can think of an electric current passing through an infinitely thin wire of a circular shape.) Heuristics and formal computations describing the motion of the fluid in this situation go back to the classical works of Helmholtz and Kelvin. The solution of the Cauchy problem can be shown to be unique in natural classes of axisymmetric solutions. (Uniqueness in more general classes remains open). It can be shown rigorously that - for the special situation at hand - the heuristic predictions of the motion of the fluid given by the local induction approximation are correct up to small error terms. We will outline the main ideas behind the proof. They are based on the construction of an approximate solution via asymptotic expansion and on establishing sufficient stability properties that make it possible for the approximate solution to stay "coherent" for a fairly long time interval. The stability analysis is based on certain geometric properties of the Euler equation that were pointed out by V. I. Arnold. The application of Arnold's ideas in our situation is not completely straightforward due to the presence of viscosity. Joint work with Thierry Gallay.

- C. Vizman
  Title: Singular vortex configurations and coadjoint orbits
  Abstract: We describe classes of coadjoint orbits of the area preserving diffeomorphism group of $\mathbb{R}^2$ that accommodate singular vorticities for ideal 2D fluids. Among them are vortex loops, pointed vortex loops, and vortex loops with dipoles. Some coadjoint orbits arise via symplectic reduction in dual pairs: the Marsden-Weinstein ideal fluid dual pair and a new variant of the Holm-Marsden EPDiff dual pair.