First homework assignment, due Sept 1 2017

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1) Show that $f(z) = z^2$ is continuous on $\mathbb{C}$

2) Prove that the sum of all the nth roots of unity is zero.

3) Show that $\frac{\partial^2}{\partial z \partial \bar{z}} = \frac{1}{4}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)$

4) If z_1, z_2 and z_3 are distinct complex numbers, show that $z_1^2 + z_2^2 + z_3^2 = z_1 z_2 + z_2 z_3 + z_1 z_3$ iff they are the vertices of an equilateral triangle. (Hint: use transformations of $\mathbb{C}$ to reduce the situation to a simple one.)

5) Show that e^z takes each value in $\mathbb{C}$ other than zero infinitely often.

6) Let P_k be the Tchebychev polynomial with $P_k(\cos\theta) = \cos k\theta$

$$(P_2(z) = 2z^2 - 1)$$

find the orbit of P_k as a transformation on $\mathbb{C}$, i.e. find

$$\{P_k^n(z) \mid n \in \mathbb{N}\}$$

For what values of z does $\lim_{n \to \infty} P_k^n(z) = \infty$.