Supplemental Appendix 1. Technical details regarding prior choices for the Bayesian SEM.

 ***Regression effects.***For all regression effects relating predictors to *IQUAL2*, priors were Normal distributions. The center of the distribution was chosen according to findings from past research. Dispersion was chosen following the recommendation of Smid et al. (2018) to define the *impossible* parameter space – parameter values that cannot occur given the scales of the variables. The indicators of the dependent variable, *IQUAL2*,ranged from 1-7, meaning it was impossible for the effect of any predictor to be <–6 or >6. We thus specified a standard deviation of 2 for these priors. Therefore, 99.7% of the distribution (i.e., +/– 3 standard deviations) lay between -6 and 6, the bounds of the possible parameter space for all regression effects. The chosen prior for the effect of experimental condition on *IQUAL2* was Normal(0, 2).

We followed a similar approach in specifying priors for the effects of the covariates on *IQUAL2.* We expected *IQUAL1* to exert a strong positive effect on *IQUAL2,* given past findings that parent-child interaction quality tends to persist across time (Hoff, 2006; Hoff-Ginsberg, 1991); therefore, we chose a center of *d* = 0.6 for this prior. For parents’ perception of play, as well as perceived role in child development, we expected weak positive effects based on prior research (Fogle & Mendez, 2006; Hembacher & Frank, 2020; Strouse et al., 2019; Wooldridge & Shapka, 2012), and centered these priors at *d* = 0.1. We expected parent’s perception of co-use with digital media and overall positive perception of digital media to have moderate positive effects based on past work (Nathanson, 2001; Strouse et al., 2019; Vaala & Takeuchi, 2012; Wood et al., 2016), and therefore centered these priors at *d* = 0.3.

***Factor loadings.*** For the factor loadings relating *IQUAL1* and *IQUAL2* to their indicators, we also specified informative priors. We again used Normal distributions, which are recommended for factor loadings (Merkle et al., 2020). Each prior was centered at 1 to reflect our expectation of strong correspondence between each indicator and the latent variable; previous studies have reported strong relationships between various pairings of the indicators (Ensor et al., 2011; Hindman & Morrison, 2012; Hirsh-Pasek et al., 2015; Mathis & Bierman, 2015; Romeo et al., 2018). Given the consistency of these results, we set the standard deviation of each prior to 5 (rather than the default 10 in *blavaan*) to reflect greater certainty that factor loadings would be strong.

***Other parameters.*** We maintained the default priors in *blavaan* for all latent variable intercepts (*N*(0,10)), observed variable intercepts (*N*(0,32)), covariances (*Beta*(1,1)), and variances (*Gamma*(1, 0.5)).

Supplemental Table 1. Demographic Details by Condition Group.

|  |  |  |
| --- | --- | --- |
|  | Control*M* (*SD*) | Experimental*M* (*SD*) |
| Age in yearsParentChildParent-child dyads Gender (Male/Female)ParentChild Recruitment locationSouthern USMidwestern USWestern USNortheast USCanada |  36.15 (7.07)4.45 (0.34) N (%)39  4/3520/1933 (84.6%)2 (5.1%)3 (7.7%)1 (2.6%)0 (0.0%) |  35.45 (6.74)4.31 (0.31) N (%)38  2/3620/1825 (65.8%)8 (22.2%)4 (10.5%)0 (0.0%)1 (2.6%) |
| Parent Education LevelLess than a 4-year degree4-year College DegreeProfessional or Graduate DegreeDid not report | 4 (10.3%)14 (35.9%)21 (53.8%)0 (0.0%) | 4 (10.5%)13 (34.2%)20 (52.6%)1 (2.6%) |
| Child Racial/Ethnic IdentityEuropean AmericanAfrican AmericanHispanic/LatinoAsianOther Race (not listed)Multiple races reportedDid not report |  29 (74.4%)0 (0.0%)0 (0.0%)0 (0.0%)1 (2.6%)9 (23.1%)0 (0.0%) |  29 (76.3%)1 (2.6%)1 (2.6%)2 (5.3%)0 (0.0%)4 (10.5%)1 (2.6%) |
| Household IncomeUnder $15,000-$45,000$45,000-$75,000$75,000-$105,000$105,000-$150,000Above $150,000 |  0 (0.0%)8 (20.5%)3 (7.7%)15 (38.5%)13 (33.3%) |  2 (5.3%)4 (10.5%)11 (28.9%)12 (31.6%)9 (23.7%) |

Supplemental Table 2. OK Play Activity Descriptions.

|  |  |  |  |
| --- | --- | --- | --- |
| **Activity type** | **Example Activity** | **General Activity Format**  | **Is co-play prompted during the activity?** |
| **Drawing** | Grow a Flower, Submarine, Ice Cream, Dance, Hug, etc. | Taking pictures, using different colors to fill in shapes, to produce a story | Yes ("Pass it to a partner.") However, not required to complete the activity |
| **How to** | How to Make an Angry Emergency, Tell a Tale, Bust A Move, etc. | How-to guides for crafts and other activities | Yes. Parents must read the instructions |
| **Stories** | Doggy Art Show, Treehouse Club,Haha Haircuts, I ♥ Candy, etc. | Create own story with drawings, pictures, songs | No |
| **Scavenger Hunt** | Give a Gift, Museum of Emotions, Fruit Emergency, etc. | Taking pictures of objects on a list | No |
| **Book Maker** | Animal Babysitter, Hero vs. Villain, Plan a Party, etc. | Recording sounds/words (similar to Mad libs) | No |
| **Movie Maker** | Question Time, Cooking Up!, Emotion Update, etc. | Recording videos | No |
| **Music Maker** | Hand Washing Song, Angry Song, Sound Bath, etc. | Recording singing and sounds | No |
| **Sparks** | Silly Word Club, Spread Joy, Mapa's Circle-Pizza, Goodnight Hike, etc. | Stories that can be paused for conversation about the reading | No |
| **Breathing Toys** | Birthday Cake, Balloons, Sunrise, Ferris Wheel, etc. | Themed breathing exercises | No |

Supplemental Table 3. Prior distributions chosen for the Bayesian SEM.

|  |  |  |
| --- | --- | --- |
| **Parameter** | **Prior** | **Description** |
| *Regressions* |  |  |
|  *IQUAL2* on Experimental Group | *Normal*(0, 2) | Informative |
|  *IQUAL2* on *IQUAL1* | *Normal*(0.6, 2) | Informative |
|  *IQUAL2* on Digital Media Involvement | *Normal*(0.3, 2) | Informative |
|  *IQUAL2* on Play as Learning | *Normal*(0.1, 2) | Informative |
|  *IQUAL2* on Role in Child Development | *Normal*(0.1, 2) | Informative |
|  *IQUAL2* on Positive Digital Media Beliefs | *Normal*(0, 2) | Informative |
|  *IQUAL2* on Time Spent with App Nudges\* | *Normal*(0.1, 2) | Informative |
|  |  |  |
| *Factor Loadings* |  |  |
|  All  | *Normal*(1, 5)  | Informative |
|  |  |  |
| *Covariances* |  |  |
|  All  | *Beta*(1, 1)  | *blavaan* default (weakly informative) |
|  |  |  |
| *Latent Variable Intercepts* |  |  |
|  All  | *Normal*(0, 10) | *blavaan* default (weakly informative) |
|  |  |  |
| *Observed Variable Intercepts* |  |  |
|  All  | *Normal*(0, 32)  | *blavaan* default (weakly informative) |
|  |  |  |
| *Variances* |  |  |
|  All  | *Gamma*(1, 0.5) | *blavaan* default (weakly informative) |

*Note.* Normal distribution is described as N(Mean, SD).

\*Time spent with app nudges was included as a predictor in a separate model run only on participants in the experimental group.

Supplemental Table 4. Prior distributions used in sensitivity analysis.

|  |  |  |  |
| --- | --- | --- | --- |
| **Parameter** | **Informative (used in original model)** | **Weakly informative (*blavaan* defaults)**  | **Noninformative/ diffuse**  |
| *Regressions* |  |  |  |
|  *IQUAL2* on Experimental Group | *Normal*(0, 2) | *Normal*(0, 10) | *Normal*(0, 1000) |
|  *IQUAL2* on *IQUAL1* | *Normal*(0.6, 2) | *Normal*(0, 10) | *Normal*(0, 1000) |
|  *IQUAL2* on Digital Media Involvement | *Normal*(0.3, 2) | *Normal*(0, 10) | *Normal*(0, 1000) |
|  *IQUAL2* on Play as Learning | *Normal*(0.1, 2) | *Normal*(0, 10) | *Normal*(0, 1000) |
|  *IQUAL2* on Role in Child Development | *Normal*(0.1, 2) | *Normal*(0, 10) | *Normal*(0, 1000) |
|  *IQUAL2* on Positive Digital Media Beliefs | *Normal*(0, 2) | *Normal*(0, 10) | *Normal*(0, 1000) |
|  *IQUAL2* on Time Spent with App Nudges\* | *Normal*(0.1, 2) | *Normal*(0, 10) | *Normal*(0, 1000) |
|  |  |  |  |
| *Factor Loadings* |  |  |  |
|  All  | *Normal*(1, 5)  | *Normal*(0, 10) | *Normal*(0, 1000) |

*Note.* In all models, weakly informative (*blavaan* default) priors were used for all covariances (*Beta*(1, 1)), latent variable intercepts (*Normal*(0, 10)), observed variable intercepts (*Normal*(0, 32)), and variances (*Gamma*(1, 0.5)).

Supplemental Table 5. Full results from the Bayesian SEM.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Parameter** | **Posterior mean** | **Posterior SD** | **HPDI** | **Gelman-Rubin Statistic** |
| *Regressions* |  |  |  |  |
|  *IQUAL2* on Experimental Group |  0.015 | 0.096 | [-0.174, 0.205] | 1.000 |
|  *IQUAL2* on *IQUAL1* |  0.699 | 0.097 | [0.528, 0.887] | 1.002 |
|  *IQUAL2* on Digital Media Involvement |  0.067 | 0.038 | [-0.007, 0.142] | 1.000 |
|  *IQUAL2* on Play as Learning |  0.015 | 0.133 | [-0.245, 0.281] | 0.999 |
|  *IQUAL2* on Role in Child Development | -0.022 | 0.171 | [-0.345, 0.319] | 0.999 |
|  *IQUAL2* on Positive Digital Media Beliefs | -0.068 | 0.044 | [-0.158, 0.018] | 1.001 |
|  |  |  |  |  |
| *Factor Loadings* |  |  |  |  |
|  *IQUAL2* to Positivity Composite (t2) | 1.000 | -- | -- | -- |
|  *IQUAL2* to Conversation Connectedness (t2) | 1.116 | 0.163 | [0.819, 1.462] | 1.001 |
|  *IQUAL2* to Coordinated Joint Engagement (t2) | 0.956 | 0.173 | [0.628, 1.333] | 1.002 |
|  *IQUAL2* to Mutuality Composite (t2) | 1.093 | 0.113 | [0.888, 1.334] | 1.003 |
|  |  |  |  |  |
|  *IQUAL1* to Positivity Composite (t1) | 1.000 | -- | -- | -- |
|  *IQUAL1* to Conversation Connectedness (t1) | 0.829 | 0.144 | [0.551, 1.110] | 0.999 |
|  *IQUAL1* to Coordinated Joint Engagement (t1) | 0.725 | 0.145 | [0.435, 1.012] | 1.001 |
|  *IQUAL1* to Mutuality Composite (t1) | 1.022 | 0.069 | [0.896, 1.162] | 1.000 |
|  |  |  |  |  |
| *Covariances* |  |  |  |  |
|  *IQUAL1* with Dig Med Inv *IQUAL1* with Play as Learning | 0.0650.072 | 0.1230.036 | [-0.179, 0.312][0.007, 0.148] | 1.0021.000 |
|  *IQUAL1* withRole in Child Dev | 0.045 | 0.026 | [-0.002, 0.100] | 1.000 |
|  *IQUAL1* withPos Dig Med Beliefs  | 0.105 | 0.108 | [-0.104, 0.329] | 1.001 |
| Dig Med Inv with Play as Learning | 0.241 | 0.095 | [0.072, 0.441] | 1.002 |
|  Dig Med Inv with Role in Child Dev | 0.029 | 0.066 | [-0.101, 0.158] | 1.002 |
| Dig Med Inv with Pos Dig Med Beliefs | 1.462 | 0.327 | [0.887, 2.173] | 1.001 |
|  |  |  |  |  |
|  Play as Learning with Role in Child Dev | 0.074 | 0.021 | [0.038, 0.119] | 1.001 |
|  Play as Learning with Pos Dig Med Beliefs | 0.182 | 0.078 | [0.043, 0.352] | 1.003 |
|  |  |  |  |  |
|  Role in Child Dev with Pos Dig Med Beliefs | 0.038 | 0.055 | [-0.069, 0.147] | 1.001 |
|  |  |  |  |  |
| *Variances* |  |  |  |  |
|  *IQUAL1* | 0.393 | 0.083 | [0.260, 0.583] | 1.002 |
|  *IQUAL2* (residual) | 0.142 | 0.035 | [0.085, 0.219] | 1.001 |
| Pos Dig Med Beliefs | 2.160 | 0.352 | [1.559, 2.945] | 1.000 |
| Role in Child Dev | 0.114 | 0.020 | [0.082, 0.160] | 1.001 |
|  Play as Learning | 0.217 | 0.037 | [0.158, 0.301] | 1.002 |
| Dig Med Inv | 2.994 | 0.481 | [2.188, 4.050] | 1.000 |
| Mutuality T1 (residual) | 0.011 | 0.012 | [0.000, 0.044] | 1.000 |
| Coordinated T1 (residual) | 0.520 | 0.088 | [0.376, 0.713] | 0.999 |
| Connectedness T1 (residual) | 0.577 | 0.102 | [0.416, 0.816] | 1.000 |
| Positivity T1 (residual) | 0.098 | 0.020 | [0.059, 0.142] | 1.001 |
| Mutuality T2 (residual) | 0.078 | 0.029 | [0.025, 0.142] | 1.001 |
| Coordinated T2 (residual) | 0.048 | 0.084 | [0.342, 0.672] | 0.999 |
| Connectedness T2 (residual) | 0.405 | 0.080 | [0.273, 0.582] | 1.000 |
| Positivity T2 (residual) | 0.157 | 0.034 | [0.100, 0.234] | 0.999 |

Supplemental Table 6. Results from sensitivity analysis.

|  |  |  |  |
| --- | --- | --- | --- |
| **Parameter** | **Estimate [HPDI] from original model (informative)** | **Estimate [HPDI] using *blavaan* defaults** **(weakly informative)** | **Estimate [HPDI] using diffuse priors (noninformative)** |
| *Regressions* |  |  |  |
|  *IQUAL2* on Experimental Group |  0.015 [-0.174, 0.205] |  0.014 [-0.171, 0.197]  |  0.016 [-0.174, 0.206] |
|  *IQUAL2* on *IQUAL1* |  0.699 [0.528, 0.887] |  0.697 [0.510, 0.895]  |  0.698 [0.519, 0.879] |
|  *IQUAL2* on Digital Media Involvement |  0.067 [-0.007, 0.142] |  0.066 [-0.003, 0.141]  |  0.065 [-0.007, 0.139] |
|  *IQUAL2* on Play as Learning |  0.015 [-0.245, 0.281] |  0.017 [-0.239, 0.267]  |  0.019 [-0.242. 0.285] |
|  *IQUAL2* on Role in Child Development | -0.022 [-0.345, 0.319] | -0.012 [-0.347, 0.351]  | -0.025 [-0.371, 0.329] |
|  *IQUAL2* on Positive Digital Media Beliefs | -0.068 [-0.158, 0.018] | -0.067 [-0.153, 0.020]  | -0.066 [-0.153, 0.020] |
|  |  |  |  |
| *Factor Loadings* |  |  |  |
|  *IQUAL2* to Positivity Composite (t2) | 1.000 | 1.000 | 1.000 |
|  *IQUAL2* to Conversation Connectedness (t2) | 1.116 [0.819, 1.462] | 1.117 [0.809, 1.455] | 1.120 [0.826, 1.459] |
|  *IQUAL2* to Coordinated Joint Engagement (t2) | 0.956 [0.628, 1.333] | 0.958 [0.643, 1.306] | 0.9645 [0.636, 1.308] |
|  *IQUAL2* to Mutuality Composite (t2) | 1.093 [0.888, 1.334] | 1.098 [0.887, 1.330] | 1.095 [0.888, 1.342] |
|  |  |  |  |
|  *IQUAL1* to Positivity Composite (t1) | 1.000 | 1.000 | 1.000 |
|  *IQUAL1* to Conversation Connectedness (t1) | 0.829 [0.551, 1.110] | 0.833 [0.552, 1.124] | 0.832 [0.548. 1.119] |
|  *IQUAL1* to Coordinated Joint Engagement (t1) | 0.725 [0.435, 1.012] | 0.728 [0.459, 1.027] | 0.732 [0.464, 1.023] |
|  *IQUAL1* to Mutuality Composite (t1) | 1.022 [0.896, 1.162] | 1.023 [0.890, 1.166] | 1.026 [0.893, 1.174] |

*Note.* All estimates are unstandardized. HPDI = Highest Posterior Density Interval.

Supplemental Table 7. Results from Regression Analysis Predicting Time Spent with Nudges.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Predictor** | ***Beta*** | **S.E.** | ***t*** | ***p*-value** |
|  Intercept | 6.01 | 17.67 | 0.34 | 0.74 |
|  Digital Media Positive Beliefs | 0.09 | 0.84 | 0.11 | 0.91 |
|  Perceived Parent Role | 2.73 | 2.91 | 0.94 | 0.35 |
|  Play as Learning | -3.09 | 2.39 | -1.29 | 0.21 |
|  Digital Media Parent Involvement | 0.44 | 0.67 | 0.66 | 0.51 |

Supplemental Figure 1. Presence of co-play across games and experimental groups at pre- and post-test.



Supplemental Figure 2. Posterior density plots for regression estimates from full Bayesian SEM.



Supplemental Figure 3. Trace plots for regression estimates from full Bayesian SEM.



Supplemental Figure 4. Results from parent feedback survey.



*Note.* 1=Never, 2=Almost never, 3=Sometimes, 4=Almost always, 5=Always.



*Note.* 1=Never, 2=Almost never, 3=Sometimes, 4=Almost always, 5=Always.



*Note.* 1=Never, 2=Almost never, 3=Sometimes, 4=Almost always, 5=Always.



*Note.* 1=Never, 2=Almost never, 3=Sometimes, 4=Almost always, 5=Always.