Compare & Discuss Problems

Topic 3: Solving Systems of Linear Equations

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Implementation Checklist

Compare	**Prepare to Compare** Students took time to understand what the problem was asking and understand both methods.	☐ Yes	☐ No
	Make Comparisons Students identified mathematical similarities and differences between the two methods.	☐ Yes	☐ No
Discuss	**Prepare to Discuss** <u>Think</u>: Students spent around 1 minute thinking independently about the worksheet prompts.	☐ Yes	☐ No
	<u>Pair</u>: Students spent around 2 minutes working in pairs or small groups discussing the worksheet prompts.	☐ Yes	☐ No
	Discuss Connections <u>Share</u>: A 3-6 minute whole-class conversation occurred where students discussed connections that included question asking and answering by the teacher and students. • Most students were involved in this whole class conversation. • The teacher asked follow-up questions in response to students' thinking, such as "Why do you think that's true?", "Do you agree or disagree? Why?", "Can you say more about that?", and "What did you like about their answer?".	☐ Yes	☐ No
	Identify the Big Idea The teacher showed the Big Idea page to the class to provide a clear, explicit statement of the Big Idea. Students identified the Big Idea and summarized it in their own words.	☐ Yes	☐ No

Timing	At least 8 minutes were spent in the Compare phase and at least 12 minutes were spent in the Discussion phase. Students spent more than half their time in the Discussion phase.	☐ Yes	☐ No

Compare & Discuss: Algebra 1 PD Institute
Discussion Resources

<u>Why have a mathematical discussion?</u>

- To deepen students' understanding of the mathematical content.
- To enhance student engagement and interest in mathematics.

<u>What should a teacher do to have a good mathematical discussion?</u>

- **<u>BEFORE</u>** the discussion starts:
  - Thoroughly **solve** the problems that will be discussed.
  - **Anticipate** student responses, errors, and difficulties.
  - **Plan** questions to ask, as well as problem extensions to use.
- **<u>DURING</u>** the discussion:
  - **Ask** lots of open-ended questions, using the following question stems to spark and continue conversation:
    - *Do you agree with Layla? Why?*
    - *Can you summarize what Riley said?*
    - *Can you give another example?*
    - *Can you describe that in more detail?*
    - *What do you mean by XXXX?*
    - *How did you do that?*
    - *What might be confusing about this example?*
  - **Re-voice** and **summarize** student contributions to keep the conversation going, saying things like:
    - *What I am hearing is XXXX. Is that what you mean?*
    - *Are you saying XXXX?*
    - *I am not sure I understand what you mean. Can you explain it again?*
  - **Manage** flow of the conversation, involving many voices from the class.
  - **Involve as many students** in the discussion as possible.
    - Be sure to **solicit** participation from students who do not have their hands raised, using *equity sticks*, *note cards*, *spinners*, or a *random name generator* for randomly selecting students to speak.
    - **Consider keeping track** of which students have spoken with a clipboard of the class roster, both to remember who has spoken and to ensure equitable participation.
  - **Hold** students accountable for listening to and understanding others' contributions, saying things like:
    - *Gloria thinks that XXXX. Tim, can you summarize what Gloria said, in your own words?*
  - **Provide** students credit for discussion participation as part of their grade.

Prepare to Compare & Discuss: Teacher Prep Checklist

For each Worked Example Pair, it is important you review the problem and its associated worksheets in advance before presenting it to the class. When prepping, keep the following checklist in mind:

- ✓ **Ensure you understand** each method in the WEP.
- ✓ **Read** the Big Idea message so you know where the discussion should lead by the conclusion of the exercise.
- ✓ **Review** the prompt on the *Discuss Connections* worksheet, and:
  - o **Add** extension questions that will push your students to dig deeper during the discussion, OR
  - o **Create** additional, supporting questions that will allow struggling students to grapple with the prompt more successfully.
- ✓ **Determine** when in the class you plan to present the WEP.
- ✓ **Make sufficient copies** of the worksheet(s) for participating students.

That's it! For each WEP, we don't anticipate more than 5-10 minutes of prep. Please remember to reach out to research staff if any questions or concerns come up during planning.

Differentiating Compare & Discuss Problems

We strongly believe, and our research supports, that Compare & Discuss problems can be an effective way to engage in mathematics for all learners. Below, you will find a general list of recommendations to keep in mind as you consider differentiating the Compare & Discuss problems to fit your students' needs.

DON'T:

- **Change** the examples such that they are a far removal from the implementation model.
- **Skip** whole chapters.
- **Change or adapt** the tests.
    o For research purposes, it is important every student takes the same test, even if content on the test was not covered in your class.
- **Eliminate** the side-by-side comparison of the solution methods.
- **Rush through/gloss over** the WEPs (don't save them for the last 5 minutes of class!).
    o If you are working with students and the Compare phase seems like it is moving quickly, that might not be a problem—it gives you more time to work on the Discuss phase, incorporating more extension questions for deeper discussion.

DO:

✓ **Plan ahead** with research staff.
✓ **Adapt** WEPs for content not covered, rather than skipping the examples altogether.
✓ **Blend** comparison types – types are not mutually exclusive (some can be both Why does it work? & Which is better?).
    o This may influence your extension questions for the Discuss phase.
✓ **Address** changes for later chapters with lower level classes (content in earlier chapters [1, 3, and 5] tends to be covered in all levels, but you may need to change/adapt content for later chapters [7, 9]).
✓ **Adapt** Which is correct?/How do they differ? WEPs for lower level classes.
    o Some students may be overwhelmed by a comparison with two different problems; others may struggle with determining which method is incorrect. Discuss with research staff ways to adapt these two comparison types for struggling students.

*Lastly, we encourage **<u>creativity</u>**! We're happy to work with you to find ways to incorporate the Compare & Discuss problems into your class as a yearlong theme (e.g. using **Holiday greeting cards, dress-up days**, etc.).*

Topic 3: Solving Systems of Linear Equations- Overview

Section	Table of Contents (Page #)	WEP Type	Suggested Use
3.1	7	Which is correct?	Mid-lesson
3.2	11	Why does it work?	Beginning of lesson
3.3	15	Which is better?	Mid-lesson
3.4	19	Which is better?	Mid-lesson
3.5	23	Which is better?	End of lesson
3.6	27	Why does it work?	Beginning of lesson
3.7	31	Why does it work?	End of lesson
3.8	35	Why does it work?	Mid-lesson
3.9	39	Which is better?	Review activity

Compare (8 minutes)

Prepare to Compare

- What is the problem asking?
- What is happening in the first method?
- What is happening in the second method?

Make Comparisons

- What are the similarities and differences between the two methods?
  - Which method is better?
  - Which method is correct?
  - Why do both methods work?
  - How do the problems differ?

Discuss (12minutes)

Prepare to Discuss (think, pair)

- How does this comparison help you understand this problem?
- How might you apply these methods to a similar problem?

Discuss Connections (share)

- What ideas would you like to share with the class?

Identify the Big Idea

- Can you summarize the Big Idea in your own words?

Topic 3.1: Solving Systems of Linear Equations by Graphing

WEP Type: Which is correct?
Suggested use: Mid-lesson
Problem: Riley and Gloria were asked if the point (-5, 6) is a solution to the linear system

$$\begin{cases} y = -3x - 9 \\ y = 2x - 3 \end{cases}$$

Phase	Guiding Discussion Questions and Implementation Notes
Prepare to Compare	**How does this shortcut work?** **Would it result in the same solution if Riley decided to graph the equations instead?** **Does it matter into which equation you plug the point?**
Make Comparisons	**Which method is correct?** **What is the same or similar about Riley's "plug into the first equation" method and Gloria's "plug into the second equation" method? What is different?** *Riley and Gloria both plugged the point into one of the equations to see if it makes the equation true. Riley chose the first equation, and Gloria chose the second equation.*
Prepare to Discuss (Think, Pair)	**Can both Riley and Gloria be correct? Can a point be both a solution and not a solution?**
Discuss Connections (Share)	*No, Riley and Gloria cannot both be correct. Riley only plugged the point into the first equation, saw that it worked, and assumed it would work the second equation as well. If he had plugged the point into the second equation, he would have seen that it does not work as a solution. So, the point does not lie on the line of the second equation, making it not a solution for the system. Gloria is correct, but Riley is not.*
Identify the Big Idea	**How did Riley's mistake happen? What is the solution to a system of equations?** *On a graph, the solution is an intersection point. This is because the solution is a point on both lines. If you plug in a point to see if it is a solution, you have to check both lines. If the point is not on one of the lines, you know it can't be a solution to the system.*

Riley and Gloria were asked if (–5, 6) is a solution to the system

$$\begin{cases} y = -3x - 9 \\ y = 2x - 3 \end{cases}$$

Riley's "plug into the first equation" way

I can just plug in (–5, 6).

$$\begin{cases} y = -3x - 9 \\ y = 2x - 3 \end{cases}$$

↓

$$y = -3x - 9$$
$$6 = -3(-5) - 9$$
$$6 = 15 - 9$$
$$6 = 6$$

↓

Yes, (-5, 6) is a solution.

Gloria's "plug into the second equation" way

I can just plug in (–5, 6).

$$\begin{cases} y = -3x - 9 \\ y = 2x - 3 \end{cases}$$

↓

$$y = 2x - 3$$
$$6 = 2(-5) - 3$$
$$6 = -10 - 3$$
$$6 = -13$$

↓

No, (-5, 6) is not a solution.

Does it matter into which equation you plug the point?

What is the same or similar about Riley's "plug into the first equation" method and Gloria's "plug into the second equation" method? What is different?

Discuss Connections

Can both Riley and Gloria be correct? Can a point be both a solution and not a solution?

Think, Pair. First, think about the question(s) above independently. Then, get with a partner and discuss your answers. After talking with your partner, what is your answer?

Think	Pair

Share. After reviewing the worksheet as a class, summarize the answer(s) your class agrees on. Was this different from your original response?

Big Idea. When your teacher tells you to do so, write what you think is the big idea of this example, in your own words.

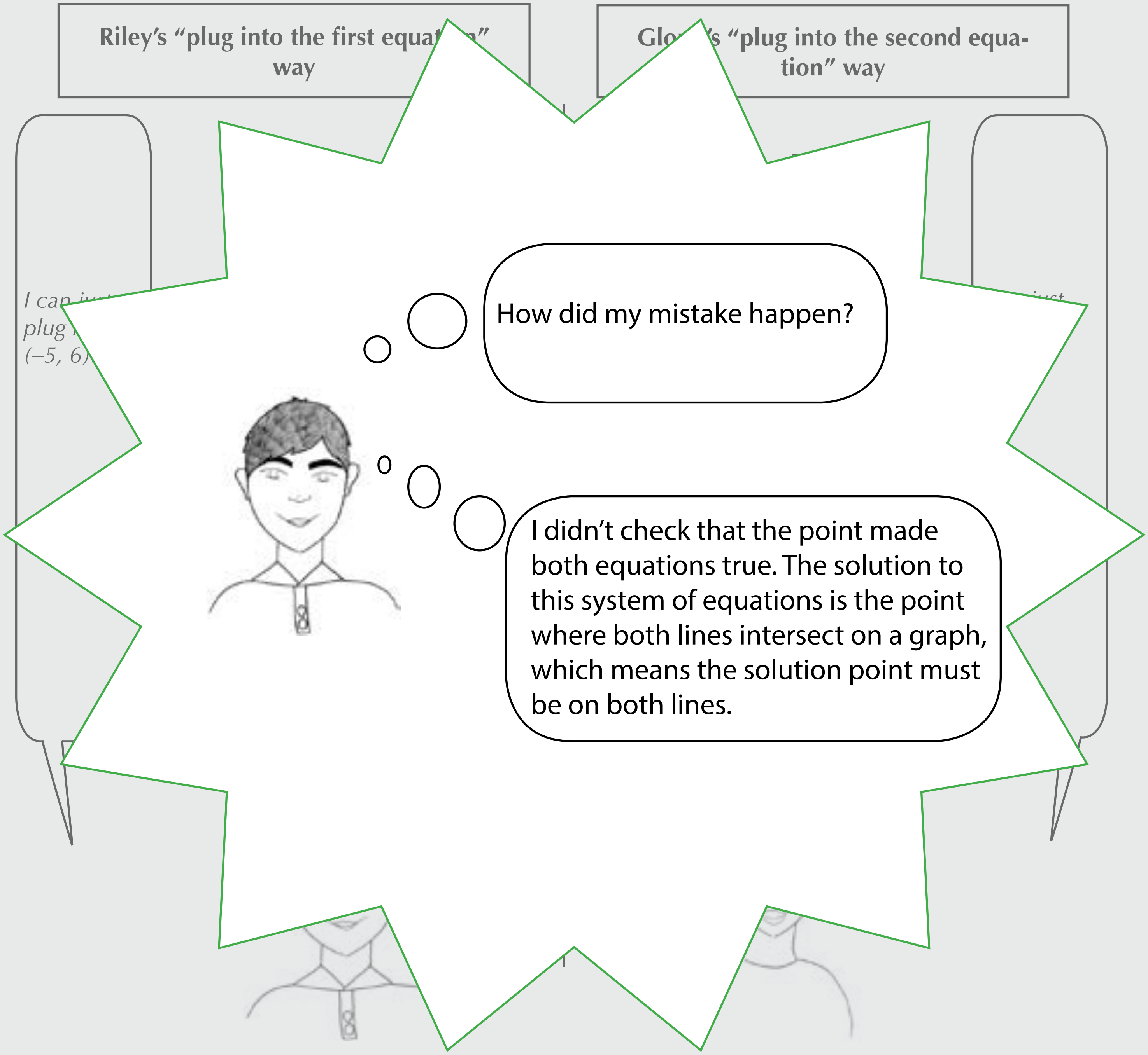
Riley and Gloria were asked if (–5, 6) is a solution to the system

$$\begin{cases} y = -3x - 9 \\ y = 2x - 3 \end{cases}$$

Riley's "plug into the first equat... way

Glo...'s "plug into the second equation" way

I can ju... plug i... (–5, 6)...



Does it matter into which equation you plug the point?

What is the same or similar about Riley and Gloria's methods? What is different?

Topic 3.2: Solving Systems of Linear Equations by Substitution

WEP Type: Why does it work?
Suggested use: Beginning of lesson
Problem: Emma and Layla were asked to solve the linear system

$$\begin{cases} x + 3y = 2 \\ 5x + y = -4 \end{cases}$$

Phase	Guiding Discussion Questions and Implementation Notes
Prepare to Compare	**Why did Emma start by solving for the x variable?** **Why did Layla start by solving for the y variable?** **What did Emma and Layla do to find the solution to the system?**
Make Comparisons	**Why do both methods work?** **Does it matter which variable you solve for first? Why or why not?** *No. Emma and Layla both followed the steps for solving a system of equations using substitution correctly: 1) solve for the variable in one equation, then 2) plug that expression into the other equation and solve; lastly, 3) plug this value back into the first equation you solved for to find the value of the next variable. Because Emma and Layla both followed these steps and just started with different equations in step 1, they arrived at the same solution.*
Prepare to Discuss (Think, Pair)	**If the problem were changed so the second equation was 5x + 2y = -4, would you use Emma's "solve for x" or Layla's "solve for y" way? Why?**
Discuss Connections (Share)	*Emma's way, because the first equation has an x with a coefficient of 1, making it easier to solve for.*
Identify the Big Idea	**How does substitution work? Is there more than one way to use substitution?** *The process of substitution allows you to eliminate one variable so that you can solve for the other. So, there is more than one way to use substitution: you can do this for either variable first, and you'll get the same answer in the end.*

Emma and Layla were asked to solve the linear system

$$\begin{cases} x + 3y = 2 \\ 5x + y = -4 \end{cases}$$

Emma's "solve for x" way

$$\begin{cases} x + 3y = 2 \\ 5x + y = -4 \end{cases}$$

↓

$x = 2 - 3y$
$5(2 - 3y) + y = -4$
$10 - 15y + y = -4$
$10 - 14y = -4$
$-14y = -14$
$y = 1$

↓

$x = 2 - 3y$
$x = 2 - 3(1)$
$x = 2 - 3$
$x = -1$

↓

The solution is (-1, 1)

I solved the first equation for x. I substituted this into the second equation, and then I solved for y.

I plugged this answer back into the equation I solved for x. I then found x. Here is my answer.

Layla's "solve for y" way

$$\begin{cases} x + 3y = 2 \\ 5x + y = -4 \end{cases}$$

↓

$y = -4 - 5x$
$x + 3(-4 - 5x) = 2$
$x - 12 - 15x = 2$
$-14x - 12 = 2$
$-14x = 14$
$x = -1$

↓

$y = -4 - 5x$
$y = -4 - 5(-1)$
$y = -4 + 5$
$y = 1$

↓

The solution is (-1, 1)

I solved the second equation for y. I substituted this into the first equation, and then I solved for x.

I plugged this answer back into the equation I solved for y. I then found y. Here is my answer.

What did Emma and Layla do to find the solution to the system?

Does it matter which variable you solve for first? Why or why not?

Discuss Connections

If the problem were changed so the second equation was $5x + 2y = -4$, **would you use Emma's "solve for x" way or Layla's "solve for y" way? Why?**

Think, Pair. First, think about the question(s) above independently. Then, get with a partner and discuss your answers. After talking with your partner, what is your answer?

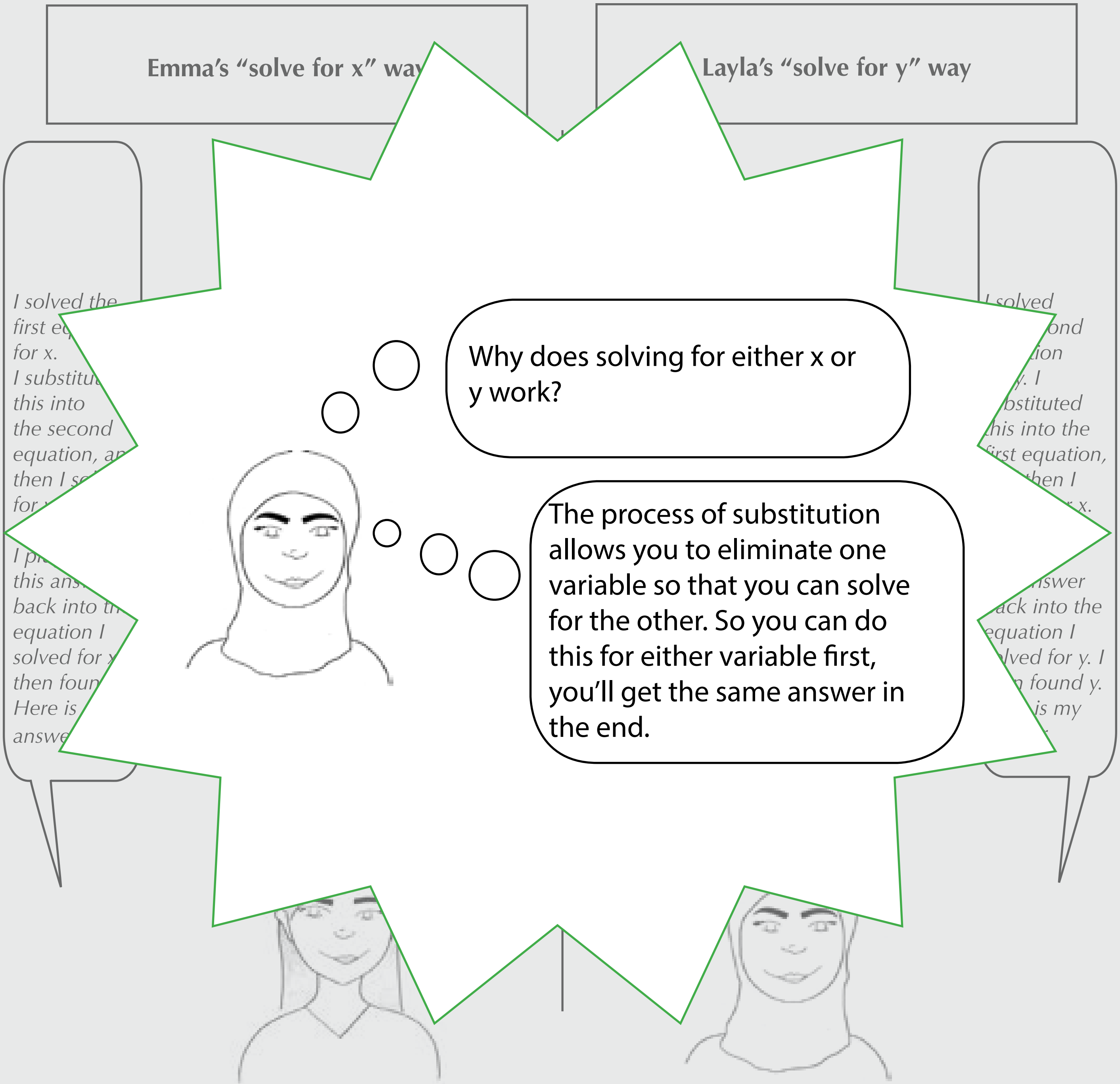
Think	Pair

Share. After reviewing the worksheet as a class, summarize the answer(s) your class agrees on. Was this different from your original response?

Big Idea. When your teacher tells you to do so, write what you think is the big idea of this example, in your own words.

Emma and Layla were asked to solve the linear system

$$\begin{cases} x + 3y = 2 \\ 5x + y = -4 \end{cases}$$



What did Emma and Layla do to find the solution to the system?

Does it matter which variable you solve for first? Why or why not?

Topic 3.3: Solving Systems of Linear Equations by Substitution

WEP Type: Which is better?
Suggested use: Mid-lesson
Problem: Gloria and Tim were asked to solve the linear system

$$\begin{cases} 4x + 6y = 4 \\ x - 2y = -6 \end{cases}$$

Phase	Guiding Discussion Questions and Implementation Notes
Prepare to Compare	**Why did Gloria choose to solve the second equation for the x variable? Why did Tim choose to solve the second equation for the y variable?** **Why did the negatives disappear when Tim solved for the y variable?** **Will both methods lead to the same solution?**
Make Comparisons	**Which method do you think is better, Gloria's "solve for x" way or Tim's "solve for y" way? Why?** *Gloria's "solve for x" method is easier because x has a coefficient of one and is therefore easier to solve for. Because y has a coefficient of 2, Tim had to take an extra step to solve for y, and it resulted in him having to use fractions in the following steps, which are more difficult to work with than the whole numbers Gloria was able to use.*
Prepare to Discuss (Think, Pair)	**If the original problem had 4x + 2y = -6 as the second equation, would you use Gloria's "solve for x" way or Tim's "solve for y" way? Why?**
Discuss Connections (Share)	*If I had to use substitution to solve this system of equations, I would use Tim's "solve for y" method of solving for the y variable in the second equation. When solving for the y variable, the expression you end up with will have only whole numbers (y = -2x -3), whereas if you use Gloria's "solve for x" way to solve for the x variable, you would end up with fractions in the expression (x = -1/2y – 3/2).*
Identify the Big Idea	**What features should you look for in a system of equations to help you decide the easiest way to use the substitution method?** *To find the easiest way, you can look at the system to see which variable will be easier to solve for in one of the equations. For example, variables with a coefficient of 1 are often easiest to solve for. If neither variable has a coefficient of 1 and you need to use substitution, you need to think about what each resulting expression would look like when you solve for one of the variables; working with whole numbers is always easier than working with fractions!*

Gloria and Tim were asked to solve the linear system

$$\begin{cases} 4x + 6y = 4 \\ x - 2y = -6 \end{cases}$$

Gloria's "solve for x" way

$$\begin{cases} 4x + 6y = 4 \\ x - 2y = -6 \end{cases}$$

I solved the second equation for x.

$$x - 2y = -6$$
$$x = 2y - 6$$

I substituted this into the first equation.

$$4(2y - 6) + 6y = 4$$

Then I solved for y.

$$8y - 24 + 6y = 4$$
$$14y - 24 = 4$$
$$14y = 28$$
$$y = 2$$

I plugged this back into the equation I solved for x. Then I found x.

$$x = 2y - 6$$
$$x = 2(2) - 6$$
$$x = 4 - 6$$
$$x = -2$$

Here is my answer.

The solution is (-2, 2)

Tim's "solve for y" way

$$\begin{cases} 4x + 6y = 4 \\ x - 2y = -6 \end{cases}$$

I solved the second equation for y.

$$x - 2y = -6$$
$$-2y = -x - 6$$
$$y = \frac{x}{2} + 3$$

I substituted this into the first equation.

$$4x + 6(\frac{x}{2} + 3) = 4$$

Then I solved for x.

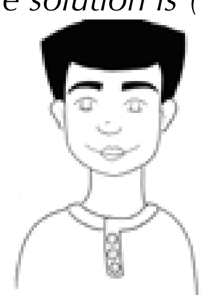
$$4x + 3x + 18 = 4$$
$$7x + 18 = 4$$
$$7x = -14$$
$$x = -2$$

I plugged this back into the equation I solved for y. Then I found y.

$$y = \frac{x}{2} + 3$$
$$y = \frac{-2}{2} + 3$$
$$y = -1 + 3$$
$$y = 2$$

Here is my answer.

The solution is (-2, 2)



Why did Gloria choose to solve the second equation for the x variable? Why did Tim choose to solve the second equation for the y variable?

Which method do you think is better, Gloria's "solve for x" or Tim's "solve for y" way? Why?

Discuss Connections

If the original problem had $4x + 2y = -6$ **as the second equation, would you use Gloria's "solve for x" way or Tim's "solve for y" way? Why?**

Think, Pair. First, think about the question(s) above independently. Then, get with a partner and discuss your answers. After talking with your partner, what is your answer?

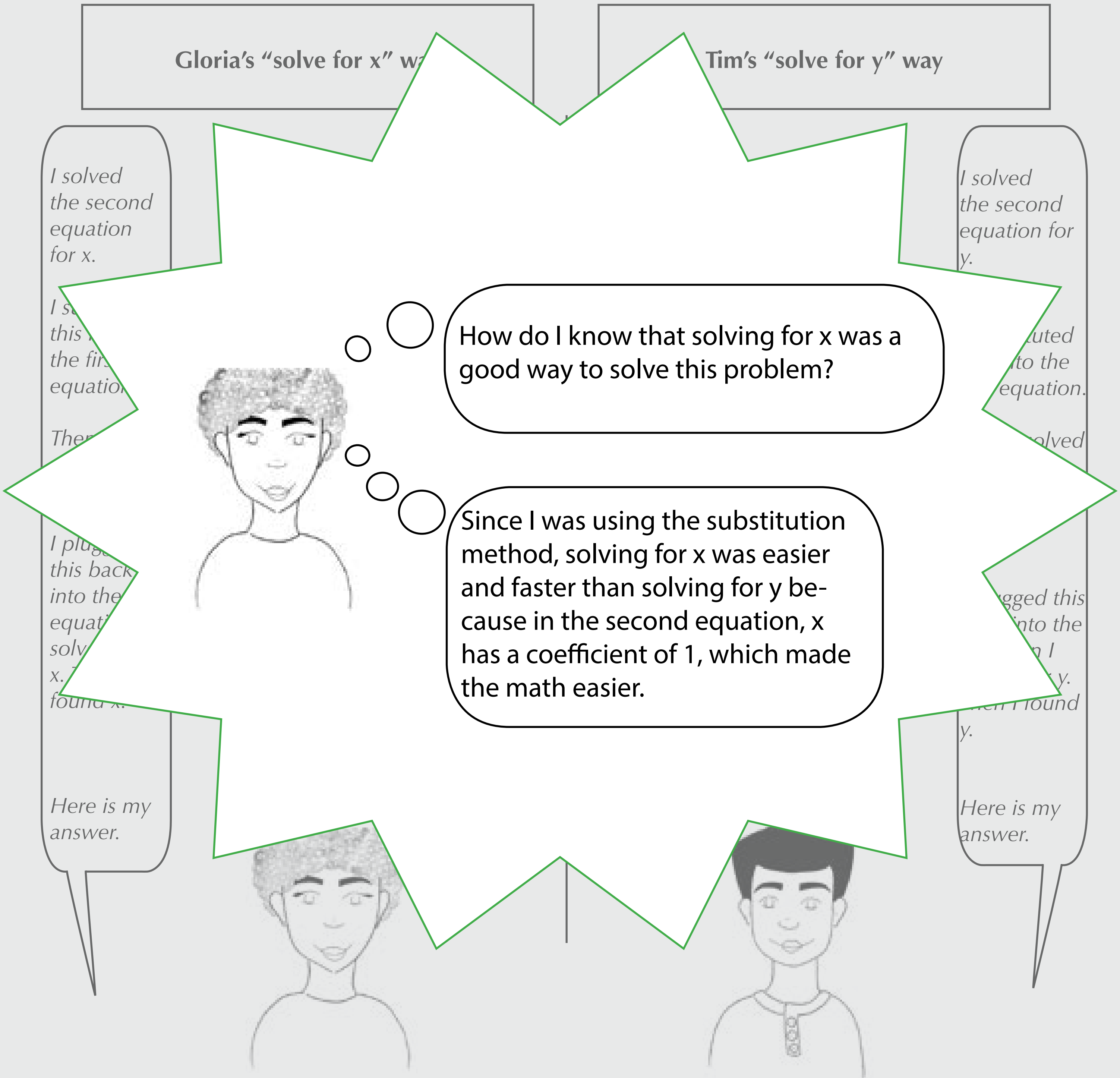
Think	Pair

Share. After reviewing the worksheet as a class, summarize the answer(s) your class agrees on. Was this different from your original response?

Big Idea. When your teacher tells you to do so, write what you think is the big idea of this example, in your own words.

Gloria and Tim were asked to solve the linear system

$$\begin{cases} 4x + 6y = 4 \\ x - 2y = -6 \end{cases}$$



Why did Gloria choose to solve the second equation for the x variable? Why did Tim choose to solve the second equation for the y variable?

Which method do you think is better, Gloria's or Tim's? Why?

Topic 3.4: Solving Systems of Linear Equations by Elimination

WEP Type: Why does it work?
Suggested use: Mid-lesson
Problem: Layla and Riley were asked to solve the linear system

$$\begin{cases} 3x + 2y = 9 \\ -3x - 6y = 3 \end{cases}$$

Phase	Guiding Discussion Questions and Implementation Notes
Prepare to Compare	**Why does Riley multiply the top equation by 3? What does Layla mean when she says she "eliminated the x variable"?** **Why did Riley plug 5 into the second equation to find y instead of the first equation?**
Make Comparisons	**Why do both methods work?** **Layla used the "eliminate the x terms" way and Riley used the "eliminate the y terms" way, yet they got the same answer. Why?** *Layla found the y-coordinate of the solution first, while Riley found the x-coordinate first. But the solution should be the same for both methods – both methods lead to finding the same point.*
Prepare to Discuss (Think, Pair)	**Is there a situation where eliminating x would be better than eliminating y, or vice versa?**
Discuss Connections (Share)	*If both x terms or both y terms have the same or opposite coefficients, it is easier to eliminate those terms first. Or if one term has coefficients that are multiples of each other, then you only need to multiply 1 equation in the first step before you eliminate. In this case, Layla's method is easier because the x terms have the same and opposite coefficients.*
Identify the Big Idea	**What did comparing these methods tell us about solving a system of equations by elimination?** *When using the elimination method, you can eliminate either the x-variable or the y-variable. In either case, you are merely changing the form of the equation to make it easier to find a point that is the solution to the system.*

Layla and Riley were asked to solve the linear system

$$\begin{cases} 3x + 2y = 9 \\ -3x - 6y = 3 \end{cases}$$

Layla's "eliminate the x terms" way

$$\begin{cases} 3x + 2y = 9 \\ -3x - 6y = 3 \end{cases}$$

$$\begin{aligned} 3x + 2y &= 9 \\ \underline{-3x - 6y} &\underline{= 3} \\ -4y &= 12 \\ y &= -3 \end{aligned}$$

First, I eliminated the x variable. Then, I solved for y.

$$\begin{aligned} 3x + 2(-3) &= 9 \\ 3x - 6 &= 9 \\ 3x &= 15 \\ x &= 5 \end{aligned}$$

I plugged this into the first equation to solve for x.

The solution is (5, -3)

This is my solution.

Riley's "eliminate the y terms" way

$$\begin{cases} 3x + 2y = 9 \\ -3x - 6y = 3 \end{cases}$$

$$3(3x + 2y = 9)$$

$$\begin{aligned} 9x + 6y &= 27 \\ \underline{-3x - 6y} &\underline{= 3} \\ 6x \quad &= 30 \\ x &= 5 \end{aligned}$$

First, I multiplied the top equation by 3. Then, I solved for x.

$$\begin{aligned} -3(5) - 6y &= 3 \\ -15 - 6y &= 3 \\ -6y &= 18 \\ y &= -3 \end{aligned}$$

I plugged this into the second equation to solve for y.

The solution is (5, -3)

This is my solution.

Why does Riley multiply the top equation by 3? What does Layla mean when she says she "eliminated the x variable"?

Layla and Riley used different ways, yet got the same answer. Why?

Discuss Connections

Is there a situation where eliminating x would be better than eliminating y, or vice versa?

Think, Pair. First, think about the question(s) above independently. Then, get with a partner and discuss your answers. After talking with your partner, what is your answer?

Think	Pair

Share. After reviewing the worksheet as a class, summarize the answer(s) your class agrees on. Was this different from your original response?

Big Idea. When your teacher tells you to do so, write what you think is the big idea of this example, in your own words.

Layla and Riley were asked to solve the linear system

$$\begin{cases} 3x + 2y = 9 \\ -3x - 6y = 3 \end{cases}$$

Layla's "…inate the x t… | **…iminate the … …ms" way**

First, I multiplied the … to… b… so…

I plu… t…

This is … soluti…

First, I multiplied the … …x.

…my

What is the best way to use the elimination method?

You can eliminate either the x-variable or the y-variable.The best way to use the elimination method is to eliminate the variable where the coefficients are the same (or opposites) in the two equations.

Why does Riley multiply the top equation by 3? What does Layla mean when she says she "eliminated the x variable"?

Layla and Riley used different ways, yet got the same answer. Why?

Topic 3.5: Solving Systems of Linear Equations by Elimination

WEP Type: Which is better?
Suggested use: End of lesson
Problem: Tim and Emma were asked to solve the linear system

$$\begin{cases} 3x + 2y = 8 \\ x - 3y = 10 \end{cases}$$

Phase	Guiding Discussion Questions and Implementation Notes
Prepare to Compare	**Why does Tim begin with his first step?** **Why did Tim choose to plug y = -2 into the second equation to find x instead of the first equation?**
Make Comparisons	**Which method is better? What are some advantages of Tim's "substitution" way and Emma's "elimination" way?** *Gloria's "solve for x" method is easier because x has a coefficient of one and is therefore easier to solve for. Because y has a coefficient of 2, Tim had to take an extra step to solve for y, and it resulted in him having to use fractions in the following steps, which are more difficult to work with than the whole numbers Gloria was able to use.*
Prepare to Discuss (Think, Pair)	**Is there a situation where substitution would be better than elimination, or vice versa?**
Discuss Connections (Share)	*If one of the equations has a variable with a coefficient of 1, that equation is easy to rearrange, so substitution might be better. If the same variable in both equations has the same or opposite coefficients, then elimination might be better.*
Identify the Big Idea	**Can you always use either substitution or elimination? Which is better?** *When solving a system of linear equations, substitution and elimination are both correct methods that will give you the same answer. Substitution might be easier if the variable has a coefficient of 1.*

Tim and Emma were asked to solve the linear system

$$\begin{cases} 3x + 2y = 8 \\ x - 3y = 10 \end{cases}$$

Tim's "substitution" way

$$\begin{cases} 3x + 2y = 8 \\ x - 3y = 10 \end{cases}$$

↓

$x = 3y + 10$

↓

$3(3y + 10) + 2y = 8$

↓

$9y + 30 + 2y = 8$
$11y + 30 = 8$
$11y = -22$
$y = -2$

↓

$x - 3(-2) = 10$
$x + 6 = 10$
$x = 4$

↓

The solution is (4, -2)

I solved the second equation for x.

I plugged this into the first equation.

I then solved for y.

I plugged y into the second equation to find x.

Emma's "elimination" way

$$\begin{cases} 3x + 2y = 8 \\ x - 3y = 10 \end{cases}$$

↓

$3x + 2y = 8$
$-3(x - 3y = 10)$

↓

$3x + 2y = 8$
$\underline{-3x + 9y = -30}$
$11y = -22$
$y = -2$

↓

$x - 3(-2) = 10$
$x + 6 = 10$
$x = 4$

↓

The solution is (4, -2)

I multiplied the bottom equation by -3.

I then used elimination and solved for y.

I plugged y into the second equation to find x.

Why did Tim choose to plug y = –2 into the second equation to find x instead of the first equation?

Which method is better? What are some advantages of Tim's "substitution" way? Of Emma's "elimination" way?

Discuss Connections

Is there a situation where substitution would be better than elimination, or vice versa?

Think, Pair. First, think about the question(s) above independently. Then, get with a partner and discuss your answers. After talking with your partner, what is your answer?

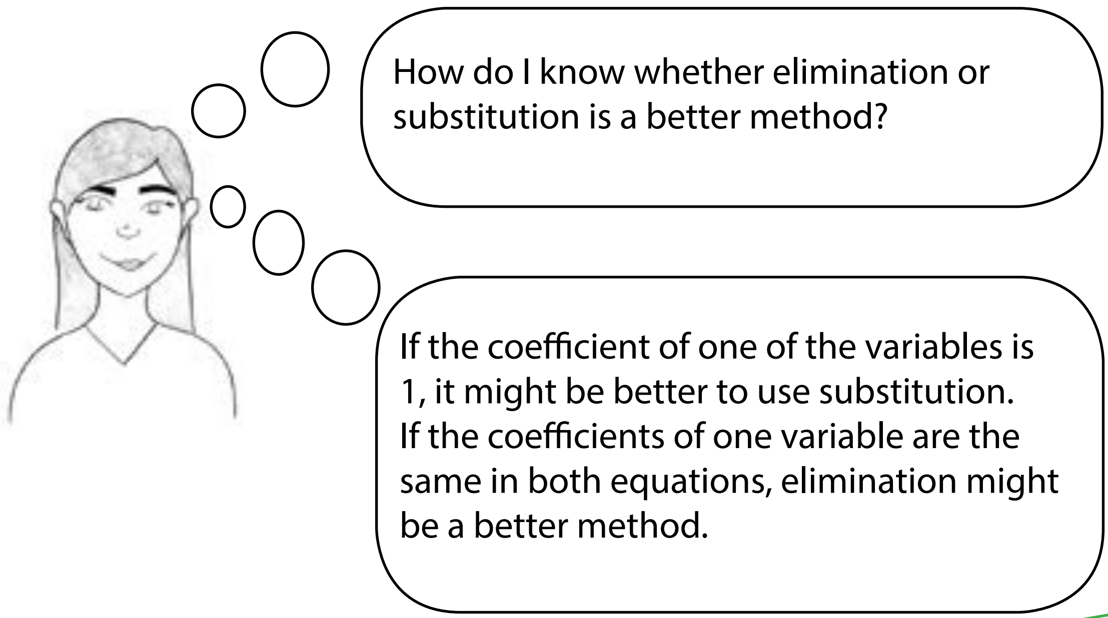
Think	Pair

Share. After reviewing the worksheet as a class, summarize the answer(s) your class agrees on. Was this different from your original response?

Big Idea. When your teacher tells you to do so, write what you think is the big idea of this example, in your own words.

Tim and Emma were asked to solve the linear system

$$\begin{cases} 3x + 2y = 8 \\ x - 3y = 10 \end{cases}$$



Why did Tim choose to plug y = –2 into the second equation to find x instead of the first equation?

Which method is better? What are some advantages of Tim's "substitution" way? Of Emma's "elimination" way?

Topic 3.6: Solving Systems of Linear Equations by Graphing & Using Substitution

WEP Type: Why does it work?
Suggested use: Beginning of lesson
Problem: Riley and Gloria were asked to solve the linear system

$$\begin{cases} -3x + 3y = -12 \\ 2x + y = 2 \end{cases}$$

Phase	Guiding Discussion Questions and Implementation Notes
Prepare to Compare	**Why did Riley find the x- and y-intercepts? How did this help him graph the line of the equation?** **Why did Gloria start by solving for the y variable?** **How did Riley know where the line for the second equation would extend beyond the x- and y- intercepts?**
Make Comparisons	**Why does solving by graphing or using substitution result in the same solution?** *Riley and Gloria are both trying to find the point that makes both equations in the system true. Riley has decided to do this graphically by finding the point where the two lines intersect, whereas Gloria has decided to do this algebraically by using substitution to solve for the x and y-coordinates of the point. Both methods will result in the same solution because in this example, there is only one point that makes the system true.*
Prepare to Discuss (Think, Pair)	**If Riley's graph of another system showed two lines that never intersect, what does this mean about the solution of the system?**
Discuss Connections (Share)	*If Riley's graph of another system showed two lines that never intersect, this means that there is no solution to the system of equations. The solution to a system of equations can be shown graphically by the intersection of the two lines. If the two lines never intersect, there is no point of the graph that satisfies both equations.*
Identify the Big Idea	**How can both graphing and algebra help you find the solution to a system of equations?** *The solution to a system of equations is the point that makes both equations true. You can use substitution to find this point with algebra, or you can graph the two equations to find where they intersect. Both methods will give you the same solution.*

Riley and Gloria were asked to check whether the solution to the linear system

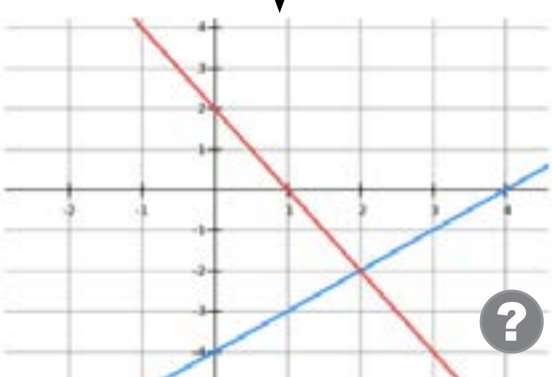
$$\begin{cases} -3x + 3y = -12 \\ 2x + y = 2 \end{cases}$$

is (2, -2).

Riley's "check by graphing" way

$$\begin{cases} -3x + 3y = -12 \\ 2x + y = 2 \end{cases}$$

-3x + 3y = -12
x-intercept = 4
y-intercept = -4
2x + y = 2
x-intercept = 1
y-intercept = 2



The solution is (2, -2)

For the first equation, I found the x- and y-intercepts. I also did this for the second equation.

Then I graphed the lines. They cross at the point (2, -2).

Gloria's "check by plugging back in" way

$$\begin{cases} -3x + 3y = -12 \\ 2x + y = 2 \end{cases}$$

-3x + 3y = -12
-3(2) + 3(-2) = -12
-6 + (-6) = -12

2x + y = 2
2(2) + (-2) = 2
4 + (-2) = 2

The solution is (2, -2)

I plugged in (2, -2) into the first equation.

It works!

Then I plugged in (2, -2) into the second equation.

It works!

Why did Riley find the x- and y-intercepts? How did this help him graph the line of the equation?

If Gloria plugged in the point (1, 0) to both equations, would it work in one equation, both equations, or neither of them? How do you know?

Discuss Connections

If Riley's graph of another system showed two lines that never intersect, what does this mean about the solution of the system?

Think, Pair. First, think about the question(s) above independently. Then, get with a partner and discuss your answers. After talking with your partner, what is your answer?

Think	Pair

Share. After reviewing the worksheet as a class, summarize the answer(s) your class agrees on. Was this different from your original response?

Big Idea. When your teacher tells you to do so, write what you think is the big idea of this example, in your own words.

Riley and Gloria were as... ed to check whethe... the solution to the linear system

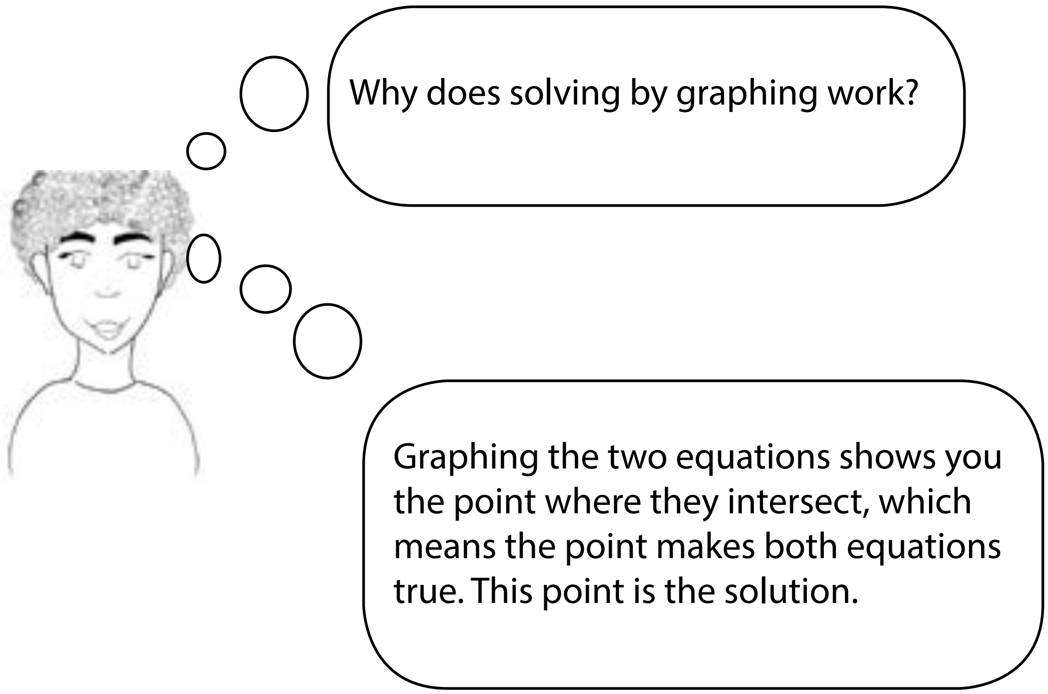
$\begin{cases} 3x + 3y \ldots \\ 2x + y \ldots \end{cases}$

Riley's ... **back in"**

For the first equatio... I... th... y-in... I also... this fo... second equatio...

I fo... solutio...

I solved ...second ...ed ...t ...tion. I ...olved ...for y. ...found



Why did Riley find the x- and y-i... cepts? How did this ... him graph the line of the equation?

Why does solving by graphing or using substitution result in the same solution?

Topic 3.7: Finding a System With No Solution by Graphing & Using Algebra

WEP Type: Why does it work?
Suggested use: End of lesson
Problem: Emma and Layla were asked to provide a second equation so that the following system of equations has no solution.

$$\{4x + 2y = 8$$

Phase	Guiding Discussion Questions and Implementation Notes
Prepare to Compare	**Why did Emma find the x- and y-intercepts? How did this help her graph the line of the equation?** **Why did Layla change the equation to slope-intercept form?** **Why did Emma graph a parallel line? How did Layla come up with her new equation?** **Why do the systems Emma and Layla came up with have no solution?**
Make Comparisons	**How did Emma and Layla come up with the same second equation? Are there other second equations that could have worked?** *Emma used graphing to draw a line that is parallel to the line of the first equation. This parallel line had the same slope, but had a different y-intercept. She decided to shift her new line over 2 units to the right on the x-axis, and then used the same slope as the first equation to find her new y-intercept of 8. Layla used algebra first to convert the equation to slope-intercept form in order to find the slope and y-intercept of the first equation. She then wrote a new equation that had the same slope, and decided to change the y-intercept to 8. Yes, other equations would have worked, as long as the second equation has the same slope and is not equivalent to the first.*
Prepare to Discuss (Think, Pair) Discuss Connections (Share)	**Can you find another equation other than the one that Emma and Layla found, so this system has no solution? To find this other equation, did you use Emma's "graphing" way or Layla's "using algebra" way?** *There are an infinite number of equations that can be generated for this system such that there is no solution. As long as the second equation is parallel to the first one – meaning that the two lines have no points of intersection – the system has no solution.*
Identify the Big Idea	**How can both graphing and algebra help you find a system of equations with no solution?** *When a system of equations has no solution, this means that there is no point that makes both equations true. Graphically, this means that the two equations are parallel lines. With algebra, this means that the two equations have the same slope but different y-intercepts.*

Emma and Layla were asked to provide a second equation so that the following system of equations has no solution.

$$\begin{cases} 4x + 2y = 8 \\ \end{cases}$$

Emma's "graphing" way

$$\begin{cases} 4x + 2y = 8 \\ \end{cases}$$

I found the x- and y-intercepts of the first equation and graphed the line.

4x + 2y = 8
x-intercept = 2
y-intercept = 4

Then, I graphed a line parallel to it.

I wrote the equation of this new line.

y = -2x + 8

$$\begin{cases} 4x + 2y = 8 \\ y = -2x + 8 \end{cases}$$

This system has no solution

Layla's "using algebra" way

$$\begin{cases} 4x + 2y = 8 \\ \end{cases}$$

I changed the first equation to slope-intercept form.

4x + 2y = 8
2y = -4x + 8
y = -2x + 4

I then found the slope and y-intercept.

m = -2
b = 4

I wrote a new equation with a different y-intercept.

y = -2x + 8

$$\begin{cases} 4x + 2y = 8 \\ y = -2x + 8 \end{cases}$$

This system has no solution

Why did Emma graph a parallel line? How did Layla come up with her new equation?

How did Emma and Layla come up with the same second equation? Are there other second equations that would have worked?

Discuss Connections

Can you find another equation other than the one that Emma and Layla found, so that this system has no solution?
To find this other equation, did you use Emma's "graphing" way or Layla's "using algebra" way?

Think, Pair. First, think about the question(s) above independently. Then, get with a partner and discuss your answers. After talking with your partner, what is your answer?

Think	Pair

Share. After reviewing the worksheet as a class, summarize the answer(s) your class agrees on. Was this different from your original response?

Big Idea. When your teacher tells you to do so, write what you think is the big idea of this example, in your own words.

Emma and Layla were asked to provide a second equation so that the following system of equations has no solution.

$$\begin{cases} 4x + 2y = 8 \\ \end{cases}$$

Em… "graphing" …

…'s "using alg…" way

I found the x- and y-intercepts of … equ… graph… line.

Then, I graphed a… line pa… to i…

I w… equati… this new li…

I changed the first equation to slope-…

…ound …ope and …ercept.

… a new …ation with …fferent …rcept.

How can graphing and algebra help us to see whether an equation has no solution?

When a system of equations has no solution, this means that there is no point that makes both equations true. Graphically, this means that the two equations are parallel lines. With algebra, this means that the two equations have the same slope but different y-intercepts.

Why did Emma graph a parallel li…? How did Layla come …p with her new equation?

How did Emma and Layla come up with the same second equation? Are there other second equations that would have worked?

Topic 3.8: Solving Systems of Linear Equations by Properties of Equality

WEP Type: Why does it work?
Suggested use: Mid-lesson
Problem: Gloria and Tim were asked to solve the linear system

$$\begin{cases} 2x + 3y = 12 \\ 5x - 3y = 9 \end{cases}$$

Phase	Guiding Discussion Questions and Implementation Notes
Prepare to Compare	**Why is it OK for Tim to rewrite the system as one equation?** **What did Tim do in order to rewrite the system as one equation?** **Why did the y variable disappear when Tim combined like terms?**
Make Comparisons	**Gloria says that she also rewrote the system as one equation when she added the equations together to get 7x = 21. Do you agree or disagree with Gloria? Why?** *Agree. Gloria used elimination to rewrite the system as one equation, which is the same as using the addition property of equality to add the left and right sides of the equations together to get one equation. Tim uses the same method, but writes it out horizontally versus Gloria's vertical representation.*
Prepare to Discuss (Think, Pair)	**If the second equation were changed to 5x – 2y = 9, would Gloria's "elimination" way work? What about Tim's "use the equal sign" way?**
Discuss Connections (Share)	*Both Gloria's "elimination" way and Tim's "use the equal sign" way will work for any system of linear equations. However, if the second equation were changed to 5x – 2y = 9, neither method would be particularly helpful in solving this system, since neither the x terms nor the y terms would get eliminated (unless one or both equations were multiplied by a constant prior to adding the equations together).*
Identify the Big Idea	**How can you use the properties of equality to solve a system of equations?** *Adding the same value to the left and right sides of an equation maintains the equality of both sides. In an equation, the left side is always equal to the right side. So if we have two equations, we can add the left to the left, and it will still be equal to the right plus the right sides. This can help you solve a system of equations by rewriting it as one equation to combine like terms and eliminate one of the variables.*

Gloria and Tim were asked to solve the linear system

$$\begin{cases} 2x + 3y = 12 \\ 5x - 3y = 9 \end{cases}$$

Gloria's "elimination" way

$$\begin{cases} 2x + 3y = 12 \\ 5x - 3y = 9 \end{cases}$$

$$\begin{array}{r} 2x + 3y = 12 \\ \underline{5x - 3y = 9} \\ 7x \quad = 21 \\ x = 3 \end{array}$$

First, I used elimination to solve for x.

$$2x + 3y = 12$$
$$2(3) + 3y = 12$$
$$6 + 3y = 12$$
$$3y = 6$$
$$y = 2$$

Then, I substituted this into the first equation to find y.

The solution is (3, 2)

Here is my answer.

Tim's "use the equal sign" way

$$\begin{cases} 2x + 3y = 12 \\ 5x - 3y = 9 \end{cases}$$

$$5x - 3y + 2x + 3y = 12 + 9$$
$$7x = 21$$
$$x = 3$$

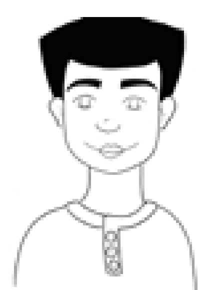
First, I rewrote the system as one equation. Then, I solved for x.

$$2x + 3y = 12$$
$$2(3) + 3y = 12$$
$$6 + 3y = 12$$
$$3y = 6$$
$$y = 2$$

I substituted this into the first equation to find y.

The solution is (3, 2)

Here is my answer.



Why is it OK for Tim to rewrite the system as one equation?

Gloria says that she also rewrote the system as one equation when she added the equations together to get $7x = 21$. Do you agree or disagree with Gloria? Why?

Discuss Connections

If the second equation were changed to $5x - 2y = 9$**, would Gloria's "elimination" way work? What about Tim's "use the equal sign" way?**

Think, Pair. First, think about the question(s) above independently. Then, get with a partner and discuss your answers. After talking with your partner, what is your answer?

Think	Pair

Share. After reviewing the worksheet as a class, summarize the answer(s) your class agrees on. Was this different from your original response?

Big Idea. When your teacher tells you to do so, write what you think is the big idea of this example, in your own words.

Gloria and Tim were asked to solve the linear system

$$\begin{cases} 2x + 3y = 12 \\ 5x - 3y = 9 \end{cases}$$

Gloria's "elimination" w

se the equal sign" way

First, I used elimination to solve for x.

The substi this int the first equation t find y.

Fi answ

First, I rewrote the system as one equation. olved

ted the tion

ny

Why does the equal sign way work?

Adding the same value to the left and right sides of an equation maintains the equality of both sides. So if we have two equations, we can add the left to the left, and it will still be equal to the right plus the right sides. This is what we are doing when we use the elimination method.

Why is it OK for Tim to rewrite t one eq

Gloria says that she also rewrote t ystem as one equat hen she added the equations together to get 7x = 21. Do you agree or disagree with Gloria? Why?

Topic 3.9: Choosing a Method to Solve Systems of Equations

WEP Type: Which is better?
Suggested use: Review activity

Problem: Find a partner. Each of you will solve $\begin{cases} 3x + 3y = 21 \\ x - 3y = 11 \end{cases}$ and the two methods you use must be *different*. Write your name in the space below, and show the work for your method.

Phase	Guiding Discussion Questions and Implementation Notes
Prepare to Compare	**Which methods did you and your partner use to solve the equation?** *Facilitator Note: Identify the methods used by asking partners to present out their work. Keep track of common methods used to guide discussion towards why that method may have been used most, and why other methods were used less frequently.*
Make Comparisons	**What are the advantages and disadvantages of each method? Which method do you think is better for solving this problem?** *Facilitator Note: Ask partners to pair-share the advantages and disadvantages to each method they used before having them share out for whole group discussion. Consider asking students to come to the board to contribute to a pros/cons T chart for different methods used.*
Prepare to Discuss (Think, Pair)	**Create a problem where Partner A's way would work better. Then create a problem where Partner B's way would work better.**
Discuss Connections (Share)	*Facilitator Note: Answers may vary significantly at this point, and that's ok! Encourage students to work with their partners to identify problems that work better for each method they used, circulating and assisting groups as needed. Ask volunteers to share their ideas with the class. For whole group discussion, ask guided questions about how groups created the problems and what characteristics of the problems lend themselves to certain methods over others.*
Identify the Big Idea	**What is the best method for solving equations like this?** *Features of the system, such as the coefficients of the terms and which side of the equation the x terms and y terms appear on, can help you determine the easiest way to solve. For problems like this, seeing same or opposite coefficients might suggest that elimination would be good; seeing a variable with a coefficient of 1 might also suggest that substitution is a good way.* *Facilitator Note: Encourage students to share their opinions by listing out all the methods they used and taking a class vote on which method is best. This is a suggested takeaway for this type of problem, though students may have other ideas! Ask volunteers to justify their responses for which method is best.*

Find a partner. Each of you will solve the following system using a *<u>different</u>* method. Write your name in the space below, and show the work for your method.

$$\begin{cases} 3x + 3y = 21 \\ x - 3y = 11 \end{cases}$$

______________________________'s way	______________________________'s way

Describe the methods used by you and your partner to solve the system.

What are the advantages and disadvantages of each method? Which method do you think is better for solving this problem?

Discussing Connections

Create a problem where Partner A's way would work better. Then create a problem where Partner B's way would work better.

Think, Pair. First, think about the question(s) above independently. Then, get with a partner and discuss your answers. After talking with your partner, what is your answer?

Think	Pair

Share. After reviewing the worksheet as a class, summarize the answer(s) your class agrees on. Was this different from your original response?

Big Idea. When your teacher tells you to do so, write what you think is the big idea of this example, in your own words.

Find a partner. Each of you will solve the following system using a *different* method. Write your name in the space below, and show the work for your method.

$$\begin{cases} 3x + 3y = 21 \\ x - 3y = 11 \end{cases}$$

______________________'s way ______________________'s way

How do I know how to choose a good way to solve this problem?

Features of the system, such as coefficients of the terms and which side of the equation the x terms and y terms appear on, can help you figure out the easiest way to solve. For this problem, seeing that y has the coefficient 3 suggests elimination is a good way; seeing x has a coefficient of 1 suggests substitution is a good way.

Describe the methods used by you and your partner to solve the system.

What are the advantages and disadvantages of each method? Which method do you think is better for solving this problem?