Magic Squares

Vanderbilt Math Circle

September 3, 2019

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1	2	3
4	5	6
7	8	9

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1	2	3
4	5	6
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Questions

- What is the sum of each row?

Magic Squares

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1	2	3
4	5	6
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Questions

- What is the sum of each row?
- Can you arrange the numbers so that the sum of each row is the same?

Magic Squares

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1	2	3
4	5	6
7	8	9

Questions

- What is the sum of each row?
- Can you arrange the numbers so that the sum of each row is the same?
- What is the sum now?

Magic Squares

Definition

An order 3 **Magic Square** is a 3×3 square grid filled with the numbers 1 through 9 without repeats so that each row, column, and diagonal sums to 15.

1	2	3
4	5	6
7	8	9

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1	2	3
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Strategies

- What combinations of 3 numbers sum to 15?

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Strategies

- What combinations of 3 numbers sum to 15?
- Is there a certain number that must go in the center?

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1	2	3
4	5	6
7	8	9

Strategies

- What combinations of 3 numbers sum to 15?
- Is there a certain number that must go in the center?
- What other patterns are there?

Solutions to magic squares

2	9	4
7	5	3
6	1	8

Solutions to magic squares

2	9	4
7	5	3
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Questions

- Given a solution, can you use it to find a different one?

Solutions to magic squares

2	9	4
7	5	3
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Questions

- Given a solution, can you use it to find a different one?
- How many solutions are there?

Magic squares with other sequences

4	5	6
7	8	9
10	11	12

Magic squares with other sequences

4	5	6
7	8	9
10	11	12

Questions

- Can you construct a magic square with the numbers 4–12 instead of 1–9?

Magic squares with other sequences

4	5	6
7	8	9
10	11	12

Questions

- Can you construct a magic square with the numbers 4–12 instead of 1–9?
- What strategies did you use before?

Magic squares with other sequences

4	5	6
7	8	9
10	11	12

Questions

- Can you construct a magic square with the numbers 4–12 instead of 1–9?
- What strategies did you use before?
- What should the sum be now?

Order 2 magic squares

1	2
3	4

Order 2 magic squares

1	2
3	4

Questions

- Do order 2 magic squares exist?

Order 2 magic squares

1	2
3	4

Questions

- Do order 2 magic squares exist?
- What if we use a different sequence of 4 consecutive numbers?

Order 2 magic squares

1	2
3	4

Questions

- Do order 2 magic squares exist?
- What if we use a different sequence of 4 consecutive numbers?
- What if we use 4 even numbers?

Higher order magic squares

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

Higher order magic squares

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

Questions

- Do order 4 magic squares exist?

Higher order magic squares

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

Questions

- Do order 4 magic squares exist?
- What strategies can we use from before?

Higher order magic squares

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

Questions

- Do order 4 magic squares exist?
- What strategies can we use from before?
- What strategies can we not use?

Challenge problems

5	10	9
12	8	4
7	6	11

3	17	7
13	9	5
11	1	15

Challenge problems

5	10	9
12	8	4
7	6	11

3	17	7
13	9	5
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Strategies

- If N is the sum of a magic square, call N a **magic number**.

Challenge problems

5	10	9
12	8	4
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13	9	5
11	1	15

Strategies

- If N is the sum of a magic square, call N a **magic number**.
- What are the order 3 magic numbers?

Challenge problems

5	10	9
12	8	4
7	6	11

3	17	7
13	9	5
11	1	15

Strategies

- If N is the sum of a magic square, call N a **magic number**.
- What are the order 3 magic numbers?
- If k is a magic number is $2k$ also a magic number?