Introduction

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36

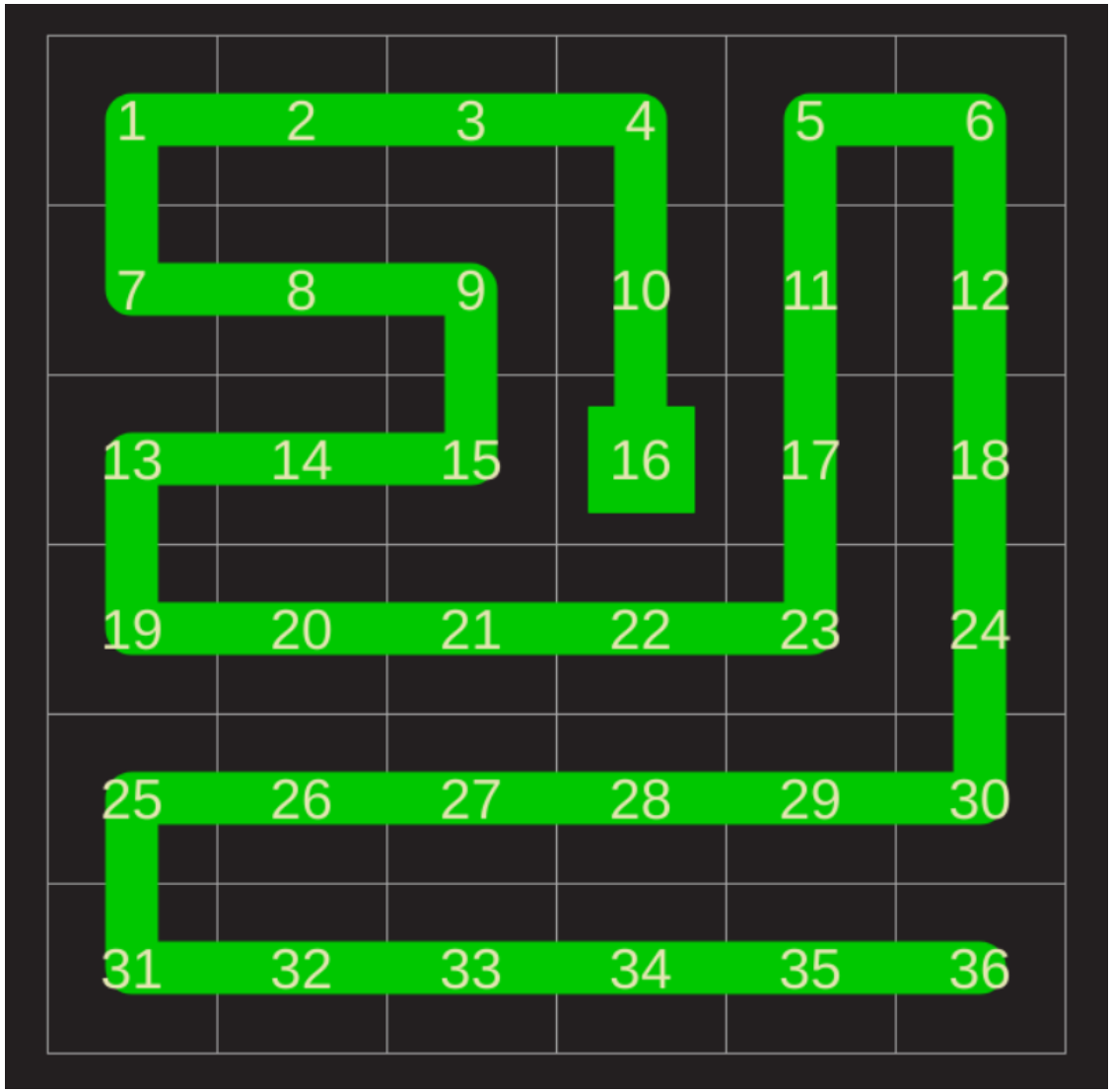
Spaghetti path rules

- We start with an n by n grid that has the numbers $1, 2, 3, 4, ..., n^2$.
- Pick any number k to start with. Then, we create a path on the grid starting at k with the following rules:
  1. At each point in the grid, we can only move left, right, up, or down.
  2. We cannot move to a square that has already been occupied by the path.
  3. If more than one square is available, we must move to the square with the smallest number. (If only one square is available, then we take that square).

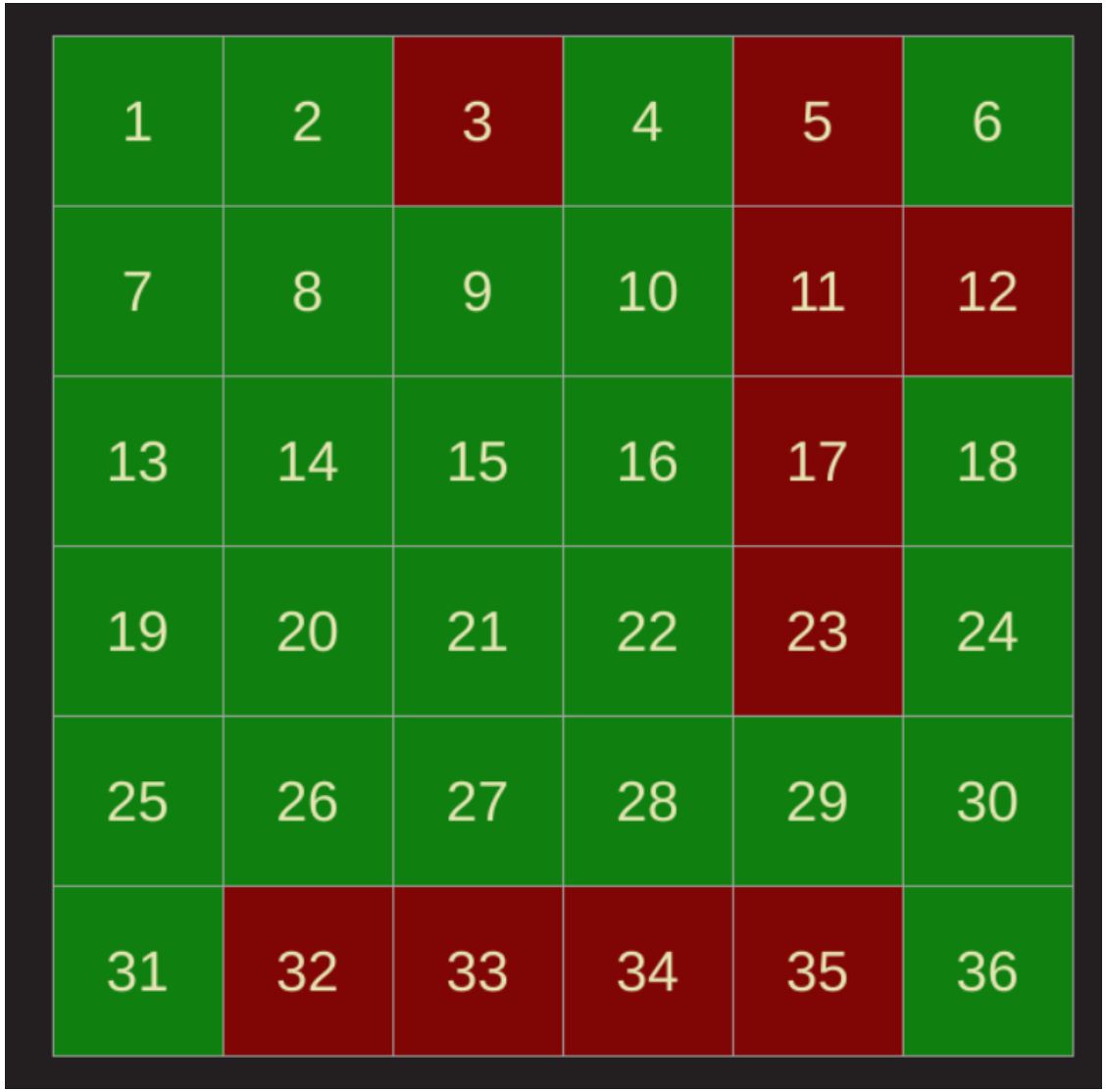
Definition 1.

A path in the grid starting at a number k will be called a **spaghetti path** if it goes through all of the squares in the grid and doesn't break any of the rules above.

For the 6 x 6 grid, which starting numbers result in a spaghetti path?



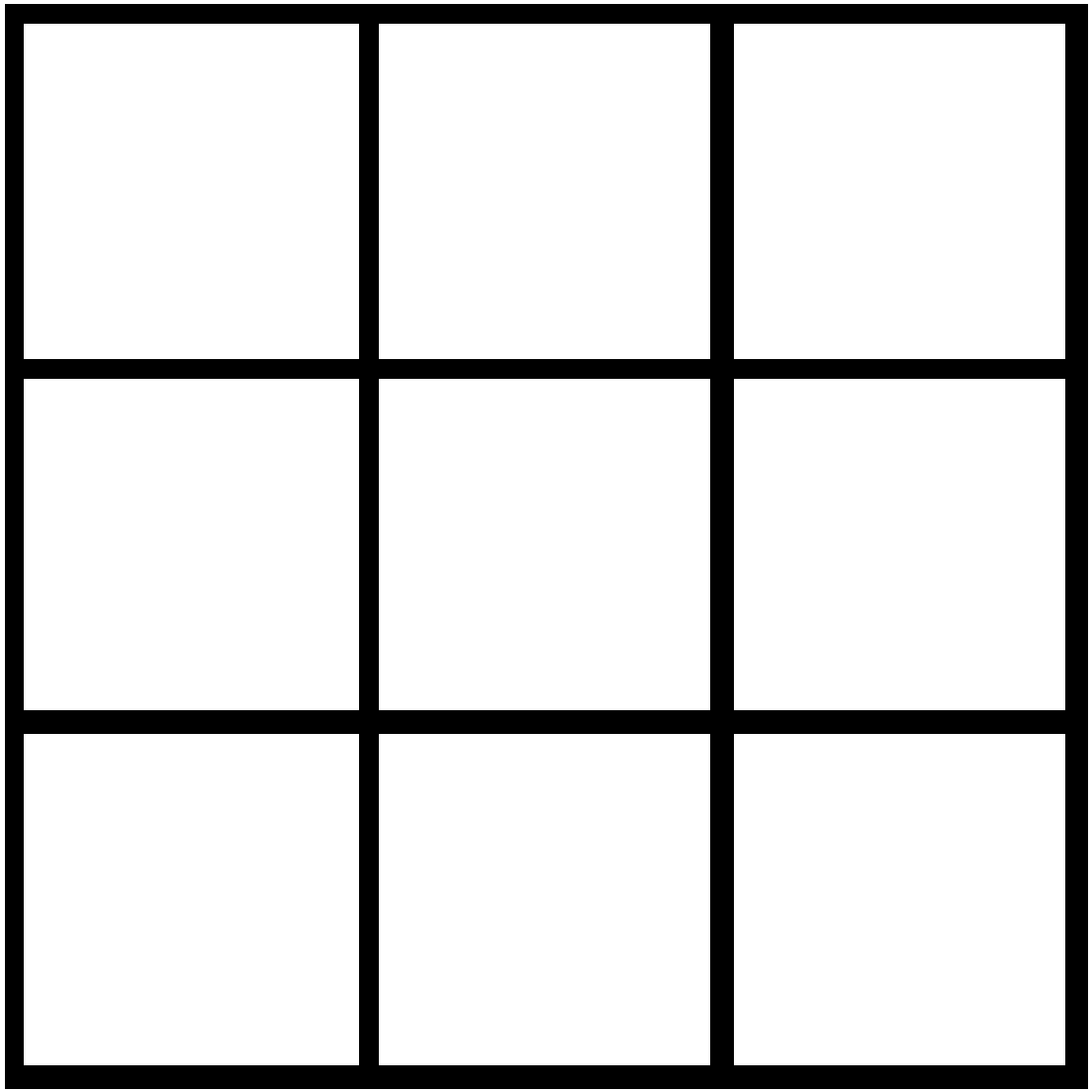
Solution for the 6 x 6 grid



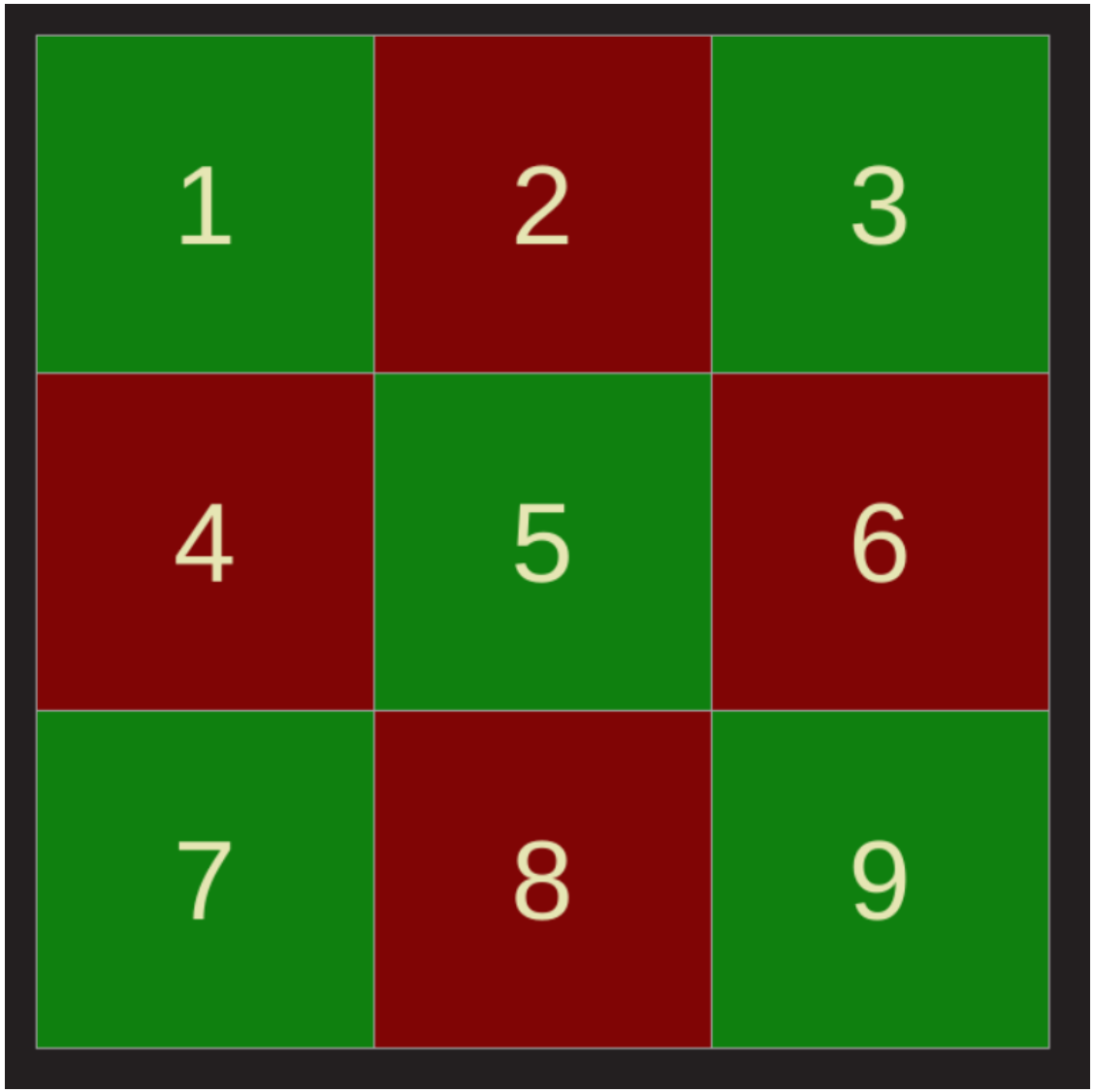
Key Question

- Let n be a natural number i.e. $1, 2, 3, \ldots$
- **Is it possible to label all the numbers in an $n \times n$ grid so that every number has a spaghetti path?**
  - What about the opposite question? Can we arrange all the numbers in the grid so that no number has a spaghetti path?
- A good strategy is to start with a much easier case.

The 3×3 grid

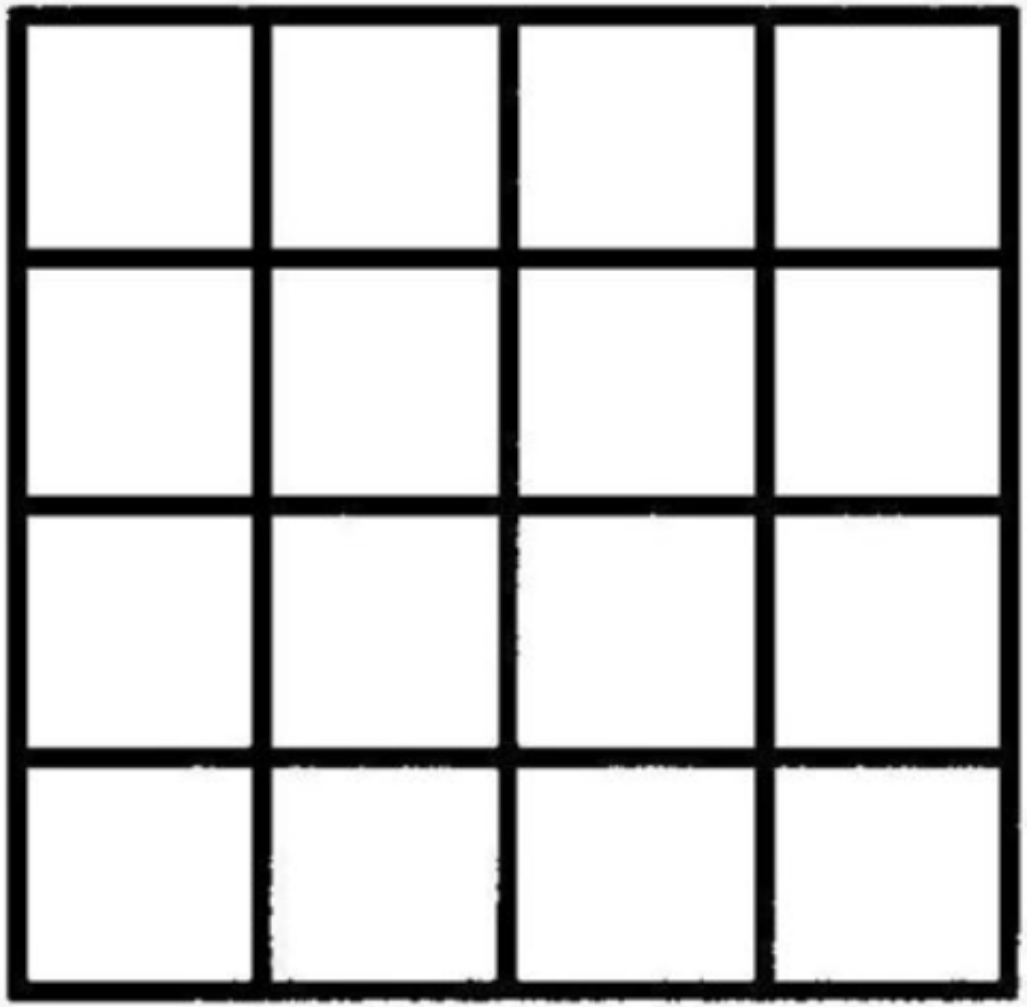


The 3×3 grid



Key observations from the 3×3 grid

The 4×4 grid



Key observations from the 4×4 grid

1. Can we label the 4×4 grid so that every number has a spaghetti path?

Key observations from the 4×4 grid

1. Can we label the 4×4 grid so that every number has a spaghetti path?
2. Can we label the 4×4 grid so that every number **does not** have a spaghetti path?

Key observations from the 4×4 grid

1. Can we label the 4×4 grid so that every number has a spaghetti path?
2. Can we label the 4×4 grid so that every number **does not** have a spaghetti path?
3. What do you think will happen for the 5×5 grid?

Key observations from the 4×4 grid

1. Can we label the 4×4 grid so that every number has a spaghetti path?
2. Can we label the 4×4 grid so that every number **does not** have a spaghetti path?

Solving the general problem

1. What do you think will happen for the 5×5 grid? Try to use the same argument from the 3×3 case.

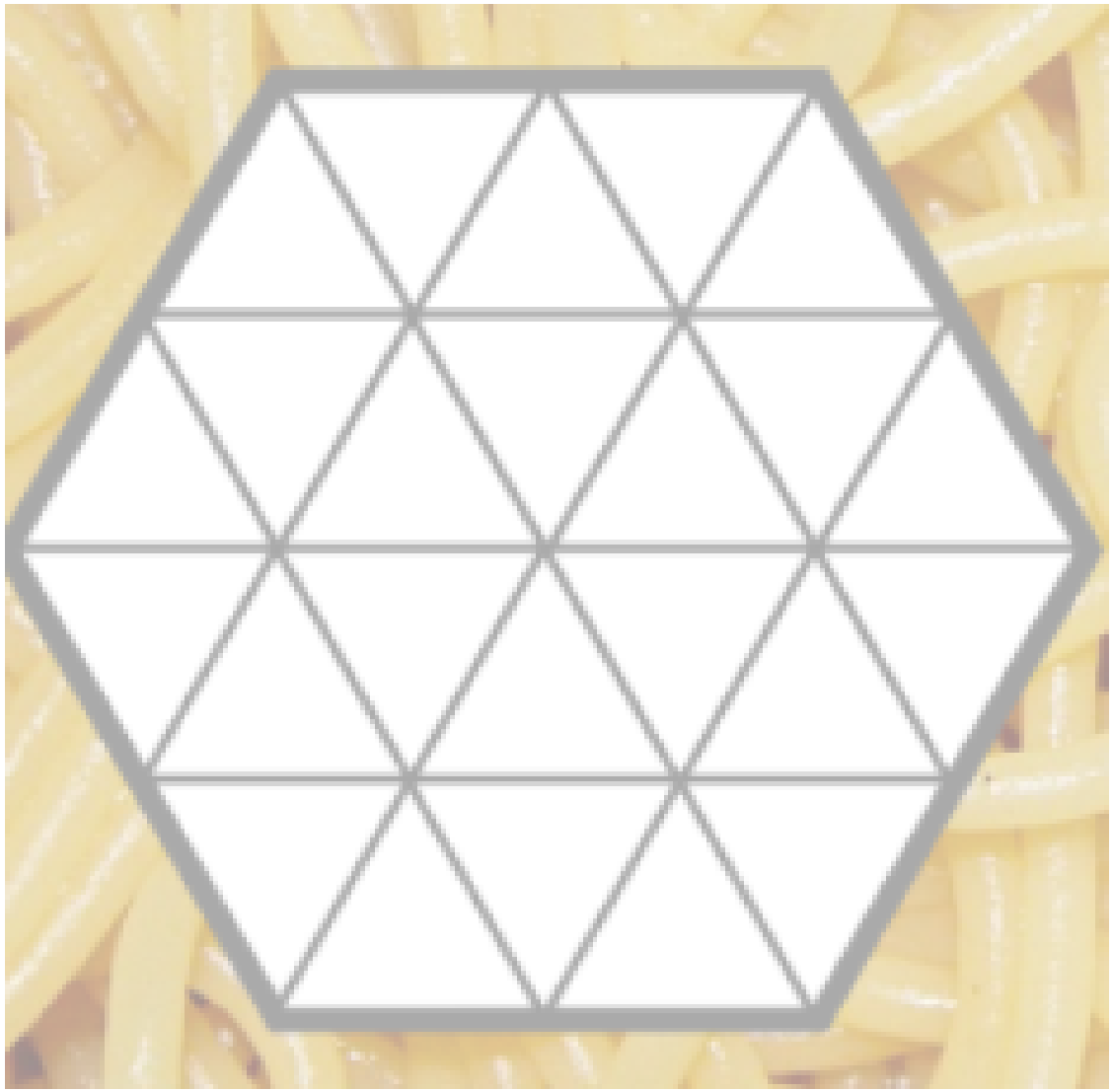
Solving the general problem

1. What do you think will happen for the 5×5 grid? Try to use the same argument from the 3×3 case.
2. What do you think will happen for the 6×6 grid? Can we arrange the numbers so that every square has a spaghetti path?

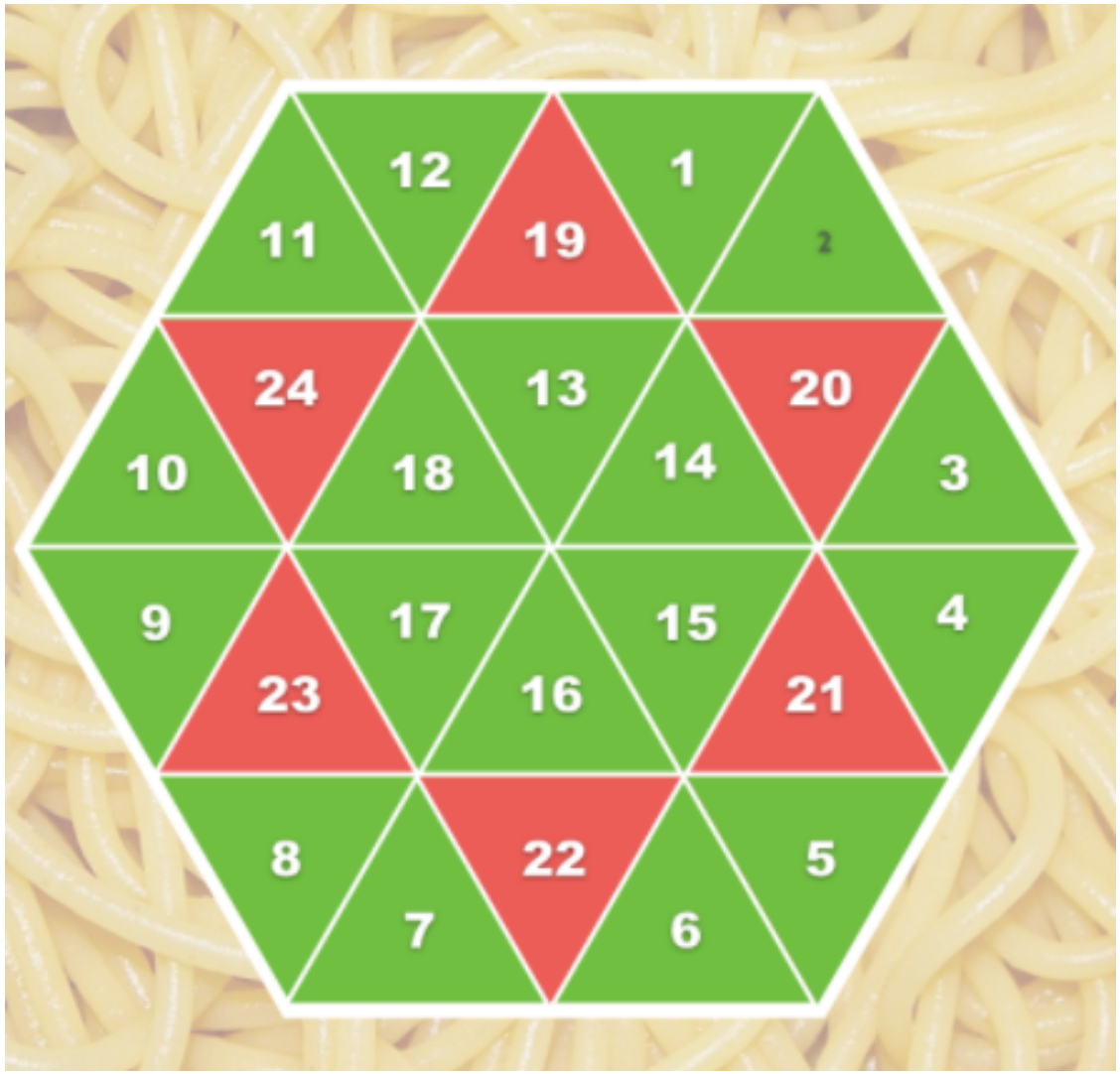
Solving the general problem

1. What do you think will happen for the 5×5 grid? Try to use the same argument from the 3×3 case.
2. What do you think will happen for the 6×6 grid? Can we arrange the numbers so that every square has a spaghetti path?
3. **What about for the $n \times n$ grid?**

The 24-triangle Hexagon



The 24-triangle Hexagon



Final Remarks

1. This problem is closely related to the area of Graph theory with Hamiltonian paths.
2. Given an arbitrary graph or polygonal grid, it is extremely hard to determine whether we can label the grid to get any spaghetti paths at all!
3. Check out the Hamiltonian path problem.