HAT PROBLEMS

Problem 1. You and your teammate both close your eyes. A hat is placed on each of your heads. Hats can be either red or green. When you open your eyes, you can see your teammate's hat but not your own.

On the count of three, the players each say a color (red or green) at the same time. If one person correctly says the color of hir own hat, you both win as a team. Otherwise, you lose as a team.

Come up with a strategy that both players can follow by which a win is guaranteed.



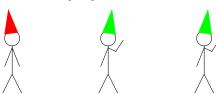
Date: February 2020.

Problem 2. Now there are 3 players on your team. Once again, you all close your eyes and hats are placed on your heads, either red or green. When you open your eyes, you can see the colors of your teammates' hats, but not your own.

Before anyone speaks, each player raises hir hand if ze sees any red hats on the heads of either of the other two players.

Now, you can speak, but only if you know the color of your own hat, in which case you should say that color.

Under what circumstances will there be a player who can correctly say the color of hir own hat? Is there ever a scenario in which no player can correctly say the color of hir own hat?



Problem 3. Now your team consists of 100 players, and you are standing in a line. You all close your eyes, and hats are placed on all of your heads. Hats are either red or green.

When you open your eyes you can see the hats on the people in front of you, but not your own hat or the hats on the people behind you.

Going down the line starting from the person in back (who can see all the hats but hir own), each person says "red," "green," or "pass" in turn. Each player hears what the other players say as they go down the line

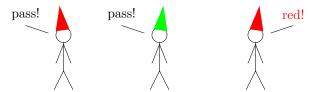
All 100 players win as a team if not everyone says pass, and if everyone who says a color says the color of hir own hat.



Problem 4. Now you are on a 3 person team again. When you close your eyes, hats are placed on your head that are either red or green. When you open your eyes you can see your teammates' hats but not your own.

On the count of three, everyone says either "red," "green," or "pass," all at once. Your team wins if not everyone says pass, and each person who says a color says the color of hir own hat.

Devise a strategy by which you win more than half the time. How often will you win by this strategy?



Problem 5. This game is for a three person team but with a new twist: there are now three colors of hats! As before, the three of you close your eyes and a hat is placed on each of your heads. Hats are red, green, or purple. When you open your eyes, you can see the color(s) of your teammates' hats but not your own.

On the count of three, you all say colors. If at least one of you says the color of the hat on your head, then you win as a team.

Come up with a strategy by which you win no matter what hats are on your heads.

