Discrete scale invariance in quantum spin chains - a proposed experiment.

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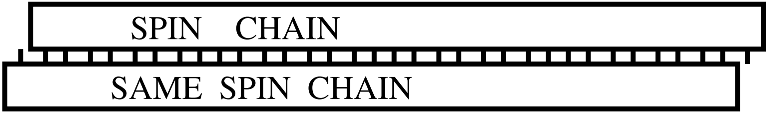
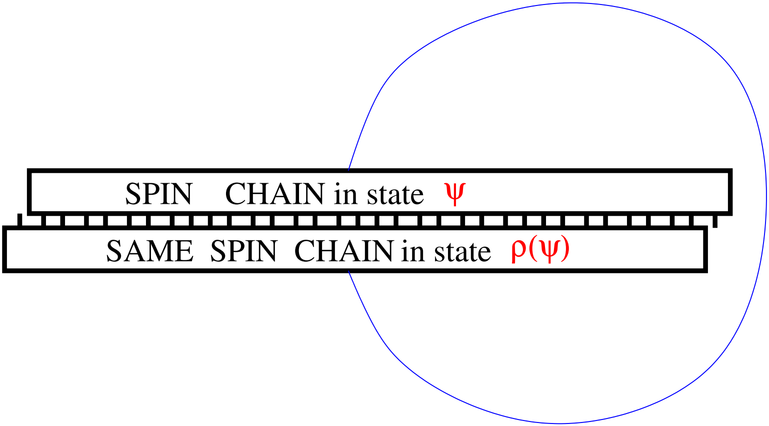
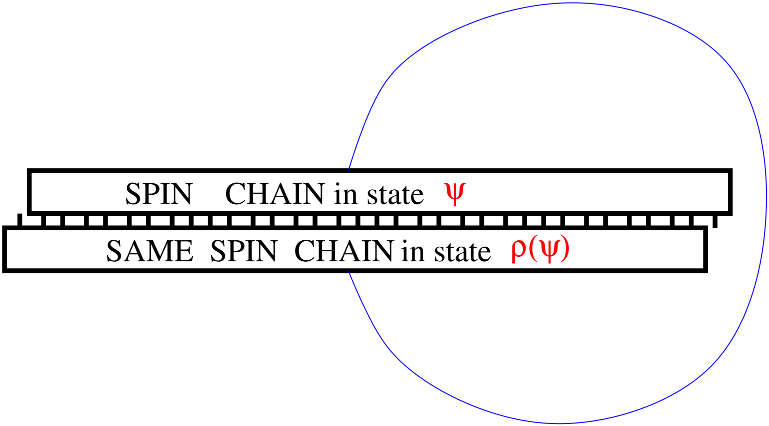
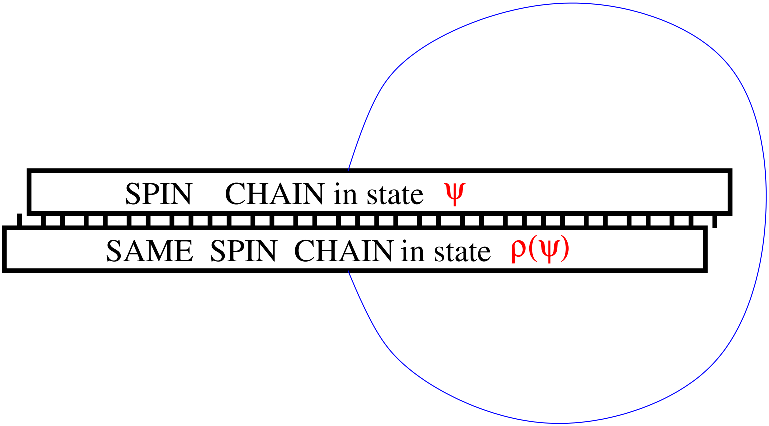



SPIN CHAIN in state ψ
SAME SPIN CHAIN in state ρ(ψ)



Correlation almost 1 away from critical point.



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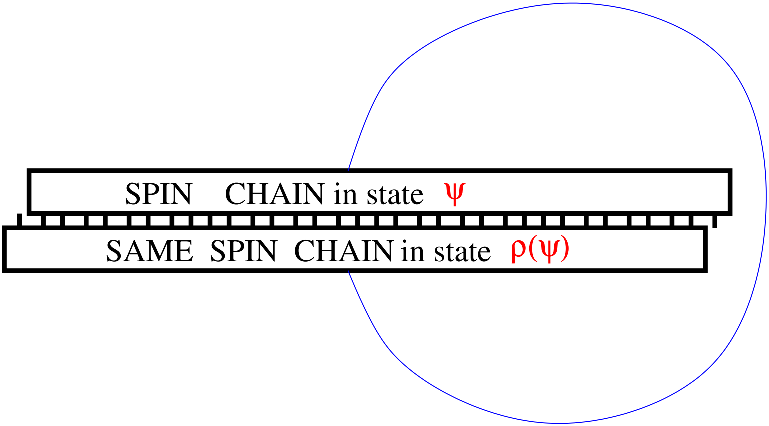

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My "prediction": the correlation should *DROP* significantly at the critical point.

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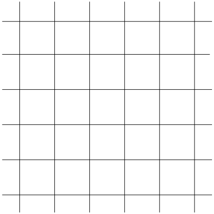
Equivalence between 1-d quantum spin chain and 2-d classical stat mech lattice system:



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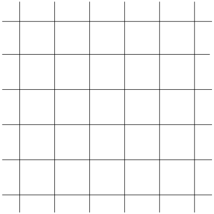
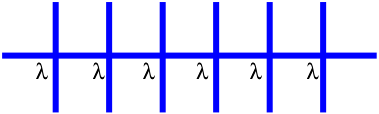


One builds the lattice states up row by row with a matrix called the "Transfer matrix" $T(\lambda)$:



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Thus: "the transfer matrix determines the infinitesimal time evolution of the chain."

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This idea, that constrained quantum systems should be described by a relative tensor product, should not be restricted to one dimension.

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There exist bimodules ${}_M\mathcal{H}_M$ *so that* $(\overset{\circlearrowleft}{\otimes})^n_M\mathcal{H}$ *is* **finite dimensional** *and grows like* $(2\cos\pi/k)^n$ *for* $k = 3, 4, 5, \cdots$.

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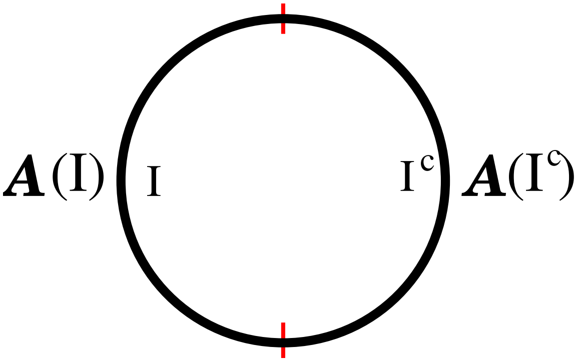
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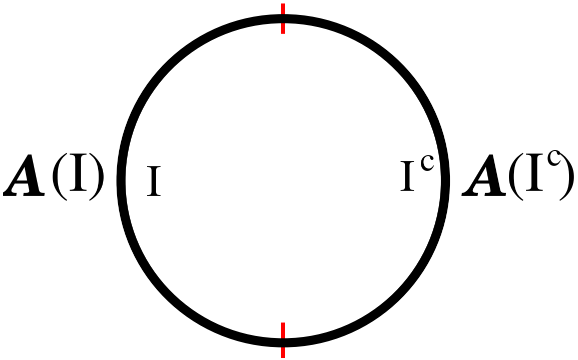

The observables in $A(I)$ commute with those in $A(I^c)$ so we have a bimodule as required for our anyonic spin chain.

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thus in particular T contains all rotations by dyadic rationals.The definition of the action is geometric but a little complicated since of necessity any scale transformations will not preserve a given finite dimensional approximation to the direct limit.

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The calculation that yields this discontinuity is however very suggestive.

It involves **iterating a classical dynamical system (which can even be a rational transformation of the Riemann sphere) in the spectral parameter space of a transfer matrix!**

To approach the continuous limit we need to investigate how the ROTATION $\rho_{\frac{1}{2^n}}$ by $\frac{1}{2^n}$, which is an element of Thompson's group T, acts on states. In particular I want to calculate the coefficients

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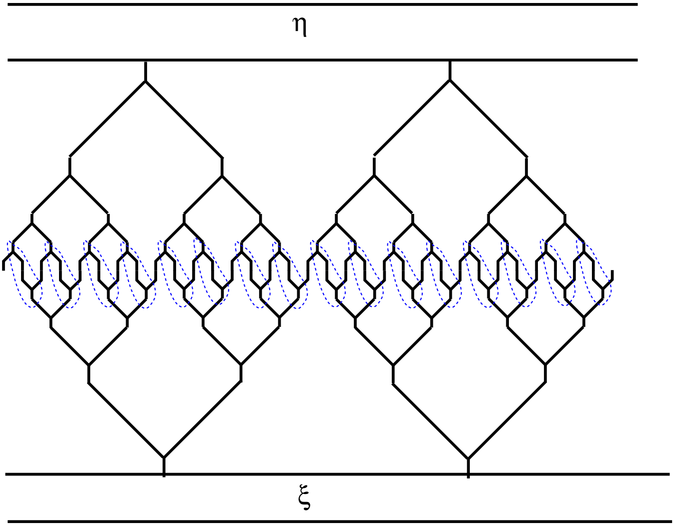
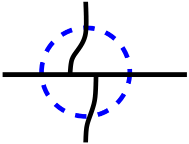
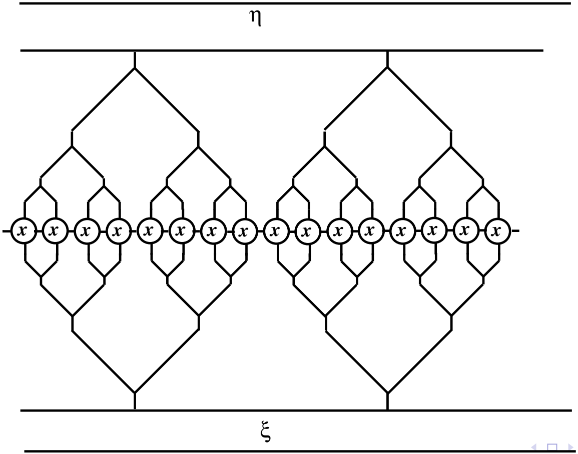

η
ξ

Now all the regions in the blue dotted circles can be deformed to look like



so if we call x the element inside the box with 4 legs, the picture becomes:



We recognise the *transfer matrix* $T_{2^{n+k}}(x)$!
Thus " The transfer matrix determines infinitesimal space translation". If we are in one dimension and time=space then we have recovered our previous mantra in a topsy turvy fashion!

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Note the resemblance between the calculation and the experiment.