Universal Quantum Computation with Gapped Boundaries

Iris Cong, Meng Cheng, Zhenghan Wang

cong@g.harvard.edu

Department of Physics
Harvard University, Cambridge, MA

May 4th, 2019

Refs.: C, Cheng, Wang, *Comm. Math. Phys.* (2017) 355: 645.
C, Cheng, Wang, *Phys. Rev. Lett.* **119**, 170504 (2017).

Introduction: Quantum Computation

- Quantum computing:
  - Feynman (1982): Use a quantum-mechanical computer to simulate quantum physics (classically intractable)
  - Encode information in qubits, gates = unitary operations
  - Applications: Factoring/breaking RSA (Shor, 1994), quantum machine learning, ...

Introduction: Quantum Computation

- Quantum computing:
  - Feynman (1982): Use a quantum-mechanical computer to simulate quantum physics (classically intractable)
  - Encode information in qubits, gates = unitary operations
  - Applications: Factoring/breaking RSA (Shor, 1994), quantum machine learning, ...
- Major challenge: local decoherence of qubits

Introduction: Topological Quantum Computation

- **Topological** quantum computing (TQC) (Kitaev, 1997; Freedman et al., 2003):
  - Encode information in *topological* degrees of freedom
  - Perform *topologically protected* operations

Topological Quantum Computation

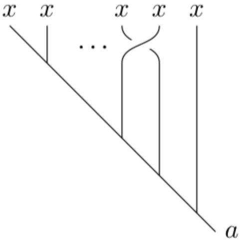
- Traditional realization of TQC: *anyons* in *topological phases of matter* (e.g. Fractional Quantum Hall - FQH)
  - Elementary quasiparticles in 2 dimensions s.t. $|\psi_1\psi_2\rangle = e^{i\phi}|\psi_2\psi_1\rangle$

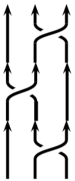
Topological Quantum Computation

- Traditional realization of TQC: *anyons* in *topological phases of matter* (e.g. Fractional Quantum Hall - FQH)
  - Elementary quasiparticles in 2 dimensions s.t. $|\psi_1\psi_2\rangle = e^{i\phi}|\psi_2\psi_1\rangle$
- Qubit encoding: degeneracy arising from the *fusion rules*
  - e.g. Toric code: $e^i m^j \otimes e^k m^l = e^{(i+k)(\text{mod } 2)} m^{(j+l)(\text{mod } 2)}$ $(i, j = 0, 1)$
  - e.g. Ising: $\sigma \otimes \sigma = 1 \oplus \psi$, $\psi \oplus \psi = 1$
  - e.g. Fibonacci: $\tau \otimes \tau = 1 \oplus \tau$

Topological Quantum Computation

- Topologically protected operations: Braiding of anyons
  - Move one anyon around another $\rightarrow$ pick up phase (due to Aharonov-Bohm)





Figures: (1) Z. Wang, *Topological Quantum Computation*.
(2) C. Nayak et al., *Non-abelian anyons and topological quantum computation*.

Problem

- Problem: Abelian anyons have no degeneracy, so no computation power $\rightarrow$ need non-abelian anyons for TQC (e.g. $\nu = 5/2, 12/5$)

Problem

- Problem: Abelian anyons have no degeneracy, so no computation power $\to$ need non-abelian anyons for TQC (e.g. $\nu = 5/2, 12/5$)
- These are difficult to realize, existence is still uncertain

Problem

- Problem: Abelian anyons have no degeneracy, so no computation power $\rightarrow$ need non-abelian anyons for TQC (e.g. $\nu = 5/2, 12/5$)
- These are difficult to realize, existence is still uncertain
- Question: Given a top. phase that supports only abelian anyons, is it possible "engineer" other non-abelian objects?

Problem

- Problem: Abelian anyons have no degeneracy, so no computation power $\rightarrow$ need non-abelian anyons for TQC (e.g. $\nu = 5/2, 12/5$)
- These are difficult to realize, existence is still uncertain
- Question: Given a top. phase that supports only abelian anyons, is it possible "engineer" other non-abelian objects?
  - Answer: Yes! We consider boundaries of the topological phase $\rightarrow$ *gapped boundaries*
  - We'll even get a universal gate set from gapped boundaries of an abelian phase (specifically, bilayer $\nu = 1/3$ FQH)

Overview

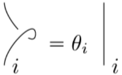
- Framework of TQC
- Introduce gapped boundaries and their framework
- Gapped boundaries for TQC
- Universal TQC with gapped boundaries in bilayer $\nu = 1/3$ FQH
- Summary and Outlook

Overview

- **Framework of TQC**
- Introduce gapped boundaries and their framework
- Gapped boundaries for TQC
- Universal TQC with gapped boundaries in bilayer $\nu = 1/3$ FQH
- Summary and Outlook

Formally, an *anyon model* $\mathcal{B}$ consists of the following data:

- Set of anyon types/labels: $\{a, b, c ...\}$, one of which should represent the vacuum $\mathbf{1}$
    - Each anyon type has a topological twist $\theta_i \in \mathrm{U}(1)$:



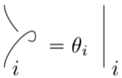
[1] More general cases exist, but are not used in this talk.

Figures: Z. Wang, *Topological Quantum Computation* (2010)

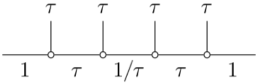
Framework of TQC

Formally, an *anyon model* $\mathcal{B}$ consists of the following data:

- Set of anyon types/labels: $\{a, b, c...\}$, one of which should represent the vacuum $\mathbf{1}$
  - Each anyon type has a topological twist $\theta_i \in \mathrm{U}(1)$:



- For each pair of anyon types, a set of fusion rules: $a \otimes b = \oplus_c N_{ab}^c c$.
  - The fusion space of $a \otimes b$ is a vector space V_{ab} with basis V_{ab}^c.[1]
  - The fusion space of $a_1 \otimes a_2 \otimes ... \otimes a_n$ to b is a vector space $V_{a_1 a_2 ... a_n}^b$ with basis given by anyon labels in intermediate segments, e.g.

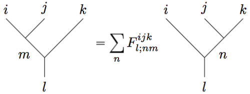


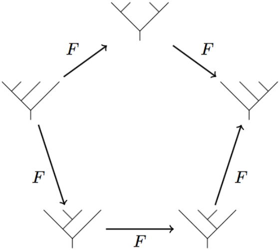
[1] More general cases exist, but are not used in this talk.

Figures: Z. Wang, *Topological Quantum Computation* (2010)

Formally, an *anyon model* $\mathcal{B}$ consists of the following data: [Cont'd]

- Associativity: for each (a, b, c, d), a set of (unitary) linear transformations $\{F^{abc}_{d;ef} : V^{abc}_d \to V^{abc}_d\}$ satisfying "pentagons"



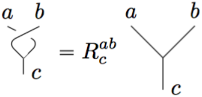


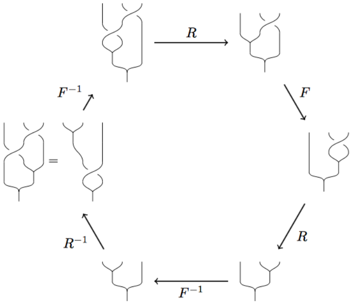
Figures: Z. Wang, *Topological Quantum Computation* (2010)

Framework of TQC

Formally, an *anyon model* $\mathcal{B}$ consists of the following data: [Cont'd]

- (Non-degenerate) braiding: For each (a, b, c) s.t. $N_{a,b}^{c} \neq 0$, a set of phases[1] $R_{ab}^{c} \in U(1)$ compatible with associativity ("hexagons"):





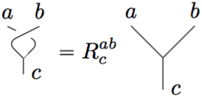
[1] More general cases exist, but are not used in this talk.

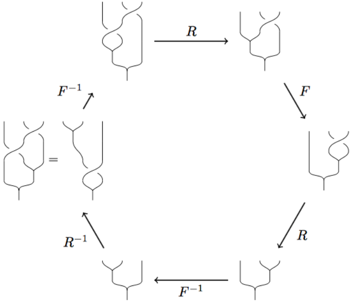
Figures: Z. Wang, *Topological Quantum Computation* (2010)

Framework of TQC

Formally, an *anyon model* $\mathcal{B}$ consists of the following data: [Cont'd]

- (Non-degenerate) braiding: For each (a, b, c) s.t. $N^c_{a,b} \neq 0$, a set of phases[1] $R^c_{ab} \in U(1)$ compatible with associativity ("hexagons"):





Mathematically, this is captured by a (unitary) *modular tensor category* (UMTC)

[1] More general cases exist, but are not used in this talk.

Figures: Z. Wang, *Topological Quantum Computation* (2010)

Framework of TQC

- In this framework, a 2D *topological phase of matter* is an equivalence class of gapped Hamiltonians $\mathcal{H} = \{H\}$ whose low-energy excitations form the same anyon model $\mathcal{B}$
- Examples (on the lattice) include Kitaev's quantum double models, Levin-Wen string-net models, ...

Framework of TQC: Example

- Physical system: Bilayer FQH system, $1/3$ Laughlin state of opposite chirality in each layer
- Equivalent to $\mathbb{Z}_3$ toric code (Kitaev, 2003)
- Anyon types: $e^a m^b$, $a, b = 0, 1, 2$
- Twist: $\theta(e^a m^b) = \omega^{ab}$ where $\omega = e^{2\pi i/3}$
- Fusion rules: $e^a m^b \otimes e^c m^d \to e^{(a+c)(\text{mod } 3)} m^{(b+d)(\text{mod } 3)}$
- F symbols all trivial (0 or 1)
- $R_{e^a m^b, e^c m^d} = e^{2\pi i bc/3}$
- UMTC: $\text{SU}(3)_1 \times \overline{\text{SU}(3)_1} \cong \mathfrak{D}(\mathbb{Z}_3) = \mathcal{Z}(\text{Rep}(\mathbb{Z}_3)) = \mathcal{Z}(\text{Vec}_{\mathbb{Z}_3})$

Overview

- Framework of TQC
- **Introduce gapped boundaries and their framework**
- Gapped boundaries for TQC
- Universal gate set with gapped boundaries in bilayer $\nu = 1/3$ FQH
- Summary and Outlook

Gapped Boundaries: Framework

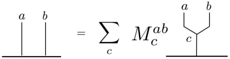
- A *gapped boundary* is an equivalence class of gapped local (commuting) extensions of $H \in \mathcal{H}$ to the boundary

Gapped Boundaries: Framework

- A *gapped boundary* is an equivalence class of gapped local (commuting) extensions of $H \in \mathcal{H}$ to the boundary
- In the anyon model: Collection of bulk bosonic ($\theta = 1$) anyons which condense to vacuum on the boundary (think Bose condensation)
  - All other bulk anyons condense to confined "boundary excitations" $\alpha, \beta, \gamma ...$
- Mathematically, Lagrangian algebra $\mathcal{A} \in \mathcal{B}$

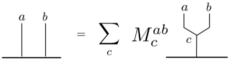
Gapped Boundaries: Framework

More rigorously, gapped boundaries come with M symbols (like F symbols for the bulk):



Gapped Boundaries: Framework

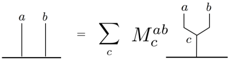
More rigorously, gapped boundaries come with M symbols (like F symbols for the bulk):



(More generally, we also define these with boundary excitations, but that is unnecessary for this talk.)

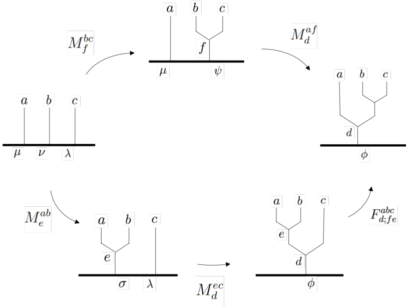
Gapped Boundaries: Framework

More rigorously, gapped boundaries come with M symbols (like F symbols for the bulk):



(More generally, we also define these with boundary excitations, but that is unnecessary for this talk.)

M symbols must be compatible with F symbols ("mixed pentagons"):



Gapped Boundaries: Example

- Physical system: Bilayer $\nu = 1/3$ FQH, equiv. to $\mathbb{Z}_3$ toric code
- Anyon types: $e^a m^b$, $a, b = 0, 1, 2$

Gapped Boundaries: Example

- Physical system: Bilayer $\nu = 1/3$ FQH, equiv. to $\mathbb{Z}_3$ toric code
- Anyon types: $e^a m^b$, $a, b = 0, 1, 2$
- Two gapped boundary types:
  - Electric charge condensate: $\mathcal{A}_1 = 1 \oplus e \oplus e^2$
  - Magnetic flux condensate: $\mathcal{A}_2 = 1 \oplus m \oplus m^2$

Gapped Boundaries: Example

- Physical system: Bilayer $\nu = 1/3$ FQH, equiv. $\mathbb{Z}_3$ toric code
- We will work mainly with $\mathcal{A}_1 = 1 \oplus e \oplus e^2$:
  - Algebraically, $\mathcal{A}_1$, $\mathcal{A}_2$ are equivalent by electric-magnetic duality
  - Easier to work with charge condensate - read-out can be done by measuring electric charge (Barkeshli, 2016)
  - It is interesting to consider *both* $\mathcal{A}_1$ and $\mathcal{A}_2$ at the same time - we do this in a separate paper[3], will briefly mention in our Outlook

[3] C, Cheng, Wang, Phys. Rev. B 96, 195129 (2017)

Gapped Boundaries: Example

- Physical system: Bilayer $\nu = 1/3$ FQH, equiv. $\mathbb{Z}_3$ toric code
- We will work mainly with $\mathcal{A}_1 = 1 \oplus e \oplus e^2$:
  - Algebraically, $\mathcal{A}_1$, $\mathcal{A}_2$ are equivalent by electric-magnetic duality
  - Easier to work with charge condensate - read-out can be done by measuring electric charge (Barkeshli, 2016)
  - It is interesting to consider *both* $\mathcal{A}_1$ and $\mathcal{A}_2$ at the same time - we do this in a separate paper[3], will briefly mention in our Outlook
- M symbols for this theory are all 0 or 1

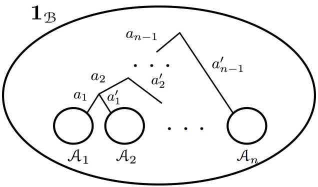
[3] C, Cheng, Wang, Phys. Rev. B 96, 195129 (2017)

Overview

- Framework of TQC
- Introduce gapped boundaries and their framework
- **Gapped boundaries for TQC**
- Universal gate set with gapped boundaries in bilayer $\nu = 1/3$ FQH
- Summary and Outlook

Qudit encoding:

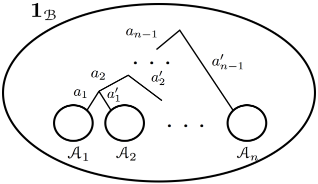
- Gapped boundaries give rise to a natural ground state degeneracy: *n* gapped boundaries on a plane, with total charge vacuum

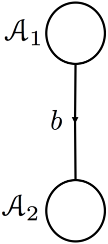


Gapped Boundaries for TQC

Qudit encoding:

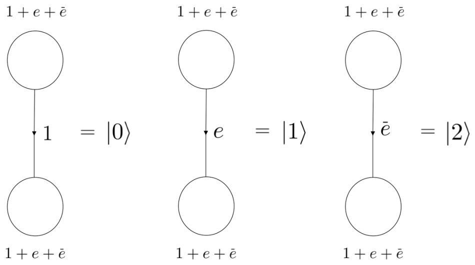
- Gapped boundaries give rise to a natural ground state degeneracy: n gapped boundaries on a plane, with total charge vacuum
- For qudit encoding: use $n = 2$





Gapped Boundaries for TQC: Example

For our bilayer $\nu = 1/3$ FQH system, we have:



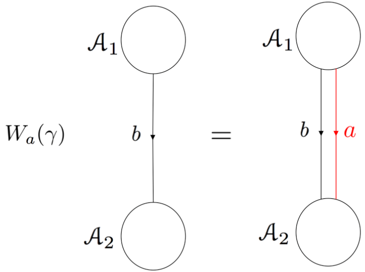
Gapped Boundaries for TQC

Topologically protected operations:

- Tunnel-*a* operations
- Loop-*a* operations
- Braiding gapped boundaries
- Topological charge measurement*

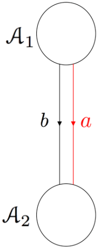
Tunnel-a Operations

Starting from state $|b\rangle$, tunnel an a anyon from $\mathcal{A}_1$ to $\mathcal{A}_2$:

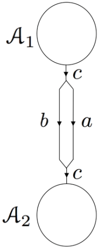


Tunnel-*a* Operations

Compute using *M*-symbols:



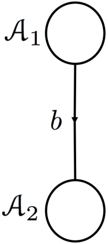
$$= \sum_c M_c^{ab}(\mathcal{A}_2)[M_c^{ab}(\mathcal{A}_1)]^\dagger$$



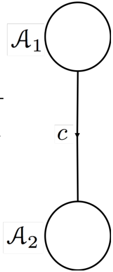
Tunnel-a Operations

Result:

$W_a(\gamma)$

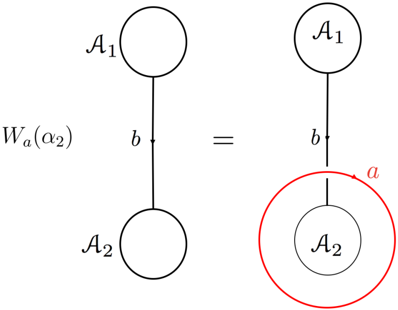


$$= \sum_c M_c^{ab}(\mathcal{A}_1)[M_c^{ab}]^\dagger(\mathcal{A}_2)\sqrt{\frac{d_a d_b}{d_c}} \quad (1)$$



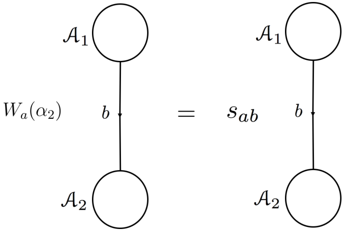
Loop-a Operations

Starting from state $|b\rangle$, loop an a anyon around one of the boundaries:

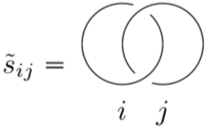


Loop-a Operations

Similar computation methods lead to the formula:

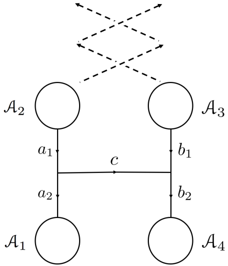


where $s_{ab} = \tilde{s}_{ab}/d_b$ is given by the modular S matrix of the theory:



Braiding Gapped Boundaries

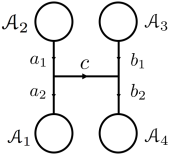
Braid gapped boundaries around each other:



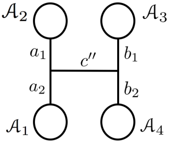
(Mathematically, this gives a representation of the (spherical) $2n$-strand pure braid group.)

Braiding Gapped Boundaries

Simplify with (bulk) R and F moves to get:

σ_2^2 

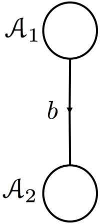
$$= \sum_{c,c'} F^{a_2 a_1 b_1}_{b_2; c' c} R^{b_1 a_1}_{c'} R^{a_1 b_1}_{c} \left(F^{a_2 a_1 b_1}_{b_2}\right)^{-1}_{c'' c'} \tag{2}$$

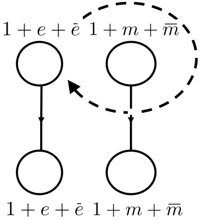


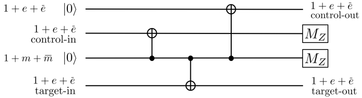
Gapped Boundaries for TQC: Example

For our bilayer $\nu = 1/3$ FQH case: $(\mathcal{A}_1 = \mathcal{A}_2 = 1 \oplus e \oplus e^2)$

- Tunnel an e anyon from $\mathcal{A}_1$ to $\mathcal{A}_2$: $W_a(\gamma)|b\rangle = |a \otimes b\rangle$
  $\rightarrow W_e(\gamma) = \sigma_3^x$, where $\sigma_3^x|i\rangle = |(i+1)(\text{mod } 3)\rangle$
- Loop an m anyon around $\mathcal{A}_2$: $W_m(\alpha_2)|e^j\rangle = \omega^j|e^j\rangle$
  $\rightarrow W_m(\alpha_2) = \sigma_3^z$
- Braid gapped boundaries: get $\wedge\sigma_3^z$



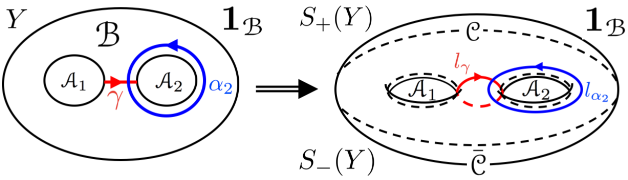




- Motivation:
  - Property F conjecture (Naidu and Rowell, 2011): Braidings alone cannot be universal for TQC for most physically plausible systems

Topological Charge Projection/Measurement

- Motivation:
  - Property F conjecture (Naidu and Rowell, 2011): Braidings alone cannot be universal for TQC for most physically plausible systems
- Topological charge projection (TCP) (Barkeshli and Freedman, 2016):
  - Doubled theories: Wilson line lifts to a loop $\rightarrow$ measure topological charge through the loop



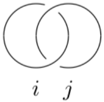
  - Resulting projection operator: $(\mathcal{O}_x(\beta) = W_{x\bar{x}}(\gamma_i)$ or $W_x(\alpha_i))$

$$P_\beta^{(a)} = \sum_{x \in \mathcal{C}} S_{0a} S_{xa}^* \mathcal{O}_x(\beta). \tag{3}$$

Topological Charge Projection/Measurement

- Topological charge projection (TCP): [Cont'd]
  - Given an anyon theory $\mathcal{C}$, its $\mathcal{S}, \mathcal{T}$ matrices

$$\mathcal{S} = \left\{ \tilde{s}_{ij} = \right.$$



$$\left. \right\}, \qquad \mathcal{T} = \text{diag}(\theta_i)$$

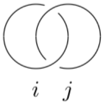
    give mapping class group representations $V_{\mathcal{C}}(Y)$ for surfaces Y.
  - Barkeshli and Freedman showed that topological charge projections generate all matrices in $V_{\mathcal{C}}(Y)$

[1] More general cases exist, but are not used in this talk.

Topological Charge Projection/Measurement

- Topological charge projection (TCP): [Cont'd]
  - Given an anyon theory $\mathcal{C}$, its $\mathcal{S}, \mathcal{T}$ matrices

$$\mathcal{S} = \left\{ \tilde{s}_{ij} = \right.$$



$$\left. \right\}, \qquad \mathcal{T} = \text{diag}(\theta_i)$$

  give mapping class group representations $V_{\mathcal{C}}(Y)$ for surfaces Y.
  - Barkeshli and Freedman showed that topological charge projections generate all matrices in $V_{\mathcal{C}}(Y)$
- General topological charge measurements (TCMs):
  - Projection operators $P_\beta^{(a)} = \sum_{x \in \mathcal{C}} S_{0a} S^*_{xa} \mathcal{O}_x(\beta) \rightarrow$ topological charge measurements perform the *complement* of $P_\beta^{(a)}$[1]
  - Not always physical, but special cases are *symmetry protected* – we examine this

[1] More general cases exist, but are not used in this talk.

Overview

- Framework of TQC
- Introduce gapped boundaries and their framework
- Gapped boundaries for TQC
- **Universal gate set with gapped boundaries in bilayer $\nu = 1/3$ FQH**
- Summary and Outlook

Universal Gate Set

Universal (*metaplectic*) gate set for the qutrit model (Cui and Wang, 2015):

1. The single-qutrit Hadamard gate H_3, defined as $H_3|j\rangle = \frac{1}{\sqrt{3}} \sum_{i=0}^{2} \omega^{ij} |i\rangle$, $j = 0, 1, 2$, $\omega = e^{2\pi i/3}$
2. The two-qutrit entangling gate SUM_3, defined as $\mathrm{SUM}_3|i\rangle|j\rangle = |i\rangle|(i+j) \bmod 3\rangle, \; i, j = 0, 1, 2.$
3. The single-qutrit generalized phase gate $Q_3 = \mathrm{diag}(1, 1, \omega)$.
4. Any nontrivial single-qutrit classical (i.e. Clifford) gate not equal to H_3^2.
5. A projection M of a state in the qutrit space $\mathbb{C}^3$ to $\mathrm{Span}\{|0\rangle\}$ and its orthogonal complement $\mathrm{Span}\{|1\rangle, |2\rangle\}$, so that the resulting state is coherent if projected into $\mathrm{Span}\{|1\rangle, |2\rangle\}$.

Universal Gate Set - Bilayer $\nu = 1/3$ FQH

Universal (*metaplectic*) gate set for the qutrit model (Cui and Wang, 2015):

1. $H_3 = \mathcal{S}$ for $\mathcal{C} = \mathrm{SU}(3)_1$ (single layer $\nu = 1/3$ FQH), so it can be implemented by TCP.

Universal Gate Set - Bilayer $\nu = 1/3$ FQH

Universal (*metaplectic*) gate set for the qutrit model (Cui and Wang, 2015):

1. $H_3 = \mathcal{S}$ for $\mathcal{C} = \mathrm{SU}(3)_1$ (single layer $\nu = 1/3$ FQH), so it can be implemented by TCP.
2. $\wedge \sigma_3^z$ can be implemented by gapped boundary braiding. Conjugate second qutrit by H_3 to get SUM_3.

Universal Gate Set - Bilayer $\nu = 1/3$ FQH

Universal (*metaplectic*) gate set for the qutrit model (Cui and Wang, 2015):

1. $H_3 = \mathcal{S}$ for $\mathcal{C} = \mathrm{SU}(3)_1$ (single layer $\nu = 1/3$ FQH), so it can be implemented by TCP.
2. $\wedge\sigma_3^z$ can be implemented by gapped boundary braiding. Conjugate second qutrit by H_3 to get SUM_3.
3. TCP can implement $\mathrm{diag}(1, \omega, \omega)$ (Dehn twist of $\mathrm{SU}(3)_1$). Follow by σ_3^x for Q_3.

Universal Gate Set - Bilayer $\nu = 1/3$ FQH

Universal (*metaplectic*) gate set for the qutrit model (Cui and Wang, 2015):

1. $H_3 = \mathcal{S}$ for $\mathcal{C} = \mathrm{SU}(3)_1$ (single layer $\nu = 1/3$ FQH), so it can be implemented by TCP.
2. $\wedge\sigma_3^z$ can be implemented by gapped boundary braiding. Conjugate second qutrit by H_3 to get SUM_3.
3. TCP can implement $\mathrm{diag}(1, \omega, \omega)$ (Dehn twist of $\mathrm{SU}(3)_1$). Follow by σ_3^x for Q_3.
4. Any nontrivial single-qutrit classical (i.e. Clifford) gate not equal to H_3^2 - we have a Pauli-X from tunneling e.

Universal Gate Set - Bilayer $\nu = 1/3$ FQH

Universal (*metaplectic*) gate set for the qutrit model (Cui and Wang, 2015):

1. $H_3 = \mathcal{S}$ for $\mathcal{C} = \mathrm{SU}(3)_1$ (single layer $\nu = 1/3$ FQH), so it can be implemented by TCP.
2. $\wedge\sigma_3^z$ can be implemented by gapped boundary braiding. Conjugate second qutrit by H_3 to get SUM_3.
3. TCP can implement $\mathrm{diag}(1, \omega, \omega)$ (Dehn twist of $\mathrm{SU}(3)_1$). Follow by σ_3^x for Q_3.
4. Any nontrivial single-qutrit classical (i.e. Clifford) gate not equal to H_3^2 - we have a Pauli-X from tunneling e.
5. Projective measurement - we use the TCM which is the complement of

$$P_\gamma^{(1)} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}. \quad (4)$$

Conjugating $1 - P_\gamma^{(1)}$ with the Hadamard gives the result.

Universal Gate Set - Bilayer $\nu = 1/3$ FQH

To get the projective measurement, we introduce a *symmetry-protected* topological charge measurement:

- Want to tune system s.t. quasiparticle tunneling along γ is enhanced $\rightarrow$ implement $H' = -tW_\gamma(e) + \text{h.c.}$
    - $t =$ (complex) tunneling amplitude, $W_\gamma(e) =$ tunnel-e operator

Universal Gate Set - Bilayer $\nu = 1/3$ FQH

To get the projective measurement, we introduce a *symmetry-protected* topological charge measurement:

- Want to tune system s.t. quasiparticle tunneling along γ is enhanced $\rightarrow$ implement $H' = -tW_\gamma(e) + \text{h.c.}$
  - $t =$ (complex) tunneling amplitude, $W_\gamma(e) =$ tunnel-e operator
- Implementing $M \leftrightarrow$ ground state of H' is doubly degenerate for $|e\rangle, |e^2\rangle \leftrightarrow t$ is real (beyond topological protection)

Universal Gate Set - Bilayer $\nu = 1/3$ FQH

To get the projective measurement, we introduce a *symmetry-protected* topological charge measurement:

- Want to tune system s.t. quasiparticle tunneling along γ is enhanced $\rightarrow$ implement $H' = -tW_\gamma(e) + \text{h.c.}$
  - t = (complex) tunneling amplitude, $W_\gamma(e)$ = tunnel-e operator
- Implementing $M \leftrightarrow$ ground state of H' is doubly degenerate for $|e\rangle, |e^2\rangle \leftrightarrow t$ is real (beyond topological protection)
- Physically, could realize in fractional quantum spin Hall state – quantum spin Hall + time-reversal symmetry (exchange two layers)
  - Topologically equiv. to $\nu = 1/3$ Laughlin, e = bound state of spin up/down quasiholes
  - e is time-reversal invariant $\rightarrow$ tunneling amplitude of e must be real $\rightarrow$ symmetry-protected TCM

Overview

- Framework of TQC
- Introduce gapped boundaries and their framework
- Gapped boundaries for TQC
- Universal gate set with gapped boundaries in bilayer $\nu = 1/3$ FQH
- **Summary and Outlook**

Summary

- Decoherence is a major challenge to quantum computing → **topological quantum computing** (TQC)
- TQC with anyons requires non-abelian topological phases (difficult to implement) → **engineer non-abelian objects** (e.g. **gapped boundaries**) from abelian phases
- We can get a **universal quantum computing gate set** from a purely abelian theory (bilayer $\nu = 1/3$ FQH), which is trivial for anyonic TQC
    - Topologically protected qudit encoding and Clifford gates
    - Symmetry-protected implementation for non-Clifford projection

Outlook

- Practical implementation of the symmetry-protected TCM
- More thorough study of symmetry-protected quantum computation
  - Amount of protection offered and computation power
- Other routes to engineer non-abelian objects
  - Boundary defects/parafermion zero modes from gapped boundaries of $\nu = 1/3$ FQH (Lindner et al., 2014)
  - Genons and symmetry defects (Barkeshli et al., 2014; C, Cheng, Wang, 2017; Delaney and Wang, 2018)
  - How would these look when combined with gapped boundaries?

Acknowledgments

- Special thanks to Cesar Galindo, Shawn Cui, Maissam Barkeshli for answering many questions
- Many thanks to Prof. Freedman and everyone at Station Q for a great summer
- None of this would have been possible without the guidance and dedication of Prof. Zhenghan Wang

Quantum Convolutional Neural Networks

Iris Cong

Soonwon Choi

Mikhail D. Lukin

arXiv:1810.03787

Why quantum machine learning?

Machine learning: interpret and process large amounts of data

Unmanned Vehicle

Genomics

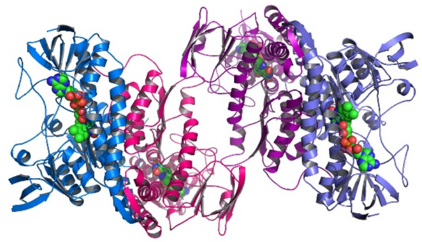
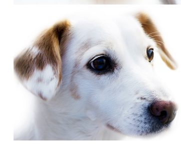


Image recognition

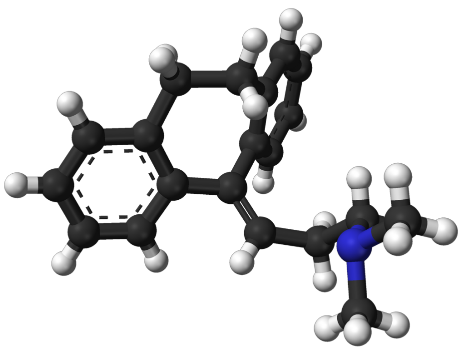
= Cat



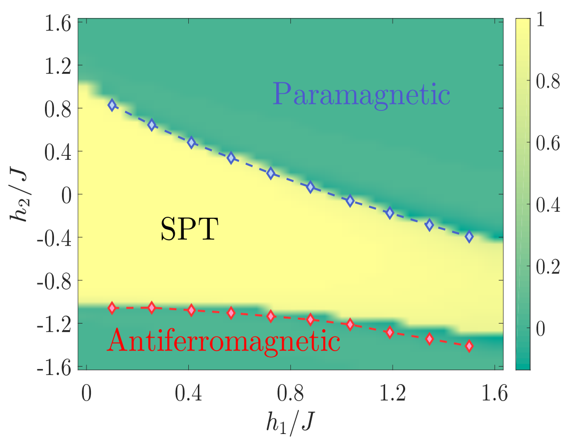
= Dog

Quantum physics: many-body interactions→ extremely large complexity

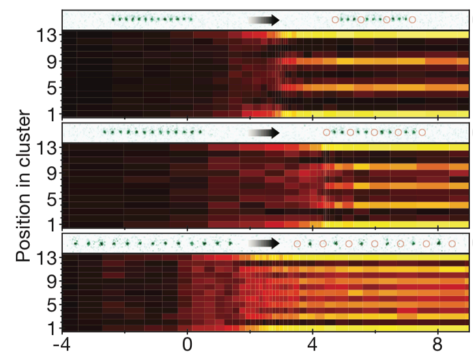
Quantum chemistry



Quantum phases of matter



Near-term quantum computers/ quantum simulators



H. Bernien et al, *Nature* (2017)

Machine learning + Quantum many-body physics = Exciting!

Quantum Machine Learning

- Using a *quantum computer* to perform machine learning tasks

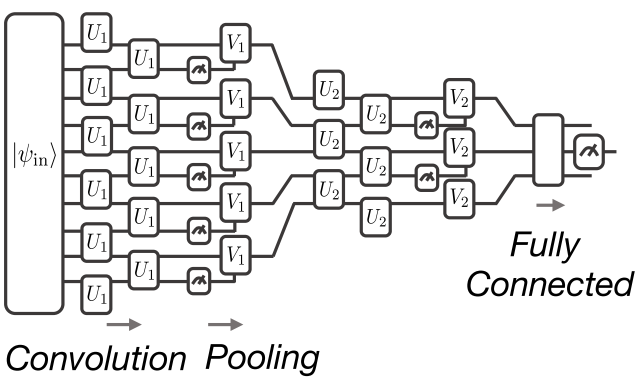
- Many open questions:
  - Why/how does quantum machine learning work?
  - Concrete circuit models suitable for near-term implementation?
  - Relationship to quantum many-body physics?
  - Relationship with quantum information theory?

➡ *Goal of our work*

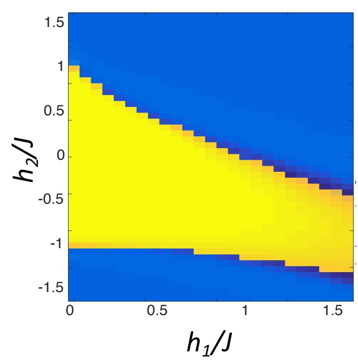
Main Contributions

Concrete + efficient circuit model for quantum classification problems

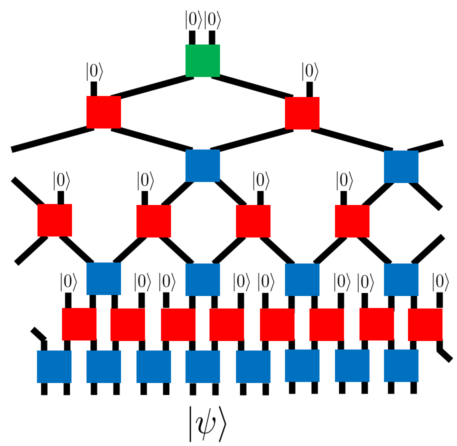
- Good connection to existing ML techniques



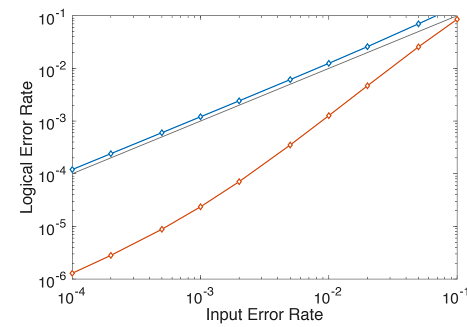
Application: Quantum Phase Recognition



Theoretical Explanation: RG Flow, MERA, Quantum Error Correction



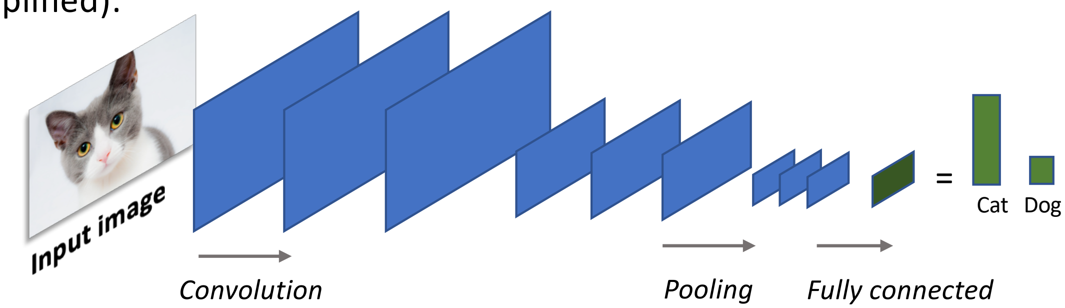
Application: Optimizing Quantum Error Correction



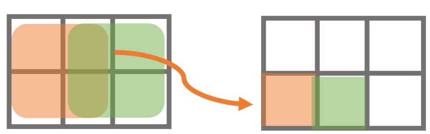
Review of (Classical) CNN

- Structured neural network: multiple *layers* of image processing

Example (simplified):

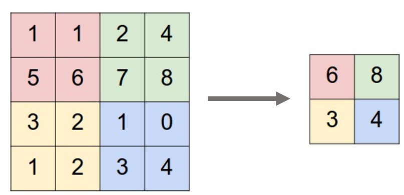


Convolution layers

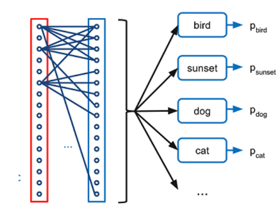


$$y_{i,j} = w_0 x_{i,j} + w_1 x_{i+,j} + w_2 x_{i,j+1} + w_3 x_{i+1,j+1}$$

Pooling layers



Fully connected

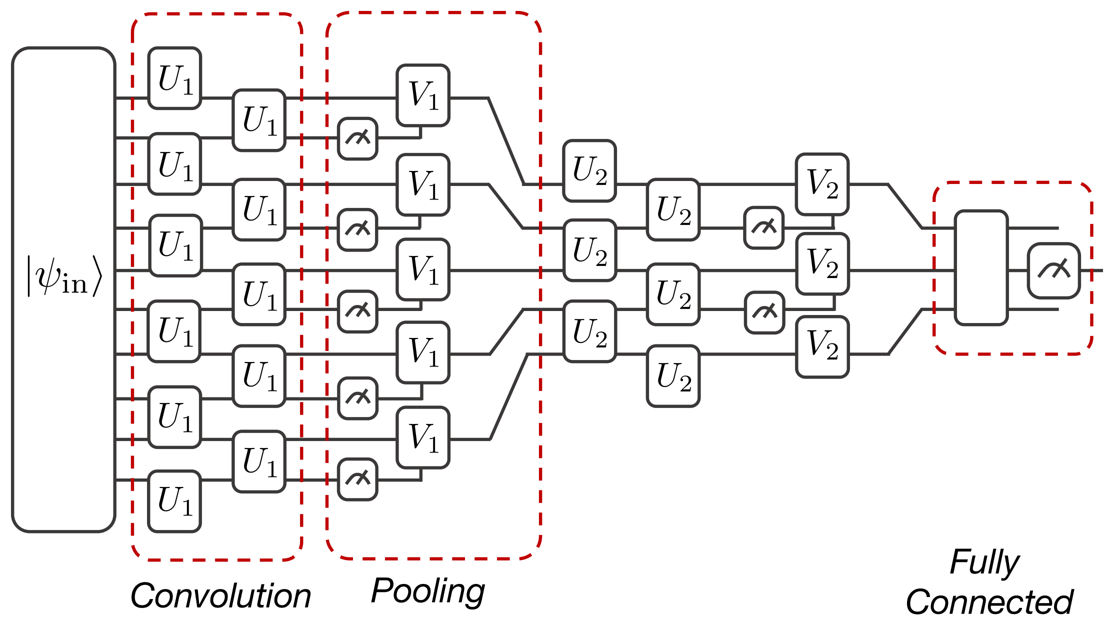


Quantum CNN Architecture

Same types of layers:

1. Convolution
   - Local unitaries, trans. inv., 1D, 2D, 3D …
2. Pooling
   - Reduce system size
   - Final unitary depends on meas. outcomes
3. Fully connected
   - Non-local measurement

Total number of parameters ~ O(log *N*)

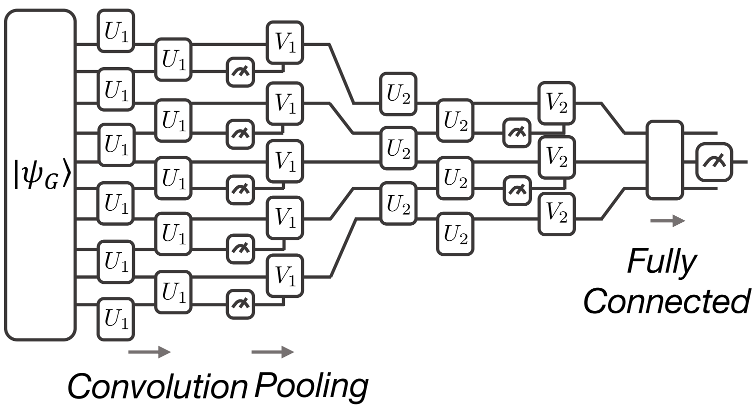


Application: Quantum Phase Recognition

Problem: Given quantum many-body system in (unknown) ground state $|\psi_G\rangle$, does $|\psi_G\rangle$ belong to a particular quantum phase $\mathcal{P}$?

Direct analog of image classification, but *intrinsically quantum problem*

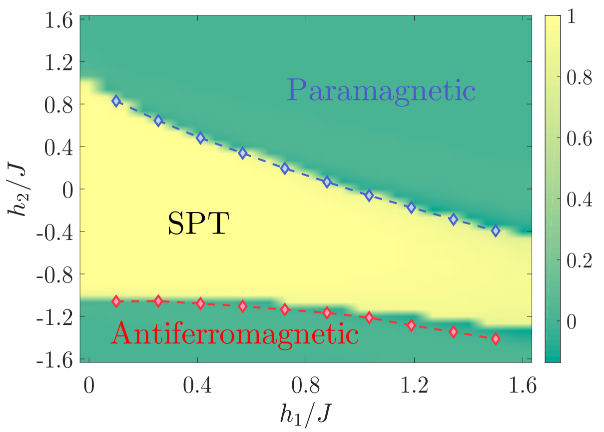
Claim: Quantum CNN is very efficient in quantum phase recognition

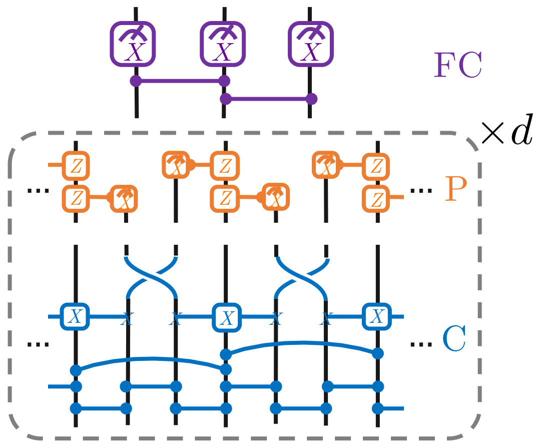


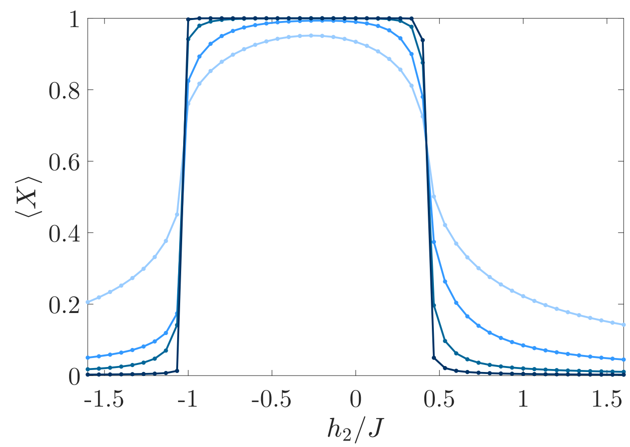
$$= \begin{cases} 1 & \text{if } |\psi_G\rangle \in \mathcal{P} \\ 0 & \text{if } |\psi_G\rangle \notin \mathcal{P} \end{cases}$$

Example: 1D ZXZ Model ($G = \mathbb{Z}_2 \times \mathbb{Z}_2$ SPT)

- SPT phase: cannot be detected by local order parameter
- $H = -J \sum_i Z_i X_{i+1} Z_{i+2} - h_1 \sum_i X_i - h_2 \sum_i X_i X_{i+1}$



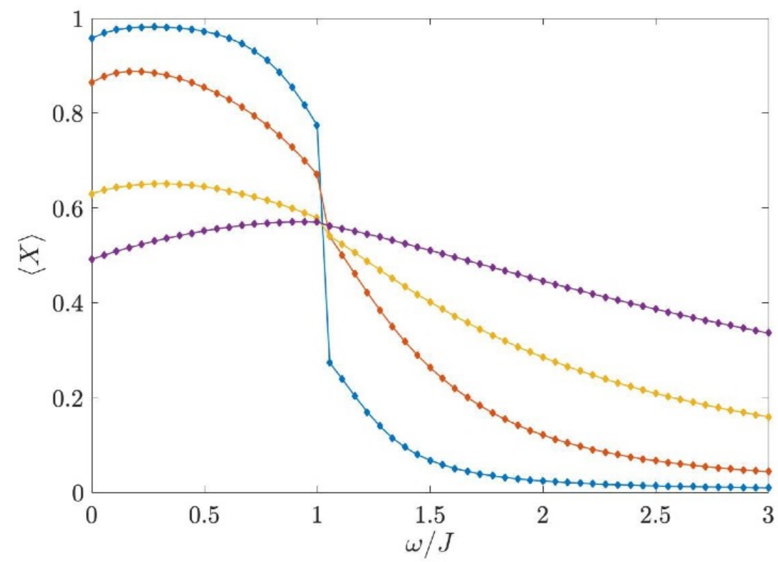




Phase diagram is obtained from iDMRG with bond dimension 150.
Input states are obtained from DMRG with system sizes 45, 135, bond dimension 130.
Circuit is performed using matrix-product state update.

Example: 1D ZXZ Model ($G = \mathbb{Z}_2 \times \mathbb{Z}_2$ SPT)

- S = 1 Haldane phase transition:
- Same phase, map spin-1 to pair of spin-1/2



Sample Complexity

- Existing approaches to detect SPT: measure nonzero expectation value of string order parameters (long operator product)
  - Problem: expectation value vanishes near phase boundary → many repetitions
  - QCNN: much sharper → fewer repetitions
- Quantify with *sample complexity*: How many copies of the input state are required to determine with 95% confidence that $|\psi_G\rangle \in \mathcal{P}$?

$$M_{\min} = \frac{1.96^2}{(\arcsin\sqrt{p} - \sqrt{\arcsin p_0})^2}$$

Sample Complexity

- Comparison with existing approaches:

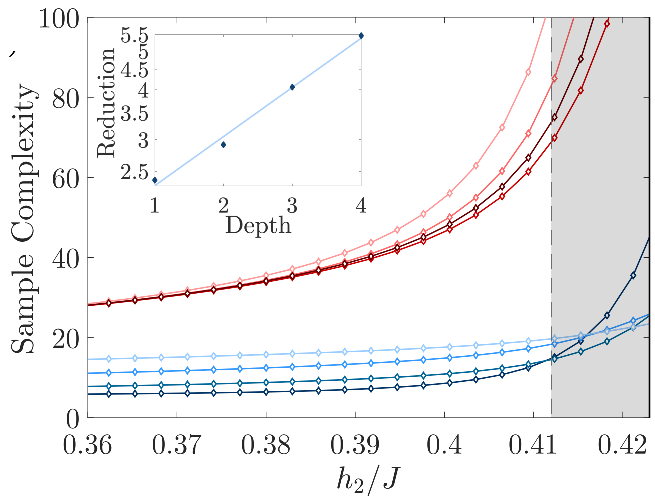
  string order parameters

- SOP (red): independent of string length

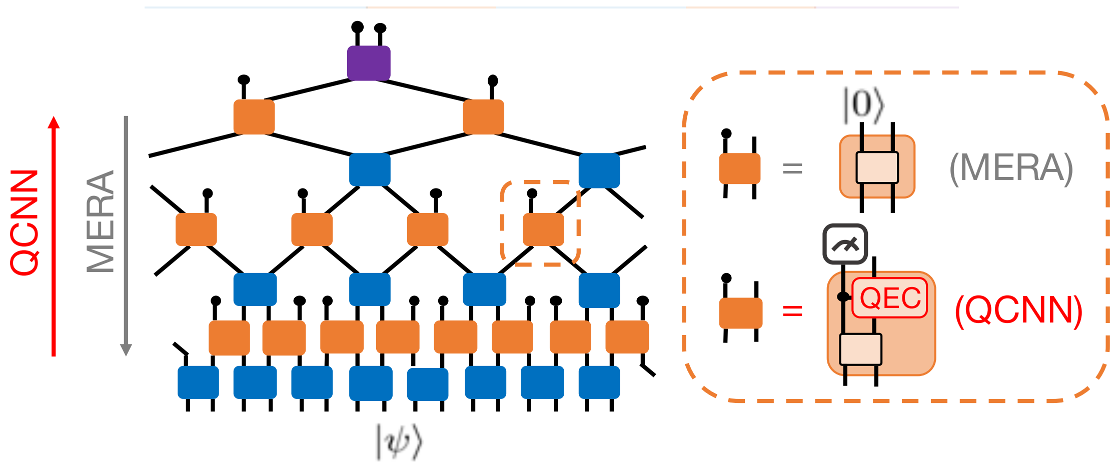
- QCNN (blue): much better and

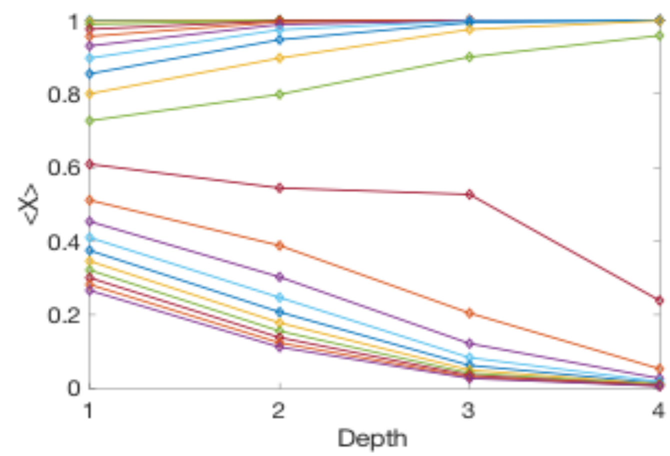
  improves with depth up to finite size

$h_1 = 0.5\ J$:



Why does it work?

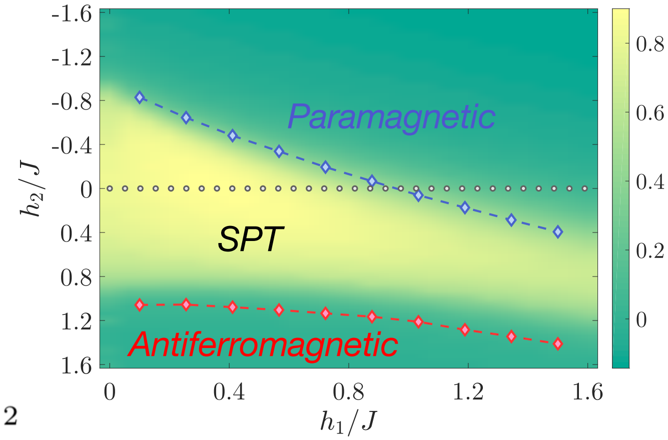




QCNN ≈ MERA + QEC ≈ "RG flow"

Training: Example

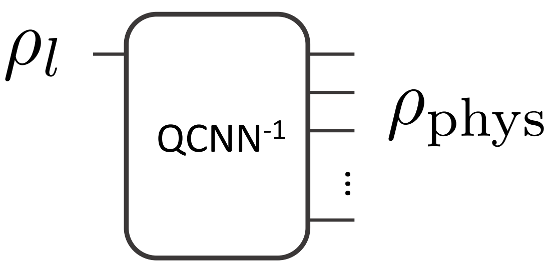
- N = 15 spins (depth 1) for simulations
- Initialize all unitaries to random values
- Train along $h_2 = 0$ (solvable)
- Gradient descent: $\mathrm{MSE} = \frac{1}{2M}\sum_{\alpha=1}^{M}(y_i - f_{\{U_i,V_j,F\}}(|\psi_\alpha\rangle))^2$
- Observation: training on 1D, solvable set can still produce the correct 2D phase diagram
- Demonstrates how QCNN structure avoids overfitting



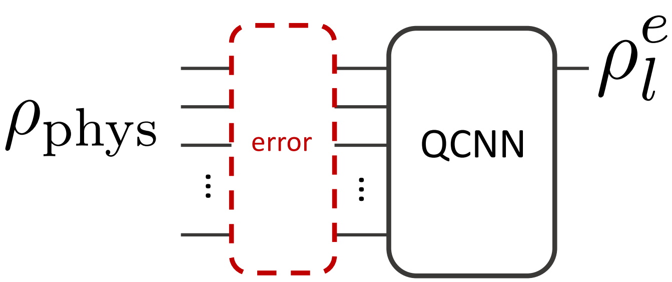
Optimizing Quantum Error Correction

Problem: Given a realistic but unknown error model, find a resource-efficient, fault-tolerant quantum error correction code to protect against these errors.

QCNN structure resembles nested quantum error correction, and can be used to simultaneously optimize encoder and decoder



Encoder



Decoder

Optimizing Quantum Error Correction

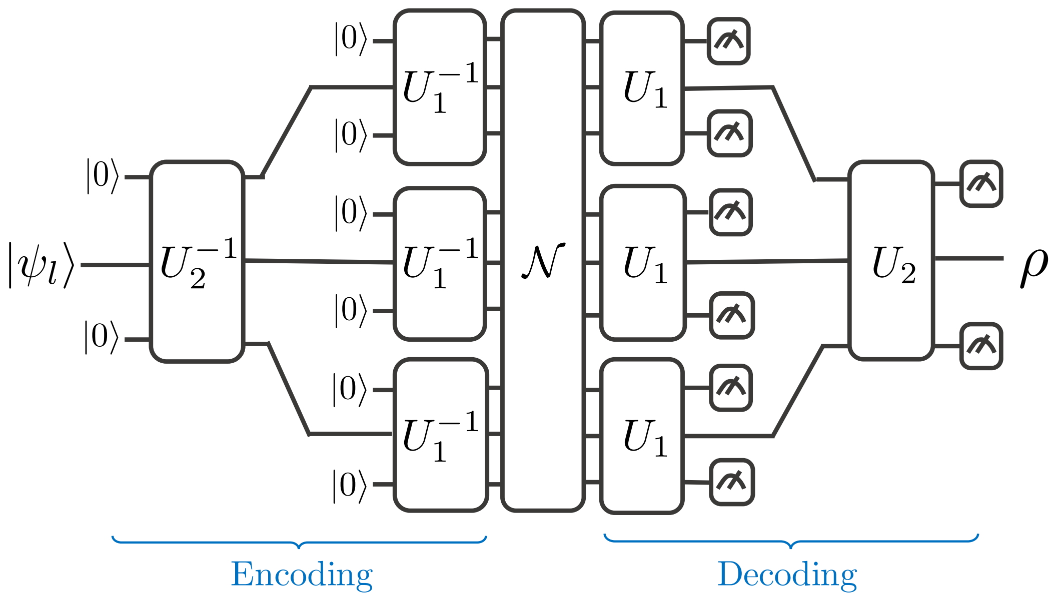
Error models:

- Isotropic depolarization ($p_x = p_y = p_z$)
- Anisotropic depolarization $\mathcal{N}_{1,i} : \rho \mapsto (1 - \sum_{\mu} p_{\mu})\rho + \sum_{\mu} p_{\mu} \sigma_i^{\mu} \rho \sigma_i^{\mu}$
- Anisotropic depolarization + correlated error ($X_i X_{i+1}$)

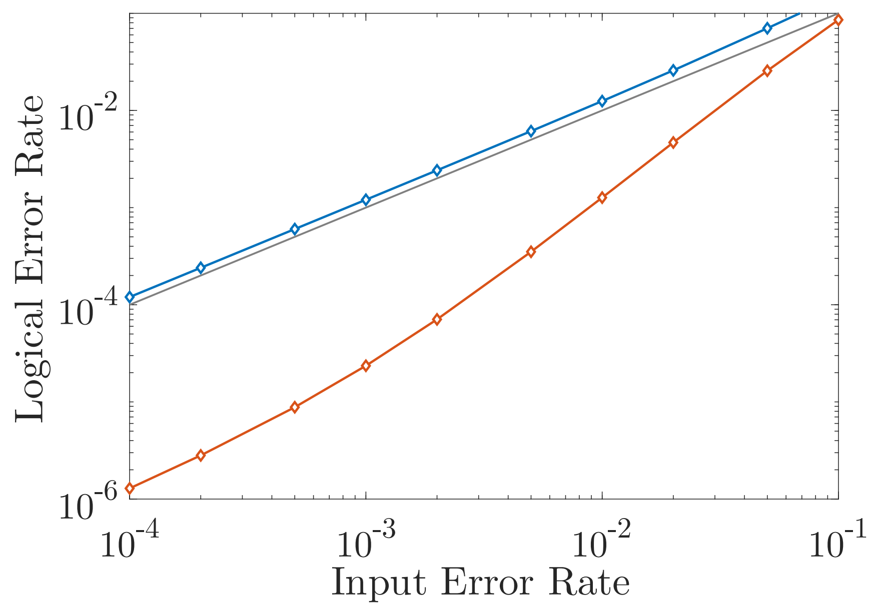
$$\mathcal{N}_{2,i} : \rho \mapsto (1 - p_{xx})\rho + p_{xx} X_i X_{i+1} \rho X_i X_{i+1}$$

Optimizing Quantum Error Correction

Circuit structure:

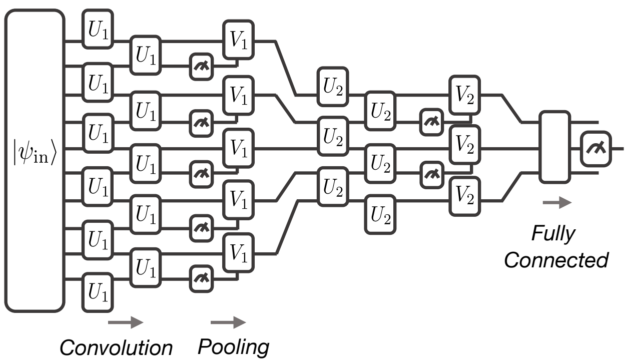


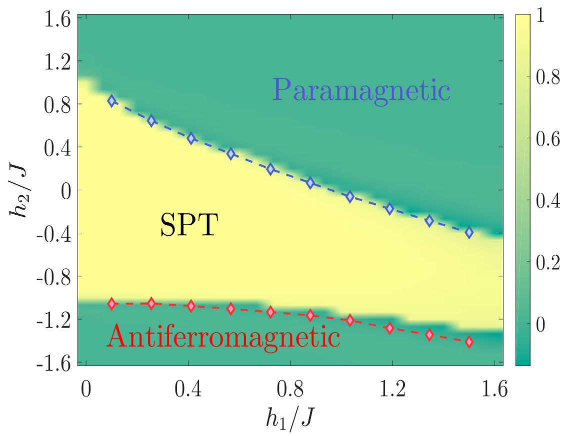
Results for correlated error:

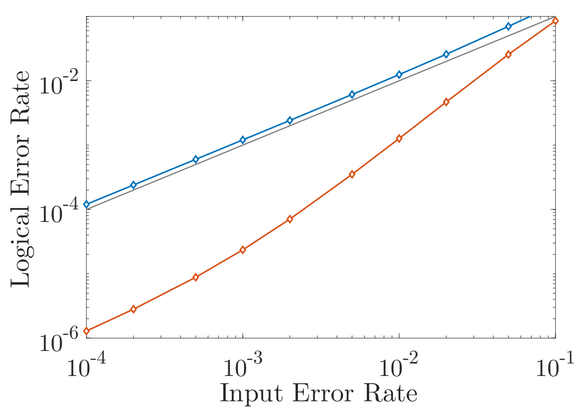


Summary

- Concrete circuit model for quantum classification
- Application to quantum phase recognition:
  - 1D SPT phase ($\mathbb{Z}_2 \times \mathbb{Z}_2$)
  - Theoretical explanation: QCNN ≈ MERA + QEC ≈ RG flow
- Optimizing Quantum Error Correction







Thanks!