Online Appendix to accompany Rights & Sterba (In press). R squared measures for multilevel models with three or more levels. *Multivariate Behavioral Research. DOI: 10.1080/00273171.2021.1985948*

Online Appendix A: Derivations of the population correspondence between pre-existing three-level MLM R^2 measures and measures from our framework

Online Appendix A provides derivations showing how the pre-existing three-level MLM R^2 measures listed in Table 2 reflect the same population quantities as measures (or sums of measures) from our framework. This involves replacing terms in the population expression for the pre-existing measures with terms from our model-implied variance expressions that reflect this same quantity in the population. Symbols not defined here were defined in the manuscript. Though all pre-existing measures listed below are equivalent in the population to measures obtained from our framework, some are additionally exactly equivalent in the sample (those not denoted with *'s below, i.e., $R^2_{NSJ(m)}$ and $R^2_{NSJ(c)}$).

Section A1: Correspondence with Snijders & Bosker's (2012) measure

The Snijders and Bosker (1999, 2012) measure was defined in manuscript Equation (2) and is reproduced here:

$$R^2_{S\&B} = 1 - \frac{\varphi_{000} + \tau_{000} + \sigma^2}{\varphi_{000(null)} + \tau_{000(null)} + \sigma^2_{(null)}} \quad \text{(OA1)}$$

The Snijders and Bosker (2012) approach requires constraining random slope variation to 0, though our framework does not impose this requirement. In the denominator of Equation (OA1), the sum of the random-intercept-only model variance components reflects the total outcome variance. Replacing this expression with our model-implied outcome variance expression in manuscript Equation (9) and simplifying due to having constrained random slope variation to 0 yields

$$\begin{aligned} R^{2*}_{S\&B} &= 1 - \frac{\varphi_{000} + \tau_{000} + \sigma^2}{\boldsymbol{\gamma}_1'\boldsymbol{\Phi}_1\boldsymbol{\gamma}_1 + \boldsymbol{\gamma}_2'\boldsymbol{\Phi}_2\boldsymbol{\gamma}_2 + \boldsymbol{\gamma}_3'\boldsymbol{\Phi}_3\boldsymbol{\gamma}_3 + \varphi_{000} + \tau_{000} + \sigma^2} \\ &= \frac{\boldsymbol{\gamma}_1'\boldsymbol{\Phi}_1\boldsymbol{\gamma}_1 + \boldsymbol{\gamma}_2'\boldsymbol{\Phi}_2\boldsymbol{\gamma}_2 + \boldsymbol{\gamma}_3'\boldsymbol{\Phi}_3\boldsymbol{\gamma}_3 + \varphi_{000} + \tau_{000} + \sigma^2}{\boldsymbol{\gamma}_1'\boldsymbol{\Phi}_1\boldsymbol{\gamma}_1 + \boldsymbol{\gamma}_2'\boldsymbol{\Phi}_2\boldsymbol{\gamma}_2 + \boldsymbol{\gamma}_3'\boldsymbol{\Phi}_3\boldsymbol{\gamma}_3 + \varphi_{000} + \tau_{000} + \sigma^2} \\ &\quad - \frac{\varphi_{000} + \tau_{000} + \sigma^2}{\boldsymbol{\gamma}_1'\boldsymbol{\Phi}_1\boldsymbol{\gamma}_1 + \boldsymbol{\gamma}_2'\boldsymbol{\Phi}_2\boldsymbol{\gamma}_2 + \boldsymbol{\gamma}_3'\boldsymbol{\Phi}_3\boldsymbol{\gamma}_3 + \varphi_{000} + \tau_{000} + \sigma^2} \\ &= \frac{\boldsymbol{\gamma}_1'\boldsymbol{\Phi}_1\boldsymbol{\gamma}_1 + \boldsymbol{\gamma}_2'\boldsymbol{\Phi}_2\boldsymbol{\gamma}_2 + \boldsymbol{\gamma}_3'\boldsymbol{\Phi}_3\boldsymbol{\gamma}_3}{\boldsymbol{\gamma}_1'\boldsymbol{\Phi}_1\boldsymbol{\gamma}_1 + \boldsymbol{\gamma}_2'\boldsymbol{\Phi}_2\boldsymbol{\gamma}_2 + \boldsymbol{\gamma}_3'\boldsymbol{\Phi}_3\boldsymbol{\gamma}_3 + \varphi_{000} + \tau_{000} + \sigma^2} \\ &= R_t^{2(f_1)} + R_t^{2(f_2)} + R_t^{2(f_3)} \end{aligned} \quad \text{(OA2)}$$

Again, this holds under Snijders and Boskers' (2012) assumption that random slope variation is 0.

Section A2: Correspondence with Johnson's (2014) extension of Nakagawa & Schielzeth's (2013) marginal measure

The linear mixed model version of Johnson's (2014) extension of Nakagawa & Schielzeth's (2013) marginal measure was defined in manuscript Equation (3) and is reproduced here:

$$R^2_{NSJ(m)} = \frac{\sigma_f^2}{\sigma_f^2 + \overline{\sigma^2_{L2}} + \overline{\sigma^2_{L3}} + \sigma^2_\varepsilon} \quad \text{(OA3)}$$

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Here, we show the correspondence between each of these terms individually with terms from our model-implied variance expression. Note first that, from the manuscript Appendix A derivation,

$$\begin{aligned}\sigma_f^2 &= \text{var}(\gamma_{000} + \mathbf{x}'_{1ijk}\boldsymbol{\gamma}_1 + \mathbf{x}'_{2jk}\boldsymbol{\gamma}_2 + \mathbf{x}'_{3k}\boldsymbol{\gamma}_3) \\ &= \boldsymbol{\gamma}'_1\boldsymbol{\Phi}_1\boldsymbol{\gamma}_1 + \boldsymbol{\gamma}'_2\boldsymbol{\Phi}_2\boldsymbol{\gamma}_2 + \boldsymbol{\gamma}'_3\boldsymbol{\Phi}_3\boldsymbol{\gamma}_3\end{aligned} \quad . \qquad \text{(OA4)}$$

Next, for terms concerning random effect variation, Johnson (2014) provides the general expression:

$$\overline{\sigma_l^2} = tr(\mathbf{ZTZ}')/N \qquad \text{(OA5)}$$

with N denoting total sample size and $\mathbf{Z}$ denoting a $N \times (p_l + 1)$ design matrix for the random effects across level-l units (i.e., a column of N 1's for the level-l intercept, and a column for each of the p_l predictors with a random slope varying across level-l) and $\mathbf{T}$ denoting the random effect covariance matrix for level-l. Using our notation, this can be computed for level-2 as

$$\begin{aligned}\overline{\sigma_{L2}^2} &= tr\left(\mathbf{Z}_{1*2}\mathbf{T}_{1*2}\mathbf{Z}'_{1*2}\right)/N \\ &= tr\left(\mathbf{T}_{1*2}\mathbf{Z}'_{1*2}\mathbf{Z}_{1*2}\right)/N \\ &= N \times tr(\mathbf{T}_{1*2}\frac{\mathbf{Z}'_{1*2}\mathbf{Z}_{1*2}}{N})/N \\ &= tr(\mathbf{T}_{1*2}\frac{\mathbf{Z}'_{1*2}\mathbf{Z}_{1*2}}{N})\end{aligned} \qquad \text{(OA6)}$$

Note that $\frac{\mathbf{Z}'_{1*2}\mathbf{Z}_{1*2}}{N}$ denotes a $(p_2+1)\times(p_2+1)$ matrix with the means of the squares of each element of $\mathbf{Z}_{1*2}$ across all N observations on the diagonal, and the means of pairwise products of each nonredundant element of $\mathbf{Z}_{1*2}$ across all N observations on the off-diagonals. We will call this matrix $\overline{\mathbf{Z}^2_{1*2}}$. Note also that, by the definition of variance, $\boldsymbol{\Sigma}_{1*2}$ (the covariance matrix of elements of $\mathbf{w}'_{1*2ijk}$) can be given as $\overline{\mathbf{Z}^2_{1*2}} - \overline{\mathbf{Z}}^2_{1*2}$, with $\overline{\mathbf{Z}}^2_{1*2}$ denoting a matrix with the squared means of each element of $\mathbf{Z}_{1*2}$ across all N observations on the diagonal and the pairwise products of means of each nonredundant element of $\mathbf{Z}_{1*2}$ across all N observations on the off-diagonals. Thus,

$$\begin{aligned}\overline{\sigma_{L2}^2} &= tr\left(\mathbf{T}_{1*2}\overline{\mathbf{Z}^2_{1*2}}\right) \\ &= tr\left(\mathbf{T}_{1*2}\left(\boldsymbol{\Sigma}_{1*2} + \overline{\mathbf{Z}}^2_{1*2}\right)\right) \\ &= tr\left(\mathbf{T}_{1*2}\boldsymbol{\Sigma}_{1*2} + \mathbf{T}_{1*2}\overline{\mathbf{Z}}^2_{1*2}\right) \\ &= tr\left(\mathbf{T}_{1*2}\boldsymbol{\Sigma}_{1*2}\right) + tr\left(\mathbf{T}_{1*2}\overline{\mathbf{Z}}^2_{1*2}\right)\end{aligned} \qquad \text{(OA7)}$$

With cluster-mean-centered level-1 predictors, $\overline{\mathbf{Z}}^2_{1*2}$ contains the intercept variance as the first element and all other elements are 0. Thus,

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$$\begin{aligned}\overline{\sigma^2_{L2}} &= tr(\mathbf{T}_{1*2}\boldsymbol{\Sigma}_{1*2}) + tr(\mathbf{T}_{1*2}\overline{\mathbf{Z}}^2_{1*2}) \\ &= tr(\mathbf{T}_{1*2}\boldsymbol{\Sigma}_{1*2}) + \tau_{00}\end{aligned} \quad \text{(OA8)}$$

For the random effects across level-3 units, we can express $\overline{\sigma^2_l}$ as

$$\begin{aligned}\overline{\sigma^2_{L3}} &= tr\left(\begin{pmatrix}\mathbf{Z}_{1*3} & \mathbf{Z}_{2*3}\end{pmatrix}\begin{pmatrix}\mathbf{T}_{1*3} & \mathbf{0}\\ \mathbf{0} & \mathbf{T}_{2*3}\end{pmatrix}\begin{pmatrix}\mathbf{Z}'_{1*3}\\ \mathbf{Z}'_{2*3}\end{pmatrix}\right)/N \\ &= tr\left(\begin{pmatrix}\mathbf{Z}_{1*3}\mathbf{T}_{1*3} & \mathbf{Z}_{2*3}\mathbf{T}_{2*3}\end{pmatrix}\begin{pmatrix}\mathbf{Z}'_{1*3}\\ \mathbf{Z}'_{2*3}\end{pmatrix}\right)/N \\ &= tr\left(\mathbf{Z}_{1*3}\mathbf{T}_{1*3}\mathbf{Z}'_{1*3} + \mathbf{Z}_{2*3}\mathbf{T}_{2*3}\mathbf{Z}'_{2*3}\right)/N \\ &= \left(tr\left(\mathbf{Z}_{1*3}\mathbf{T}_{1*3}\mathbf{Z}'_{1*3}\right) + tr(\mathbf{Z}_{2*3}\mathbf{T}_{2*3}\mathbf{Z}'_{2*3})\right)/N \\ &= N \times \left(tr\left(\mathbf{T}_{1*3}\frac{\mathbf{Z}'_{1*3}\mathbf{Z}_{1*3}}{N}\right) + tr\left(\mathbf{T}_{2*3}\frac{\mathbf{Z}'_{2*3}\mathbf{Z}_{2*3}}{N}\right)\right)/N \\ &= tr\left(\mathbf{T}_{1*3}\frac{\mathbf{Z}'_{1*3}\mathbf{Z}_{1*3}}{N}\right) + tr\left(\mathbf{T}_{2*3}\frac{\mathbf{Z}'_{2*3}\mathbf{Z}_{2*3}}{N}\right)\end{aligned} \quad \text{(OA9)}$$

Using the same steps and logic as shown above for $\overline{\sigma^2_{L2}}$,

$$\begin{aligned}\overline{\sigma^2_{L3}} &= tr\left(\mathbf{T}_{1*3}\overline{\mathbf{Z}^2_{1*3}}\right) + tr\left(\mathbf{T}_{2*3}\overline{\mathbf{Z}^2_{2*3}}\right) \\ &= tr(\mathbf{T}_{1*3}\boldsymbol{\Sigma}_{1*3}) + tr(\mathbf{T}_{2*3}\boldsymbol{\Sigma}_{2*3}) + \varphi_{000}\end{aligned} \quad \text{(OA10)}$$

Lastly, σ^2_ε is the same as our σ^2. Thus, we can express $R^2_{NSJ(m)}$ as

$$\begin{aligned}R^2_{NSJ(m)} &= \frac{\boldsymbol{\gamma}'_1\boldsymbol{\Phi}_1\boldsymbol{\gamma}_1 + \boldsymbol{\gamma}'_2\boldsymbol{\Phi}_2\boldsymbol{\gamma}_2 + \boldsymbol{\gamma}'_3\boldsymbol{\Phi}_3\boldsymbol{\gamma}_3}{\boldsymbol{\gamma}'_1\boldsymbol{\Phi}_1\boldsymbol{\gamma}_1 + \boldsymbol{\gamma}'_2\boldsymbol{\Phi}_2\boldsymbol{\gamma}_2 + \boldsymbol{\gamma}'_3\boldsymbol{\Phi}_3\boldsymbol{\gamma}_3 + tr(\boldsymbol{\Sigma}_{1*2}\mathbf{T}_{1*2}) + tr(\boldsymbol{\Sigma}_{1*3}\mathbf{T}_{1*3}) + tr(\boldsymbol{\Sigma}_{2*3}\mathbf{T}_{2*3}) + \varphi_{000} + \tau_{000} + \sigma^2} \\ &= R_t^{2(f_1)} + R_t^{2(f_2)} + R_t^{2(f_3)}\end{aligned} \quad \text{(OA11)}$$

Section A3: Correspondence with Johnson's (2014) extension of Nakagawa & Schielzeth's (2013) conditional measure

The linear mixed model version of Johnson's (2014) extension of Nakagawa & Schielzeth's (2013) conditional measure was defined in manuscript Equation (4) and is reproduced here:

$$R^2_{NSJ(c)} = \frac{\sigma^2_f + \overline{\sigma^2_{L2}} + \overline{\sigma^2_{L3}}}{\sigma^2_f + \overline{\sigma^2_{L2}} + \overline{\sigma^2_{L3}} + \sigma^2_\varepsilon} \quad \text{(OA12)}$$

Each of these terms was shown in Online Appendix Section A2 to be equivalent to certain terms from our variance partitioning. We can thus express $R^2_{NSJ(c)}$ as

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$$R^2_{NSJ(c)} = \frac{\boldsymbol{\gamma}_1'\boldsymbol{\Phi}_1\boldsymbol{\gamma}_1 + \boldsymbol{\gamma}_2'\boldsymbol{\Phi}_2\boldsymbol{\gamma}_2 + \boldsymbol{\gamma}_3'\boldsymbol{\Phi}_3\boldsymbol{\gamma}_3 + tr(\boldsymbol{\Sigma}_{1*2}\mathbf{T}_{1*2}) + tr(\boldsymbol{\Sigma}_{1*3}\mathbf{T}_{1*3}) + tr(\boldsymbol{\Sigma}_{2*3}\mathbf{T}_{2*3}) + \varphi_{000} + \tau_{000}}{\boldsymbol{\gamma}_1'\boldsymbol{\Phi}_1\boldsymbol{\gamma}_1 + \boldsymbol{\gamma}_2'\boldsymbol{\Phi}_2\boldsymbol{\gamma}_2 + \boldsymbol{\gamma}_3'\boldsymbol{\Phi}_3\boldsymbol{\gamma}_3 + tr(\boldsymbol{\Sigma}_{1*2}\mathbf{T}_{1*2}) + tr(\boldsymbol{\Sigma}_{1*3}\mathbf{T}_{1*3}) + tr(\boldsymbol{\Sigma}_{2*3}\mathbf{T}_{2*3}) + \varphi_{000} + \tau_{000} + \sigma^2}$$
$$= R_t^{2(f_1)} + R_t^{2(f_2)} + R_t^{2(f_3)} + R_t^{2(v_{1*2})} + R_t^{2(v_{1*3})} + R_t^{2(v_{2*3})} + R_t^{2(m_2)} + R_t^{2(m_3)} \quad \text{(OA13)}$$

Section A4: Correspondence with Raudenbush & Bryk's (2002) level-1 measure

Raudenbush & Bryk's (2002) (see also Bryk & Raudenbush, 1992) level-1 measure was defined in manuscript Equation (5) and is reproduced here:

$$R^2_{R\&B,1} = \frac{\sigma^2_{(null)} - \sigma^2}{\sigma^2_{(null)}}. \quad \text{(OA14)}$$

Here, the level-1 residual variance from a random-intercept-only model, $\sigma^2_{(null)}$, reflects the outcome variance at level-1. Replacing this term with our model-implied level-1 variance, this becomes

$$R^{2*}_{R\&B,1} = \frac{(\boldsymbol{\gamma}_1'\boldsymbol{\Phi}_1\boldsymbol{\gamma}_1 + tr(\boldsymbol{\Sigma}_{1*2}\mathbf{T}_{1*2}) + tr(\boldsymbol{\Sigma}_{1*3}\mathbf{T}_{1*3}) + \sigma^2) - \sigma^2}{\boldsymbol{\gamma}_1'\boldsymbol{\Phi}_1\boldsymbol{\gamma}_1 + tr(\boldsymbol{\Sigma}_{1*2}\mathbf{T}_{1*2}) + tr(\boldsymbol{\Sigma}_{1*3}\mathbf{T}_{1*3}) + \sigma^2}$$
$$= \frac{\boldsymbol{\gamma}_1'\boldsymbol{\Phi}_1\boldsymbol{\gamma}_1 + tr(\boldsymbol{\Sigma}_{1*2}\mathbf{T}_{1*2}) + tr(\boldsymbol{\Sigma}_{1*3}\mathbf{T}_{1*3})}{\boldsymbol{\gamma}_1'\boldsymbol{\Phi}_1\boldsymbol{\gamma}_1 + tr(\boldsymbol{\Sigma}_{1*2}\mathbf{T}_{1*2}) + tr(\boldsymbol{\Sigma}_{1*3}\mathbf{T}_{1*3}) + \sigma^2}. \quad \text{(OA15)}$$
$$= R_1^{2(f_1)} + R_1^{2(v_{1*2})} + R_1^{2(v_{1*3})}$$

Section A5: Correspondence with Raudenbush & Bryk's (2002) level-2 measure

Raudenbush & Bryk's (2002) (see also Bryk & Raudenbush, 1992) level-2 measure was defined in manuscript Equation (6) and is reproduced here:

$$R^2_{R\&B,2} = \frac{\tau_{000(null)} - \tau_{000}}{\tau_{000(null)}} \quad \text{(OA16)}$$

Here, the level-2 residual variance from a random-intercept-only model, $\tau_{000(null)}$, reflects the outcome variance at level-2. Replacing this term with our model-implied level-2 variance, this becomes

$$R^2_{R\&B,2} = \frac{(\boldsymbol{\gamma}_2'\boldsymbol{\Phi}_2\boldsymbol{\gamma}_2 + tr(\boldsymbol{\Sigma}_{2*3}\mathbf{T}_{2*3}) + \tau_{000}) - \tau_{000}}{\boldsymbol{\gamma}_2'\boldsymbol{\Phi}_2\boldsymbol{\gamma}_2 + tr(\boldsymbol{\Sigma}_{2*3}\mathbf{T}_{2*3}) + \tau_{000}}$$
$$= \frac{\boldsymbol{\gamma}_2'\boldsymbol{\Phi}_2\boldsymbol{\gamma}_2 + tr(\boldsymbol{\Sigma}_{2*3}\mathbf{T}_{2*3})}{\boldsymbol{\gamma}_2'\boldsymbol{\Phi}_2\boldsymbol{\gamma}_2 + tr(\boldsymbol{\Sigma}_{2*3}\mathbf{T}_{2*3}) + \tau_{000}} \quad \text{(OA17)}$$
$$= R_2^{2(f_2)} + R_2^{2(v_2)}$$

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Section A6: Correspondence with Raudenbush & Bryk's (2002) level-3 measure

Raudenbush & Bryk's (2002) (see also Bryk & Raudenbush, 1992) level-3 measure was defined in manuscript Equation (7) and is reproduced here:

$$R^2_{R\&B,3} = \frac{\varphi_{000(null)} - \varphi_{000}}{\varphi_{000(null)}} \tag{OA18}$$

Here, the level-3 residual variance from a random-intercept-only model, $\varphi_{000(null)}$, reflects the outcome variance at level-3. Replacing this term with our model-implied level-3 variance, this becomes

$$\begin{aligned} R^{2*}_{R\&B,3} &= \frac{(\boldsymbol{\gamma}_3'\boldsymbol{\Phi}_3\boldsymbol{\gamma}_3 + \varphi_{000}) - \varphi_{000}}{\boldsymbol{\gamma}_3'\boldsymbol{\Phi}_3\boldsymbol{\gamma}_3 + \varphi_{000}} \\ &= \frac{\boldsymbol{\gamma}_3'\boldsymbol{\Phi}_3\boldsymbol{\gamma}_3}{\boldsymbol{\gamma}_3'\boldsymbol{\Phi}_3\boldsymbol{\gamma}_3 + \varphi_{000}} \\ &= R_3^{2(f_3)} \end{aligned} \tag{OA19}$$

Section A7: Correspondence with Singer & Willett's (2003) and Peugh & Heck's (2017) measure

Singer & Willett's (2003) measure (see also Peugh & Heck, 2017) was defined in manuscript Equation (8) and is reproduced here:

$$R^2_{S\&W} = corr(\hat{y}^{(marg)}_{ijk}, y_{ijk})^2 \tag{OA20}$$

with $\hat{y}^{(marg)}_{ijk}$ denoting a marginal predicted score for level-1 unit *i*, level-2 unit *j*, and level-3 unit *k*, i.e., $\hat{y}^{(marg)}_{ijk} = \gamma_{000} + \mathbf{x}'_{1ijk}\boldsymbol{\gamma}_1 + \mathbf{x}'_{2jk}\boldsymbol{\gamma}_2 + \mathbf{x}'_{3k}\boldsymbol{\gamma}_3$. For simplicity, here we set $\mathbf{x}' = \begin{pmatrix} 1 & \mathbf{x}'_{1ijk} & \mathbf{x}'_{2jk} & \mathbf{x}'_{3k} \end{pmatrix}$ and $\boldsymbol{\gamma}' = \begin{pmatrix} \gamma_{000} & \boldsymbol{\gamma}'_1 & \boldsymbol{\gamma}'_2 & \boldsymbol{\gamma}'_3 \end{pmatrix}$ so that $\hat{y}^{(marg)}_{ijk} = \mathbf{x}'\boldsymbol{\gamma}$.

We can re-express $R^2_{S\&W}$ as

$$\begin{aligned} R^2_{S\&W} &= corr(\hat{y}^{(marg)}_{ijk}, y_{ijk})^2 \\ &= \frac{\text{cov}(\hat{y}^{(marg)}_{ijk}, y_{ijk})^2}{\text{var}(\hat{y}^{(marg)}_{ijk})\,\text{var}(y_{ijk})} \end{aligned} \tag{OA21}$$

Note that, in the population,

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$$\begin{aligned}
\text{cov}(\hat{y}_{ijk}^{(marg)}, y_{ijk}) &= E[\hat{y}_{ijk}^{(marg)} y_{ijk}] - E[\hat{y}_{ijk}^{(marg)}]E[y_{ijk}] \\
&= E[(\mathbf{x}'\boldsymbol{\gamma})(\mathbf{x}'\boldsymbol{\gamma} + \mathbf{w}'\mathbf{u} + e_{ijk})] - E[\mathbf{x}'\boldsymbol{\gamma}]E[\mathbf{x}'\boldsymbol{\gamma} + \mathbf{w}'\mathbf{u} + e_{ijk}] \\
&= E[(\mathbf{x}'\boldsymbol{\gamma})^2 + (\mathbf{x}'\boldsymbol{\gamma})(\mathbf{w}'\mathbf{u}) + (\mathbf{x}'\boldsymbol{\gamma})e_{ijk}] - E[\mathbf{x}'\boldsymbol{\gamma}](E[\mathbf{x}'\boldsymbol{\gamma}] + E[\mathbf{w}'\mathbf{u}] + E[e_{ijk}]) \\
&= E[(\mathbf{x}'\boldsymbol{\gamma})^2] + E[(\mathbf{x}'\boldsymbol{\gamma})(\mathbf{w}'\mathbf{u})] + E[(\mathbf{x}'\boldsymbol{\gamma})e_{ijk}] - E[\mathbf{x}'\boldsymbol{\gamma}]E[\mathbf{x}'\boldsymbol{\gamma}] \\
&= E[(\mathbf{x}'\boldsymbol{\gamma})^2] + E[(\mathbf{x}'\boldsymbol{\gamma})]E[(\mathbf{w}'\mathbf{u})] + E[(\mathbf{x}'\boldsymbol{\gamma})]E[e_{ijk}] - E[\mathbf{x}'\boldsymbol{\gamma}]^2 \\
&= E[(\mathbf{x}'\boldsymbol{\gamma})^2] - E[\mathbf{x}'\boldsymbol{\gamma}]^2 \\
&= \text{var}(\hat{y}_{ijk}^{(marg)})
\end{aligned} \tag{OA22}$$

Thus,

$$\begin{aligned}
R_{S\&W}^{2*} &= \frac{\text{cov}(\hat{y}_{ijk}^{(marg)}, y_{ijk})^2}{\text{var}(\hat{y}_{ijk}^{(marg)})\,\text{var}(y_{ijk})} \\
&= \frac{\text{var}(\hat{y}_{ijk}^{(marg)})^2}{\text{var}(\hat{y}_{ijk}^{(marg)})\,\text{var}(y_{ijk})} \\
&= \frac{\text{var}(\hat{y}_{ijk}^{(marg)})}{\text{var}(y_{ijk})}
\end{aligned} \tag{OA23}$$

Using our model-implied variances, per the derivation in manuscript Appendix A this becomes

$$\begin{aligned}
R_{S\&W}^{2*} &= \frac{\text{var}(\mathbf{x}'\boldsymbol{\gamma})}{\text{var}(\mathbf{x}'\boldsymbol{\gamma} + \mathbf{w}'\mathbf{u} + e_{ijk})} \\
&= \frac{\boldsymbol{\gamma}_1'\boldsymbol{\Phi}_1\boldsymbol{\gamma}_1 + \boldsymbol{\gamma}_2'\boldsymbol{\Phi}_2\boldsymbol{\gamma}_2 + \boldsymbol{\gamma}_3'\boldsymbol{\Phi}_3\boldsymbol{\gamma}_3}{\boldsymbol{\gamma}_1'\boldsymbol{\Phi}_1\boldsymbol{\gamma}_1 + \boldsymbol{\gamma}_2'\boldsymbol{\Phi}_2\boldsymbol{\gamma}_2 + \boldsymbol{\gamma}_3'\boldsymbol{\Phi}_3\boldsymbol{\gamma}_3 + tr(\boldsymbol{\Sigma}_{1*2}\mathbf{T}_{1*2}) + tr(\boldsymbol{\Sigma}_{1*3}\mathbf{T}_{1*3}) + tr(\boldsymbol{\Sigma}_{2*3}\mathbf{T}_{2*3}) + \varphi_{000} + \tau_{000} + \sigma^2} \\
&= R_t^{2(f_1)} + R_t^{2(f_2)} + R_t^{2(f_3)}
\end{aligned} \tag{OA24}$$

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Online Appendix B: Derivation of analytic relationships between total and level-specific R^2 measures

In this online appendix, we formally derive the relationship between total and level-specific measures that have the same source of explained variance (denoted generically as *s*). We show that they have a conditionally linear, positive relationship that is *moderated* by the amount of cluster dependency, i.e., the proportion of total variance that is across clusters, or intraclass correlation [ICC]. Preliminarily, we note that the proportion of total variance that is across both level-2 and level-3 units, or ICC_{23}, can be computed with our variance decomposition as:

$$\begin{aligned} ICC_{23} &= \frac{\text{level-2 variance} + \text{level-3 variance}}{\text{total variance}} \\ &= \frac{\boldsymbol{\gamma}_2'\boldsymbol{\Phi}_2\boldsymbol{\gamma}_2 + \boldsymbol{\gamma}_3'\boldsymbol{\Phi}_3\boldsymbol{\gamma}_3 + tr(\boldsymbol{\Sigma}_{2*3}\mathbf{T}_{2*3}) + \varphi_{000} + \tau_{000}}{\boldsymbol{\gamma}_1'\boldsymbol{\Phi}_1\boldsymbol{\gamma}_1 + \boldsymbol{\gamma}_2'\boldsymbol{\Phi}_2\boldsymbol{\gamma}_2 + \boldsymbol{\gamma}_3'\boldsymbol{\Phi}_3\boldsymbol{\gamma}_3 + tr(\boldsymbol{\Sigma}_{1*2}\mathbf{T}_{1*2}) + tr(\boldsymbol{\Sigma}_{1*3}\mathbf{T}_{1*3}) + tr(\boldsymbol{\Sigma}_{2*3}\mathbf{T}_{2*3}) + \varphi_{000} + \tau_{000} + \sigma^2} \end{aligned} \quad \text{(OB1)}$$

The proportion of total variance that is specifically at level-2 is

$$\begin{aligned} ICC_{2} &= \frac{\text{level-2 variance}}{\text{total variance}} \\ &= \frac{\boldsymbol{\gamma}_2'\boldsymbol{\Phi}_2\boldsymbol{\gamma}_2 + tr(\boldsymbol{\Sigma}_{2*3}\mathbf{T}_{2*3}) + \tau_{000}}{\boldsymbol{\gamma}_1'\boldsymbol{\Phi}_1\boldsymbol{\gamma}_1 + \boldsymbol{\gamma}_2'\boldsymbol{\Phi}_2\boldsymbol{\gamma}_2 + \boldsymbol{\gamma}_3'\boldsymbol{\Phi}_3\boldsymbol{\gamma}_3 + tr(\boldsymbol{\Sigma}_{1*2}\mathbf{T}_{1*2}) + tr(\boldsymbol{\Sigma}_{1*3}\mathbf{T}_{1*3}) + tr(\boldsymbol{\Sigma}_{2*3}\mathbf{T}_{2*3}) + \varphi_{000} + \tau_{000} + \sigma^2} \end{aligned} \quad \text{(OB2)}$$

And the proportion of total variance that is at level-3 is

$$\begin{aligned} ICC_{3} &= \frac{\text{level-3 variance}}{\text{total variance}} \\ &= \frac{\boldsymbol{\gamma}_3'\boldsymbol{\Phi}_3\boldsymbol{\gamma}_3 + \varphi_{000}}{\boldsymbol{\gamma}_1'\boldsymbol{\Phi}_1\boldsymbol{\gamma}_1 + \boldsymbol{\gamma}_2'\boldsymbol{\Phi}_2\boldsymbol{\gamma}_2 + \boldsymbol{\gamma}_3'\boldsymbol{\Phi}_3\boldsymbol{\gamma}_3 + tr(\boldsymbol{\Sigma}_{1*2}\mathbf{T}_{1*2}) + tr(\boldsymbol{\Sigma}_{1*3}\mathbf{T}_{1*3}) + tr(\boldsymbol{\Sigma}_{2*3}\mathbf{T}_{2*3}) + \varphi_{000} + \tau_{000} + \sigma^2} \end{aligned} \quad \text{(OB3)}$$

A level-1 measure, $R_1^{2(s)}$, and a total measure, $R_t^{2(s)}$, are related purely through the degree of higher level clustering, or ICC_{23}:

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$$\begin{aligned}
R_1^{2(s)} &= \frac{\theta_s}{\boldsymbol{\gamma}_1'\boldsymbol{\Phi}_1\boldsymbol{\gamma}_1 + tr(\boldsymbol{\Sigma}_{1*2}\mathbf{T}_{1*2}) + tr(\boldsymbol{\Sigma}_{1*3}\mathbf{T}_{1*3}) + \sigma^2} \\
&= \frac{\theta_s / \text{var}(y_{ijk})}{(\boldsymbol{\gamma}_1'\boldsymbol{\Phi}_1\boldsymbol{\gamma}_1 + tr(\boldsymbol{\Sigma}_{1*2}\mathbf{T}_{1*2}) + tr(\boldsymbol{\Sigma}_{1*3}\mathbf{T}_{1*3}) + \sigma^2) / \text{var}(y_{ijk})} \\
&= \frac{R_t^{2(s)}}{(\boldsymbol{\gamma}_1'\boldsymbol{\Phi}_1\boldsymbol{\gamma}_1 + tr(\boldsymbol{\Sigma}_{1*2}\mathbf{T}_{1*2}) + tr(\boldsymbol{\Sigma}_{1*3}\mathbf{T}_{1*3}) + \sigma^2) / \text{var}(y_{ijk})} \\
&= \frac{R_t^{2(s)}}{(\text{var}(y_{ijk}) - (\boldsymbol{\gamma}_2'\boldsymbol{\Phi}_2\boldsymbol{\gamma}_2 + \boldsymbol{\gamma}_3'\boldsymbol{\Phi}_3\boldsymbol{\gamma}_3 + tr(\boldsymbol{\Sigma}_{2*3}\mathbf{T}_{2*3}) + \varphi_{000} + \tau_{000})) / \text{var}(y_{ijk})} \\
&= \frac{R_t^{2(s)}}{\text{var}(y_{ijk}) / \text{var}(y_{ijk}) - (\boldsymbol{\gamma}_2'\boldsymbol{\Phi}_2\boldsymbol{\gamma}_2 + \boldsymbol{\gamma}_3'\boldsymbol{\Phi}_3\boldsymbol{\gamma}_3 + tr(\boldsymbol{\Sigma}_{2*3}\mathbf{T}_{2*3}) + \varphi_{000} + \tau_{000}) / \text{var}(y_{ijk})} \\
&= \frac{R_t^{2(s)}}{1 - ICC_{23}}
\end{aligned} \quad \text{(OB4)}$$

A level-2 measure, $R_2^{2(s)}$, and a total measure, $R_t^{2(s)}$, are related purely through the degree of clustering at level-2, or ICC_2:

$$\begin{aligned}
R_2^{2(s)} &= \frac{\theta_s}{\boldsymbol{\gamma}_2'\boldsymbol{\Phi}_2\boldsymbol{\gamma}_2 + tr(\boldsymbol{\Sigma}_{2*3}\mathbf{T}_{2*3}) + \tau_{000}} \\
&= \frac{\theta_s / \text{var}(y_{ijk})}{(\boldsymbol{\gamma}_2'\boldsymbol{\Phi}_2\boldsymbol{\gamma}_2 + tr(\boldsymbol{\Sigma}_{2*3}\mathbf{T}_{2*3}) + \tau_{000}) / \text{var}(y_{ijk})} \\
&= \frac{R_t^{2(s)}}{ICC_2}
\end{aligned} \quad \text{(OB5)}$$

A level-3 measure, $R_3^{2(s)}$, and a total measure, $R_t^{2(s)}$, are related purely through the degree of clustering at level-3, or ICC_3:

$$\begin{aligned}
R_3^{2(s)} &= \frac{\theta_s}{\boldsymbol{\gamma}_3'\boldsymbol{\Phi}_3\boldsymbol{\gamma}_3 + \varphi_{000}} \\
&= \frac{\theta_s / \text{var}(y_{ijk})}{(\boldsymbol{\gamma}_3'\boldsymbol{\Phi}_3\boldsymbol{\gamma}_3 + \varphi_{000}) / \text{var}(y_{ijk})} \\
&= \frac{R_t^{2(s)}}{ICC_3}
\end{aligned} \quad \text{(OB6)}$$

Online Appendix to accompany Rights & Sterba (In press). R squared measures for multilevel models with three or more levels. *Multivariate Behavioral Research. DOI: 10.1080/00273171.2021.1985948*

Online Appendix C: Derivation showing that R-squared measures obtained from a cluster-mean-centered model and those obtained from a model that instead centers predictors by a constant value will be equivalent when predictors have variance at only one level

Here we compare the full set of total and level-specific R-squared measures obtained from a cluster-mean-centered model (using the formulas described in manuscript Table 1) to the full set of total and level-specific R-squared measures obtained from a model in which predictors are instead centered by an arbitrary constant (using the formulas described in manuscript Table 5). In particular, we show that, when each raw predictor has variance at only one level, the full set of R-squared measures will be the same whether predictors are centered by cluster means (which, in this case, would be equivalent to the grand means) or by any other arbitrary constant. The general idea is that centering predictors by a constant will always yield a likelihood-equivalent model, and as such, the model-implied total and level-specific variances will not change, nor will the proportion of variance attributable to each individual source.

We start with a three-level model in which we predict the outcome, y_{ijk} (with *i* denoting observation within-cluster, *j* denoting level-2 cluster within level-3 cluster, and *k* denoting level-3 cluster) with a level-1 predictor x_{ijk}, a level-2 predictor w_{jk}, and a level-3 predictor z_k.

$$y_{ijk} = \gamma_0 + \gamma_1 x_{ijk} + \gamma_2 w_{jk} + \gamma_3 z_k + u_{0jk} + u_{0k} + u_{1jk} x_{ijk} + u_{1k} x_{ijk} + u_{2k} w_{jk} + e_{ijk}$$

$$e_{ijk} \sim N(0,\sigma^2) \qquad \text{(OC1)}$$

$$\begin{bmatrix} u_{0jk} \\ u_{1jk} \\ u_{0k} \\ u_{1k} \\ u_{2k} \end{bmatrix} \sim MVN\left(\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \tau_{00*2} & & & & \\ \tau_{01*2} & \tau_{11*2} & & & \\ 0 & 0 & \tau_{00*3} & & \\ 0 & 0 & \tau_{01*3} & \tau_{11*3} & \\ 0 & 0 & \tau_{02*3} & \tau_{12*3} & \tau_{22*3} \end{bmatrix} \right)$$

As shown in Equation OC1, the model contains a random intercept that varies across level-2 and level-3 units, a random slope of x_{ijk} that varies across level-2 and level-3 units, a random slope of w_{jk} that varies across level-3 units, and a fixed slope of z_k. Here we will assume that each predictor has variance exclusively at its respective level, and each is centered such that it has a mean of 0. Thus, we will term this model in Equation OC1 the *mean-centered model* (noting that x_{ijk} and w_{jk} can be considered to be either grand-mean-centered or cluster-mean-centered, as these two centering options are equivalent in this context). For this model, we can thus use the formulas in manuscript Equations 10-18 (for cluster-mean-centered models) to compute the total variance attributable to each source.

The total variance attributable to source f_1 in the *mean-centered model* is given as

$$\begin{aligned} \boldsymbol{\gamma}_1' \boldsymbol{\Phi} \boldsymbol{\gamma}_1 &= \left[\gamma_{100}\right]\left[\text{var}(x_{ijk})\right]\left[\gamma_{100}\right] \\ &= \gamma_{100}^2 \, \text{var}(x_{ijk}) \end{aligned} \qquad \text{(OC2)}$$

The total variance attributable to source v_{1*2} is

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$$tr(\mathbf{\Sigma}_{1*2}\mathbf{T}_{1*2}) = tr\left(\begin{bmatrix} 0 & \\ 0 & \text{var}(x_{ijk}) \end{bmatrix}\begin{bmatrix} \tau_{00*2} & \\ \tau_{01*2} & \tau_{11*2} \end{bmatrix}\right) = tr\left(\begin{bmatrix} 0 & 0 \\ 0 & \text{var}(x_{ijk})\tau_{11*2} \end{bmatrix}\right) = \text{var}(x_{ijk})\tau_{11*2} \quad \text{(OC3)}$$

The total variance attributable to source v_{1*3} is

$$tr(\mathbf{\Sigma}_{1*3}\mathbf{T}_{1*3}) = tr\left(\begin{bmatrix} 0 & \\ 0 & \text{var}(x_{ijk}) \end{bmatrix}\begin{bmatrix} \tau_{00*3} & \\ \tau_{01*3} & \tau_{11*3} \end{bmatrix}\right) = tr\left(\begin{bmatrix} 0 & 0 \\ 0 & \text{var}(x_{ijk})\tau_{11*3} \end{bmatrix}\right) = \text{var}(x_{ijk})\tau_{11*3} \quad \text{(OC4)}$$

The total variance attributable to source v_{2*3} is

$$tr(\mathbf{\Sigma}_{2*3}\mathbf{T}_{2*3}) = tr\left(\begin{bmatrix} \text{var}(w_{jk}) \end{bmatrix}\begin{bmatrix} \tau_{22*3} \end{bmatrix}\right) = tr\left(\begin{bmatrix} \text{var}(w_{jk})\tau_{22*3} \end{bmatrix}\right) = \text{var}(w_{jk})\tau_{22*3} \quad \text{(OC5)}$$

And the total variance attributable to source m_2 is given simply as τ_{00*2}, to source m_3 as τ_{00*3}, and to level-1 residuals as σ^2 (note that we are using modified notation for the variance components, for instance, in that φ_{000} in manuscript equation 17 is listed as τ_{00*3} here in Online Appendix C; this is simply to facilitate a direct comparison between parameters from the *mean-centered model* to the later-defined model with alternative centering).

Adding each of these individual variances together yields the following the model-implied total variance for the *mean-centered model*:

$$\text{var}(y_{ijk}) = \gamma_1^2 \text{var}(x_{ijk}) + \gamma_2^2 \text{var}(w_{jk}) + \gamma_3^2 \text{var}(z_k) + \tau_{11*2} \text{var}(x_{ijk}) + \tau_{11*3} \text{var}(x_{ijk}) + \tau_{22*3} \text{var}(w_{jk}) + \tau_{00*2} + \tau_{00*3} + \sigma^2 \quad \text{(OC6)}$$

Adding the level-1 variances yields the model-implied level-1 outcome variance:

$$\text{var}_{i|jk}(y_{ijk}) = \gamma_1^2 \text{var}(x_{ijk}) + \tau_{11*2} \text{var}(x_{ijk}) + \tau_{11*3} \text{var}(x_{ijk}) + \sigma^2 \quad \text{(OC7)}$$

Adding the level-2 variances yields the model-implied level-2 outcome variance:

$$\text{var}(y_{j|k}) = \gamma_2^2 \text{var}(w_{jk}) + \tau_{22*3} \text{var}(w_{jk}) + \tau_{00*2} \quad \text{(OC8)}$$

And adding the level-3 variances yields the model-implied level-3 outcome variance:

$$\text{var}_k(y_{ijk}) = \gamma_3^2 \text{var}(z_k) + \tau_{00*3} \quad \text{(OC9)}$$

We can thus form the following total R-squared measures for the *mean-centered model* as such, using the formulas provided for cluster-mean-centered models in Table 1:

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$$R_t^{2(f_1)} = \frac{\gamma_1^2 \text{ var}(x_{ijk})}{\text{Eqn OC6}}$$

$$R_t^{2(f_2)} = \frac{\gamma_2^2 \text{ var}(w_{jk})}{\text{Eqn OC6}}$$

$$R_t^{2(f_3)} = \frac{\gamma_3^2 \text{ var}(z_k)}{\text{Eqn OC6}}$$

$$R_t^{2(v_{1*2})} = \frac{\tau_{11*2} \text{ var}(x_{ijk})}{\text{Eqn OC6}}$$

$$R_t^{2(v_{1*3})} = \frac{\tau_{11*3} \text{ var}(x_{ijk})}{\text{Eqn OC6}}$$

$$R_t^{2(v_{2*3})} = \frac{\tau_{22*3} \text{ var}(w_{jk})}{\text{Eqn OC6}}$$

$$R_t^{2(m_2)} = \frac{\tau_{00*2}}{\text{Eqn OC6}}$$

$$R_t^{2(m_3)} = \frac{\tau_{00*3}}{\text{Eqn OC6}} \qquad \text{(OC10)}$$

As well as the following level-specific measures:

$$R_1^{2(f_1)} = \frac{\gamma_1^2 \text{ var}(x_{ijk})}{\text{Eqn OC7}}$$

$$R_2^{2(f_2)} = \frac{\gamma_2^2 \text{ var}(w_{jk})}{\text{Eqn OC8}}$$

$$R_3^{2(f_3)} = \frac{\gamma_3^2 \text{ var}(z_k)}{\text{Eqn OC9}}$$

$$R_1^{2(v_{1*2})} = \frac{\tau_{11*2} \text{ var}(x_{ijk})}{\text{Eqn OC7}}$$

$$R_1^{2(v_{1*3})} = \frac{\tau_{11*3} \text{ var}(x_{ijk})}{\text{Eqn OC7}}$$

$$R_2^{2(v_{2*3})} = \frac{\tau_{22*3} \text{ var}(w_{jk})}{\text{Eqn OC8}}$$

$$R_2^{2(m_2)} = \frac{\tau_{00*2}}{\text{Eqn OC8}}$$

$$R_3^{2(m_3)} = \frac{\tau_{00*3}}{\text{Eqn OC9}} \qquad \text{(OC11)}$$

We will next consider an alternative model specification wherein we center x_{ijk}, w_{jk}, and z_k each by their own arbitrary constant (*a*, *b*, and *c*, respectively).

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$$\begin{aligned} y_{ijk} &= \gamma_0^* + \gamma_1^*(x_{ijk} - a) + \gamma_2^*(w_{jk} - b) + \gamma_3^*(z_k - c) \\ &\quad + u_{0jk}^* + u_{0k}^* + u_{1jk}^*(x_{ijk} - a) + u_{1k}^*(x_{ijk} - a) + u_{2k}^*(w_{jk} - b) + e_{ijk}^* \end{aligned} \tag{OC12}$$

Here we use asterisks to distinguish the parameters from this *centered-by-constant model* to those from the *mean-centered model*. Because these are likelihood-equivalent models, however, they are simply reparameterizations of each other. As such, we can re-express the *centered-by-constant* model in the same form of the *mean-centered model* (i.e., as regression of *y* on *x*, *w*, and *z*) to establish equivalencies between the model expressions (following similar procedures to that in Kreft, de Leeuw, & Aiken [1995] and Brincks et al. [2017]).

$$\begin{aligned} y_{ijk} &= (\gamma_0^* - \gamma_1^* a - \gamma_2^* b - \gamma_3^* c) + \gamma_1^* x_{ijk} + \gamma_2^* w_{jk} + \gamma_3^* z_k \\ &\quad + (u_{0jk}^* - u_{1jk}^* a) + (u_{0k}^* - u_{1k}^* a - u_{2k}^* b) + u_{1jk}^* x_{ijk} + u_{1k}^* x_{ijk} + u_{2k}^* w_{jk} + e_{ijk}^* \end{aligned} \tag{OC13}$$

This re-expression reveals the following equalities between the *centered-by-constant* model and the *mean-centered model*:

$$\begin{aligned} \gamma_0 &= \gamma_0^* - \gamma_1^* a - \gamma_2^* b - \gamma_3^* c \\ \gamma_1 &= \gamma_1^* \\ \gamma_2 &= \gamma_2^* \\ \gamma_3 &= \gamma_3^* \\ u_{0jk} &= u_{0jk}^* - u_{1jk}^* a \\ u_{0k} &= u_{0k}^* - u_{1k}^* a - u_{2k}^* b \\ u_{1jk} &= u_{1jk}^* \\ u_{1k} &= u_{1k}^* \\ u_{2k} &= u_{2k}^* \\ e_{ijk} &= e_{ijk}^* \end{aligned} \tag{OC14}$$

These further imply the following:

$$\begin{aligned} \gamma_0^* &= \gamma_0 + \gamma_1 a + \gamma_2 b + \gamma_3 c \\ \gamma_1^* &= \gamma_1 \\ \gamma_2^* &= \gamma_2 \\ \gamma_3^* &= \gamma_3 \\ u_{0jk}^* &= u_{0jk} + u_{1jk} a \\ u_{0k}^* &= u_{0k} + u_{1k} a + u_{2k} b \\ u_{1jk}^* &= u_{1jk} \\ u_{1k}^* &= u_{1k} \\ u_{2k}^* &= u_{2k} \\ e_{ijk}^* &= e_{ijk} \end{aligned} \tag{OC15}$$

Using the equivalencies established in Equation OC15, we can rewrite the *centered-by-constant model* (with a regression of *y* on *x* – *a*, *w* – *b*, and *z* – *c*) as

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$$\begin{aligned} y_{ijk} &= (\gamma_0 + \gamma_1 a + \gamma_2 b + \gamma_3 c) + \gamma_1 (x_{ijk} - a) + \gamma_2 (w_{jk} - b) + \gamma_3 (z_k - c) + (u_{0jk} + u_{1jk} a) \\ &+ (u_{0k} + u_{1k} a + u_{2k} b) + u_{1jk} (x_{ijk} - a) + u_{1k} (x_{ijk} - a) + u_{2k} (w_{jk} - b) + e_{ijk} \end{aligned} \quad \text{(OC16)}$$

Using the formulas in Appendix B for non-cluster-mean-centered models, we can then compute the variance attributable to each individual source. To simplify final computations, we will define each individual matrix involved in the variance formulas (noting that all symmetric matrices will include only the unique elements, i.e., the diagonal and lower triangle). The vector of fixed components is thus

$$\boldsymbol{\gamma} = \begin{bmatrix} \gamma_0 + \gamma_1 a + \gamma_2 b + \gamma_3 c \\ \gamma_1 \\ \gamma_2 \\ \gamma_3 \end{bmatrix} \quad \text{(OC17)}$$

The $\boldsymbol{\Phi}$ matrices are

$$\begin{aligned} \boldsymbol{\Phi}_1 &= \begin{bmatrix} 0 & & & \\ 0 & \text{var}((x_{ijk} - a) - (x_{\bullet jk} - a)) & & \\ 0 & 0 & \text{var}((w_{jk} - a) - (w_{jk} - a)) & \\ 0 & 0 & 0 & \text{var}((z_k - a) - (z_k - a)) \end{bmatrix} \\ &= \begin{bmatrix} 0 & & & \\ 0 & \text{var}(x_{ijk}) & & \\ 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{aligned} \quad \text{(OC18)}$$

$$\begin{aligned} \boldsymbol{\Phi}_2 &= \begin{bmatrix} 0 & & & \\ 0 & \text{var}((x_{\bullet jk} - a) - (x_{\bullet\bullet k} - a)) & & \\ 0 & 0 & \text{var}((w_{jk} - a) - (w_{\bullet k} - a)) & \\ 0 & 0 & 0 & \text{var}((z_k - a) - (z_k - a)) \end{bmatrix} \\ &= \begin{bmatrix} 0 & & & \\ 0 & 0 & & \\ 0 & 0 & \text{var}(w_{jk}) & \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{aligned} \quad \text{(OC19)}$$

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$$\mathbf{\Phi}_3 = \begin{bmatrix} 0 & & & \\ 0 & \text{var}(x_{\bullet\bullet k} - a) & & \\ 0 & 0 & \text{var}(w_{\bullet k} - a) & \\ 0 & 0 & 0 & \text{var}(z_k - a) \end{bmatrix} = \begin{bmatrix} 0 & & & \\ 0 & \text{var}(x_{\bullet\bullet k}) & & \\ 0 & 0 & \text{var}(w_{\bullet k}) & \\ 0 & 0 & 0 & \text{var}(z_k) \end{bmatrix} = \begin{bmatrix} 0 & & & \\ 0 & 0 & & \\ 0 & 0 & 0 & \\ 0 & 0 & 0 & \text{var}(z_k) \end{bmatrix} \quad \text{(OC20)}$$

The $\mathbf{\Sigma}$ matrices are then

$$\mathbf{\Sigma}_{1*2} = \begin{bmatrix} 0 & \\ 0 & \text{var}((x_{ijk} - a) - (x_{\bullet jk} - a)) \end{bmatrix} = \begin{bmatrix} 0 & \\ 0 & \text{var}(x_{ijk}) \end{bmatrix} \quad \text{(OC21)}$$

$$\mathbf{\Sigma}_{2*2} = \begin{bmatrix} 0 & \\ 0 & \text{var}((x_{\bullet jk} - a) - (x_{\bullet\bullet k} - a)) \end{bmatrix} = \begin{bmatrix} 0 & \\ 0 & 0 \end{bmatrix} \quad \text{(OC22)}$$

$$\mathbf{\Sigma}_{3*2} = \begin{bmatrix} 0 & \\ 0 & \text{var}(x_{\bullet\bullet k} - a) \end{bmatrix} = \begin{bmatrix} 0 & \\ 0 & 0 \end{bmatrix} \quad \text{(OC23)}$$

$$\mathbf{\Sigma}_{1*3} = \begin{bmatrix} 0 & & \\ 0 & \text{var}((x_{ijk} - a) - (x_{\bullet jk} - a)) & \\ 0 & 0 & \text{var}((w_{jk} - b) - (w_{jk} - b)) \end{bmatrix} = \begin{bmatrix} 0 & & \\ 0 & \text{var}(x_{ijk}) & \\ 0 & 0 & 0 \end{bmatrix} \quad \text{(OC24)}$$

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$$\mathbf{\Sigma}_{2*3} = \begin{bmatrix} 0 & & \\ 0 & \text{var}((x_{\bullet jk} - a) - (x_{\bullet\bullet k} - a)) & \\ 0 & 0 & \text{var}((w_{jk} - b) - (w_{\bullet k} - b)) \end{bmatrix}$$
$$= \begin{bmatrix} 0 & & \\ 0 & 0 & \\ 0 & 0 & \text{var}(w_{jk}) \end{bmatrix} \tag{OC25}$$

$$\mathbf{\Sigma}_{3*3} = \begin{bmatrix} 0 & & \\ 0 & \text{var}(x_{\bullet\bullet k} - a) & \\ 0 & 0 & \text{var}(w_{\bullet k} - a) \end{bmatrix}$$
$$= \begin{bmatrix} 0 & & \\ 0 & \text{var}(x_{\bullet\bullet k}) & \\ 0 & 0 & \text{var}(w_{\bullet k}) \end{bmatrix} \tag{OC26}$$
$$= \begin{bmatrix} 0 & & \\ 0 & 0 & \\ 0 & 0 & 0 \end{bmatrix}$$

The $\mathbf{T}$ matrices are

$$\mathbf{T}_2 = \begin{bmatrix} \text{var}(u_{0jk} + u_{1jk}a) & \\ \text{cov}(u_{0jk} + u_{1jk}a, u_{1jk}) & \text{var}(u_{1jk}) \end{bmatrix}$$
$$= \begin{bmatrix} \text{var}(u_{0jk}) + a^2\,\text{var}(u_{1jk}) + 2a\,\text{cov}(u_{0jk}, u_{1jk}) & \\ \text{cov}(u_{0jk}, u_{1jk}) + a\,\text{cov}(u_{1jk}, u_{1jk}) & \text{var}(u_{1jk}) \end{bmatrix} \tag{OC27}$$
$$= \begin{bmatrix} \tau_{00*2} + a^2\tau_{11*2} + 2a\tau_{01*2} & \\ \tau_{01*2} + a\tau_{11*2} & \tau_{11*2} \end{bmatrix}$$

$$\mathbf{T}_3 = \begin{bmatrix} \text{var}(u_{0k} + u_{1k}a + u_{2k}b) & & \\ \text{cov}(u_{0k} + u_{1k}a + u_{2k}b, u_{1k}) & \text{var}(u_{1k}) & \\ \text{cov}(u_{0k} + u_{1k}a + u_{2k}b, u_{2k}) & \text{cov}(u_{1k}, u_{2k}) & \text{var}(u_{2k}) \end{bmatrix}$$
$$= \begin{bmatrix} \text{var}(u_{0k}) + a^2\,\text{var}(u_{1k}) + b^2\,\text{var}(u_{2k}) + 2a\,\text{cov}(u_{0k}, u_{1k}) + 2b\,\text{cov}(u_{0k}, u_{2k}) + 2ab\,\text{cov}(u_{1k}, u_{2k}) & & \\ \text{cov}(u_{0k}, u_{1k}) + a\,\text{cov}(u_{1k}, u_{1k}) + b\,\text{cov}(u_{2k}, u_{1k}) & \text{var}(u_{1k}) & \\ \text{cov}(u_{0k}, u_{2k}) + a\,\text{cov}(u_{1k}, u_{2k}) + b\,\text{cov}(u_{2k}, u_{2k}) & \text{cov}(u_{1k}, u_{2k}) & \text{var}(u_{2k}) \end{bmatrix}$$
$$= \begin{bmatrix} \tau_{00*3} + a^2\tau_{11*3} + b^2\tau_{22*3} + 2a\tau_{01*3} + 2b\tau_{02*3} + 2ab\tau_{12*3} & & \\ \tau_{01*3} + a\tau_{11*3} + b\tau_{12*3} & \tau_{11*3} & \\ \tau_{02*3} + a\tau_{12*3} + b\tau_{22*3} & \tau_{12*3} & \tau_{22*3} \end{bmatrix} \tag{OC28}$$

And the $\mathbf{m}$ vectors are

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$$\mathbf{m}_{1*2} = \begin{bmatrix} 1-1 \\ E[(x_{ijk}-a)-(x_{\bullet jk}-a)] \end{bmatrix} = \begin{bmatrix} 0 \\ E[x_{ijk}-x_{\bullet jk}] \end{bmatrix} = \begin{bmatrix} 0 \\ E[x_{ijk}]-E[x_{\bullet jk}] \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{OC29}$$

$$\mathbf{m}_{2*2} = \begin{bmatrix} 1-1 \\ E[(x_{\bullet jk}-a)-(x_{\bullet\bullet k}-a)] \end{bmatrix} = \begin{bmatrix} 0 \\ E[x_{\bullet jk}-x_{\bullet\bullet k}] \end{bmatrix} = \begin{bmatrix} 0 \\ E[x_{\bullet jk}]-E[x_{\bullet\bullet k}] \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{OC30}$$

$$\mathbf{m}_{3*2} = \begin{bmatrix} 1 \\ E[x_{\bullet\bullet k}-a] \end{bmatrix} = \begin{bmatrix} 1 \\ E[x_{\bullet\bullet k}]-a \end{bmatrix} = \begin{bmatrix} 1 \\ -a \end{bmatrix} \tag{OC31}$$

$$\mathbf{m}_{1*3} = \begin{bmatrix} 1-1 \\ E[(x_{ijk}-a)-(x_{\bullet jk}-a)] \\ E[(w_{jk}-b)-(w_{jk}-b)] \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{OC32}$$

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$$\mathbf{m}_{2*3} = \begin{bmatrix} 1-1 \\ E[(x_{\bullet jk} - a) - (x_{\bullet\bullet k} - a)] \\ E[(w_{jk} - b) - (w_{\bullet k} - b)] \end{bmatrix} = \begin{bmatrix} 0 \\ E[x_{\bullet jk} - x_{\bullet\bullet k}] \\ E[w_{jk} - w_{\bullet k}] \end{bmatrix} = \begin{bmatrix} 0 \\ E[x_{\bullet jk}] - E[x_{\bullet\bullet k}] \\ E[w_{jk}] - E[w_{\bullet k}] \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{(OC33)}$$

$$\mathbf{m}_{3*3} = \begin{bmatrix} 1 \\ E[x_{\bullet\bullet k} - a] \\ E[w_{\bullet k} - b] \end{bmatrix} = \begin{bmatrix} 1 \\ E[x_{\bullet\bullet k}] - a \\ E[w_{\bullet k}] - b \end{bmatrix} = \begin{bmatrix} 1 \\ -a \\ -b \end{bmatrix} \quad \text{(OC34)}$$

Using these matrices, the total variance attributable to source f_1 in the *centered-by-constant model* is thus given as

$$\begin{aligned} \boldsymbol{\gamma}'\boldsymbol{\Phi}_1\boldsymbol{\gamma} &= \begin{bmatrix} \gamma_0 + \gamma_1 a + \gamma_2 b + \gamma_3 c & \gamma_1 & \gamma_2 & \gamma_3 \end{bmatrix} \begin{bmatrix} 0 & & & \\ 0 & \text{var}(x_{ijk}) & & \\ 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \gamma_0 + \gamma_1 a + \gamma_2 b + \gamma_3 c \\ \gamma_1 \\ \gamma_2 \\ \gamma_3 \end{bmatrix} \\ &= \begin{bmatrix} 0 & \gamma_1 \text{var}(x_{ijk}) & 0 & 0 \end{bmatrix} \begin{bmatrix} \gamma_0 + \gamma_1 a + \gamma_2 b + \gamma_3 c \\ \gamma_1 \\ \gamma_2 \\ \gamma_3 \end{bmatrix} \\ &= \gamma_1^2 \text{var}(x_{ijk}) \end{aligned} \quad \text{(OC35)}$$

The total variance attributable to f_2 is

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$$\boldsymbol{\gamma}'\boldsymbol{\Phi}_2\boldsymbol{\gamma} = \begin{bmatrix} \gamma_0+\gamma_1 a+\gamma_2 b+\gamma_3 c & \gamma_1 & \gamma_2 & \gamma_3 \end{bmatrix} \begin{bmatrix} 0 & & & \\ 0 & 0 & & \\ 0 & 0 & \text{var}(w_{jk}) & \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \gamma_0+\gamma_1 a+\gamma_2 b+\gamma_3 c \\ \gamma_1 \\ \gamma_2 \\ \gamma_3 \end{bmatrix}$$
$$= \begin{bmatrix} 0 & 0 & \gamma_2\,\text{var}(w_{jk}) & 0 \end{bmatrix} \begin{bmatrix} \gamma_0+\gamma_1 a+\gamma_2 b+\gamma_3 c \\ \gamma_1 \\ \gamma_2 \\ \gamma_3 \end{bmatrix}$$
$$= \gamma_2^2\,\text{var}(w_{jk}) \tag{OC36}$$

The total variance attributable to f_3 is

$$\boldsymbol{\gamma}'\boldsymbol{\Phi}_3\boldsymbol{\gamma} = \begin{bmatrix} \gamma_0+\gamma_1 a+\gamma_2 b+\gamma_3 c & \gamma_1 & \gamma_2 & \gamma_3 \end{bmatrix} \begin{bmatrix} 0 & & & \\ 0 & 0 & & \\ 0 & 0 & 0 & \\ 0 & 0 & 0 & \text{var}(z_j) \end{bmatrix} \begin{bmatrix} \gamma_0+\gamma_1 a+\gamma_2 b+\gamma_3 c \\ \gamma_1 \\ \gamma_2 \\ \gamma_3 \end{bmatrix}$$
$$= \begin{bmatrix} 0 & 0 & 0 & \gamma_3\,\text{var}(z_j) \end{bmatrix} \begin{bmatrix} \gamma_0+\gamma_1 a+\gamma_2 b+\gamma_3 c \\ \gamma_1 \\ \gamma_2 \\ \gamma_3 \end{bmatrix}$$
$$= \gamma_3^2\,\text{var}(z_j) \tag{OC37}$$

The total variance attributable to v_{1*2} is

$$tr(\boldsymbol{\Sigma}_{1*2}\mathbf{T}_2) = tr\left(\begin{bmatrix} 0 & \\ 0 & \text{var}(x_{ijk}) \end{bmatrix} \begin{bmatrix} \tau_{00*2}+a^2\tau_{11*2}+2a\tau_{01*2} & \\ \tau_{01*2}+a\tau_{11*2} & \tau_{11*2} \end{bmatrix} \right)$$
$$= tr\left(\begin{bmatrix} 0 & 0 \\ \text{var}(x_{ijk})\tau_{01*2}+\text{var}(x_{ijk})a\tau_{11*2} & \text{var}(x_{ijk})\tau_{11*2} \end{bmatrix} \right) \tag{OC38}$$
$$= \text{var}(x_{ijk})\tau_{11*2}$$

The total variance attributable to v_{1*3} is

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$$tr(\boldsymbol{\Sigma}_{1*3}\mathbf{T}_3) = tr\left(\begin{bmatrix} 0 & & \\ 0 & \text{var}(x_{ijk}) & \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} \tau_{00*3} + a^2\tau_{11*3} + b^2\tau_{22*3} & & \\ \tau_{01*3} + a\tau_{11*3} + b\tau_{12*3} & \tau_{11*3} & \\ \tau_{02*3} + a\tau_{12*3} + b\tau_{22*3} & \tau_{12*3} & \tau_{22*3} \end{bmatrix}\right)$$
$$= tr\left(\begin{bmatrix} 0 & 0 & 0 \\ \text{var}(x_{ijk})(\tau_{01*3} + a\tau_{11*3} + b\tau_{12*3}) & \text{var}(x_{ijk})\tau_{11*3} & \text{var}(x_{ijk})(\tau_{02*3} + a\tau_{12*3} + b\tau_{22*3}) \\ 0 & 0 & 0 \end{bmatrix}\right)$$
$$= \text{var}(x_{ijk})\tau_{11*3} \tag{OC39}$$

The total variance attributable to v_{2*2} is

$$tr(\boldsymbol{\Sigma}_{2*2}\mathbf{T}_2) = tr\left(\begin{bmatrix} 0 & \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \tau_{00*2} + a^2\tau_{11*2} + 2a\tau_{01*2} & \\ \tau_{01*2} + a\tau_{11*2} & \tau_{11*2} \end{bmatrix}\right)$$
$$= tr\left(\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\right) \tag{OC40}$$
$$= 0$$

The total variance attributable to v_{2*3} is

$$tr(\boldsymbol{\Sigma}_{2*3}\mathbf{T}_3) = tr\left(\begin{bmatrix} 0 & & \\ 0 & 0 & \\ 0 & 0 & \text{var}(w_{jk}) \end{bmatrix}\begin{bmatrix} \tau_{00*3} + a^2\tau_{11*3} + b^2\tau_{22*3} & & \\ \tau_{01*3} + a\tau_{11*3} + b\tau_{12*3} & \tau_{11*3} & \\ \tau_{02*3} + a\tau_{12*3} + b\tau_{22*3} & \tau_{12*3} & \tau_{22*3} \end{bmatrix}\right)$$
$$= tr\left(\begin{bmatrix} 0 & & \\ 0 & 0 & \\ \text{var}(w_{jk})(\tau_{02*3} + a\tau_{12*3} + b\tau_{22*3}) & \text{var}(w_{jk})\tau_{12*3} & \text{var}(w_{jk})\tau_{22*3} \end{bmatrix}\right) \tag{OC41}$$
$$= \text{var}(w_{jk})\tau_{22*3}$$

The total variance attributable to v_{3*2} is

$$tr(\boldsymbol{\Sigma}_{3*2}\mathbf{T}_2) = tr\left(\begin{bmatrix} 0 & \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \tau_{00*2} + a^2\tau_{11*2} + 2a\tau_{01*2} & \\ \tau_{01*2} + a\tau_{11*2} & \tau_{11*2} \end{bmatrix}\right)$$
$$= tr\left(\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\right) \tag{OC42}$$
$$= 0$$

The total variance attributable to v_{3*3} is

Online Appendix to accompany Rights & Sterba (In press). R squared measures for multilevel models with three or more levels. *Multivariate Behavioral Research. DOI: 10.1080/00273171.2021.1985948*

$$
\begin{aligned}
tr(\boldsymbol{\Sigma}_{3*3}\mathbf{T}_3) &= tr\left(\begin{bmatrix} 0 & & \\ 0 & 0 & \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} \tau_{00*3}+a^2\tau_{11*3}+b^2\tau_{22*3} & & \\ \tau_{01*3}+a\tau_{11*3}+b\tau_{12*3} & \tau_{11*3} & \\ \tau_{02*3}+a\tau_{12*3}+b\tau_{22*3} & \tau_{12*3} & \tau_{22*3} \end{bmatrix}\right) \\
&= tr\left(\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\right) \\
&= 0
\end{aligned} \qquad \text{(OC43)}
$$

The total variance attributable to m_2 is

$$
\begin{aligned}
\mathbf{m}'_{1*2}\mathbf{T}_2\mathbf{m}_{1*2}+\mathbf{m}'_{2*2}\mathbf{T}_2\mathbf{m}_{2*2}+\mathbf{m}'_{3*2}\mathbf{T}_2\mathbf{m}_{3*2} &= \begin{bmatrix} 0 & 0 \end{bmatrix}\begin{bmatrix} \tau_{00*2}+a^2\tau_{11*2}+2a\tau_{01*2} & \\ \tau_{01*2}+a\tau_{11*2} & \tau_{11*2} \end{bmatrix}\begin{bmatrix} 0 \\ 0 \end{bmatrix} \\
&\quad + \begin{bmatrix} 0 \\ 0 \end{bmatrix}\begin{bmatrix} \tau_{00*2}+a^2\tau_{11*2}+2a\tau_{01*2} & \\ \tau_{01*2}+a\tau_{11*2} & \tau_{11*2} \end{bmatrix}\begin{bmatrix} 0 \\ 0 \end{bmatrix} \\
&\quad + \begin{bmatrix} 1 & -a \end{bmatrix}\begin{bmatrix} \tau_{00*2}+a^2\tau_{11*2}+2a\tau_{01*2} & \\ \tau_{01*2}+a\tau_{11*2} & \tau_{11*2} \end{bmatrix}\begin{bmatrix} 1 \\ -a \end{bmatrix} \\
&= 0+0+\begin{bmatrix} \tau_{00*2}+a^2\tau_{11*2}+2a\tau_{01*2}-a\tau_{01*2}-a^2\tau_{11*2} & \tau_{01*2}+a\tau_{11*2}-a\tau_{11*2} \end{bmatrix}\begin{bmatrix} 1 \\ -a \end{bmatrix} \\
&= \tau_{00*2}+a^2\tau_{11*2}+2a\tau_{01*2}-a\tau_{01*2}-a^2\tau_{11*2}-a\tau_{01*2}-a^2\tau_{11*2}+a^2\tau_{11*2} \\
&= \tau_{00*2}+(a^2-a^2-a^2+a^2)\tau_{11*2}+(2a-a-a)\tau_{01*2} \\
&= \tau_{00*2}
\end{aligned} \qquad \text{(OC44)}
$$

And the total variance attributable to m_3 is

$$
\begin{aligned}
\mathbf{m}'_{1*3}\mathbf{T}_3\mathbf{m}_{1*3}+\mathbf{m}'_{2*3}\mathbf{T}_3\mathbf{m}_{2*3}+\mathbf{m}'_{3*3}\mathbf{T}_3\mathbf{m}_{3*3} &= \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} \tau_{00*3}+a^2\tau_{11*3}+b^2\tau_{22*3}+2a\tau_{01*3}+2b\tau_{02*3}+2ab\tau_{12*3} & & \\ \tau_{01*3}+a\tau_{11*3}+b\tau_{12*3} & \tau_{11*3} & \\ \tau_{02*3}+a\tau_{12*3}+b\tau_{22*3} & \tau_{12*3} & \tau_{22*3} \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \\
&\quad + \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} \tau_{00*3}+a^2\tau_{11*3}+b^2\tau_{22*3}+2a\tau_{01*3}+2b\tau_{02*3}+2ab\tau_{12*3} & & \\ \tau_{01*3}+a\tau_{11*3}+b\tau_{12*3} & \tau_{11*3} & \\ \tau_{02*3}+a\tau_{12*3}+b\tau_{22*3} & \tau_{12*3} & \tau_{22*3} \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \\
&\quad + \begin{bmatrix} 1 & -a & -b \end{bmatrix}\begin{bmatrix} \tau_{00*3}+a^2\tau_{11*3}+b^2\tau_{22*3}+2a\tau_{01*3}+2b\tau_{02*3}+2ab\tau_{12*3} & & \\ \tau_{01*3}+a\tau_{11*3}+b\tau_{12*3} & \tau_{11*3} & \\ \tau_{02*3}+a\tau_{12*3}+b\tau_{22*3} & \tau_{12*3} & \tau_{22*3} \end{bmatrix}\begin{bmatrix} 1 \\ -a \\ -b \end{bmatrix} \\
&= 0+0+\begin{bmatrix} \tau_{00*3}+a^2\tau_{11*3}+b^2\tau_{22*3}+2a\tau_{01*3}+2b\tau_{02*3}+2ab\tau_{12*3}-a(\tau_{01*3}+a\tau_{11*3}+b\tau_{12*3})-b(\tau_{02*3}+a\tau_{12*3}+b\tau_{22*3}) \\ \tau_{01*3}+a\tau_{11*3}+b\tau_{12*3}-a\tau_{11*3}-b\tau_{12*3} \\ \tau_{02*3}+a\tau_{12*3}+b\tau_{22*3}-a\tau_{12*3}-b\tau_{22*3} \end{bmatrix}'\begin{bmatrix} 1 \\ -a \\ -b \end{bmatrix} \\
&= \tau_{00*3}+a^2\tau_{11*3}+b^2\tau_{22*3}+2a\tau_{01*3}+2b\tau_{02*3}+2ab\tau_{12*3}-a(\tau_{01*3}+a\tau_{11*3}+b\tau_{12*3})-b(\tau_{02*3}+a\tau_{12*3}+b\tau_{22*3}) \\
&\quad -a(\tau_{01*3}+a\tau_{11*3}+b\tau_{12*3}-a\tau_{11*3}-b\tau_{12*3})-b(\tau_{02*3}+a\tau_{12*3}+b\tau_{22*3}-a\tau_{12*3}-b\tau_{22*3}) \\
&= \tau_{00*3}+a^2\tau_{11*3}+b^2\tau_{22*3}+2a\tau_{01*3}+2b\tau_{02*3}+2ab\tau_{12*3}-a\tau_{01*3}-a^2\tau_{11*3}-ab\tau_{12*3}-b\tau_{02*3}-ab\tau_{12*3}-b^2\tau_{22*3} \\
&\quad -a\tau_{01*3}-a^2\tau_{11*3}-ab\tau_{12*3}+a^2\tau_{11*3}+ab\tau_{12*3}-b\tau_{02*3}-ab\tau_{12*3}-b^2\tau_{22*3}+ab\tau_{12*3}+b^2\tau_{22*3} \\
&= \tau_{00*3}+(a^2-a^2-a^2+a^2)\tau_{11*3}+(b^2-b^2-b^2+b^2)\tau_{22*3}+(-a-a+2a)\tau_{01*3}+(-b-b+2b)\tau_{02*3}+(-ab-ab-ab+ab-ab+ab+2ab)\tau_{12*3} \\
&= \tau_{00*3}
\end{aligned} \qquad \text{(OC45)}
$$

Adding each of these individual variances together yields the following the model-implied total variance for the *centered-by-constant model*:

Online Appendix to accompany Rights & Sterba (In press). R squared measures for multilevel models with three or more levels. *Multivariate Behavioral Research. DOI: 10.1080/00273171.2021.1985948*

$$\begin{aligned}\text{var}(y_{ijk}) &= \gamma_1^2 \text{ var}(x_{ijk}) + \gamma_2^2 \text{ var}(w_{jk}) + \gamma_3^2 \text{ var}(z_k) \\ &+ \tau_{11*2} \text{ var}(x_{ijk}) + \tau_{11*3} \text{ var}(x_{ijk}) + \tau_{22*3} \text{ var}(w_{jk}) + \tau_{00*2} + \tau_{00*3} + \sigma^2\end{aligned} \tag{OC46}$$

Adding the level-1 variances yields the model-implied level-1 outcome variance:

$$\text{var}_{i|jk}(y_{ijk}) = \gamma_1^2 \text{ var}(x_{ijk}) + \tau_{11*2} \text{ var}(x_{ijk}) + \tau_{11*3} \text{ var}(x_{ijk}) + \sigma^2 \tag{OC47}$$

Adding the level-2 variances yields the model-implied level-2 outcome variance:

$$\text{var}_{j|k}(y_{ijk}) = \gamma_2^2 \text{ var}(w_{jk}) + \tau_{22*3} \text{ var}(w_{jk}) + \tau_{00*2} \tag{OC48}$$

And adding the level-3 variances yields the model-implied level-3 outcome variance:

$$\text{var}_k(y_{ijk}) = \gamma_3^2 \text{ var}(z_k) + \tau_{00*3} \tag{OC49}$$

We can thus form the following total R-squared measures for the *centered-by-constant model* as such, using the formulas provided for non-cluster-mean-centered models in Table 5 (here we used asterisks to distinguish these R-squared measures from the measures computed for the *mean-centered model*, with the latter denoted without an asterisk):

$$\begin{aligned}
R_t^{2(f_1)*} &= \frac{\gamma_1^2 \text{ var}(x_{ijk})}{\text{Eqn OC46}} = R_t^{2(f_1)} \\
R_t^{2(f_2)*} &= \frac{\gamma_2^2 \text{ var}(w_{jk})}{\text{Eqn OC46}} = R_t^{2(f_2)} \\
R_t^{2(f_3)*} &= \frac{\gamma_3^2 \text{ var}(z_k)}{\text{Eqn OC46}} = R_t^{2(f_3)} \\
R_t^{2(v_{1*2})*} &= \frac{\tau_{11*2} \text{ var}(x_{ijk})}{\text{Eqn OC46}} = R_t^{2(v_{1*2})} \\
R_t^{2(v_{1*3})*} &= \frac{\tau_{11*3} \text{ var}(x_{ijk})}{\text{Eqn OC46}} = R_t^{2(v_{1*3})} \\
R_t^{2(v_{2*2})*} &= 0 \\
R_t^{2(v_{2*3})*} &= \frac{\tau_{22*3} \text{ var}(w_{jk})}{\text{Eqn OC46}} = R_t^{2(v_{2*3})} \\
R_t^{2(v_{3*2})*} &= 0 \\
R_t^{2(v_{3*3})*} &= 0 \\
R_t^{2(m_2)*} &= \frac{\tau_{00*2}}{\text{Eqn OC46}} = R_t^{2(m_2)} \\
R_t^{2(m_3)*} &= \frac{\tau_{00*3}}{\text{Eqn OC46}} = R_t^{2(m_3)}
\end{aligned} \tag{OC50}$$

We can similarly form the following level-specific measures:

Online Appendix to accompany Rights & Sterba (In press). R squared measures for multilevel models with three or more levels. *Multivariate Behavioral Research. DOI: 10.1080/00273171.2021.1985948*

$$
\begin{aligned}
R_1^{2(f_1)*} &= \frac{\gamma_1^2 \operatorname{var}(x_{ijk})}{\text{Eqn OC47}} = R_1^{2(f_1)} \\
R_2^{2(f_2)*} &= \frac{\gamma_2^2 \operatorname{var}(w_{jk})}{\text{Eqn OC48}} = R_2^{2(f_2)} \\
R_3^{2(f_3)*} &= \frac{\gamma_3^2 \operatorname{var}(z_k)}{\text{Eqn OC49}} = R_3^{2(f_3)} \\
R_1^{2(v_{1*2})*} &= \frac{\tau_{11*2} \operatorname{var}(x_{ijk})}{\text{Eqn OC47}} = R_1^{2(v_{1*2})} \\
R_1^{2(v_{1*3})*} &= \frac{\tau_{11*3} \operatorname{var}(x_{ijk})}{\text{Eqn OC47}} = R_1^{2(v_{1*3})} \\
R_2^{2(v_{2*2})*} &= 0 \\
R_2^{2(v_{2*3})*} &= \frac{\tau_{22*3} \operatorname{var}(w_{jk})}{\text{Eqn OC48}} = R_2^{2(v_{2*3})} \\
R_3^{2(v_{3*2})*} &= 0 \\
R_3^{2(v_{3*3})*} &= 0 \\
R_2^{2(m_2)*} &= \frac{\tau_{00*2}}{\text{Eqn OC48}} = R_2^{2(m_2)} \\
R_3^{2(m_3)*} &= \frac{\tau_{00*3}}{\text{Eqn OC49}} = R_3^{2(m_3)}
\end{aligned}
\qquad \text{(OC51)}
$$

Hence, each measure computed for the *mean-centered model* is exactly equivalent to the corresponding measure computed for the *centered-by-constant model*, while each measure computed for the *centered-by-constant model* that is not applicable to the *mean-centered model* (i.e., $R_t^{2(v_{2*2})*}$, $R_t^{2(v_{3*2})*}$, $R_t^{2(v_{2*3})*}$, $R_t^{2(v_{3*3})*}$) is equal to 0.

Online Appendix to accompany Rights & Sterba (In press). R squared measures for multilevel models with three or more levels. *Multivariate Behavioral Research. DOI: 10.1080/00273171.2021.1985948*

Online Appendix D: *r2mlm3* R function

Here we describe and provide the R code for the *r2mlm3* function. This function has now also been inserted into the *r2mlm* R package (Shaw, Rights, Sterba, & Flake, 2020) on CRAN.

***r2mlm3* R function Description:**

This function reads in raw data and three-level multilevel model (MLM) parameter estimates and outputs all relevant R^2 measures as well as an accompanying bar chart. That is, when entering results from a cluster-mean-centered model (i.e., a model in which all level-1 and level-2 predictors are cluster-mean-centered, using the method for cluster-mean-centering described in the manuscript), this function outputs all total and level-specific measures in Table 1 with an accompanying barchart (e.g., Figure 1). When entering *non*-cluster-mean-centered model results (i.e., results from a model in which *at least one* level-1 or level-2 predictor is *not* cluster-mean-centered), the function outputs all total and level-specific measures from Table 5 with an accompanying bar chart. For each type of model, any number of predictors is supported. Any of the predictors at level-1 or any intermediate level (i.e., levels below the highest level) can have random slopes across any higher-level unit.

This *r2mlm3* function can also accommodate two-level models. To input results for two-level models, set the following arguments equal to NULL: *l3_covs*, *random_covs13*, *random_covs23*, *gamma_3*, *Tau13*, *Tau23*.

***r2mlm3* R function Input:**

data – Dataset with rows denoting observations and columns denoting variables

clustermeancentered – By default, this argument is set to TRUE, indicating that cluster-mean-centered model results are being inputted. When instead entering non-cluster-mean-centered model results, set this argument to FALSE. Additionally, for non-cluster-mean-centered model results, random effect variances/covariances are to be entered in arguments *Tau2_noncmc* and *Tau3_noncmc* (defined below), rather than in the *Tau12*, *Tau13*, and *Tau23* arguments used for cluster-mean-centered model results. Additionally, when entering non-cluster-mean-centered model results, user must specify *l2clusterID_noncmc* and *l3clusterID_noncmc* (neither of which are necessary for cluster-mean-centered model results). Function input is otherwise the same for cluster-mean-centered and non-cluster-mean-centered model results.

l1_covs – Vector of numbers (or variable names) corresponding to the columns in the dataset of the level-1 predictors used in the MLM (if none used, set to NULL)

l2_covs – Vector of numbers (or variable names) corresponding to the columns in the dataset of the level-2 predictors used in the MLM (if none used, set to NULL)

l3_covs – Vector of numbers (or variable names) corresponding to the columns in the dataset of the level-3 predictors used in the MLM (if none used, set to NULL)

random_covs12 – Vector of numbers (or variable names) corresponding to the columns in the dataset of the level-1 predictors that have random slopes across level-2 units in the MLM (if no such random slopes, set to NULL)

random_covs13 – Vector of numbers (or variable names) corresponding to the columns in the dataset

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of the level-1 predictors that have random slopes across level-3 units in the MLM (if no such random slopes, set to NULL)

random_covs23 – Vector of numbers (or variable names) corresponding to the columns in the dataset of the level-2 predictors that have random slopes across level-3 units in the MLM (if no such random slopes, set to NULL)

gamma_1 – Vector of fixed slope estimates for all level-1 predictors, to be entered in the order of the predictors listed by *l1_covs* (if none, set to NULL)

gamma_2 – Vector of fixed slope estimates for all level-2 predictors, to be entered in the order listed by *l2_covs* (if none, set to NULL)

gamma_3 – Vector of fixed slope estimates for all level-3 predictors, to be entered in the order listed by *l3_covs* (if none, set to NULL)

Tau12 – for cluster-mean-centered model results (set to NULL if entering non-cluster-mean-centered model results), this is the random effect covariance matrix with the first row/column denoting the intercept variance and covariances across level-2 units and each subsequent row/column denotes a given level-1 predictor's random slope variance and covariances across level-2 units (to be entered in the order listed by *random_covs12*; if none, set to NULL)

Tau13 – for cluster-mean-centered model results (set to NULL if entering non-cluster-mean-centered model results), this is the random effect covariance matrix with the first row/column denoting the intercept variance and covariances across level-3 units and each subsequent row/column denotes a given level-1 predictor's random slope variance and covariances across level-3 units (to be entered in the order listed by *random_covs13*; if none, set to NULL)

Tau23 – for cluster-mean-centered model results (set to NULL if entering non-cluster-mean-centered model results), this is the random effect covariance matrix with each row/column denoting a given level-2 predictor's random slope variance and covariances across level-3 units (to be entered in the order listed by *random_covs23*; if none, set to NULL)

Tau2_noncmc – For non-cluster-mean-centered model results, this is the level-2 random effect covariance matrix; the first row/column denotes the intercept variance and covariances across level-2 units and each subsequent row/column denotes a given predictor's random slope variance and covariances across level-2 units (to be entered in the order listed by *randomcovsl2_noncmc*; by default, this argument is set to NULL).

Tau3_noncmc – For non-cluster-mean-centered model results, this is the level-3 random effect covariance matrix; the first row/column denotes the intercept variance and covariances across level-3 units and each subsequent row/column denotes a given predictor's random slope variance and covariances across level-3 units (to be entered in the order listed by *randomcovsl2_noncmc*; by default, this argument is set to NULL).

l2clusterID_noncmc – For non-cluster-mean-centered model results, this is the number (or variable name) corresponding to the column in the dataset containing the level-2 cluster identification (function assumes that each level-2 cluster ID is unique; by default, this argument is set to NULL)

l3clusterID_ noncmc – For non-cluster-mean-centered model results, this is the number (or variable name) corresponding to the column in the dataset containing the level-3 cluster identification (function assumes that each level-3 cluster ID is unique; by default, this argument is set to

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NULL)

sigma2 – level-1 residual variance

r2mlm3 R function Code:

```
r2mlm3<- function(data,l1_covs,l2_covs,l3_covs,random_covs12,random_covs13,random_covs23,
                gamma_1,gamma_2,gamma_3,Tau12,Tau13,Tau23,sigma2,
                clustermeancentered=TRUE, Tau2_noncmc=NULL,Tau3_noncmc=NULL,
                l2clusterID_noncmc=NULL,l3clusterID_noncmc=NULL){

  if(clustermeancentered==TRUE){

   ##compute phis
  if(is.null(l1_covs)==T){
   phi_1 <- 0
   gamma_1 <- 0
  }else{
   phi_1 <- var(data[,l1_covs],na.rm=T)}
  if(is.null(l2_covs)==T){
   phi_2 <- 0
   gamma_2 <- 0
  }else{
   phi_2 <- var(data[,l2_covs],na.rm=T)}
  if(is.null(l3_covs)==T){
   phi_3 <- 0
   gamma_3 <- 0
  }else{
   phi_3 <- var(data[,l3_covs],na.rm=T)}

  ##compute variances/covariances of random components

  if(is.null(Tau13)==TRUE) Tau13 <- matrix(c(0),1,1)

  if(is.null(random_covs12)==F){
  var_randomcovs12 <- var(cbind(1,data[,c(random_covs12)]),na.rm=T)
  psi12 <- matrix(c(diag(Tau12)),ncol=1)
  kappa12 <- matrix(c(Tau12[lower.tri(Tau12)==TRUE]),ncol=1)
  v12 <- matrix(c(diag(var_randomcovs12)),ncol=1)
  r12 <- matrix(c(var_randomcovs12[lower.tri(var_randomcovs12)==TRUE]),ncol=1)
  }else{
   var_randomcovs12 <- 0
   psi12 <- 0
   kappa12 <- 0
   v12 <- 0
   r12 <- 0
  }

  if(is.null(random_covs13)==F){
  var_randomcovs13 <- var(cbind(1,data[,c(random_covs13)]),na.rm=T)
  psi13 <- matrix(c(diag(Tau13)),ncol=1)
  kappa13 <- matrix(c(Tau13[lower.tri(Tau13)==TRUE]),ncol=1)
  v13 <- matrix(c(diag(var_randomcovs13)),ncol=1)
  r13 <- matrix(c(var_randomcovs13[lower.tri(var_randomcovs13)==TRUE]),ncol=1)
  }else{
   var_randomcovs13 <- 0
```

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```
  psi13 <- 0
  kappa13 <- 0
  v13 <- 0
  r13 <- 0
}

if(is.null(random_covs23)==F){
var_randomcovs23 <- var(cbind(data[,c(random_covs23)]),na.rm=T)
psi23 <- matrix(c(diag(Tau23)),ncol=1)
kappa23 <- matrix(c(Tau23[lower.tri(Tau23)==TRUE]),ncol=1)
v23 <- matrix(c(diag(var_randomcovs23)),ncol=1)
r23 <- matrix(c(var_randomcovs23[lower.tri(var_randomcovs23)==TRUE]),ncol=1)
}else{
  var_randomcovs23 <- 0
  psi23 <- 0
  kappa23 <- 0
  v23 <- 0
  r23 <- 0
}

##compute level-specific and total variance

l1var <- (t(gamma_1)%*%phi_1%*%gamma_1) + (t(v12)%*%psi12 + 2*(t(r12)%*%kappa12)) + (t(v13)%*%psi13 +
           2*(t(r13)%*%kappa13))+ sigma2

l2var <-(t(gamma_2)%*%phi_2%*%gamma_2) + (t(v23)%*%psi23 + 2*(t(r23)%*%kappa23)) + Tau12[1,1]

l3var <- (t(gamma_3)%*%phi_3%*%gamma_3) + Tau13[1,1]

totalvar <- l1var + l2var + l3var

#total R2 measures

  R2_f1_t <- (t(gamma_1)%*%phi_1%*%gamma_1)/totalvar
  R2_f2_t <- (t(gamma_2)%*%phi_2%*%gamma_2)/totalvar
  R2_f3_t <- (t(gamma_3)%*%phi_3%*%gamma_3)/totalvar
  R2_v12_t <- (t(v12)%*%psi12 + 2*(t(r12)%*%kappa12)) /totalvar
  R2_v13_t <- (t(v13)%*%psi13 + 2*(t(r13)%*%kappa13))/totalvar
  R2_v23_t <- (t(v23)%*%psi23 + 2*(t(r23)%*%kappa23)) /totalvar
  R2_m2_t <- Tau12[1,1] /totalvar
  R2_m3_t <- Tau13[1,1]/totalvar

    #l1 R2 measures
    R2_f1_1 <- (t(gamma_1)%*%phi_1%*%gamma_1) /l1var
    R2_v12_1 <- (t(v12)%*%psi12 + 2*(t(r12)%*%kappa12)) /l1var
    R2_v13_1 <- (t(v13)%*%psi13 + 2*(t(r13)%*%kappa13))/l1var

    #l2 R2 measures
    R2_f2_2 <-(t(gamma_2)%*%phi_2%*%gamma_2)/l2var
    R2_v23_2 <- (t(v23)%*%psi23 + 2*(t(r23)%*%kappa23)) /l2var
    R2_m2_2 <- Tau12[1,1]/l2var

    #l3 R2 measures
     R2_f3_3 <-  (t(gamma_3)%*%phi_3%*%gamma_3)/l3var
    R2_m3_3 <- Tau13[1,1]/l3var

    R2_table <- matrix(c(R2_f1_t,R2_f2_t,R2_f3_t,R2_v12_t,R2_v13_t,R2_v23_t,R2_m2_t,R2_m3_t,
                    R2_f1_1,"NA","NA",R2_v12_1,R2_v13_1,"NA","NA","NA",
                    "NA",R2_f2_2,"NA","NA","NA",R2_v23_2,R2_m2_2,"NA",
                    "NA","NA",R2_f3_3,"NA","NA","NA","NA",R2_m3_3)
                    ,ncol=4)
  rownames(R2_table) <- c("f1","f2","f3","v12","v13","v23","m2","m3")
```

Online Appendix to accompany Rights & Sterba (In press). R squared measures for multilevel models with three or more levels. *Multivariate Behavioral Research. DOI: 10.1080/00273171.2021.1985948*

```
  colnames(R2_table) <- c("total","l1","l2","l3")

  ##barchart

  contributions_stacked <-
      matrix(c(R2_f1_t,R2_f2_t,R2_f3_t,R2_v12_t,R2_v13_t,R2_v23_t,R2_m2_t,R2_m3_t,sigma2/totalvar,
               R2_f1_1,0,0,R2_v12_1,R2_v13_1,0,0,0,sigma2/l1var,
               0,R2_f2_2,0,0,0,R2_v23_2,R2_m2_2,0,0,
               0,0,R2_f3_3,0,0,0,0,R2_m3_3,0),nrow=9,ncol=4)
  colnames(contributions_stacked) <- c("total","level-1","level-2","level-3")
  rownames(contributions_stacked) <- c("f1","f2","f3",
                "v12","v13","v23",
                "m2","m3","resid")

  barplot(contributions_stacked, main="Decomposition", horiz=FALSE,

      ylim=c(0,1),col=c("darkred","steelblue","darkgoldenrod1","darkred","darkred","midnightblue","midnightblue","darkgoldenrod1","white"),
    ylab="proportion of variance",
    density=c(NA,NA,NA,20,40,20,40,20,NA),angle=c(0,0,0,45,45,135,135,0,0))

      legend(4.89,.7,title="source",legend=rownames(contributions_stacked),fill=c("darkred","steelblue","darkgoldenrod1","darkred","darkred","midnightblue","midnightblue","darkgoldenrod1","white"),
    cex=.7, pt.cex = 1,xpd=TRUE,density=c(NA,NA,NA,30,50,30,50,30,NA),angle=c(0,0,0,45,45,135,135,0,0))

Output <- list(noquote(R2_table))
names(Output) <- c("R2s")
}

if(clustermeancentered==FALSE){

  allfixedcovs_noncmc<- c(l1_covs,l2_covs,l3_covs)
  allgamma_noncmc <- c(gamma_1,gamma_2,gamma_3)
  randomcovsl2_noncmc <- random_covs12
  randomcovsl3_noncmc <- c(random_covs13,random_covs23)

  ##create version of dataset that decomposes predictors into level-specific portions

  data_temp <- data

    if(is.null(allfixedcovs_noncmc)==F){
     data_fixed_centered <- matrix(NA,nrow(data),length(allfixedcovs_noncmc)*3)
     colnames(data_fixed_centered) <- c(paste0("p",seq(length(allfixedcovs_noncmc)),"_l1"),
                paste0("p",seq(length(allfixedcovs_noncmc)),"_l2"),
                paste0("p",seq(length(allfixedcovs_noncmc)),"_l3"))


     if(length(l2_covs)>0){
       data_fixed_centered[,(length(l1_covs)+1):(length(allfixedcovs_noncmc))] <- 0
     }

     if(length(l3_covs)>0){

      data_fixed_centered[,(length(allfixedcovs_noncmc)+length(l1_covs)+length(l2_covs)+1):(2*length(allfixedcovs_noncmc))] <- 0
       data_fixed_centered[,(length(l1_covs)+1):(length(allfixedcovs_noncmc))] <- 0
     }
    }

  if(is.null(randomcovsl2_noncmc)==F){
  data_randoml2_centered <- matrix(NA,nrow(data),length(randomcovsl2_noncmc)*3)
  colnames(data_randoml2_centered) <- c(paste0("p",seq(length(randomcovsl2_noncmc)),"_l1"),
```

Online Appendix to accompany Rights & Sterba (In press). R squared measures for multilevel models with three or more levels. *Multivariate Behavioral Research. DOI: 10.1080/00273171.2021.1985948*

```
                              paste0("p",seq(length(randomcovsl2_noncmc)),"_l2"),
                              paste0("p",seq(length(randomcovsl2_noncmc)),"_l3"))
}

if(is.null(randomcovsl3_noncmc)==F){
data_randoml3_centered <- matrix(NA,nrow(data),length(randomcovsl3_noncmc)*3)
colnames(data_randoml3_centered) <- c(paste0("p",seq(length(randomcovsl3_noncmc)),"_l1"),
                              paste0("p",seq(length(randomcovsl3_noncmc)),"_l2"),
                              paste0("p",seq(length(randomcovsl3_noncmc)),"_l3"))
}


##loop through level-2 clusters
for(clus2 in seq(max(as.numeric(data[,l2clusterID_noncmc])))){

  for(i in seq(nrow(data))){

    if(data[i,l2clusterID_noncmc]==clus2){

      if(is.null(allfixedcovs_noncmc)==F){
      for(ncov in seq(length(allfixedcovs_noncmc))){
        ##compute level-1 portion of predictor
        data_fixed_centered[i,paste0("p",ncov,"_l1")] <- data[i,allfixedcovs_noncmc[ncov]]-
          mean(data_temp[which(data_temp[,l2clusterID_noncmc]==clus2),allfixedcovs_noncmc[ncov]],na.rm=T)
        ##compute level-2 portion of predictor
        data_fixed_centered[i,paste0("p",ncov,"_l2")] <-
          mean(data_temp[which(data_temp[,l2clusterID_noncmc]==clus2),allfixedcovs_noncmc[ncov]],na.rm=T)
      }
      }

      if(is.null(randomcovsl2_noncmc)==F){
      for(ncov in seq(length(randomcovsl2_noncmc))){
        ##compute level-1 portion of predictor
        data_randoml2_centered[i,paste0("p",ncov,"_l1")] <- data[i,randomcovsl2_noncmc[ncov]]-
          mean(data_temp[which(data_temp[,l2clusterID_noncmc]==clus2),randomcovsl2_noncmc[ncov]],na.rm=T)
        ##compute level-2 portion of predictor
        data_randoml2_centered[i,paste0("p",ncov,"_l2")] <-
          mean(data_temp[which(data_temp[,l2clusterID_noncmc]==clus2),randomcovsl2_noncmc[ncov]],na.rm=T)
      }
      }

      if(is.null(randomcovsl3_noncmc)==F){
      for(ncov in seq(length(randomcovsl3_noncmc))){
        ##compute level-1 portion of predictor
        data_randoml3_centered[i,paste0("p",ncov,"_l1")] <- data[i,randomcovsl3_noncmc[ncov]]-
          mean(data_temp[which(data_temp[,l2clusterID_noncmc]==clus2),randomcovsl3_noncmc[ncov]],na.rm=T)
        ##compute level-2 portion of predictor
        data_randoml3_centered[i,paste0("p",ncov,"_l2")] <-
          mean(data_temp[which(data_temp[,l2clusterID_noncmc]==clus2),randomcovsl3_noncmc[ncov]],na.rm=T)
      }
      }
    }
  }
}

##loop through level-3 clusters
for(clus3 in seq(max(as.numeric(data[,l3clusterID_noncmc])))){

  for(i in seq(nrow(data))){

      if(data[i,"schoolid"]==clus3){
```

Online Appendix to accompany Rights & Sterba (In press). R squared measures for multilevel models with three or more levels. *Multivariate Behavioral Research. DOI: 10.1080/00273171.2021.1985948*

```
      if(is.null(allfixedcovs_noncmc)==F){
      for(ncov in seq(length(allfixedcovs_noncmc))){
       ##compute level-3 portion of predictor
            data_fixed_centered[i,paste0("p",ncov,"_l3")] <-
         mean(data_temp[which(data_temp[,l3clusterID_noncmc]==clus3),allfixedcovs_noncmc[ncov]],na.rm=T)
      }
      }

      if(is.null(randomcovsl2_noncmc)==F){
      for(ncov in seq(length(randomcovsl2_noncmc))){
       ##compute level-3 portion of predictor
       data_randoml2_centered[i,paste0("p",ncov,"_l3")] <-
         mean(data_temp[which(data_temp[,l3clusterID_noncmc]==clus3),randomcovsl2_noncmc[ncov]],na.rm=T)
      }
      }

      if(is.null(randomcovsl3_noncmc)==F){
      for(ncov in seq(length(randomcovsl3_noncmc))){
       ##compute level-3 portion of predictor
        data_randoml3_centered[i,paste0("p",ncov,"_l3")] <-
         mean(data_temp[which(data_temp[,l3clusterID_noncmc]==clus3),randomcovsl3_noncmc[ncov]],na.rm=T)
      }
      }

    }
  }
}

if(is.null(allfixedcovs_noncmc)==F){
for(ncov in seq(length(allfixedcovs_noncmc))){
 ##cluster-mean-center level-2 predictor
data_fixed_centered[,paste0("p",ncov,"_l2")] <- data_fixed_centered[,paste0("p",ncov,"_l2")] -
         data_fixed_centered[,paste0("p",ncov,"_l3")]
}
}

if(is.null(randomcovsl2_noncmc)==F){
for(ncov in seq(length(randomcovsl2_noncmc))){
 ##cluster-mean-center level-2 predictor
data_randoml2_centered[,paste0("p",ncov,"_l2")] <- data_randoml2_centered[,paste0("p",ncov,"_l2")] -
         data_randoml2_centered[,paste0("p",ncov,"_l3")]
}
}

if(is.null(randomcovsl3_noncmc)==F){
for(ncov in seq(length(randomcovsl3_noncmc))){
##cluster-mean-center level-2 predictor
data_randoml3_centered[,paste0("p",ncov,"_l2")] <- data_randoml3_centered[,paste0("p",ncov,"_l2")] -
         data_randoml3_centered[,paste0("p",ncov,"_l3")]
}
}

##make lower-level portions of cluster-level predictors equal to 0
if(is.null(allfixedcovs_noncmc)==F){
   if(length(l2_covs)>0){
   data_fixed_centered[,(length(l1_covs)+1):(length(allfixedcovs_noncmc))] <- 0
  }

  if(length(l3_covs)>0){

         data_fixed_centered[,(length(allfixedcovs_noncmc)+length(l1_covs)+length(l2_covs)+1):(2*length(allfixedcovs_no
         ncmc))] <- 0
```

Online Appendix to accompany Rights & Sterba (In press). R squared measures for multilevel models with three or more levels. *Multivariate Behavioral Research. DOI: 10.1080/00273171.2021.1985948*

```
    data_fixed_centered[,(length(l1_covs)+1):(length(allfixedcovs_noncmc))] <- 0
  }
}


if(is.null(allfixedcovs_noncmc)==F){
##compute phis
phi1 <- var(data_fixed_centered[,seq(length(allfixedcovs_noncmc))],na.rm=T)
phi2 <- var(data_fixed_centered[,c(length(allfixedcovs_noncmc)+seq(length(allfixedcovs_noncmc)))],na.rm=T)
phi3 <- var(data_fixed_centered[,c(2*length(allfixedcovs_noncmc)+seq(length(allfixedcovs_noncmc)))],na.rm=T)

##compute variance explained by fs
f1 <- t(allgamma_noncmc)%*%phi1%*%allgamma_noncmc
f2 <- t(allgamma_noncmc)%*%phi2%*%allgamma_noncmc
f3 <- t(allgamma_noncmc)%*%phi3%*%allgamma_noncmc
}else{
 f1 <-f2<-f3<-0
}

##make lower-level portions of cluster-level predictors equal to 0
if(is.null(randomcovsl3_noncmc)==F){
 if(length(random_covs23)>0){
   data_randoml3_centered[,(length(random_covs13)+1):(length(randomcovsl3_noncmc))] <- 0
 }
}

if(is.null(randomcovsl2_noncmc)==F){

##compute Sigmas
Sigma12 <- var(cbind(0,data_randoml2_centered[,seq(length(randomcovsl2_noncmc))]),na.rm=T)
 Sigma22 <-
      var(cbind(0,data_randoml2_centered[,c(length(randomcovsl2_noncmc)+seq(length(randomcovsl2_noncmc)))]),na.r
      m=T)
  Sigma32 <-
      var(cbind(0,data_randoml2_centered[,c(2*length(randomcovsl2_noncmc)+seq(length(randomcovsl2_noncmc)))]),n
      a.rm=T)

 ##compute variance explained by vs
 v12<- as.numeric(sum(diag(Sigma12%*%Tau2_noncmc)))
  v22<- as.numeric(sum(diag(Sigma22%*%Tau2_noncmc)))
  v32<- as.numeric(sum(diag(Sigma32%*%Tau2_noncmc)))

 ##compute m vectors

 m_mat12 <-
      matrix(c(as.numeric(colMeans(cbind(0,data_randoml2_centered[,seq(length(randomcovsl2_noncmc))]),na.rm=T))),
      ncol=1)
 m_mat22 <-
      matrix(c(as.numeric(colMeans(cbind(0,data_randoml2_centered[,c(length(randomcovsl2_noncmc)+seq(length(rand
      omcovsl2_noncmc)))]),na.rm=T))),ncol=1)
 m_mat32 <-
      matrix(c(as.numeric(colMeans(cbind(1,data_randoml2_centered[,c(2*length(randomcovsl2_noncmc)+seq(length(ra
      ndomcovsl2_noncmc)))]),na.rm=T))),ncol=1)

##compute variance explained by m

 m2 <-
      (t(m_mat12)%*%Tau2_noncmc%*%m_mat12+t(m_mat22)%*%Tau2_noncmc%*%m_mat22+t(m_mat32)%*%Ta
      u2_noncmc%*%m_mat32)

}else{
  v12 <- v22 <- v32 <- 0
```

Online Appendix to accompany Rights & Sterba (In press). R squared measures for multilevel models with three or more levels. *Multivariate Behavioral Research. DOI: 10.1080/00273171.2021.1985948*

```
   m2 <- Tau2_noncmc[1,1]
  }

  if(is.null(randomcovsl3_noncmc)==F){

   ##compute Sigmas
   Sigma13 <- var(cbind(0,data_randoml3_centered[,seq(length(randomcovsl3_noncmc))]),na.rm=T)
   Sigma23 <-
        var(cbind(0,data_randoml3_centered[,c(length(randomcovsl3_noncmc)+seq(length(randomcovsl3_noncmc)))]),na.r
        m=T)
   Sigma33 <-
        var(cbind(0,data_randoml3_centered[,c(2*length(randomcovsl3_noncmc)+seq(length(randomcovsl3_noncmc)))]),n
        a.rm=T)

   ##compute variance explained by vs
   v13<- as.numeric(sum(diag(Sigma13%*%Tau3_noncmc)))
   v23<- as.numeric(sum(diag(Sigma23%*%Tau3_noncmc)))
   v33<- as.numeric(sum(diag(Sigma33%*%Tau3_noncmc)))

   ##compute m vectors
   m_mat13 <-
        matrix(c(as.numeric(colMeans(cbind(0,data_randoml3_centered[,seq(length(randomcovsl3_noncmc))]),na.rm=T))),
        ncol=1)
   m_mat23 <-
        matrix(c(as.numeric(colMeans(cbind(0,data_randoml3_centered[,c(length(randomcovsl3_noncmc)+seq(length(rand
        omcovsl3_noncmc)))]),na.rm=T))),ncol=1)
   m_mat33 <-
        matrix(c(as.numeric(colMeans(cbind(1,data_randoml3_centered[,c(2*length(randomcovsl3_noncmc)+seq(length(ra
        ndomcovsl3_noncmc)))]),na.rm=T))),ncol=1)

   ##compute variance explained by m
   m3 <-
        (t(m_mat13)%*%Tau3_noncmc%*%m_mat13+t(m_mat23)%*%Tau3_noncmc%*%m_mat23+t(m_mat33)%*%Ta
        u3_noncmc%*%m_mat33)

  }else{
   v13 <- v23 <- v33 <- 0
   m3 <- Tau3_noncmc[1,1]
  }

  ##compute R2 measures

  totvar <- sum(f1,f2,f3,v12,v13,v22,v23,v32,v33,m2,m3,sigma2)
  l1var <- sum(f1,v12,v13,sigma2)
  l2var <- sum(f2,v22,v23,m2)
  l3var <- sum(f3,v32,v33,m3)

  R2_f1_t <- f1/totvar
  R2_f2_t <- f2/totvar
  R2_f3_t <- f3/totvar
  R2_v12_t <- v12/totvar
  R2_v13_t <- v13/totvar
  R2_v22_t <- v22/totvar
  R2_v23_t <- v23/totvar
  R2_v32_t <- v32/totvar
  R2_v33_t <- v33/totvar
  R2_m2_t <- m2/totvar
  R2_m3_t <- m3/totvar

  R2_f1_1 <- f1/l1var
  R2_f2_2 <- f2/l2var
  R2_f3_3 <- f3/l3var
```

Online Appendix to accompany Rights & Sterba (In press). R squared measures for multilevel models with three or more levels. *Multivariate Behavioral Research. DOI: 10.1080/00273171.2021.1985948*

```
  R2_v12_1 <- v12/l1var
  R2_v13_1 <- v13/l1var
  R2_v22_2 <- v22/l2var
  R2_v23_2 <- v23/l2var
  R2_v32_3 <- v32/l3var
  R2_v33_3 <- v33/l3var
  R2_m2_2 <- m2/l2var
  R2_m3_3 <- m3/l3var

 ##create table of measures

  R2_table <-
        matrix(c(R2_f1_t,R2_f2_t,R2_f3_t,R2_v12_t,R2_v13_t,R2_v22_t,R2_v23_t,R2_v32_t,R2_v33_t,R2_m2_t,R2_m3
        _t,
             R2_f1_1,"NA","NA",R2_v12_1,R2_v13_1,"NA","NA","NA","NA","NA","NA",
             "NA",R2_f2_2,"NA","NA","NA",R2_v22_2,R2_v23_2,"NA","NA",R2_m2_2,"NA",
             "NA","NA",R2_f3_3,"NA","NA","NA","NA",R2_v32_3,R2_v33_3,"NA",R2_m3_3)
            ,ncol=4)
  rownames(R2_table) <- c("f1","f2","f3","v12","v13","v22","v23","v32","v33","m2","m3")
  colnames(R2_table) <- c("total","l1","l2","l3")

  ##barchart

  contributions_stacked <-
        matrix(c(R2_f1_t,R2_f2_t,R2_f3_t,R2_v12_t,R2_v13_t,R2_v22_t,R2_v23_t,R2_v32_t,R2_v33_t,R2_m2_t,R2_m3
        _t,sigma2/totvar,
                    R2_f1_1,0,0,R2_v12_1,R2_v13_1,0,0,0,0,0,0,sigma2/l1var,
                    0,R2_f2_2,0,0,0,R2_v22_2,R2_v23_2,0,0,R2_m2_2,0,0,
                    0,0,R2_f3_3,0,0,0,0,R2_v32_3,R2_v33_3,0,R2_m3_3,0),nrow=12,ncol=4)
  colnames(contributions_stacked) <- c("total","level-1","level-2","level-3")
  rownames(contributions_stacked) <- c("f1","f2","f3",
                     "v12","v13","v22","v23","v32","v33",
                     "m2","m3","resid")

  barplot(contributions_stacked, main="Decomposition", horiz=FALSE,
      ylim=c(0,1),col=c("darkred","steelblue","darkgoldenrod1","darkred","darkred","midnightblue","midnightblue",
              "darkgoldenrod1","darkgoldenrod1","midnightblue","darkgoldenrod1","white"),
      ylab="proportion of variance",
      density=c(NA,NA,NA,20,40,40,20,40,20,40,20,NA),angle=c(0,0,0,45,45,90,135,90,90,135,0,0))

        legend(4.89,.7,title="source",legend=rownames(contributions_stacked),fill=c("darkred","steelblue","darkgoldenrod
        1","darkred","darkred","midnightblue","midnightblue",

        "darkgoldenrod1","darkgoldenrod1","midnightblue","darkgoldenrod1","white"),
     cex=.7, pt.cex =
        1,xpd=TRUE,density=c(NA,NA,NA,30,50,50,30,50,30,50,30,NA),angle=c(0,0,0,45,45,90,135,90,90,135,0,0))

  Output <- list(noquote(R2_table))
  names(Output) <- c("R2s")
 }

 return(Output)
}
```

Online Appendix to accompany Rights & Sterba (In press). R squared measures for multilevel models with three or more levels. *Multivariate Behavioral Research. DOI: 10.1080/00273171.2021.1985948*

Online Appendix E: Walkthrough of empirical example predicting math achievement for students nested within teachers nested within schools

Online Appendix E provides a step-by-step demonstration of how to implement the R^2 framework using our empirical example in which we predict student math achievement from student-level, teacher-level, and school-level predictors. Specifically, we walk though how to (1) download the dataset; (2) read the dataset in R; (3) center the dataset and compute cluster means; (4) fit the model in R using *lmer*; and (5) compute all R^2 measures using the model output and the *r2mlm3* function.

Step 1. Download data at https://www.stata-press.com/data/mlmus2.html

This link contains the support materials for Multilevel and Longitudinal Modeling Using Stata, Second Edition (Rabe-Hesketh & Skrondal, 2005). At this website, there will be a link to a zip file titled "mlmus2.zip." After clicking this and downloading it, the appropriate dataset will be a DTA file in the zip folder titled "instruction.dta".

Step 2. Read in data

This data can be read using the read.dta function in the *foreign* package. See below for R code.

```
##install package, if not already installed
install.packages("foreign")

##load foreign package
library(foreign)

##read in data, here naming the dataset "instruction"
#replace ~ with the actual file path
instruction <- read.dta("~/instruction.dta")
```

Step 3. Center data and compute cluster means

In our analysis, we cluster-mean-centered each level-1 predictor and each level-2 predictor using the method described in the manuscript. We additionally included cluster-means of level-1 variables and level-2 variables. This can be accomplished using the following R code:

```
##create version of dataset to be centered
instruction_centered <- instruction

##create temporary version of dataset (used in computing means below)
instruction_centered_temp <- instruction

##loop through level-2 units (classrooms)
for(clus2 in seq(max(instruction_centered[,"classid"]))){

##loop through observations
  for(i in seq(nrow(instruction_centered))){

##determine if observation belongs to the current level-2 unit
```

Online Appendix to accompany Rights & Sterba (In press). R squared measures for multilevel models with three or more levels. *Multivariate Behavioral Research. DOI: 10.1080/00273171.2021.1985948*

```
  if(instruction_centered[i,"classid"]==clus2){

##center each level-1 predictor by classroom mean
      instruction_centered[i,"mathkind"] <- instruction_centered[i,"mathkind"]-
mean(instruction_centered_temp[which(instruction_centered_temp[,"classid"]==clus2),"mathkind"],na.rm=T)
      instruction_centered[i,"girl"] <- instruction_centered[i,"girl"]-
mean(instruction_centered_temp[which(instruction_centered_temp[,"classid"]==clus2),"girl"],na.rm=T)
      instruction_centered[i,"ses"] <- instruction_centered[i,"ses"]-
mean(instruction_centered_temp[which(instruction_centered_temp[,"classid"]==clus2),"ses"],na.rm=T)



      }
  }
}

##loop through level-3 units (schools)
for(clus3 in seq(max(instruction_centered[,"schoolid"]))){

##loop through observations
for(i in seq(nrow(instruction_centered))){

##determine if observation belongs to the current level-3 unit
    if(instruction_centered[i,"schoolid"]==clus3){

##school-mean center each level-2 predictor
      instruction_centered[i,"mathprep"] <- instruction_centered[i,"mathprep"]-
mean(instruction_centered_temp[which(instruction_centered_temp[,"schoolid"]==clus3),"mathprep"],na.rm=T)
      instruction_centered[i,"mathknow"] <- instruction_centered[i,"mathknow"]-
mean(instruction_centered_temp[which(instruction_centered_temp[,"schoolid"]==clus3),"mathknow"],na.rm=T)
      instruction_centered[i,"yearstea"] <- instruction_centered[i,"yearstea"]-
mean(instruction_centered_temp[which(instruction_centered_temp[,"schoolid"]==clus3),"yearstea"],na.rm=T)

##compute school-means for each level-1 and level-2 predictor
      instruction_centered[i,"mathprep_schmean"] <-
mean(instruction_centered_temp[which(instruction_centered_temp[,"schoolid"]==clus3),"mathprep"],na.rm=T)
      instruction_centered[i,"mathknow_schmean"] <-
mean(instruction_centered_temp[which(instruction_centered_temp[,"schoolid"]==clus3),"mathknow"],na.rm=T)
      instruction_centered[i,"mathkind_schmean"] <-
mean(instruction_centered_temp[which(instruction_centered_temp[,"schoolid"]==clus3),"mathkind"],na.rm=T)
      instruction_centered[i,"girl_schmean"] <-
mean(instruction_centered_temp[which(instruction_centered_temp[,"schoolid"]==clus3),"girl"],na.rm=T)
      instruction_centered[i,"ses_schmean"] <-
mean(instruction_centered_temp[which(instruction_centered_temp[,"schoolid"]==clus3),"ses"],na.rm=T)
      instruction_centered[i,"yearstea_schmean"] <-
mean(instruction_centered_temp[which(instruction_centered_temp[,"schoolid"]==clus3),"yearstea"],na.rm=T)

        }
  }
}

#grand-mean-center level-3 predictors
instruction_centered[,"mathprep_schmean"] <- instruction_centered[,"mathprep_schmean"]-
mean(instruction_centered[,"mathprep_schmean"],na.rm=T)
instruction_centered[,"mathknow_schmean"] <- instruction_centered[,"mathknow_schmean"]-
mean(instruction_centered[,"mathknow_schmean"],na.rm=T)
```

Online Appendix to accompany Rights & Sterba (In press). R squared measures for multilevel models with three or more levels. *Multivariate Behavioral Research. DOI: 10.1080/00273171.2021.1985948*

```
instruction_centered[,"mathkind_schmean"] <- instruction_centered[,"mathkind_schmean"]-
mean(instruction_centered[,"mathkind_schmean"],na.rm=T)
instruction_centered[,"girl_schmean"] <- instruction_centered[,"girl_schmean"]-
mean(instruction_centered[,"girl_schmean"],na.rm=T)
instruction_centered[,"ses_schmean"] <- instruction_centered[,"ses_schmean"]-
mean(instruction_centered[,"ses_schmean"],na.rm=T)
instruction_centered[,"yearstea_schmean"] <- instruction_centered[,"yearstea_schmean"]-
mean(instruction_centered[,"yearstea_schmean"],na.rm=T)
```

Step 4. Fit model in lmer

This model can be fit using the *lmer* function in the *lme4* package:

```
##install packages, if not already installed
install.packages("lme4")
install.packages("lmerTest")

##load packages
library(lme4)
library(lmerTest)

##fit model
fit_mathgain<-lmer(mathgain~mathkind+girl+ses+
            mathprep+mathknow +yearstea+
            mathkind_schmean+girl_schmean+ses_schmean+mathprep_schmean+mathknow_schmean+yearstea_schmean +
            (mathkind|schoolid/classid), data=instruction_centered,control=lmerControl(optimizer="Nelder_Mead"))

##output results
summary(fit_mathgain)
```

The summary() function will output the following results:

```
Linear mixed model fit by REML. t-tests use Satterthwaite's method ['lmerModLmerTest']
Formula: mathgain ~ mathkind + girl + ses + mathprep + mathknow + yearstea +      mathkind_schmean + girl_sc
hmean + ses_schmean + mathprep_schmean +
    mathknow_schmean + yearstea_schmean + (mathkind | schoolid/classid)
   Data: instruction_centered
Control: lmerControl(optimizer = "Nelder_Mead")

REML criterion at convergence: 10365

Scaled residuals:
    Min      1Q  Median      3Q     Max
-2.9466 -0.6090 -0.0364  0.5634  4.0661

Random effects:
 Groups           Name        Variance   Std.Dev.   Corr
 classid:schoolid (Intercept)  2.043e+02  14.29467
                  mathkind     4.414e-02   0.21010  -0.22
 schoolid         (Intercept ) 3.623e+01   6.01887
                  mathkind     5.302e-03   0.07281  -0.44
 Residual                      6.758e+02  25.99570
Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105

Fixed effects:
```

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```
                   Estimate  Std. Error          df    t value      Pr(>|t|)
(Intercept)        57.76571   1.36791    92.64551     42.229      < 2e-16 ***
mathkind           -0.43074    0.03281    83.34343    -13.128      < 2e-16 ***
girl               -2.11203    1.89558   754.50922     -1.114     0.265554
ses                 5.80394    1.45095   748.37460      4.000     6.96e-05 ***
mathprep            1.68821    1.57092   166.37160      1.075     0.284082
mathknow            5.93540    1.72194   162.63173      3.447     0.000722 ***
yearstea           -0.05250    0.16777   166.48060     -0.313     0.754730
mathkind_schmean   -0.32474    0.06726   114.08571     -4.828     4.32e-06 ***
girl_schmean        2.07370    8.43397   123.38818      0.246     0.806188
ses_schmean         5.02485    4.02446   102.71088      1.249     0.214658
mathprep_schmean    2.45981    2.28142   102.65648      1.078     0.283474
mathknow_schmean   -1.40308    1.81452   110.99929     -0.773     0.441015
yearstea_schmean    0.40647    0.20773   121.69303      1.957     0.052674 .
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Step 5. Enter parameter estimates and data into *r2mlm3* function

The *r2mlm3* function, and descriptions of the input, can be found in Online Appendix C, and we have now also made it available in the *r2mlm* R package on CRAN.

```
##call function
r2mlm3(

  ##specify dataset
  data=instruction_centered,

  ##specify columns of dataset containing level-1 predictors
  l1_covs=c("mathkind","girl","ses"),

  ##specify columns of dataset containing level-2 predictors
  l2_covs=c("mathprep","mathknow","yearstea"),

  ##specify columns of dataset containing level-3 predictors
  l3_covs=c("mathkind_schmean","girl_schmean","ses_schmean","mathprep_schmean","mathknow_schmean","y
earstea_schmean"),

  ##specify columns of dataset containing level-1 predictors whose slope vary across level-2 units
  random_covs12=c("mathkind"),

  ##specify columns of dataset containing level-1 predictors whose slope vary across level-3 units
  random_covs13=c("mathkind"),

  ##specify columns of dataset containing level-2 predictors whose slope vary across level-3 units
  #not included in above model
  random_covs23=NULL,

  ##insert fixed component estimates for each level-1 predictor
  #(mathkind, girl, and ses in above output)
  gamma_1=c(-0.43074,-2.11203,5.80394),

  ##insert fixed component estimates for each level-2 predictor
```

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```
  #(mathprep, mathknow, and yearstea in above output)
  gamma_2=c(1.68821,5.93540,-0.05250),

  ##insert fixed component estimates for each level-3 predictor
  #(each predictor ending in "_schmean" in above output)
  gamma_3=c(-0.32474,2.07370,5.02485,2.45981,-1.40308,0.40647),

  ##specify random effects associated with the level-2 random intercept and the level-1 predictor random slopes a
cross level-2 units
  #found under "Random effects" in the group labeled "classid:schoolid"
  Tau12=matrix(c(14.29467^2,-0.22*0.21010*14.29467,-0.22*0.21010*14.29467,0.21010^2),2,2),

  ##specify random effects associated with the level-3 random intercept and the level-1 predictor random slopes a
cross level-3 units
  #found under "Random effects" in the group labeled "schoolid"
  Tau13=matrix(c(6.01887^2,-0.44*0.07281*6.01887,-0.44*0.07281*6.01887,0.07281^2),2,2),

  ##specify random effects associated with the level-2 predictor random slopes
  #not included in above model
  Tau23=NULL,

  ##specify residual variance
  #found under "Random effect" in group labeled "Residual"
  sigma2=25.99570^2)
```

The *r2mlm3* function will output the following results:

```
$R2s
      total                 l1                    l2                    l3
f1    0.144035123153756     0.194417853603958     NA                    NA
f2    0.0150373994173882    NA                    0.081824772703193     NA
f3    0.0454555673340852    NA                    NA                    0.603091013329324
v12   0.0346170944859092    0.0467259725308216    NA                    NA
v13   0.00415738812055641   0.00561162067487076   NA                    NA
v23   0                     NA                    0                     NA
m2    0.168738233812099     NA                    0.918175227296807     NA
m3    0.0299154236597132    NA                    NA                    0.396908986670676
```

And will also output the following plot:

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