29th Cumberland Conference on Combinatorics, Graph Theory and Computing

Vanderbilt University, Nashville, Tennessee, U.S.A., 20–21 May 2017

Saturday, 20 May 2017

8:00	Registration, SC4 lobby	
8:20	Opening remarks	
8:30	1. Cranston, SC 4309	
	SC 4309	*SC 4327*
9:30	2. Santana	3. Brauch
9:55	4. Rolek	5. Atici
10:20	Coffee break	
	SC 4309	*SC 4327*
10:45	6. Jiang	7. Nastase
11:10	8. Cameron	9. Kuhl
11:35	10. Rorabaugh	11. Bahmanian
12:00	Lunch (on your own)	
1:45	12. Randall, SC 4309	
	SC 4309	*SC 4327*
2:45	13. Loeb	14. Fahrbach
3:10	15. Brandt	16. Sullivan
3:35	Coffee break	
	SC 4309	*SC 4327*
4:00	17. Zhang	18. Bóna
4:25	19. McCarty	20. Godbole
4:50	21. Kanno	22. McKee
5:15	23. Xu	24. Lewis
5:40	25. Wei	26. Mominul Haque

Sunday, 21 May 2017

8:30	27. Sanità, SC 4309	
	SC 4309	*SC 4327*
9:30	28. Ye	29. Slilaty
9:55	30. Plummer	31. Goodrich
10:20	Coffee break	
	SC 4309	*SC 4327*
10:45	32. Chen	33. Fife
11:10	34. Li	35. Gershkoff
11:35	36. Gu	37. Grace
12:00	Lunch (provided)	
1:00	38. Král', SC 4309	
	SC 4309	*SC 4327*
2:00	39. Pór	40. Shull
2:25	41. Zha	42. Hoyer
2:50	Conference ends	

Plenary talks are 50 minutes with 5 minutes for questions and a 5 minute break at the end.

Contributed talks are 18 minutes with 2 minutes for questions and a 5 minute break at the end.

We gratefully acknowledge support from the National Science Foundation and the Shanks Endowment of the Department of Mathematics at Vanderbilt University.

Speaker index by talk number

1. SC 4309

Edge-coloring of graphs and multigraphs

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We survey some beautiful theorems and conjectures in edge-coloring, focusing on graph classes where an obvious lower bound on the chromatic index holds with equality. One of these problems leads us to Tashkinov trees, which are a powerful recoloring tool for edge-coloring multigraphs.

2. SC 4309

Strong chromatic index of graphs with maximum degree four

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A strong edge-coloring is a coloring of the edges of a graph such that every color class forms an induced matching. In 1985, Erdős and Nešetřil conjectured that every graph with maximum degree Δ has a strong edge-coloring using at most $\frac{5}{4}\Delta^2$ colors. While this conjecture has inspired a wide range of work in the area of strong edge-colorings, only one nontrivial case (for graphs with maximum degree three) has been verified (due to Anderson, and independently, Horák, Qing, and Trotter). In this talk we will discuss our recent work that shows 21 colors suffice for graphs with maximum degree four, extending a previous result of Cranston. We will also present several additional problems in this area.

3. SC 4327

Obstacle Numbers of Some Ptolemaic Graphs

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An obstacle representation of a graph is a visibility (line of sight) graph in which the vertices are points in the plane with a set of polygonal obstacles. Two vertices are adjacent if the straight line between the two points does not intersect an obstacle. The obstacle number of a graph G, denoted $obs(G)$ is an embedding with the fewest number of polygons that realizes the graph. Ptolemaic graphs are graphs that can be built from K_1 using three operations: [1] adding a pendent vertex, [2] adding a true twin vertex, [3] adding a false twin vertex to a vertex whose neighborhood is a clique. This presentation will find the obstacle numbers for a subclass of the Ptolemaic graphs that can be built from a single vertex using only the first two operations, adding a pendent vertex and adding a true twin.

4. SC 4309

Double-critical graph conjecture for claw-free graphs

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A connected graph G with chromatic number t is $d$ouble-critical if $G\backslash\{x,y\}$ is $(t-2)$-colorable for each edge $xy \in E(G)$. The complete graphs are the only known examples of double-critical graphs. A long-standing conjecture of Erdős and Lovász from 1966, which is referred to as the Double-Critical Graph Conjecture, states that there are no other double-critical graphs. That is, if a graph G with chromatic number t is double-critical, then G is the complete graph on t vertices. This has been verified for $t \leq 5$, but remains open for $t \geq 6$. In this paper, we first prove that if G is a non-complete, double-critical graph with chromatic number $t \geq 6$, then no vertex of degree $t+1$ is adjacent to a vertex of degree $t+1$, $t+2$, or $t+3$ in G. We then use this result to show that the Double-Critical Graph Conjecture is true for double-critical graphs G with chromatic number $t \leq 8$ if G is claw-free.

5. SC 4327

Vector Space based Secret Sharing Scheme for non-complete multipartite graphs

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Some time it is not safe to give secure key K to an individual. One needs to find a way to give shares of information about the secure key K to certain authorized group so that when the group pool their share, then they can figure out the key K. If one or more people are missing from the group, rest of the group member is unable to find the key K. There are some construction for complete multipartite graph based authorization. If authorized group is no longer forming a multipartite graph, then there is no know sharing algorithm. We will present algorithm for some special cases.

6. SC 4309

Property of an edge-chromatic critical graph : Hamiltonicity

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An edge-Δ-critical graph G is a simple connected graph of maximum degree Δ such that edge chromatic number $\chi'(G) = \Delta + 1$ and $\chi'(G-e) = \Delta$ for each edge e of G. In 1965, Vizing conjectured that every edge-Δ-critical graph with chromatic index at least 3 contains a 2-factor. Chen and Shan verified Vizing's 2-factor conjecture for $\Delta \geq n/2$. Obviously, if a graph is Hamiltonian, then it contains a 2-factor. Let G be an edge-Δ-critical graph of order n. Luo and Zhao proved that G is Hamiltonian when $\Delta \geq 6n/7$. And Luo, Miao and Zhao showed that G is Hamiltonian when $\Delta \geq 4n/5$. Recently, Chen, Chen and Zhao showed that G is Hamiltonian if $\Delta \geq 3n/4$. Using a new method, we proved that every edge chromatic critical graph of order n with $\Delta \geq 2n/3 + 13$ is Hamiltonian.

7. SC 4327

The maximum size of a partial spread in a finite vector space

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Let $V(n,q)$ denote the vector space of dimension n over the finite field with q elements. A *partial t-spread* of $V(n,q)$ is a set of t-dimensional subspaces of $V(n,q)$ such that any two of them have trivial intersection. Let $r \equiv n \pmod t$. We prove that if $t > (q^r-1)/(q-1)$, then the maximum size, i.e., cardinality, of a partial t-spread of $V(n,q)$ is $(q^n - q^{t+r})/(q^t-1)+1$. This essentially settles a longstanding open problem in this area. Prior to this result, this maximum size was only known for $r=1$ and for $r=q=2$. In particular, this result also determines the clique number of the q-Kneser graph.

8. SC 4309

A $(5,5)$-coloring of K_n with few colors

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For fixed integers p and q, let $f(n,p,q)$ denote the minimum number of colors needed to color all of the edges of the complete graph K_n such that no clique of p vertices spans fewer than q distinct colors. Any edge-coloring with this property is known as a (p,q)-coloring. In this talk, I will discuss an explicit $(5,5)$-coloring that shows that $f(n,5,5) \leq n^{1/3+o(1)}$ as $n \to \infty$. This improves upon the best known probabilistic upper bound of $O\left(n^{1/2}\right)$ given by Erdős and Gyárfás, and comes close to matching the best known lower bound $\Omega\left(n^{1/3}\right)$.

9. SC 4327

Completing Some Partial Latin Squares

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A partial latin square can be completed if there is a latin square of the same order containing it. Let $r, c, s \in \{1, 2, \ldots, n\}$ and P be a partial latin square of order n in which each nonempty cell lies in row r, column c, or contains symbol s. We will show that if $n \notin \{3,4,5\}$ and row r, column c, and symbol s can be completed in P, then a completion of P exists. We will also show that it is always possible to complete partial latin squares with two filled rows and two filled columns, except for a few small counterexamples.

10. SC 4309

Arc Graphs and Posets

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The arc graph $\delta(G)$ of a digraph G is the digraph with the set of arcs of G as vertex-set, where the arcs of $\delta(G)$ join consecutive arcs of G. In 1981, Poljak and Rödl characterised the chromatic number of $\delta(G)$ in terms of the chromatic number of G when G is symmetric (i.e., undirected). In contrast, directed graphs with equal chromatic numbers can have arc graphs with distinct chromatic numbers. Even though the arc graph of a symmetric graph is not symmetric, we show that the chromatic number of the iterated arc graph $\delta^k(G)$ still only depends on the chromatic number of G when G is symmetric. arXiv:1610.01259 [math.CO]

11. SC 4327

On the Existence of Generalized Designs

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A set S of q-subsets of an n-set X is a design with parameters (n,q,r,λ) if every r-subset of X belongs to exactly λ elements of S. In other words, a design with parameters (n,q,r,λ) is an n-vertex q-uniform hypergraph in which every r-subset of the vertex set belongs to exactly λ edges. The existence of a design with parameters (n,q,r,λ) is equivalent to a K_q^r-decomposition of λK_n^r (the complete λ-fold r-uniform hypergraph of order n). By Keevash's Theorem (2014), λK_n^r can be decomposed into K_q^r when some obvious divisibility conditions are satisfied and n is sufficiently large. In this talk, I will discuss a "multipartite" version of Keevash's Theorem.

12. SC 4309

Sampling Paths, Partitions and Permutations

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Random sampling is ubiquitous across the sciences and engineering as a means of studying properties of large sets. Often these sets are exponentially large, and to be useful we need to be able to sample elements from a target distribution in polynomial time. We will look at how a simple mountain-valley Markov chain on monotonic lattice paths can be used as the basis for sampling algorithms arising in self-assembly, combinatorics and computing. Examples we will discuss include sampling lozenge tilings, biased permutations, and integer partitions.

13. SC 4309

Entire Colorability for a Class of Plane Graphs

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A plane graph G is *entirely k-colorable* if every element in the set of vertices, edges, and faces of G can be colored from $1, \ldots, k$ so that every two adjacent or incident elements have distinct colors. In 2011, Wang and Zhu asked if every simple plane graph G, other than K_4, is entirely $(\Delta(G) + 3)$-colorable. In 2012, Wang, Mao, and Miao answered in the affirmative for simple plane graphs with $\Delta(G) \geq 8$. We show that every loopless plane multigraph with $\Delta(G) = 7$, no 2-faces, and no two 3-faces sharing an edge is entirely 10-colorable.

14. SC 4327

Efficient Boltzmann samplers for weighted partitions and selections

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Boltzmann sampling is commonly used to sample from large combinatorial sets by biasing the distributions to favor samples of a particular size. For the approach to be effective, one needs to prove that the sampling procedure is efficient and samples of the desired size will be generated with sufficiently high probability. We use this approach to provide a provably efficient sampling algorithm for a class of weighted integer partitions that gives the first rigorous solution to a sampling problem related to Bose–Einstein condensation from statistical physics. Our sampling algorithm uses a probabilistic interpretation of the ordinary generating function for these objects. Other approaches using generating functions have been previously considered, but without rigorous bounds on the rejection rates. We use the Khintchine–Meinardus probabilistic method to provide such a bound through a singularity analysis of the associated Dirichlet generating function.

15. SC 4309

Additive coloring planar graphs with girth at least 5

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The *additive coloring number* of G, denoted $\chi_\Sigma(G)$, is the least integer k for which G has a labeling of its vertices from $\{1, 2, \ldots, k\}$ such that two adjacent vertices have distinct sums of labels on their neighbors. In 2009, Czerwiński, Grytczuk, and Żelazny conjectured that $\chi_\Sigma(G) \leq \chi(G)$, where $\chi(G)$ is the chromatic number of G. This conjecture remains open even for bipartite graphs, for which no constant bound is currently known. In this talk, we discuss known bounds on the additive coloring number and present improved bounds for planar graphs with girth at least five. Our proof uses the discharging method and the Combinatorial Nullstellensatz to obtain results on a list version of additive coloring.

16. SC 4327

Graph Minors: When Being Shallow is Hard

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Identifying dense substructures is a frequent task in analyzing real-world graphs, with a rich history of results characterizing its computational complexity for various notions of substructure. For example, one can find the densest subgraph in polynomial time using flow-based methods, yet finding the densest clique or graph minor is NP-complete. We show that in some sense, finding dense substructures which are just slightly 'less local' than subgraphs seems to be intrinsically difficult.

Specifically, we consider r-shallow minors, which naturally intermediate between the local nature of subgraphs ($r = 0$) and the global notion of minors ($r = \infty$). Finding densest 0-shallow minors is in P, but Densest 1-Shallow Minor is NP-complete, so we focus on substructures that fall between 0- and 1-shallow. Specifically, we prove that Densest $r/2$-Shallow Topological Minor and Densest r-Subdivision are NP-complete already in sub-cubic apex-graphs for $r \geq 1$, and that neither problem can be solved in time $O(2^{o(n)})$ unless the Exponential Time Hypothesis (ETH) fails. Further, for Densest 1-Shallow Topological Minor, we show the problem is FPT for bounded treewidth, but no algorithm with running time $O(2^{o(tw(G)^2)}n)$ can exist unless the ETH fails.

17. SC 4309

Planar graphs with girth at least 5 are (3, 4)-colorable

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A graph is $(d_1, d_2, \ldots, d_k)$-colorable if its vertex set can be partitioned into k nonempty subsets so that the subgraph induced by the ith part has maximum degree at most d_i for each $i \in \{1, \ldots, k\}$. It was previously known that planar graphs with girth at least 5 are $(3, 5)$-colorable and $(4, 4)$-colorable. We improve both results by showing that planar graphs with girth at least 5 are $(3, 4)$-colorable.

18. SC 4327

Balanced vertices in rooted labeled trees

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A vertex v in a rooted tree is called balanced if all descending paths from v to a leaf have the same length. For a number of tree varieties, we will compute the limiting probability that a random vertex of a random tree is balanced as the tree size goes to infinity. In some cases, we can prove that the mentioned probability decreases as the size of the tree increases.

19. SC 4309

The extremal function and Colin de Verdière parameter

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For a graph G, the Colin de Verdière graph parameter $\mu(G)$ is the maximum corank of any matrix in a certain family of generalized adjacency matrices of G. Given a non-negative integer t, the family of graphs with Colin de Verdière parameter no more than t is minor-closed. A graph G is planar if and only if $\mu(G) \leq 3$. Colin de Verdière conjectured that the chromatic number $\chi(G)$ of a graph satisfies $\chi(G) \leq \mu(G) + 1$. For graphs with $\mu(G) \leq 3$ this is the Four Color Theorem.

We conjecture that if G has at least t vertices and $\mu(G) \leq t$, then $|E(G)| \leq t|V(G)| - t(t+1)/2$. If this conjecture is true, then $\chi(G) \leq 2\mu(G)$. We show that this conjecture is related to the graph complement conjecture for the Colin de Verdière parameter, and prove that the conjecture is true for every graph G such that: either $\mu(G) \leq 7$, or $\mu(G) \geq |V(G)| - 6$, or the complement of G is chordal, or G is chordal.

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20. SC 4327

Expected Number of Distinct Subsequences in Randomly Generated Binary Strings

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When considering binary strings, it is natural to ask how many distinct subsequences might exist in a given string. Given that there is an existing algorithm which provides a straightforward way to compute the number of distinct subsequences in a fixed string, we will be interested in the expected number of distinct subsequences in random strings. This expected value is already known for random binary strings where each letter in the string is equally likely to be a 1 or a 0. We first generalize this result to random strings where the letter 1 appears with probability $\alpha \in [0, 1]$, and then to the case of non-uniform letter generation from an alphabet of size d. Finally, we identify subsequences that are the most “typical” in the sense that they occur an “average” number of times.

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21. SC 4309

Quasi-surfaces: Chromatic Numbers and Euler’s Formula

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Euler’s formula is the foundation for many results in graph theory. Knowing this universal constant associated with graph embeddings on any surface has allowed us to advance our understanding of many foundational concepts in the discipline. In this talk, we describe a quasi-surface, a generalization of both the k-book space and the 2-sphere, for the first time. Natural questions related to graph embeddings such as the chromatic number, and whether or not there is an equivalent to the Euler formula for a quasi-surface are investigated.

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22. SC 4327

New characterizations of Gallais i-triangulated graphs

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In 1962, Tibor Gallai defined the oddly-named ”i-triangulated graphs.” In 1993, Elias Dahlhaus named the oddly-defined ”good generalized strongly chordal graphs.” I ’ll give some modern characterizations of each and a link between them.

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23. SC 4309

An Interactive Proof Study of Erdős-Szekeres Conjecture

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In 1935, Erdős and Szekeres proved that for every integer $n \geq 3$, there is a minimal integer $ES(n)$ such that any set of $ES(n)$ points in the plane in general position contains n points in convex position. They showed that $ES(n) \geq 2^{n-2}+1$ and conjectured this to be sharp.

There are many different variations of Erdős-Szekeres Conjecture. Babai and Moran said:"Since the creation of formal systems, the element of *interaction* in the proof process has been ignored in mathematics." In this paper, for the first time, the *Interactive Proof* Method was introduced to investigate the Erdős-Szekeres Conjecture. The connection of this method to the design of randomized and deterministic algorithms to find the convex hull of n points for a given point set is also discussed.

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24. SC 4327

On Vertex-Edge and Edge-Vertex Domination and Degrees

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In this talk we will introduce vertex-edge (ve) and edge-vertex (ev) domination and degrees. Amongst other things, we will present some results regarding the ve and ev-regularity of graphs, the ve and ev-irregularity of graphs, and some connections with chemical graphs.

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25. SC 4309

Some new progress in decycling number

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In this talk we consider the effects of spanning trees on several graphical invariants such as decycling number, independence number, large induced forest and covering number of a graph and present several more equivalent conditions for such numbers. Based on the result in cubic graph, an application is provided too.

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26. SC 4327

Irregular Total Labellings of Knödel Graphs $W_{3,n}$

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The total edge irregularity strength $tes(G)$ and total vertex irregularity strength $tvs(G)$ are invariants analogous to irregular strength $s(G)$ of a graph G for total labellings. Bača et al. determined the bounds and precise values for some families of graphs concerning these parameters. In this paper, we show the exact values of the total edge irregularity strength and total vertex irregularity strength of Knödel graphs $W_{3,n}$.

Keywords: *Irregular total labelling; Knödel graphs; Total labelling*

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27. SC 4309

Matchings, covers, and network games

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Several interesting game theory problems are defined on networks, where the vertices represent players and the edges model the way players can interact with each other. In such games, studying the structure of the underlying graph that describes the network setting is important to identify the existence of stable outcomes for the corresponding games. Prominent examples are cooperative matching games and network bargaining games. A key role in such games is played by stable graphs, which are graphs where the cardinality of a maximum matching equals the size of a minimum fractional vertex-cover. In fact, stable graphs characterize instances of such games that admit the existence of stable outcomes. In this talk, we will discuss properties of stable graphs, and discuss the algorithmic problem of turning a given graph into a stable one, via edge- and vertex-removal operations, highlighting both the graph theory and the game theory aspects of the problem.

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28. SC 4309

On Perfect Matchings in Matching-Covered Graphs

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Let G be a matching-covered graph, i.e., every edge is contained in a perfect matching. An edge subset X of G is feasible if there exists two perfect matchings M_1 and M_2 such that $|M_1 \cap X| \not\equiv |M_2 \cap X| \pmod 2$. Lukot'ka and Rollová proved that an edge subset X of a regular bipartite graph is not feasible if and only if X is switching-equivalent to $\emptyset$, and they further ask whether a non-feasible set of a regular graph of class 1 is always switching-equivalent to either $\emptyset$ or $E(G)$? Two edges of G are equivalent to each other if a perfect matching M of G either contains both of them or contains none of them. An equivalent class of G is an edge subset K with at least two edges such that the edges of K are mutually equivalent. An equivalent class is not a feasible set. Lovász proved that an equivalent class of a brick has size 2. In this paper, we show that, for every integer $k \geq 3$, there exist infinitely many k-regular graphs of class 1 with an arbitrarily large equivalent class K such that K is not switching-equivalent to either $\emptyset$ or $E(G)$, which provides a negative answer to the problem proposed by Lukot'ka and Rollová. Further, we characterize bipartite graphs with equivalent class, and characterize matching-covered bipartite graphs of which every edge is removable.

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29. SC 4327

Biased graphs and Gain Graphs

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A theta graph consists of three internally disjoint paths sharing the same endpoints. Note that there are exactly three cycles in a theta graph. A biased graph is a pair (G, B) in which G is a graph and B is a collection of cycles such that any theta subgraph does not have exactly two cycles from B.

Given a group Γ and a graph G, let φ be a function from the set of orientations of the edges of G to Γ such that $\varphi(e^{-1}) = \varphi(e)^{-1}$. The pair (G, φ) is called a gain graph. (In topological graph theory this is often called a voltage graph.) If we let B_φ be the collection of cycles C of G whose φ-values along C compose to the identity element of Γ, then (G, B_φ) is a biased graph. This is, in fact, the canonical example of a biased graph.

Biased graphs and gain graphs have many applications in linear programming, algebraic topology, topological graph theory, and matroid theory. In this talk we will present some current results concerning biased graphs and gain graphs.

30. SC 4309

Matching Extension in Prism Graphs

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If G is any graph, the *prism graph* of G, denoted $P(G)$, is the cartesian product of G with a single edge, or equivalently, the graph obtained by taking two copies of G, say G_1 and G_2, with the same vertex labelings and joining each vertex of G_1 to the vertex of G_2 having the same label by an edge. A connected graph G has property $E(m,n)$ (or more briefly "G is $E(m,n)$") if for every pair of disjoint matchings M and N in G with $|M| = m$ and $|N| = n$ respectively, there is a perfect matching F in G such that $M \subseteq F$ and $N \cap F = \emptyset$. In this paper, we begin the study of the $E(m,n)$ properties of the prism graph $P(G)$ when G is an arbitrary graph, as well as the more special situations when, in addition, G is bipartite or bicritical.

31. SC 4327

Exploiting Bipartite Structure for Practical Graph Minor Embedding

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Recent advances in adiabatic quantum computing have required fast and effective algorithms for assigning a program's logical qubits to a set of physical qubits in a hardware fabric. Formally, this assignment can be viewed as a graph minor. Computing graph minor embeddings is difficult in general, therefore efficient algorithms rely on local domain knowledge such as hardware fabric topology and program structure; we examine how to exploit bipartite structure in both of these graphs. We first show how to compute graph bipartization (i.e. Minimum Odd Cycle Transversal) exactly for series-parallel graphs in linear time, and provide an approximation ratio for the general case. We additionally show an asymptotically-sharp upper bound on the bipartite edit distance for minors of the 'Chimera' hardware class (recently implemented by D-Wave Systems). Experimentally, we find that heuristics utilizing these results can construct minor embeddings for larger programs than previous methods, while also using a smaller hardware resource footprint.

32. SC 4309

Resonance Polynomials of Cata-Condensed Hexagonal Systems

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Coauthors: Dong Ye and Xiaoya Zha

A hexagonal system is a finite 2-connected plane bipartite graph in which every interior face is bounded by a regular hexagon. A hexagonal system is called cata-condensed if it is outer planar. A set of disjoint hexagons H of a hexagonal system G is a forcing resonant set if a subgraph consisting of deleting all vertices of H from G has a unique perfect matching. The forcing resonance polynomial of G is defined as $f(x) = \sum_{i=0}^{cl(G)} a_i x^i$ where a_i is the number of distinct forcing resonant set of size i and $cl(G)$ is the Clar number of G. The polynomial can be used to enumerate the forcing resonant sets of hexagonal systems. In this paper, we compute the forcing resonance polynomial of cata-condensed hexagonal system G. Our computation results demonstrate that an isomer with larger coefficient vectors of forcing resonance polynomial has larger HOMO-LUMO gap. In other words, an isomer with larger coefficient vector is more stable.

33. SC 4327

Nested Matroids and Laminar Matroids

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A matroid is a finite set with a collection of independent sets that behave like linearly independent sets in a vector space. The rank, $r(X)$, of a set X is the size of a largest independent subset of X, and the closure, $cl(X)$, of X is $\{x : r(X \cup \{x\} = r(X)\}$. A laminar family is a collection of sets such that if two sets intersect, one is contained in the other. A laminar matroid is defined in terms of a laminar family $\mathscr{A}$ and a capacity function $c : \mathscr{A} \to \mathbb{N}$, a set I being independent if $|I \cap A| \leq c(A)$ for all $A \in \mathscr{A}$. It is not hard to show that the class of laminar matroids is minor closed. This talk will describe the set of excluded minors for this class.

34. SC 4309

A kind of conditional connectivity in networks

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The concept of conditional connectivity proposed by Harary, it provided an interesting measures for fault-tolerance in networks. The conditional connectivity of G with respect to some property P is the smallest cardinality of a set S of vertices, if any, such that every component of the disconnected graph $G - S$ has property P. In this talk, we consider the property $\delta \geq h$ in some networks, report recent results about this topic, and also propose several problems.

35. SC 4327

A notion of minor-based matroid connectivity

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A matroid M is N-connected if, for every pair of elements $\{e, f\}$ of $E(M)$, there is an N-minor of M using $\{e, f\}$. We prove some equivalent characterizations of N-connectivity for certain matroids N. In particular, $U_{2,3}$-connected is connected and simple, $M(\mathcal{W}_2)$-connected is connected and non-uniform, and $U_{0,1} \oplus U_{1,1}$ means that M has no clones. We will also provide a decomposition theorem for when N is 3-connected.

36. SC 4309

A property on reinforcing edge-disjoint spanning hypertrees in uniform hypergraphs

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Suppose that H is a simple uniform hypergraph satisfying $|E(H)| = k(|V(H)| - 1)$. A k-partition $\pi = (X_1, X_2, \cdots, X_k)$ of $E(H)$ such that $|X_i| = |V(H)| - 1$ for $1 \leq i \leq k$ is a uniform k-partition. Let $P_k(H)$ be the collection of all uniform k-partitions of $E(H)$ and define $\varepsilon(\pi) = \sum_{i=1}^{k} c(H(X_i)) - k$, where $c(H)$ denotes the number of maximal partition-connected sub-hypergraphs of H. Let $\varepsilon(H) = \min_{\pi \in P_k(H)} \varepsilon(\pi)$. Then $\varepsilon(H) \geq 0$ with equality holds if and only if H is a union of k edge-disjoint spanning hypertrees. The parameter $\varepsilon(H)$ is used to measure how close H is being from a union of k edge-disjoint spanning hypertrees. We prove that if H is a simple uniform hypergraph with $|E(H)| = k(|V(H)| - 1)$ and $\varepsilon(H) > 0$, then there exist $e \in E(H)$ and $e' \in E(H^c)$ such that $\varepsilon(H - e + e') < \varepsilon(H)$. The result iteratively defines a finite ε-decreasing sequence of uniform hypergraphs $H_0, H_1, H_2, \cdots, H_m$ such that $H_0 = H$, H_m is the union of k edge-disjoint spanning hypertrees, and such that two consecutive hypergraphs in the sequence differ by exactly one hyperedge.

37. SC 4327

Templates for minor-closed classes of binary matroids

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Matroids conforming to a binary frame template are obtained by altering a graphic matroid in a certain way. We introduce a preorder on the set of binary frame templates and a list of minimal nontrivial templates with respect to this preorder. The 1-flowing property for matroids is a generalization of the max-flow min-cut property of graphs. An application of our main result is that all highly connected 1-flowing matroids of sufficient size are either graphic or cographic.

The classes of even-cycle matroids and even-cut matroids each have hundreds of excluded minors. Another application of our main result is that the number of excluded minors for these classes can be drastically reduced if we consider only the highly connected matroids of sufficient size.

38. SC 4309

Elusive problems in extremal graph theory

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We study the uniqueness of optimal configurations in extremal combinatorics. Empirical experience suggests that optimal solutions to extremal graph theory problems can be made asymptotically unique by introducing additional constraints. Lovász conjectured that this phenomenon is true in general: every finite feasible set of subgraph density constraints can be extended further by a finite set of density constraints such that the resulting set is satisfied by an asymptotically unique graph. We will present a counterexample to this conjecture and discuss related results.

39. SC 4309

Universal vector sequences and universal Tverberg partitions

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We show a Ramsey-type result about Tverberg partition. Namely that for every $d, n, r > 1$ integers there exists N such that every sequence of points in R^d of length N contains a subsequence of length n such that the Tverberg partitions of any $T(d, r)$-tuple are exactly the rainbow-partitions. Here $T(d, r) = (r-1)(d+1)+1$ is the Tverberg number and the rainbow partitions are a generalization of the alternating partition. This can be viewed as a generalization of order-type homogeneous sequences.

A fast growing sequence of real number is a positive sequence where each element is at least twice as large as the previous one. As part of the proof we show a higher dimensional generalization of Rosenthal's result that is a Ramsey-type result: every sequence of distinct real numbers contains a "long" subsequence that is the affine image of a fast growing sequence.

Rosenthal's result was used by Bukh and Matoušek in Erdős-Szekeres-type statements: Ramsey function and decidability in dimension 1 (2014).

40. SC 4327

On A Conjecture on Spanning Trees with few Branch Vertices

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A branch vertex of a tree is a vertex with degree at least three. Matsuda, Ozeki, and Yamashita conjectured that, if n and k are non-negative integers and G is a connected claw-free graph of order n, there is either an independent set on 2k+3 vertices whose degrees add up to at most n-3, or a spanning tree with at most k branch vertices. The authors of this conjecture proved it for k=1; we prove it for k=2.

41. SC 4309

Connectivity and W_v-Paths in Polyhedral Maps on Surfaces

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The W_v-path Conjecture (or Nonrevisiting-path Conjecture) due to Klee and Wolfe states that any two vertices of a simple polytope can be joined by a path that does not revisit any facet. This is equivalent to the well-known Hirsch Conjecture. Klee conjectured even more, namely that the W_v-path Conjecture is true for all general cell complexes. Klee proved that the W_v-path Conjecture is true for 3-polytopes (3-connected plane graphs). Later, the general W_v-path Conjecture was verified for polyhedral maps on the projective plane and the torus by Barnette, and on the Klein bottle by Pulapaka and Vince. Recently, however, Santos proved that the Hirsch conjecture is false in general.

In this talk, we show that the the W_v-path problem is closely related to (i) the local connectivity $\kappa_G(x, y)$ (i.e. the number of disjoint (x, y)-paths), (ii) the number of different homotopy classes of (x, y)-paths, and (iii) the number of (x, y)-paths in each homotopy class. For a given surface Σ, we give quantitative conditions for the existence of a W_v-path between x and y. We also provide more systematic counterexamples with high number (linear in the genus of the surface) of paths between x and y but without any W_v-path between them. These results show the importance of topological properties of embeddings of underlying graphs on the surfaces for this W_v-path geometric setting problem.

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42. SC 4327

Edge-Independent Spanning Tree Conjecture for k=4

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For a graph G, a set of subtrees of G are edge-independent with root $r \in V(G)$ if, for every vertex $v \in V(G)$, the paths between v and r in each tree are edge-disjoint. A set of k such trees represent a set of redundant broadcasts from r which can withstand $k - 1$ edge failures. It is easy to see that k-edge-connectivity is a necessary condition for the existence of a set of k edge-independent spanning trees for all possible roots. Itai and Rodeh have conjectured that this condition is also sufficient. This has been proven for $k = 2, 3$. We prove the case $k = 4$ using a decomposition of the graph similar to an ear decomposition.
