Question: Show that the statement: $P_{i+1} - P_i = (\frac{q}{p})^i P_1$ is true for all i. Remember from the video that P_i represents the probability of the droid reaching the end assuming it starts at position i. The underlined portions in the solution represents the steps needed to complete a prrof by induction.

Recall from the proof toolkit, if you need to show something is true for all natural numbers or a well-ordered set, it is probably a good idea to use proof by induction.

Define *I(n)*: $P_{i+1} - P_i = (\frac{q}{p})^i P_1$

Show that the base case (using i = *1*) is true, that is, show: $P_2 - P_1 = (\frac{q}{p})P_1$

We know: $P_2 - P_1 = \frac{q}{p}(P_1 - P_0)$ based on the derivation from the video (Recall: $P_{i+1} - P_i = (\frac{q}{p})(P_i - P_{i-1})$)

Now P_0 represents the probability of reaching the end given that you start in position 0, in this case, the crater. Since the crater is an absorbing state then the droid will never reach the end if it starts in the crater $\Rightarrow P_0 = 0$

$\Rightarrow P_2 - P_1 = \frac{q}{p}P_1$ Thus the base case has been proven!

Now show that: given the k^{th} case is true (i.e. - $P_{k+1} - P_k = (\frac{q}{p})^k P_1$), the $k+1^{th}$ case is true (i.e. - $P_{(k+1)+1} - P_{k+1} = (\frac{q}{p})^k P_1$)

$P_{(k+1)+1} - P_{(k+1)} = \frac{q}{p}(P_{k+1} - P_{(k+1)-1})$ Again we know this is true based on the derivation from the video

Notice that the left-hand side of the above equation looks like the left-hand side of the equation we want to show.

So we just need the right-hand side of the above equation to look like the right-hand side of what we want to show.

$\Rightarrow P_{(k+1)+1} - P_{(k+1)} = (\frac{q}{p})(\frac{q}{p})^k P_1$ (We can make this substitution because we know the k^{th} case is true.)

$\Rightarrow P_{(k+1)+1} - P_{(k+1)} = (\frac{q}{p})^{k+1} P_1$

Thus we have shown: given that the k^{th} case is true, the $k+1^{th}$ is true

By induction we can conclude that the statement *I(n)*: $P_{i+1} - P_i = (\frac{q}{p})^i P_1$ is true