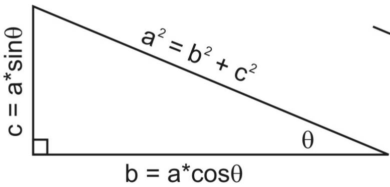

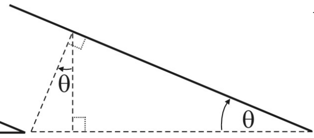

Roots of quadratic equations:

$ax^2 + bx + c = 0 \quad \Rightarrow \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\cos a + \cos b = 2\cos\left(\tfrac{1}{2}(a+b)\right)\cos\left(\tfrac{1}{2}(a-b)\right)$

$\sin a + \sin b = 2\sin\left(\tfrac{1}{2}(a+b)\right)\cos\left(\tfrac{1}{2}(a-b)\right)$

$\tan\theta = \sin\theta / \cos\theta \qquad \sin^2\theta + \cos^2\theta = 1$

$\vec{A} \bullet \vec{B} = |A_{\parallel \text{ to } B}||B|$ or $|A||B_{\parallel \text{ to } A}|$ or $|A||B|\cos\theta$ or $A_xB_x + A_yB_y + A_zB_z$

$\vec{A} \times \vec{B} = |A_{\perp \text{ to } B}||B|$ or $|A||B_{\perp \text{ to } A}|$ or $|A||B|\sin\theta$, with direction via right-hand rule

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$v_x = \frac{dx}{dt} \qquad v_{x,avg} = \frac{\Delta x}{\Delta t} \qquad x = x_0 + \int_{t_0}^{t} v_x dt$

$a_x = \frac{dv_x}{dt} \qquad a_{x,avg} = \frac{\Delta v_x}{\Delta t} \qquad v_x = v_{x0} + \int_{t_0}^{t} a_x dt$

If a_x is constant:

$x = x_0 + v_{x0}\Delta t + \tfrac{1}{2}a_x(\Delta t)^2$

$v_x = v_{x0} + a_x \Delta t$

$v_x^2 = v_{x0}^2 + 2a_x \Delta x$

Similar equations apply for translational motion along y and z.

Similar equations apply for rotational motion with substitutions: $x \to \theta$, $v \to \omega$, and $a \to \alpha$.

$\vec{a}_{rad} = \frac{v^2}{R}(-\hat{r})$, where $-\hat{r}$ points toward the center of a circular trajectory

$s = \theta R$, $\vec{v}_{\tan} = \omega R\hat{e}$, $\vec{a}_{\tan} = \alpha R\hat{e}$, where $\hat{e}$ is tangent to a circular trajectory

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$\sum \vec{F} = \vec{F}_{net} = \frac{d\vec{p}}{dt}$; if m = constant: $\vec{F}_{net} = m\vec{a} \qquad \vec{J} = \int_{t_i}^{t_f} \vec{F}_{net}\, dt = \vec{F}_{net,avg}\Delta t = \Delta\vec{p}$

$W = mg \qquad f_s \le \mu_s N \qquad f_k = \mu_k N \qquad \vec{F}_{spring} = -k(\Delta\vec{x})$

$f_{drag} = bv$ at low speed; $f_{drag} = \tfrac{1}{2}\rho A\Gamma v^2$ at high speed

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$\sum \vec{\tau} = \vec{\tau}_{net} = I\vec{\alpha}$, where $\vec{\tau} = \vec{r} \times \vec{F}$ and $I = \sum_i m_i r_i^2$ (Table of I for specific shapes below)

slender rod (length L) through center	$I = \frac{1}{12}ML^2$	slender rod (length L) through end	$I = \frac{1}{3}ML^2$
rectangular plate (a by b), through center	$I = \frac{1}{12}M(a^2 + b^2)$	rectangular plate (a by b), along edge b	$I = \frac{1}{3}Ma^2$
solid cylinder/disk, (radius R)	$I = \frac{1}{2}MR^2$	thin-walled hollow cylinder, (radius R)	$I = MR^2$
solid sphere (radius R)	$I = \frac{2}{5}MR^2$	thin-walled hollow sphere (radius R)	$I = \frac{2}{3}MR^2$
hollow cylinder (inner radius R_1, outer R_2)	$I = \frac{1}{2}M\left(R_1^2 + R_2^2\right)$	any, around a parallel axis	$I_{\parallel \text{ axis}} = I_{CM} + mr^2$

$W_c = -\Delta U$, $W_{NC} = -\Delta U_{\text{int}}$, and $W_{total} = \Delta K$, where $W = \int_{s_i}^{s_f} \vec{F} \bullet d\vec{s}$

$U_{\text{grav}} = mgh$, $U_{elastic} = \tfrac{1}{2}k(\Delta x)^2$; $K_{trans} = \tfrac{1}{2}mv^2$, $K_{rot} = \tfrac{1}{2}I\omega^2$

$\Delta K + \Delta U + \Delta U_{\text{int}} = 0$ or $K_i + U_i + W_{NC} = K_f + U_f$

$P = \frac{dW}{dt} = \vec{F} \bullet \vec{v} \qquad F_x(x) = -\frac{\partial U}{\partial x}$ with similar equations for F_y and F_z.

Simple harmonic oscillator: $x(t) = A\cos(\omega t + \phi)$ where $\omega = 2\pi f = 2\pi / T$

$\omega = \sqrt{k/m}$ (spring-mass), $\omega = \sqrt{g/L}$ (simple pendulum), $\omega = \sqrt{mgh/I}$ (physical pendulum)

Slightly damped oscillator: $x(t) = A_0 e^{-t/\tau}\cos(\omega_1 t + \phi)$ where $\tau = \dfrac{2m}{b}$ and $\omega_1 = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$

Driven, damped oscillator: $A(\omega) = F_0 \Big/ \left(m\sqrt{(\omega^2 - \omega_0^2)^2 + b^2\omega^2/m^2}\right)$

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tensile/compression: $\dfrac{F}{A} = Y\dfrac{\Delta L}{L_0}$ and $k = Y\dfrac{A}{L_0}$; volume: $P = \dfrac{F}{A} = -B\dfrac{\Delta V}{V_0}$; shear: $\dfrac{F_\parallel}{A} = S\dfrac{x}{h}$

$pV = nRT = Nk_BT$ $\langle K_{trans}\rangle = \left\langle \frac{1}{2}mv^2\right\rangle = \frac{1}{2}mv_{\text{rms}}^2 = \frac{3}{2}k_BT$ u_{int} (per d.o.f. per atom or molecule) $= \frac{1}{2}k_BT$

$\lambda = V\Big/\left(4\pi\sqrt{2}\right)r^2N$ $K_{\text{trans,total}} = \frac{3}{2}Nk_BT = \frac{3}{2}nRT$ $\Delta L = \alpha L_0 \Delta T$ and $\Delta V = \beta V_0 \Delta T$ (for solids: $\beta = 3\alpha$)

$Q = mc\Delta T$ or $Q = \pm mL_{\text{F or V}}$ or $Q_{V \text{ or } P} = nC_{V \text{ or } P}\Delta T$ where $C_V = (\#\text{ d.o.f.})R/2$ and for an ideal gas $C_p = C_V + R$

conduction: $H = \frac{dQ}{dt} = kA(T_H - T_C)/L$ radiation: $H = \frac{dQ}{dt} = \sigma\varepsilon A T^4$ and $H_{NET} = \sigma\varepsilon A\left(T^4 - T_s^4\right)$

$\Delta U = Q - W$, $\Delta U = nC_V\Delta T$, $W = \int_{V_1}^{V_2} P\,dV$ so, $W_{isothermal} = nRT\ln\left(V_f/V_i\right)$, $W_{isobaric} = P_0\left(V_f - V_i\right)$, $W_{isochoric} = 0$

adiabatic process $Q = 0$; $PV^\gamma = \text{constant}$, where $\gamma = C_p/C_V$, so $W_{adiabatic} = P_iV_i^\gamma\left(V_f^{1-\gamma} - V_i^{1-\gamma}\right)\Big/(1-\gamma)$

$e = \dfrac{W}{Q_H} = 1 - \left|\dfrac{Q_C}{Q_H}\right|$ $e_{\text{Carnot}} = 1 - \dfrac{T_C}{T_H}$ $CP = \dfrac{|Q_C|}{|W|} = \dfrac{|Q_C|}{|Q_H| - |Q_C|}$ $CP_{\text{Carnot}} = \dfrac{T_C}{T_H - T_C}$

$\Delta S = Q/T$ $S = k_B\ln\Omega$, so $\Delta S = k_B\ln\left[\Omega_f/\Omega_i\right]$ free energies: $A = U - TS$, $G = U + PV - TS$

$J_x = -D\dfrac{dn}{dx}$ $\left\langle (x - x_0)^2\right\rangle = \Delta x_{rms}^2 = 2Dt$ or $\left\langle (r - r_0)^2\right\rangle = \Delta r_{rms}^2 = 2dDt$ where d = # dimensions

Similar equations apply along y and z. $D = kT/b$ and $b = 6\pi\eta r$ for spheres at low Reynolds number

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$\rho = m/V$ $\vec{F} = P\vec{A}$ $P = P_0 + \rho g d$ $F_B = m_{fl,disp}\,g = \rho_{fl}V_{fl,disp}\,g$

$\Phi = \frac{dV}{dt} = Av = \text{constant}$ $P + \rho g y + \frac{1}{2}\rho v^2 = \text{constant}$ $F_{viscous} = \eta vA/L$ $\Phi_{\text{laminar}} = \dfrac{\pi r^4\Delta P}{8\eta L}$

$D(x,t) = A\cos\left[2\pi\left(\dfrac{x \mp vt}{\lambda}\right)\right] = A\cos\left[2\pi\left(\dfrac{x}{\lambda} \mp \dfrac{t}{T}\right)\right] = A\cos(kx \mp \omega t)$ where $k = \dfrac{2\pi}{\lambda}$, $\omega = \dfrac{2\pi}{T}$ and $v = \dfrac{\lambda}{T} = \lambda f$

transverse, string/spring: $v = \sqrt{T/\mu}$ longitudinal, fluid or bulk solid: $v = \sqrt{B/\rho}$

$D(x,t) = 2A\sin(kx)\sin(\omega t)$ (standing wave) $D(x,t) = 2A\sin(kx - \omega t)\cos\left(\dfrac{\Delta k}{2}x - \dfrac{\Delta\omega}{2}t\right)$ (beats) $f_{beat} = |f_1 - f_2|$

$f_n = n\frac{v}{2L} = nf_1$ $\lambda_n = 2L/n$ where n = 1,2,3,… (nodes or anti-nodes on both ends)

$f_n = n\frac{v}{4L} = nf_1$ $\lambda_n = 4L/n$ where n = 1,3,5,… (one node and one anti-node on ends)

$I = 2\pi^2\rho v_p f^2 s_{\max}^2$ $I = P/A$ $I(r) = P_0/(4\pi r^2)$ $\beta = (10\text{ dB})\log\left(\dfrac{I}{10^{-12}\text{ W/m}^2}\right)$ $f_{obs} = \left(\dfrac{v_s \pm v_{obs}}{v_s \mp v_{src}}\right)f$

$g = 9.81\text{ m/s}^2$	$k_B = 1.38 \times 10^{-23}$ J / K	T (K) = T (°C) + 273.15 K
1 kg weighs 2.205 lbs	$N_A = 6.02 \times 10^{23}$ mol^{-1}	1 L = 10^{-3} m^3 and 1 mL = 1 cm^3
$\sigma = 5.67 \times 10^{-8}$ W / m^2·K^4	$R = 8.31$ J / mol·K	1 Calorie = 1 kcal = 4186 J = 4186 N m
1 W = 1 J/s	$v_{\text{s in air}}$ (@ 25 °C) = 344 m/s	$v_{\text{s in water}}$ (@ 25 °C) = 1484 m/s

p_{atm} = 1 atm = 1.013×10^5 Pa = 1.013×10^5 N/m^2 = 1.013 bar = 760 mmHg = 14.70 lb/in^2