Web Appendix

Proof of Proposition 1.

(a) It is asserted in the text that $\pi_i^{F*} > \pi_i^{CN*} > \pi_i^{L*}$. Direct comparison and simplification yields:

$\pi_i^{F*} > \pi_i^{L*}$ if and only if (1) $128b^6 - 128b^5d - 64b^4d^2 + 80b^3d^3 - 12bd^5 + 3d^6 > 0$.

$\pi_i^{F*} > \pi_i^{CN*}$ if and only if (2) $-(32b^4 - 24b^2d^2 + d^4) < 0$.

$\pi_i^{CN*} > \pi_i^{L*}$ if and only if (3) $1024b^{10} - 2048b^8d^2 + 128b^7d^3 + 1408b^6d^4 - 192b^5d^5 - 384b^4d^6 + 88b^3d^7 + 34b^2d^8 - 12bd^9 + d^{10} > 0$.

Claim 1. The inequalities (1), (2) and (3) hold for all $d \in (-b, b)$.

Proof of Claim 1. First consider inequality (1). Set $d = tb$, where $t \in (-1, 1)$. Then the expression above reduces to $g_1(t) = 128 - 128t - 64t^2 + 80t^3 - 12t^5 + 3t^6$. It is clear that $g_1(-1) > 0$, $g_1(0) > 0$ and $g_1(1) > 0$. First we prove the claim for $t < 0$. For $t \in (-1, 0)$, write $g_1(t) = \{128 - 64t^2\} - t\{128 - 80t^2\} - 12t^5 + 3t^6$. Both expressions in curly brackets are positive, $-t$ is positive, $-12t^5$ is positive and $3t^6$ is positive. Hence $g_1(t) > 0$ for $t \in (-1, 0)$. Now consider $t \in (0, 1)$. Since $g_1(0) > 0$, $g_1(1) > 0$ and $g_1'(t) = -128 - 128t + 240t^2 - 60t^4 + 18t^5 < 0$ for all $t \in (0, 1)$, it follows that $g_1(t) > 0$ for all $t \in (0, 1)$. Thus, inequality (1) holds for all $d \in (-b, b)$. Inequality (2) clearly holds for all $d \in (-b, b)$. Finally, consider inequality (3). Again, set $d = tb$, where $t \in (-1, 1)$. Then the expression above reduces to $g_2(t) = 1024 - 2048t^2 + 128t^3 + 1408t^4 - 192t^5 - 384t^6 + 88t^7 + 34t^8 - 12t^9 + t^{10}$. Notice that $g_2(0) = 1024$, $g_2(1) = 47$ and $g_2(-1) = 23$. For $t \in (0, 1)$, $g_2(t)$ can be written as a combination of four expressions, each of which is itself positive for $t \in (0, 1)$: $g_2(t) = h_1(t) + t^3h_2(t) + t^4h_3(t) + t^8h_4(t)$, where $h_1(t) = 1024 - 2048t^2 + 1024t^4$, $h_2(t) = 128 - 192t^2 + 88t^4$, $h_3(t) = 384 - 384t^2$ and $h_4(t) = 34 - 12t + t^2$. The expression $h_1(t) = 1024(1 - t^2)^2 > 0$ for $t \in (0, 1)$. The expression $h_2(t) = 128 - 192t^2 + 88t^4$ has $h_2(0) = 128$ and $h_2'(t) = -384t + 352t^2 < 0$. Thus, $h_2(t)$ starts at 128 and decreases over the interval $t \in (0, 1)$; since it is still positive at $t = 1$, where $h_2(1) = 24$, it follows that $h_2(t) > 0$ for all $t \in (0, 1)$. The expression $h_3(t)$ is clearly positive for all $t \in (0, 1)$. Finally, the expression $h_4(t)$ has $h_4(0) = 34$ and $h_4'(t) = -12 + 2t < 0$. Thus, $h_4(t)$ starts at 34 and decreases over the interval $t \in (0, 1)$; since it is still positive at $t = 1$, where $h_4(1) = 23$, it follows that $h_4(t) > 0$ for all $t \in (0, 1)$. Combining these results implies that $g_2(t) = h_1(t) + t^3h_2(t) + t^4h_3(t) + t^8h_4(t) > 0$ for all $t \in (0, 1)$.

For $t \in (-1, 0)$, $g_2(t)$ can again be written as a combination of four expressions, each of which is itself positive for $t \in (-1, 0)$ (however, these are different expressions): $g_2(t) = h_5(t) + h_6(t) + t^4h_7(t) + t^4h_8(t)$, where $h_5(t) = 896 - 2048t^2 + 1175t^4$, $h_6(t) = 128 + 128t^3$, $h_7(t) = 192 - 192t - 384t^2$ and $h_8(t) = 41 + 88t^3 + 34t^4 - 12t^5 + t^6$. The expression $h_5(t) = 896 - 2048t^2 + 1175t^4$ has $h_5(0) = 896$ and $h(-1) = 23$. Moreover, since $h_5'(t) = - 4096t + 4700t^3$ and $h_5''(t) = - 4096 + 14100t^2$, this function has a maximum at $t = 0$ and a minimum at $t = -(4096/4700)^{½}$. Evaluating $h_5(t)$ at $t = -(4096/4700)^{½}$ yields $h_5(-(4096/4700)^{½}) = 3.6 > 0$. Thus, $h_5(t) > 0$ for all $t \in (-1, 0)$. The expression $h_6(t) = 128 + 128t^3$ is clearly positive for all $t \in (-1, 0)$. The expression $h_7(t) = 192 - 192t - 384t^2$ is also clearly positive for all $t \in (-1, 0)$. Finally, the expression $h_8(t) = 41 + 88t^3 + 34t^4 - 12t^5 + t^6$ has $h_8(0) = 41$ and $h_8(-1) = 0$. Moreover, $h_8'(t) = t^2[264 + 136t - 60t^2 + 6t^3] > 0$; thus, $h_8(t) > 0$ for all $t \in (-1, 0)$. Combining these results implies that $g_2(t) = h_5(t) + h_6(t) + t^4h_7(t) + t^4h_8(t) > 0$ for all $t \in (-1, 0)$. QED: Claim 1.

(b) It is asserted in the text that $w_i^{F*} > w_i^{CN*} > w_i^{L*}$ for $d > 0$ and $w_i^{L*} > w_i^{F*} > w_i^{CN*}$ for $d < 0$. Direct comparison and simplification yields:

$w_i^{F*} > w_i^{L*}$ if and only if (4) $d(16b^3 - 16b^2 - 4bd^2 + 5d^3) > 0$.

$w_i^{F*} > w_i^{CN*}$ if and only if (5) $d^2 - 4b^2 < 0$.

$w_i^{CN*} > w_i^{L*}$ if and only if (6) $d^5(64b^5 - 64b^3d^2 + 12bd^4 - d^5) > 0$.

<u>Claim 2</u>. Inequality (4) holds if and only if $d > 0$; inequality (5) holds for all $d \in (-b, b)$; and

inequality (6) holds if and only if $d > 0$.

Proof of Claim 2. First consider inequality (4). To verify that $16b^3 - 16b^2d - 4bd^2 + 5d^3 > 0$ for all $d \in (-b, b)$, set $d = tb$, where $t \in (-1, 1)$. Then the expression above reduces to $g_3(t) = 16 - 16t - 4t^2 + 5t^3$. It is clear that $g_3(-1) > 0$, $g_3(0) > 0$ and $g_3(1) > 0$. First we prove the claim for $t < 0$. For $t \in (-1, 0)$, write $g_3(t) = \{16 - 4t^2\} - t\{16 - 5t^2\}$. Both expressions in curly brackets are positive and $-t$ is positive for $t \in (-1, 0)$. Hence $g_3(t) > 0$ for $t \in (-1, 0)$. Now consider $t \in (0, 1)$. Since $g_3(0) > 0$, $g_3(1) > 0$ and $g_3'(t) = -16 - 8t + 15t^2 < 0$ for all $t \in (0, 1)$, it follows that $g_3(t) > 0$ for all $t \in (0, 1)$. Thus, inequality (4) holds if and only if $d > 0$. Inequality (5) clearly holds for all $d \in (-b, b)$. Finally, consider inequality (6). To see that $64b^5 - 64b^3d^2 + 12bd^4 - d^5 > 0$ for all $d \in (-b, b)$, set $d = tb$, where $t \in (-1, 1)$. Then the expression above reduces to $g_4(t) = 64 - 64t^2 + 12t^4 - t^5$. It is clear that $64 - 64t^2 > 0$ for all $t \in (-1, 1)$ and $12t^4 - t^5 > 0$ for all $t \in (-1, 1)$. Thus, inequality (6) holds if and only if $d > 0$. QED: Claim 2. QED: Proposition 1.

Proof of Proposition 2.

(a) It is asserted in the text that $\Pi^{L*} > \pi^{L*}$ if $d > 0$ and $\Pi^{L*} < \pi^{L*}$ if $d < 0$. Direct comparison and simplification yields: $\Pi^{L*} > \pi^{L*}$ if and only if the expression $d^3(16b^3 - 16b^2d - 4bd^2 + 5d^3) > 0$. But we have already shown (see Claim 2 above) that the term in parentheses is positive. It is asserted in the text that $\pi^{F*} > \Pi^{F*}$ for all $d \in (-b, b)$. Direct comparison and simplification yields: $\pi^{F*} > \Pi^{F*}$ if and only if the expression $d^2(5d^2 - 8b^2) < 0$, which is clearly true for all $d \in (-b, b)$.

(b) It is asserted in the text that $w^{F*} > W^{F*}$ for all $d \in (-b, b)$. Direct comparison and simplification yields $w^{F*} > W^{F*}$ if and only if $d^2 - 4b^2 < 0$, which clearly holds. QED: Proposition 2.

Proof of Proposition 3. It is asserted in the text that $W^{L*} + W^{F*} > w^{L*} + w^{F*}$ for the case of substitutes ($d \in (0, b)$). Direct comparison and simplification (using $d = tb$ for $t \in (0, 1)$) yields W^{L*}

$+ W^{F*} > w^{L*} + w^{F*}$ if and only if $g_5(t) = 256 - 192t - 352t^2 + 256t^3 + 152t^4 - 104t^5 - 22t^6 + 13t^7 > 0$.

<u>Claim 3</u>. $g_5(t) = 256 - 192t - 352t^2 + 256t^3 + 152t^4 - 104t^5 - 22t^6 + 13t^7 > 0$ for all $t \in (0, 1)$.

<u>Proof of Claim 3</u>: Since $g_5(0) = 256$, $g_5(1) = 7$ and $g_5'(t) < 0$ for all $t \in (0, 1)$, it follows that $g_5(t) > 0$ for all $t \in (0, 1)$. To see that $g_5'(t) < 0$ for all $t \in (0, 1)$, note that $g_5'(t) = -192 - 704t + 768t^2 + 608t^3 - 520t^4 - 132t^5 + 91t^6$. This can be written as a combination of three functions, each of which is itself negative for $t \in (0, 1)$: $g_5'(t) = h_9(t) + h_{10}(t) + t^5h_{11}(t)$, where $h_9(t) = -192 + 192t^2 - 576t + 576t^2$, $h_{10}(t) = - 128t + 608t^3 - 520t^4$ and $h_{11}(t) = -132 + 91t$. It is clear that $h_9(t) < 0$ and $h_{11}(t) < 0$ for all $t \in (0, 1)$. To see that $h_{10}(t) = - 128t + 608t^3 - 520t^4 < 0$ for all $t \in (0, 1)$, notice that $h_{10}(t) = -2t\{64 - 304t^2 + 260t^3\} = -2tH(t)$, where $H(t) = 64 - 304t^2 + 260t^3 > 0$. To see this, note that $H(0) = 64$ and $H(1) = 20$. $H'(t) = -608t + 720t^2 = 0$ at $t = 0$ and $t = 608/720$, the latter of which provides a minimum of $H(t)$ since $H''(t) = [720t - 608] + t720 > 0$ at $t = 608/720$. Moreover, $H(608/720) = 2.42984 > 0$. Thus $H(t) > 0$, which implies that $h_{10}(t) < 0$, which implies that $g_5'(t) < 0$, which implies that $g_5(t) > 0$, for all $t \in (0, 1)$. QED: Claim 3. QED: Proposition 3.

<u>Proof of Proposition 4.</u> Proposition 4 follows from the assertion in the text that, for the case of substitutes ($d \in (0, b)$), $\Pi^{CN*} > \pi^{L*}$. Direct comparison and simplification (using $d = tb$ for $t \in (0, 1)$) yields $\Pi^{CN*} > \pi^{L*}$ if and only if $-t[32 - 40t - 8t^2 + 24t^3 - 8t^4 + t^5] < 0$.

<u>Claim 4</u>. $g_6(t) = 32 - 40t - 8t^2 + 24t^3 - 8t^4 + t^5 > 0$ for all $t \in (0, 1)$.

<u>Proof of Claim 4.</u> First, $g_6(0) = 32$ and $g_6(1) = 1$. Next, $g_6(t)$ can be re-written as $g_6(t) = 8(1 - t^2 + t^3 - t^4) + h_{12}(t)$, where $h_{12}(t) = 24 - 40t + 16t^3 + t^5$. The expression in parentheses is positive, as is $h_{12}(t)$, for all $t \in (0, 1)$. To see this, note that $h_{12}(0) = 24$, $h_{12}(1) = 1$ and $h_{12}(t)$ is convex on $(0, 1)$, achieving its minimum at $t = .88$, where $h_{12}(.88) = .23 > 0$. QED: Claim 4. Since the bracketed term is positive, $\Pi^{CN*} > \pi^{L*}$ if $t > 0$; that is, if the goods are substitutes. QED: Proposition 4.

Claim 5. When the firms compete in prices, and the potential leader can choose either the Leader role or the Cournot role: when the goods are substitutes, firm 1 will choose the Leader role, and the trade regime will involve (negative) subsidies; when the goods are complements, firm 1 will choose the Cournot role, and the trade regime will be free trade.

Proof of Claim 5. First consider the case of substitute goods. If firm 1 chooses Leader, then the equilibrium in the one-shot strategic trade game involves a subsidy regime with payoffs of (w^{L*}, w^{F*}) for the governments. On the other hand, if firm 1 chooses Cournot, then the equilibrium in the one-shot strategic trade game involves a subsidy regime with payoffs of (w^{CN*}, w^{CN*}) for the governments. Moreover, both governments prefer these outcomes to free trade (see Table 4), so the equilibria are the same under repeated play. Thus, if firm 1 chooses Leader, then it anticipates a payoff of π^{L*}, while if firm 1 chooses Cournot, it anticipates a payoff of π^{CN*}. Since $\pi^{L*} > \pi^{CN*}$ (see Table 3), firm 1 chooses Leader, and the trade regime involves (negative) subsidies.

Now consider the case of complementary goods. If firm1 chooses Leader, then the equilibrium in the one-shot strategic trade game involves a subsidy regime with payoffs of (w^{L*}, w^{F*}) for the governments. Moreover, firm 2's government prefers this to free trade (see Table 4), so the equilibrium is the same under repeated play. Thus, if firm 1 chooses Leader, then it anticipates a payoff of π^{L*}. On the other hand, if firm 1 chooses Cournot, then the equilibrium in the one-shot strategic trade game involves a subsidy regime with payoffs of (w^{CN*}, w^{CN*}) for the governments. However, now both governments prefer free trade, and will employ trigger strategies in the repeated game to support it. Thus, if firm 1 chooses Cournot, then it anticipates a payoff of Π^{CN*}. Since $\Pi^{CN*} > \pi^{L*}$ for the case of complementary goods and price strategies, firm 1 chooses Cournot, and the trade regime involves free trade.